Squeezed quantum states of a harmonic oscillator in the problem of gravitational-wave detection

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A quantum harmonic oscillator subjected to a resonant force action and to a parametric action is considered. Account is taken of the coupling of the oscillator with the thermostat (and with the zero-point oscillations). Explicit solutions of the Heisenberg equations are obtained for the creation and annihilation operators. Various methods of producing squeezed states in the oscillator are considered, and the most suitable for the registration of a weak (gravitational) force are chosen. Concrete estimates are obtained of the improvement of the sensitivity through the use of squeezed states and are obtained. The advantages and shortcomings of the squeezed-states technique and of quantum nondemolition measurements are compared. Possible uses of the squeezed-states technique are discussed.

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The expected gravitational-wave signals of cosmic or laboratory origin are so weak that an analysis of detection properties with allowance for their quantum-mechanical character has become urgently necessary.[1] The usual observation criterion is taken to be an excess of the signal S over the noise N, i.e., $S/N \gtrsim 1$. Even if the thermal noise is neglected and there are no other imperfections in the detecting apparatus, a contribution to the noise is made by the quantum-mechanical variance of the measured quantity. A recent analysis of the so-called quantum nondemolition measurements (QND) consists precisely of a demonstration that for certain observable quantities (Hermitian operators) the variance can be made equal to zero and remains equal to zero, despite the action of the signal that leads to a change in the average value of the measured quantity. It assumed that the vanishing of the variance is due to the act of the first measurement. The successive continuous tracking of the measured quantities does not change its variance, since the quantity is QND-variable. In this idealized case the sensitivity of the detector is formally equal to infinity. The corresponding Hermitian operators, suitable for nondemolition measurement of a force F or of parametric action P were respectively named QNDF (Ref. 2) and QNDP operators (Ref. 3).

The studies of this subject have the following shortcomings. First, as a rule, they have ignored the presence of thermal noise, i.e., no account was taken of the fact that the variance of the QND variable, even when equal to zero at the initial instant of time, must inevitably increase with time because of the interaction of the oscillator with the thermostat. Second, it is not quite clear how to realize in practice instruments corresponding to the rather abstract QNDF and QNDP operators.

In the present paper we change somewhat the approach of the problem. We start from the fact that what is measured is a well defined quantity, not necessarily QND-variable. Such a quantity can be, e.g., the energy. As for the decrease of the noise N, i.e., of the variance of the measured quantity and (or) of the amplification of the signal S, i.e., the response of the detector to an external action, these are reached by a special "preparation" of the quantum state of the oscillator with the aid of really existing laboratory devices. We consider concretely the development and use of what are called squeezed states. Thus, an arriving gravitational signal interacts with an oscillator that is in a squeezed state and not, say, in an n-quantum or coherent state.

In quantum theory of light, squeezed states are known as "two-proton coherent states" and under other names.[4] In connection with detection of gravitational waves, squeezed states were considered by Caves,[5] who has shown that their use makes it possible to increase the sensitivity of a laser interferometer without increasing the laser power. It was also noted[6] that the use of squeezed states is promising in laboratory gravitation-wave experiments.

In the present paper we take consistent account of the connection between the oscillator and the thermostat. This enables us, in particular to generalize the theory of QND measurements to include the case of the presence of thermal noise and to compare the results with the method proposed here of using squeezed states.

§1. FUNDAMENTAL EQUATIONS AND THEIR SOLUTION

An arbitrary harmonic oscillator, mechanical or electrostatic, will be described in standard fashion with the aid of the creation and annihilation operators:

$$a^+=\left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{q}-i\frac{\hat{p}}{m\omega}\right),\quad a=\left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(q+i\frac{\hat{p}}{m\omega}\right),$$

$$[a,a^+]=1.$$

It is convenient also to introduce the operators X_1 and X_2 of the complex amplitude of the oscillator, in accordance with the definition

$$X_1=l(e^{-i\omega t}a^+ + e^{i\omega t}a),\qquad X_2=il(e^{-i\omega t}a^+ - e^{i\omega t}a),$$

where $l=(\hbar/2m\omega)^{1/2}$.

It is known that a coherent state $|\alpha\rangle \equiv D(\alpha)|0\rangle$ is defined with the aid of the displacement operator

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$D(\alpha)=\exp(\alpha a^+ - \alpha^* a)$, where α is a complex number. Participating in the construction of the squeezed state $|\alpha,\beta\rangle \equiv D(\alpha)S(\beta)|0\rangle$ is also the squeezing operator

$$S(\beta)=\exp\left[{}^1/_2(\beta^* a^2-\beta a^{+2})\right],$$

where β is a complex number. A squeezed state is thus characterized by two complex numbers.

