Fluctuation resistance of Josephson junctions

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The influence of superconducting fluctuations on the single-particle current in various types of tunnel junctions is considered. It is shown that there are two different fluctuation contributions to the current. The first is due to the fluctuations of the density of states in the superconductors that make up the junction, and the second to the interaction of the fluctuations through the barrier. Both contributions decrease the current flowing through a Josephson junction, but manifest themselves at different voltage scales. The type of the extrema that appear in the dependence of the differential resistance on the voltage is determined by the sign of the electron-electron interaction constant in the metals that make up the junction.

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1. INTRODUCTION

The study of fluctuation phenomena in Josephson structures was initiated by Kulik.[1] He has shown that above the superconducting-transition temperature T_c fluctuations appear in the superconducting current and produce in it rapidly oscillating increments, and as a consequence, fluctuation emission of electromagnetic waves by the junction. Owing to the junction resistance, however, these fluctuations do not manifest themselves, since their average contribution to the current flowing through the junction is zero.

This paper deals with the influence of superconducting fluctuations above T_c on the single-particle current of a Josephson junction between two amorphous or sufficiently dirty films. It turns out that allowance for the interaction of the fluctuations through the barrier produces *st* temperatures close to T_c a junction-resistance increment that depends strongly on the temperature. It is interesting that the sign of such a fluctuation contribution depends on the character of the effective interelectron interaction in both metals making up the junctions. If both metals are superconductors (with corresponding effective interaction constants $g_{s1}<0$ and $g_{s2}<0$), allowance for the interaction of the fluctuations through the barrier increases the junction resistance at low voltages. If one of the metals is normal ($g_{n1}>0$) the correction reverses sign and the effect considered decreases the resistance.

The obtained differential-resistance corrections depend not only on the temperature (at temperatures $T-T_c \ll T_c$) but also on the voltage applied to the junction, and manifest themselves as singularities of the junction current-voltage characteristic (CVC) at low voltages (the characteristic scale of variation of these corrections is $eV \sim T-T_c$). The relative value of the contribution of the effect considered depends substantially on the electron mean free path and on the presence of pair-breaking mechanisms.

2. ANALYSIS OF DIAGRAMS

We consider the influence of superconducting fluctuations on the single-particle current in a Josephson junction within the framework of the temperature diagram technique. To this end it is necessary first to represent this current in the form of a correlator of temperature Green's functions. To describe the junction we use the model of the tunnel Hamiltonian[2]

$$\hat{T}=\sum_{\mathbf{p},\mathbf{k}}(T_{\mathbf{pk}}\hat{a}_{\mathbf{p}}^{+}\hat{b}_{\mathbf{k}}+T_{\mathbf{pk}}^{*}\hat{b}_{\mathbf{k}}^{+}\hat{a}_{\mathbf{p}}),\qquad(1)$$

where $\hat{a}_p^+$ and $\hat{b}_k$ are the creation and annihilation operators in the half-spaces on the left and right of the barrier, and T_{pk} is the matrix element corresponding to the electron tunneling from the state **p** in the left half-space to the state **k** in the right.

The total current through a junction was first calculated using a Hamiltonian in this form by Ambegaokar and Baratov.[3] Their result was formulated in the form of a sum of correlators of time-dependent Green's functions of the right and left half-spaces. This sum contains both the normal quasiparticle current and the superconducting Josephson current. This approach is widely used[4] and explains many phenomena in the theory of Josephson junctions, but is not convenient for the analysis of fluctuation phenomena. The superconducting fluctuations are simplest to take into account by using the temperature diagram technique, and we must therefore represent also the single-particle tunnel current in the corresponding form.

It turns out that in the temperature diagram technique the tunnel current flowing through the junction can be represented as the imaginary part of the diagram shown in Fig. 1 (see Appendix A). The thick lines in the figure denote the total Green's functions of the right and left half-spaces, and at the vertices are the matrix element of the tunnel Hamil-

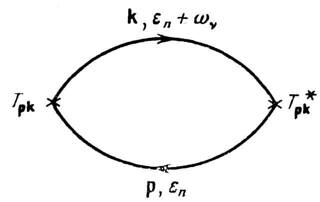

FIG. 1. Diagram for tunnel current flowing through a junction.

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tonian T_{pk}, and the common factor in front of the diagram is $-e$. Since the electron momentum is generally speaking not conserved in tunneling through the barrier, the summation over the electron states **p** and **k** is carried out independently. For this reason, the expression corresponding to the diagram contains in fact Green's functions integrated with respect to energy and independent of the value of the chemical potential in the considered half-space.

The tunnel current itself, however, is determined just by the difference between the chemical potentials on the right and left of the barrier; this difference $\Delta\mu$ is taken into account in the diagram of Fig. 1 in the following manner. The matrix elements T_{pk} at the vertices of the diagrams not only transfer to the electron from state **p** in one half-space to state **k** in the other, but impart to the electron a certain boson frequency $\omega_\nu = 2\pi T\nu$ ($\nu = 1, 2, 3, \ldots$), with respect to which an analytic continuation is effected into the upper complex-frequency plane, with the substitution $i\omega_\nu \to \omega = eV$ (V is the potential difference across the junction).

Thus, the single-particle current I_T of a Josephson junction is defined by the expression

$$I_T(V)=-e\,\mathrm{Im}\,K^R(eV), \tag{2}$$

where $K_R(\omega)$ is the analytic continuation, into the upper half-plane, of the complex frequency of the correlator of the Green's functions $K(\omega_\nu)$, which corresponds to the diagram of Fig. 1 and is defined on Matsubara frequencies $\omega_\nu > 0$:

$$K(\omega_\nu)=T\sum_{\varepsilon_n}\sum_{\mathbf{p},\mathbf{k}}|T_{\mathbf{pk}}|^2 G_{\mathrm{I}}(\mathbf{p},\varepsilon_n)G_{\mathrm{II}}(\mathbf{k},\varepsilon_n+\omega_\nu), \tag{3}$$

$\varepsilon_n = 2\pi T(n+\frac{1}{2})$ is the fermion frequency, and the indices of the total Green's functions G identify the half spaces to which they belong.

We can now take into account the influence of the superconducting fluctuations on the tunnel current in the usual manner.[5] To this end we expand the total Green's functions in terms of the fluctuation propagator that takes into account the fluctuation pairing of the electrons in each of the half-spaces, up to second order. The expansion yields the diagrams shown in Fig. 2a. A wavy line denotes a fluctuation propagor L defined by the expression

$$L(\mathbf{q},\Omega_k)=-\rho^{-1}\left[\ln\frac{T}{T_c}+\psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi\left(\frac{1}{2}\right)\right]^{-1}, \tag{4}$$

where ρ is the density of states, $\psi(x)$ is the logarithmic derivative of the Γ function, $\alpha_{\mathbf{q}} = \tau\langle(\mathbf{vq})^2\rangle/4\pi T$, and angle brackets denote averaging over the Fermi surface.

We note that the diagrams shown do not contain the anomalous Maki-Thompson diagram[6,7] or diagrams of the Aslamazov-Larkin type.[5] In our problem the processes corresponding to these diagrams are possible only in second order in the barrier transparency (their contributions to the tunnel current is $\propto |T_{pk}|^4$). Two such diagrams are shown in Fig. 2(b). In the considered case of low barrier transparency, the contributions of these diagrams are small and are disregarded here.

We discuss now the question of allowance for impurity

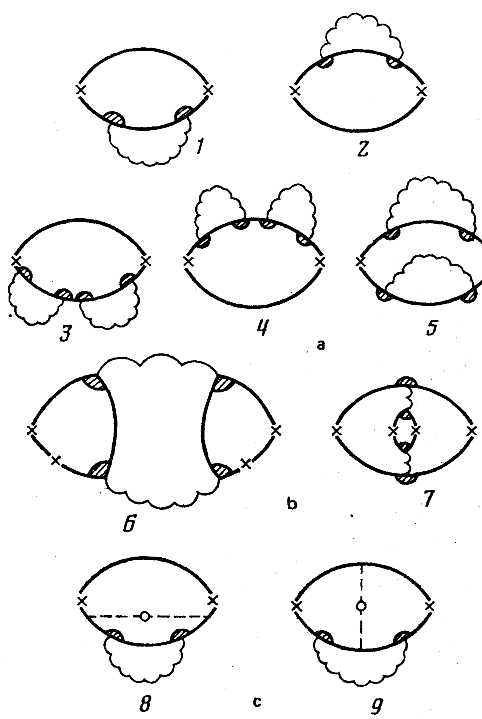

FIG. 2. Diagrams for the tunnel current with allowance for the fluctuation pairing of the electrons and for scattering from impurities. The solid lines denote the Green's functions of the normal metal with allowance for electron scattering by the impurities, the dashed lines denote scattering by impurities, the wavy lines correspond to the fluctuation propagator, and the shaded three-prong sections correspond to the vertices $\Lambda(\mathbf{q},\omega_1,\omega_2)$.

scattering. The Green's functions in the diagrams of Fig. 2 are assumed to be already averaged over the impurity positions, and the shaded three-prong vertices $\Lambda(\mathbf{q},\omega_1,\omega_2)$ are sums of ladder diagrams that occur when account is taken of the interference of the Green's functions. In the cases of interest to us, when the impurity density is high enough ($\varepsilon_F \gg \tau^{-1} \gg T$) we have for the vertex Λ the expression[8]

$$\Lambda(\mathbf{q},\omega_1,\omega_2)=\frac{|\tilde{\omega}_1-\tilde{\omega}_2|}{|\omega_1-\omega_2|+\tau\langle(\mathbf{vq})^2\rangle\theta(-\omega_1\omega_2)}, \tag{5}$$

where

$$\tilde{\omega}_n=\omega_n+\frac{1}{2\tau}\,\mathrm{sgn}\,\omega_n,$$

τ is the time between the electron-impurity collisions, and $\theta(x)$ is the Heaviside theta function.

For the diagrams of type 8 in Fig. 2(c), which do not reduce to the diagrams of Fig. 2(a), it is easy to verify by direct calculation that they are equal to zero. Finally, we can draw formally also the diagrams of type 9 [Fig. 2(c)]. As regards their meaning, however, they correspond to scattering of an electron in the left and right half-spaces by one and the same impurity, which is impossible, and their contribution is also zero. For the same reason we do not renormalize the vertices for the matrix elements T_{pk}.

As shown in Ref. 3, when considering a Josephson junction the true function of a semi-infinite metal can be replaced by the Green's function (averaged in our case over the impurity positions) of a homogenous metal:

$G(\mathbf{p}, \varepsilon_n)=[i\tilde{\varepsilon}_n-\xi(\mathbf{p})]^{-1}$.

Thus, in the case of low barrier transparency, the contribution of the superconducting fluctuations to the tunnel current is determined, accurate to second order in the superconducting fluctuations in the system, by the five diagrams of Fig. 2(a). The first two diagrams are corrections to the tunnel current and are due to the change of the density of the single-particle states in the presence of superconducting fluctuations. Diagrams 3 and 4 are the next-order corrections of the same type and are small, since they are the next terms of the expansion in the same small parameter as diagrams 1 and 2. Diagram 5 of Fig. 2(a) corresponds to the interaction of the fluctuations through the barrier. Its contribution turns out to be quite important in the proposed theory.

3. SYMMETRIC JOSEPHSON JUNCTION

We consider a Josephson tunnel junction of identical superconductors having a critical temperature T_c, at a temperature somewhat higher than the superconducting-transition temperature ($T - T_c \ll T_c$).

For the first diagram, taking into account summation over the spin indices, we have

$$K_1(\omega_\nu)=4T\sum_{\varepsilon_n}\sum_{\mathbf{p},\mathbf{k}}|T_{\mathbf{pk}}|^2 G_{II}(\mathbf{k},\varepsilon_n+\omega_\nu)\,T\sum_{\Omega_k}\int\frac{d^d\mathbf{q}}{(2\pi)^2}L(\mathbf{q},\Omega_k)$$
$$\times\Lambda^2(\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n)\,G_I^2(\mathbf{p},\varepsilon_n)\,G_I(\mathbf{q}-\mathbf{p},\Omega_k-\varepsilon_n). \quad (6)$$

Assuming the matrix elements T_{pk} to depend little on the energy near the Fermi level, we change in the usual manner[4] from summation over the electron states to integration with respect to energy:

$$e\sum_{\mathbf{p},\mathbf{k}}|T_{\mathbf{pk}}|^2(\ldots)=\frac{1}{4\pi eR_n}\int d\xi_{\mathbf{p}}\int d\xi_{\mathbf{q}}(\ldots), \quad (7)$$

where R_n is the normal resistance of the junction.

Using this transformation we obtain after simple calculations of the integrals

$$K_1(\omega_\nu)=-\frac{1}{4\pi e^2R_n}\sum_{\Omega_k=-\omega_\nu}^{\infty}\int\frac{d^2\mathbf{q}}{(2\pi)^2}L(\mathbf{q},\Omega_k)\left[\psi'\left(\frac{1}{2}+\frac{\Omega_k}{4\pi T}+\frac{\omega_\nu}{2\pi T}+\alpha_{\mathbf{q}}\right)-\psi'\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\alpha_{\mathbf{q}}\right)\right]. \quad (8)$$

In the case considered we are interested only in the terms that are most singular with respect to proximity to the transition point. It can be seen from (4) that near T_c, where $\varepsilon=(T-T_c)/T_c \ll 1$, it suffices to retain only the term with $\Omega_k=0$ in the entire sum over Ω_k. Carrying out the remaining integration and continuing analytically the expression obtained for $K_1(\omega_\nu)$ into the upper frequency half-plane, $i\omega_\nu \to eV$, we obtain

$$I_1(V)=\frac{1}{4\pi eR_n}\langle L(\mathbf{q},0)\rangle\,\mathrm{Im}\,\psi'\left(\frac{1}{2}-\frac{ieV}{2\pi T}\right), \quad (9)$$

where

$$\langle L(\mathbf{q},0)\rangle=\int\frac{d^d\mathbf{q}}{(2\pi)^d}L(\mathbf{q},0)$$

and d is the dimensionality of space.

In the considered case of a two-dimensional film (when the film thickness is smaller than the dimension of the Cooper pair) we have with logarithmic accuracy:

$$\langle L(\mathbf{q},0)\rangle=-\frac{4T}{\pi(\varepsilon_F\tau)}\ln\left(\frac{T_c}{T-T_c}\right). \quad (10)$$

Thus, at low voltages on the junction the contribution of the first diagram to the single-particle current is given by

$$I_1(V)=-\frac{7\zeta(3)}{\pi^3(\varepsilon_F\tau)}\left(\frac{V}{R_n}\right)\ln\left(\frac{T_c}{T-T_c}\right),\quad eV\ll T_c, \quad (11)$$

where $\zeta(x)$ is the Riemann zeta function.

Obviously, in the case of a symmetric junction the contribution of the second diagram is similar, and we obtain for the correction to the differential resistance of the junction, in first order in the fluctuations in the system,

$$\frac{\Delta R_{fl}{}^{I}}{R_n}=\frac{14\zeta(3)}{\pi^3(\varepsilon_F\tau)}\ln\left(\frac{T_c}{T-T_c}\right). \quad (12)$$

The contribution made to the tunnel current by the first two diagrams is thus negative at low voltages, so that the differential resistance of the junction is increased. The value of this positive increment increases quite weakly, logarithmically, as the superconducting transition is approached. On the other hand $\Delta R_{fl}{}^{I}$ becomes dependent on the voltage only at $ev \gtrsim T_c$. It is just this weak voltage and temperature dependence of the correction to the junction resistance, obtained in first order in the fluctuations, which causes the contribution from the next-order diagram to assume the principal role in the temperature dependence of the differential resistance at low voltages on the junction.

Diagram 5 of Fig. 2(a) takes into account the interaction of the fluctuations through the barrier. Corresponding to it is the analytic expression

$$K_5(\omega_\nu)=\frac{1}{\pi e^2R_n}T\sum_{\Omega_{k1}}\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}L(\mathbf{q}_1,\Omega_{k1})$$
$$\times T\sum_{\Omega_{k2}}\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}L(\mathbf{q}_2,\Omega_{k2})\,T\sum_{\varepsilon_n}\Lambda^2(\mathbf{q}_1,\varepsilon_{n+\nu},\Omega_{k1}-\varepsilon_{n+\nu})$$
$$\times\Lambda^2(\mathbf{q}_2,\varepsilon_n,\Omega_{k2}-\varepsilon_n)\,I(\varepsilon_n,\Omega_{k2})\,I(\varepsilon_{n+\nu},\Omega_{k1}), \quad (13)$$

where

$$I(\varepsilon_n,\Omega_k)=\int_{-\infty}^{\infty}\frac{d\xi}{(\xi-i\tilde{\varepsilon}_n)^2(\xi-i(\widetilde{\Omega_k+\varepsilon_n}))}. \quad (14)$$

Carrying out the integration in the block of the Green's functions (14) and substituting the explicit expression for the vertices Λ, we obtain from (5)

$$K_5^{an}(\omega_\nu)=\frac{1}{16\pi^2e^2R_n}\sum_{\Omega_{k1}}\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}L(\mathbf{q}_1,\Omega_{k1})\sum_{\Omega_{k2}}\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}L(\mathbf{q}_2,\Omega_{k2})\frac{\partial^2}{\partial\alpha_{q_1}\partial\alpha_{q_2}}\left\{\left[\alpha_{q_1}+\alpha_{q_2}+\frac{\omega_\nu}{2\pi T}+\frac{\Omega_{k1}-\Omega_{k2}}{4\pi T}\right]^{-1}\right.$$

$$\left.\times\sum_{n=-\nu}^{-1}\theta(\varepsilon_{n+\nu}-\Omega_{k1})\theta(\Omega_{k2}-\varepsilon_n)\left[\frac{1}{2\varepsilon_{n+\nu}-\Omega_{k1}+4\pi T\alpha_{q_1}}-\frac{1}{2\varepsilon_n-\Omega_{k2}-4\pi T\alpha_{q_2}}\right]\right\} \tag{15}$$

Since we are considering temperatures close to T_c, it suffices to retain in the sums over Ω_{k1} and Ω_{k2} in thisexpression only the term with $\Omega_{k1}=\Omega_{k2}=0$, for which both propagators at small momenta have a singularity; calculating the remaining sum over ε_n, we obtain

$$K_5^{an}(\omega_\nu)=\frac{1}{2^6\pi^3Te^2R_n}\int L(\mathbf{q}_1,0)\frac{d^2\mathbf{q}_1}{(2\pi)^2}\int L(\mathbf{q}_2,0)\frac{d^2\mathbf{q}_2}{(2\pi)^2}\frac{\partial^2}{\partial\alpha_{q_1}\partial\alpha_{q_2}}\left\{\left[\alpha_{q_1}+\alpha_{q_2}+\frac{\omega_\nu}{2\pi T}\right]^{-1}\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\alpha_{q_1}\right)\right.\right.$$

$$\left.\left.-\psi\left(\frac{1}{2}+\alpha_{q_1}\right)+\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\alpha_{q_2}\right)-\psi\left(\frac{1}{2}+\alpha_{q_2}\right)\right]\right\}. \tag{16}$$

Continuing now analytically this expression in the usual manner, expanding it in terms of the parameter $eV/T\ll1$, and separating its imaginary part, we have ultimately

$$I_{fl}^{II}(V)=\left(\frac{\pi}{8}\right)^2\frac{1}{(\varepsilon_F\tau)^2}\frac{V}{R_n}\varphi\left(\frac{\pi eV}{4T_c},\frac{T-T_c}{T_c}\right), \tag{17}$$

where the function $\varphi(\alpha,\varepsilon)$ is defined as

$$\varphi(\alpha,\varepsilon)=-\frac{1}{2}\frac{\partial^2}{\partial\alpha^2}\operatorname{Re}\int_0^\infty\frac{dx}{x+\varepsilon}\int_0^\infty\frac{dy}{y+\varepsilon}\frac{1}{x+y-i\alpha}. \tag{18}$$

The behavior of this function is investigated in detail in Appendix B, and the following asymptotic expressions are obtained in the limiting cases

$$\varphi\left(\frac{\pi eV}{4T_c},\frac{T-T_c}{T_c}\right)=-\begin{cases}\left(\dfrac{T_c}{T-T_c}\right)^3\ln\dfrac{4(T-T_c)}{\pi eV}, & eV\ll T-T_c,\\[2ex] \left(\dfrac{8}{\pi}\right)^2\left(\dfrac{T_c}{eV}\right)^3\ln\dfrac{\pi eV}{4(T-T_c)}, & eV\gg T-T_c.\end{cases} \tag{19}$$

The logarithmic divergence of this expression at low voltages is of the same origin as the divergence, known in the theory of superconducting fluctuations, of the anomalous Maki-Thompson contribution[6,7] to the conductivity of a superconducting film above T_c. This singularity is eliminated by taking into account any pair-breaking mechanism or the three-dimensional character of the electron motion.[8] Equations (19) are therefore meaningful at voltages $eV\gtrsim\Delta$, where Δ is the characteristic energy of the pair-breaking mechanism ($\Delta=\pi/8\tau_p$ in the presence of paramagnetic impurities, $\Delta=eH/mc$ when a magnetic field is turned on). Thus, the fluctuation addition to the resistance of a symmetric Josephson junction, due to the interaction of the fluctuations through the junction, is of the form

$$\frac{\Delta R_{fl}^{II}}{R_n}=\left(\frac{\pi}{8}\right)^2\frac{1}{(\varepsilon_F\tau)^2}\times\begin{cases}\left(\dfrac{T_c}{T-T_c}\right)^3\ln\dfrac{4(T-T_c)}{\pi eV}, & eV\ll T-T_c,\\[2ex] \dfrac{2^7}{\pi^2}\left(\dfrac{T_c}{eV}\right)^3\ln\dfrac{4(T-T_c)}{\pi eV}, & eV\gg T-T_c.\end{cases} \tag{20}$$

4. ASYMMETRIC JOSEPHSON JUNCTION

We consider now fluctuations in an asymmetric Josephson junction of two superconducting films with critical temperatures T_{c1} and T_{c2} ($T_{c1}<T_{c2}$); we are interested in temperatures $T>T_{c2}$ ($T-T_{c2}\ll T_{c2}$).

The contribution made to the tunnel current by the fluctuation change of the state density is described as before by the first two diagrams of Fig. 2(a). The second diagram, which corresponds to the fluctuation of the state density in the second half-space, makes the same contribution to the current (11) as before. The contribution of the first diagram, however, can now differ from (11), since T_{c1} can differ considerably from T and expression (11) for the current does not hold in this case.

If the critical temperatures of the films are so close that $T-T_{c1}\ll T_{c_1}$, the first-diagram contribution to the tunnel current is determined by the same expression (11), but with T_c replaced by T_{c1} (or $T_c\to T_{c2}$ for the second diagram). In this case the fluctuation increment to the resistance of the asymmetric junction from the first-order diagrams takes at low voltages the form

$$\frac{\Delta R_{fl}^{I}}{R_n}=\frac{7\zeta(3)}{\pi^3}\left[\frac{1}{\varepsilon_{F1}\tau_1}\ln\frac{T_{c1}}{T-T_{c1}}+\frac{1}{\varepsilon_{F2}\tau_2}\ln\frac{T_{c2}}{T-T_{c2}}\right]. \tag{21}$$

If, however, T_{c1} and T_{c2} differ significantly, it no longer suffices to retain only the term with $k=0$ in expression (8), when calculating the first diagram, and it is necessary to carry out complete summation over all the frequencies in the fluctuation propagator. In the case $T_{c1}\ll T_{c2}$ the corresponding calculations, in analogy with Ref. 8, can be carried out analytically. It turns out as a result that the corresponding correction to the junction resistance is practically constant in the temperature region near T_{c2}, and the total contribution to the resistance, connected with the state-density fluctuations, is determined by the expression

$$\frac{\Delta R_{fl}^{I}}{R_n}=\frac{7\zeta(3)}{\pi^3}\frac{1}{(\varepsilon_{F2}\tau_2)}\ln\left(\frac{T_{c2}}{T-T_{c2}}\right)+\frac{1}{2\pi(\varepsilon_{F1}\tau_1)}\left\{\ln\ln\frac{1}{T_{c1}\tau}-\ln\ln\frac{T}{T_{c1}}\right\}. \tag{22}$$

When account is taken of the interaction of the fluctuations, diagram 5 of Fig. 2(a) remains the principal one as before. It is calculated in analogy with the case of a symmetric junction, up to the derivation of (16). The differences between the superconducting-transition temperatures, the Fermi energies, and the times τ of the right and left half-spaces lead to a more complicated expression for the increment to the tunnel current:

$$I_{fl}^{\rm II}=\frac{\pi^2}{2^7(\varepsilon_{F1}\tau_1)(\varepsilon_{F2}\tau_2)}\left(\frac{V}{R_n}\right)\chi\left(\frac{\pi eV}{4T_{c2}},\frac{T-T_{c2}}{T_{c2}},\ln\frac{T}{T_{c1}}\right), \tag{23}$$

where the function χ is defined by the integral

$$\chi(\alpha,\varepsilon,\gamma)=-\frac{1}{2}\frac{\partial^2}{\partial\alpha^2}\operatorname{Re}\int_0^\infty\frac{dx}{x+\varepsilon}\int_0^\infty\frac{dy}{y+\gamma}\frac{1}{x+y-i\alpha}, \tag{24}$$

and the parameters α, ε, and γ correspond to the arguments of the function χ in (23). Depending on the ratio of T_{c1} and T_{c2}, the quantity $\gamma=\ln(T/T_{c1})$ can be either small or large, but always $\gamma\gg\varepsilon=(T-T_{c2})/T_{c2}$. The behavior of χ is investigated in Appendix B at different relations between its arguments. The asymptotic relations obtained there yield an explicit expression for the increment made to the resistance of an asymmetric junction by the interaction of the fluctuations:

$$\frac{\Delta R_{fl}^{\rm II}}{R_n}=\frac{\pi^2}{2^7(\varepsilon_{F1}\tau_1)(\varepsilon_{F2}\tau_2)}\times\begin{cases}\dfrac{\varepsilon+\gamma}{\varepsilon^2\gamma^2}\ln\dfrac{\varepsilon}{\alpha}, & \alpha\ll\varepsilon,\gamma,\\ \dfrac{1}{\alpha^2\gamma}\ln\dfrac{\varepsilon}{\alpha}, & \varepsilon\lesssim\alpha\lesssim\gamma,\\ \dfrac{2\pi}{\alpha^3}\ln\dfrac{\varepsilon\gamma}{\alpha^2}, & \varepsilon,\gamma\lesssim\alpha\ll1.\end{cases} \tag{25}$$

In the case $T_{c1}=T_{c2}$ these equations go over, naturally, into (20) (there is simply no intermediate voltage region in this case). If the critical temperatures differ greatly, it can be seen from (25) that the temperature dependence of $\Delta R_{fl}^{\rm II}$ near $V=0$ is weaker than in the case of a symmetric junction.

5. JUNCTION OF A NORMAL METAL AND A SUPERCONDUCTOR

We consider the case of a tunnel junction of two films, one a superconductor and the other a normal metal. At $T>T_c$ the interelectron interaction in the superconducting part of the junction leads to the formation of superconducting fluctuations. Such an interaction takes place also in the normal part of the junction, where the electron-electron interaction constant g_n is positive. When considering the effective electron-electron interaction, just as in the case of a superconductor, it is possible to separate and sum a ladder sequence of diagrams in the Cooper interaction channel.[9] The obtained vertex part of the interelectron interaction corresponds to the fluctuation propagator in the case of a superconductor, where the effective interelectron interaction constant is $g_s<0$, and takes formally the same form (4). Now, however, the critical temperature $T_c\sim\omega_0\exp[1/\rho g_s]$ is replaced by some large (of the order of the Fermi energy) temperature $T_0\sim\varepsilon_F\exp[1/\rho g_n]$, so that

$$L(\mathbf{q},\Omega_k)=\rho^{-1}\left[\ln\frac{T_0}{T}-\psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\alpha_{\mathbf{q}}\right)+\psi\left(\frac{1}{2}\right)\right]^{-1}. \tag{26}$$

Since $T_0\gtrsim\varepsilon_F$, we have $\ln(T_0/T)\gg1$ for the temperatures considered by us.

The current increment due to the fluctuations of the state density in the superconducting part of the junction remains the same as before, and is determined by (11). Fluctuations of this type in the normal part of the junction will be described as before by diagram 2, but the wavy line in it must now be taken to mean the vertex part (26). The contribution of this diagram to the tunnel current at temperatures $T-T_c\ll T_c$ and at voltages $eV\lesssim T_c$ is positive and is practically independent of these variables. The total change $\Delta R_{fl}^{\rm I}$ of the junction resistance, due to the fluctuations of the state density, is given by

$$\frac{\Delta R_{fl}^{\rm I}}{R_n}=\frac{7\zeta(3)}{\pi^3}\frac{1}{(\varepsilon_{F2}\tau_2)}\ln\left(\frac{T_c}{T-T_c}\right)-\frac{1}{2\pi(\varepsilon_{F1}\tau_1)}\left[\ln\ln\frac{T_0}{T}-\ln\ln(T_0\tau)\right] \tag{27}$$

and can either be positive or negative.

An interesting phenomenon results in the case of an *n-s* junction from the interaction through the barrier, described as before by diagram 5. Formally the calculation coincides fully with the one in the preceding section for the case $\ln(T/T_{c1})\gg1$, but now $\gamma=\ln(T/T_{c1})>0$ must be replaced by $\ln(T/T_0)<0$, and according to (25) the interaction of the fluctuations through the barrier leads in this case to a decrease of the junction resistance, i.e., the corresponding correction reverses sign compared with the usual Josephson junction. Thus, repulsion of the electrons in the normal part of the junction leads to the appearance of a minimum in the dependence of the differential resistance on the voltage at $V=0$ (Fig. 3b); the dependence of $\Delta R_{fl}^{\rm II}$ on the voltage and on the temperature is determined by the formulas

$$\frac{\Delta R_{fl}^{\rm II}}{R_n}=-\frac{\pi^2}{2^7}\frac{1}{(\varepsilon_{F1}\tau_1)(\varepsilon_{F2}\tau_2)}\frac{1}{\ln(T_0/T)}\times\begin{cases}\left(\dfrac{T_c}{T-T_c}\right)^2\ln\dfrac{4(T-T_c)}{\pi eV}, & eV\ll T-T_c,\\ \left(\dfrac{4T_c}{\pi eV}\right)^2\ln\dfrac{4(T-T_c)}{\pi eV}, & T-T_c\lesssim eV\ll T_c.\end{cases} \tag{28}$$

6. DISCUSSION OF RESULTS

The results show that superconducting fluctuations in a tunnel junction lead to the appearance of two contributions, of different character, to the current flowing through it. The first is due to fluctuations of the state density in each of the superconductors, and the second corresponds to interaction of the fluctuations through the barrier. Both effects decrease the total current flowing through the Josephson junction, and when account is taken of the fluctuation effects the junction *CVC* acquires even at $T>T_c$ features typical of the *CVC*

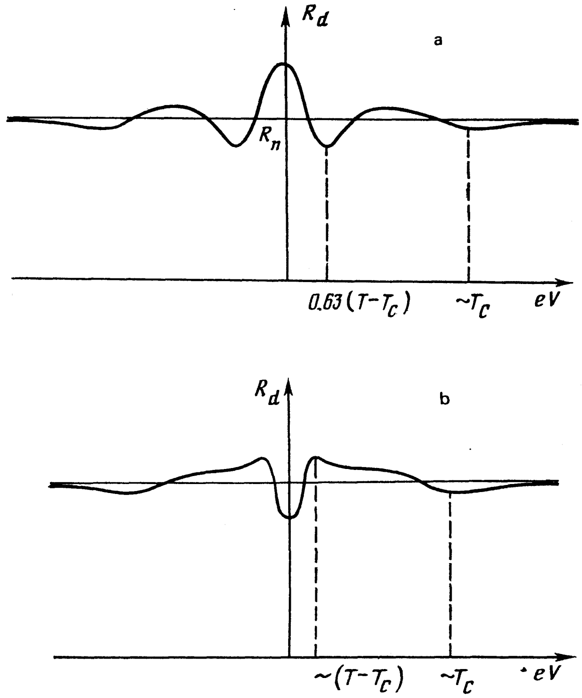

FIG. 3. Voltage dependence of the differential resistance R_d of the junction near the superconducting-transition temperature.

of a Josephson junction between two superconductors at $T<T_c$.

Let us discuss the dependence of the differential resistance of the junction on the voltage at temperatures close to T_c, using a symmetric Josephson as an example (Fig. 3a). The fluctuations of the single-particle states near T_c lead, according to (9), to a decrease of the current flowing through the junction. At $eV\sim T_c$ the corresponding correction to the resistance is positive and is determined by expression (12). Its temperature dependence has a weak (logarithmic) singularity with respect to the proximity to T_c. With increasing voltage ($T-T_c \ll eV \lesssim T_c$) $\Delta R_{fl}{}^{\rm I}$ begins to decrease, reverses sign, and after reaching a minimum in the region $eV\sim T_c$ it tends to zero with further increase of voltage. Thus, the fluctuations in the density of the single-particle states introduce into the junction differential resistance a correction with a variation scale $eV\sim T_c$.

The effect of the interaction of the fluctuations through a barrier leads to singularities in the *CVC* of the junction at much lower voltages, on the order of $eV \lesssim T-T_c$. At low voltages $eV\lesssim T-T_c$, the dependence of the differential resistance on the voltage, $R_d(V)$, is logarithmic [see (20)]. The correction at the maximum for $V=0$ is determined by the pair-breaking mechanisms in the superconductors that make up the junction, and depends also substantially on the proximity to T_c. One of the pair-breaking mechanisms may be a magnetic field (or paramagnetic impurities), whose increase causes the maximum of the junction resistance to decrease rapidly (typical fields that suppress the effects are given by $eH/mc\sim T-T_c$). The dependence of the correction on the proximity to T_c is according to (20) quite strong, $\sim(T_c/T-T_c{}^3$. In the presence pair-breaking mechanisms, however, this dependence becomes weaker, for in this case the true temperature T_c of the superconducting transition decreases, and the quantity T_c in (20) is replaced by $T_c+\delta$ (δ is the critical-temperature shift due to the pair-breaking mechanisms).

Since the considered fluctuation interaction corresponds to taking the system fluctuations into account in second-order perturbation theory, $\Delta R_{fl}{}^{\rm II}$ is of order of smallness $(\varepsilon_F\tau)^{-2}$ compared with R_n, and predominates over $\Delta R_{fl}{}^{\rm I}$ at low voltages ($eV\lesssim T-T_c$) only at temperatures $T-T_c\lesssim T_c(\varepsilon_F\tau)^{-1/3}$. In the immediate vicinity of the transition temperatures the interactions of the fluctuations in higher orders of perturbation theory become significant, and at temperatures $T-T_c\lesssim T_c(\varepsilon_F\tau)^{-1/2}$ their contribution becomes comparable with $\Delta R_{fl}{}^{\rm II}$. The temperature region $T-T_c\lesssim T_c(\varepsilon_F\tau)^{-1/2}$ is thus critical, and our analysis is not valid in it.

When the voltage is raised, $\Delta R_{fl}{}^{\rm II}$ decreases, reverses sign, and reaches its minimum at $eV=0.63(T-T_c)$. Further increase of the voltage leads to a faster (power-law) decrease of the correction $\Delta R_{fl}{}^{\rm II}$ [see (20)], which becomes negative in the temperature region $T-T_c\lesssim eV \ll T_c$. In this voltage range, however, the correction $\Delta R_{fl}{}^{\rm I}$ is still positive. Therefore when both considered effects are taken into account $R_d(V)$ assumes a peculiar shape with a maximum a zero, a plateau, and two minima of different origin (see Fig. 3a).

We note that on going farther from the superconducting-transition temperature (but remaining in the region $T-T_c \ll T_c$) the position of the first minimum shifts to the right along the voltage axis, whereas the position of the second minimum is almost unchanged.

Recent experiments by Jerome's group[10] on tunneling in the organic superconductor $(TMSTSF)_2PF_6$ have shown that the differential resistance of such a junction is a maximum at $V=0$ and exhibits a characteristic minimum at $V=0$. The position of this minimum in the superconducting phase determines the size of the gap in the spectrum. Its existence at $T>T_c$, on the other hand, is understandable, since it results from an interaction of superconducting fluctuations that take place in compounds with decreased dimensionality of the electron spectrum, such as the compound $(TMTSF)_2PF_6$ (Ref. 11).

We consider now the singularities of the CVC of the *n-s* junction (Fig. 3c). In this case the correction due to the interaction of the fluctuations through the barrier has, because of the electron repulsion in the normal part of the junction, a sign opposite that of a junction of two superconductors. This causes a minimum of $R_d(V)$ at $V=0$ and a maximum at $eV\sim T-T_c$ (see Fig. 3b).

Thus, even from only the form of the $R_d(V)$ it is possible to assess the sign of the effective interelectron-interaction constant in the investigated sample; this is a particularly attractive property in cases when the coupling constant is small and its sign cannot be judged by other measurements.

We note that the singularities of $R_d(V)$ considered by us are connected with the interelectron interaction in the Cooper channel. In addition corrections necessitated by the

Coulomb interaction of the electrons must be made to the differential resistance at voltages $eV \gtrsim T$.

In conclusion, the authors are deeply grateful to A. A. Abrikosov for suggesting the topic, and L. G. Aslamazov and B. L. Al'tshuler for valuable discussions.

APPENDIX A

We calculate the imaginary part of the diagram of Fig. 1 in general form. To this end we use the Lehmann expansion of Green's functions[13]:

$$G(\mathbf{p}, \varepsilon_k) = -\sum_{m,n} \exp[\beta(\Omega+\mu N_n - E_n)] \times |(\hat{a}_\mathbf{p}^+)_{mn}|^2 \frac{1+e^{-\beta\omega_{mn}}}{\omega_{mn} - i\varepsilon_k}, \qquad \text{(A.1)}$$

where

$$\beta = 1/T, \qquad \omega_{mn} = (E_m - \mu N_m) - (E_n - \mu N_n).$$

Substituting in this form the Green's functions of the left and right half-spaces in the expression (3) for $K(\omega_\nu)$, we have

$$K(\omega_\nu) = 4 \sum_{\mathbf{k},\mathbf{p}} |T_{\mathbf{pk}}|^2 \sum_{\substack{m_1,n_1\\m_2,n_2}} \exp[\beta(\Omega_1+\mu_1 N_{n_1} - E_{n_1})]$$
$$\times \exp[\beta(\Omega_2+\mu_2 N_{n_2} - E_{n_2})] |(\hat{a}_\mathbf{k}^+)_{m_1n_1}|^2 |(\hat{b}_\mathbf{p}^+)_{m_2n_2}|^2$$
$$\times \frac{(1+\exp[-\beta\omega_{m_1n_1}])(1+\exp[-\beta\omega_{m_2n_2}])}{2(\omega_{m_2n_2} - \omega_{m_1n_1} - i\omega_\nu)}$$
$$\times \left(\operatorname{th}\frac{\omega_{m_1n_1}}{2T} - \operatorname{th}\frac{\omega_{m_2n_2} - i\omega_\nu}{2T}\right). \qquad \text{(A.2)}$$

Shifting the argument of the hyperbolic tangent by a period $i\omega_\nu/2T$, followed by the analytic continuation $i\omega_\nu \to \Delta\mu = \text{eV}$, we obtain ultimately

$$I(V) = 4\pi e \sum_{\mathbf{k},\mathbf{p}} |T_{\mathbf{pk}}|^2 \sum_{\substack{m_1,n_1\\m_2,n_2}} \exp[\beta(\Omega_1+\mu_1 N_{n_1} - E_{n_1})]$$
$$\times \exp[\beta(\Omega_2+\mu_2 N_{n_2} - E_{n_2})] |(\hat{a}_\mathbf{k}^+)_{m_1n_1}|^2 |(\hat{b}_\mathbf{p}^+)_{m_2n_2}|^2$$
$$\times (\exp[-\beta\omega_{m_2n_2}] - \exp[-\beta\omega_{m_1n_1}])\, \delta(\omega_{m_1n_1} - \omega_{m_2n_2} + eV), \qquad \text{(A.3)}$$

which agrees with the known expression for the tunnel current.[14]

APPENDIX B

We investigate the function

$$\chi(\alpha, \varepsilon, \gamma) = -\frac{1}{2}\frac{\partial^2}{\partial\alpha^2} J(\alpha, \varepsilon, \gamma), \qquad \text{(B.1)}$$

where $J(\alpha, \varepsilon, \gamma)$ is determined by the real part of the integral

$$J(\alpha, \varepsilon, \gamma) = \operatorname{Re} \int_0^\infty \frac{dx}{x+\varepsilon} \int_0^\infty \frac{dy}{y+\gamma} \frac{1}{x+y-i\alpha}$$
$$= \operatorname{Re} \int_0^\infty \frac{dx}{x+\varepsilon} \frac{\ln(x/\gamma - i\alpha/\gamma)}{x - \gamma - i\alpha}$$
$$= \operatorname{Re} \frac{1}{\gamma} \int_{-i\tilde{\alpha}}^{\infty - i\tilde{\alpha}} \frac{dz}{z+\tilde{\varepsilon}+i\tilde{\alpha}} \frac{\ln z}{z-1}; \qquad \text{(B.2)}$$

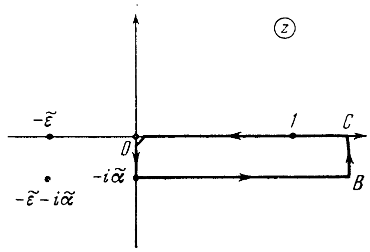

FIG. 4. Integration contour for the calculation of the integral in (B.2). The pole $z = -\tilde{\varepsilon} - i\tilde{\alpha}$ is not contained in the integration contour; $z = 1$ is a removable singular point.

here

$$\tilde{\alpha} = \alpha/\gamma, \quad \tilde{\varepsilon} = \varepsilon/\gamma, \quad \varepsilon \leqslant \gamma.$$

To calculate the integral in (B.2) we use the Cauchy theorem for the contour shown in Fig. 4. The integrals along the small arc and the remote segment BC are small, and for the integral of interest to us we have

$$J(\alpha, \varepsilon, \gamma) = \operatorname{Re} \frac{1}{\gamma} \left[\int_0^{-i\tilde{\alpha}} \frac{\ln z\, dz}{(z+\tilde{\varepsilon}+i\tilde{\alpha})(z-1)} + \int_0^\infty \frac{\ln z\, dz}{(z+\tilde{\varepsilon}+i\tilde{\alpha})(z-1)} \right]$$
$$= \operatorname{Re} \frac{1}{\gamma} \left[\frac{\pi^2 + \ln^2(\tilde{\varepsilon}+i\tilde{\alpha})}{2(1+\tilde{\varepsilon}+i\tilde{\alpha})} + \int_0^{-i\tilde{\alpha}} \frac{\ln z\, dz}{(z+\tilde{\varepsilon}+i\tilde{\alpha})(z-1)} \right]. \qquad \text{(B.3)}$$

In the case $\varepsilon \ll \alpha \ll \gamma$ the denominator of the integrand in the first integral of (B.3) can be expanded in powers of z and $z + i\tilde{\alpha}$, after which, integrating and separating the real part, we obtain

$$\chi(\alpha, \varepsilon, \gamma) = -\frac{\varepsilon+\gamma}{2\varepsilon^2\gamma^2} \ln\frac{\varepsilon}{\alpha}. \qquad \text{(B.4)}$$

In the intermediate case $\varepsilon \ll \alpha \ll \gamma$ the integral builds up on the upper limit, and we obtain

$$\chi(\alpha, \varepsilon, \gamma) = -\frac{1}{2\varepsilon^2\gamma} \ln\frac{\alpha}{\varepsilon}. \qquad \text{(B.5)}$$

For $\alpha \gg \varepsilon, \gamma$, separating the first term of the expansion in reciprocal powers of α, we have

$$\chi(\alpha, \varepsilon, \gamma) = -\frac{\pi}{2\alpha^3} \ln\left(\frac{\alpha^2}{\varepsilon\gamma}\right). \qquad \text{(B.6)}$$

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