A coherent state results from the vacuum state $|0\rangle$ under the influence of a force, i.e., on account of a term of the type $F(t)(a+a^+)$ in the Hamiltonian. One of the possible methods of obtaining a squeezed state is the use of a degenerate parametric amplifier,[4,7] i.e., of a term of the type $P(t)(a^{+2}+a^2)$ in the Hamiltonian. If the parametric pumping is effected at the frequency 2ω and is of the form

$$P(t)=|p|\sin(2\omega t+\psi),$$

the term of interest to us in the Hamiltonian can be rewritten in the form

$$P(t)\,(a^2+a^{+2})$$

$$=\frac{1}{2i}(pe^{i2\omega t}a^{+2}-p^*e^{-i2\omega t}a^{+2}+pe^{i2\omega t}a^2-p^*e^{-i2\omega t}a^2), \qquad (1)$$

$$p=|p|e^{i\psi}$$

inasmuch as for free evolution of the oscillator we have

$$a(t)=a(0)e^{-i\omega t} \text{ and } a^+(t)=a^+(0)e^{i\omega t}.$$

The first and fourth terms in (1) are rapidly oscillating and can be neglected compared with the resonant, second and third, terms. Similarly, retaining only the terms resonant at the frequency ω, the force term in the Hamiltonian can be written in the form

$$F(t)\,(a+a^+)=fe^{-i\omega t}a^++f^*e^{i\omega t}a, \qquad f=|f|e^{i\varphi}.$$

We shall thus work with a Hamiltonian

$$H=\hbar\omega a^+a+{}^1/_2 ik\omega\,(p^*e^{i2\omega t}a^2-pe^{-i2\omega t}a^{+2})$$
$$+\hbar\omega\,(fe^{-i\omega t}a^++f^*e^{i\omega t}a), \qquad (2)$$

where p and f are arbitrary complex numbers.

We note that the Hamiltonian (2) is among those that describe resonant action of a gravitational wave on a mode of the electromagnetic oscillations in the resonator.[3] To be sure, in the case of a gravitational signal the coupling constants $|p_g|$ and $|f_g|$ are determined by the amplitude of a gravitational wave h and are quite small under real conditions, whereas the coupling constants with laboratory generators, $|p_l|$ and $|f_l|$, depend to a considerable degree on the experimenter. In the general case, it can be assumed that $p=p_l+p_g$ and $f=f_l+f_g$, but hereafter we shall deal for simplicity only with estimates of observing a gravitational force and therefore put $p_g=0$ and $f_g\neq 0$.

The interaction of an oscillator with the thermostat can be described by adding to the Heisenberg equations of motion the term that takes the damping into account, and the operator of the random force (see, e.g., Ref. 8):

$$i\hbar\frac{da}{dt}=[a,H]-i\hbar\frac{\gamma}{2}a+i\hbar\theta. \qquad (3)$$

The constant $\gamma>0$ is connected with the relaxation time τ^* by the relation $\gamma^{-1}=\tau^*$. The random-force operator $\theta(t)$ satisfies the commutation relation $[\theta(t),\theta^+(t')]=\gamma\delta(t-t')$. The equation for a^+ is obtained from (3) by Hermitian conjugation.

Substituting in (3) the Hamiltonian (2) we obtain the equations of motion in explicit form

$$\frac{da}{dt}=-i\omega a-\omega pa^+e^{-i2\omega t}-i\omega fe^{-i\omega t}-\frac{\gamma}{2}a+\theta,$$
$$\frac{da^+}{dt}=i\omega a^+-\omega p^*ae^{i2\omega t}+i\omega f^*e^{i\omega t}-\frac{\gamma}{2}a^++\theta^+.$$

To solve them it is convenient to eliminate the free evolution by using the substitution

$$a(t)=e^{-i\omega t}e^{-\gamma t/2}A(t), \qquad a^+(t)=e^{i\omega t}e^{-\gamma t/2}A^+(t)$$

and carry out the Bogolyubov transformation

$$b(t)=e^{-i\psi/2}\operatorname{ch} rtA(t)+e^{i\psi/2}\operatorname{sh} rtA^+(t),$$
$$b^+(t)=e^{i\psi/2}\operatorname{ch} rtA^+(t)+e^{-i\psi/2}\operatorname{sh} rtA(t),$$

where $r=|p|\omega$. We then obtain for $b(t)$ the simple equation

$$db/dt=-i\omega e^{\gamma t/2}(f\operatorname{ch} rte^{-i\psi/2}-f^*\operatorname{sh} rte^{i\psi/2})$$
$$+e^{\gamma t/2}(e^{i(\omega t-\psi/2)}\operatorname{ch} rt\theta+e^{-i(\omega t-\psi/2)}\operatorname{sh} rt\theta^+).$$

The final solution for $a(t)$ is

$$a(t)=e^{-i\omega t}e^{-\gamma t/2}[m(t)+\Phi(t)+T(t)], \qquad (4)$$

where

$$m(t)=a_0\operatorname{ch} rt-a_0^+e^{i\psi}\operatorname{sh} rt,$$

$$\Phi(t)=-i\omega\int_0^t e^{\gamma\tau/2}[f\operatorname{ch} r(\tau-t)-f^*e^{i\psi}\operatorname{sh} r(\tau-t)]d\tau,$$

$$T(t)$$
$$=\int_0^t e^{\gamma\tau/2}[\theta(\tau)e^{i\omega\tau}\operatorname{ch} r(\tau-t)+\theta^+(\tau)e^{-i(\omega\tau-\psi)}\operatorname{sh} r(\tau-t)]d\tau.$$

An expression for $a^+(t)$ is obtained from (4) by Hermitian conjugation.

To calculate the mean values and the variances it is necessary to average using the density matrix. The complete oscillator + thermostat system is described by the direct product of the oscillator density matrix ρ and the thermostat density matrix ρ_T. The averaging of the operator $\theta(t)$ has the following properties[8] (the angle brackets $\langle\ \rangle$ denote averaging over the total density matrix, but the operator θ is in fact averaged over ρ_T):

$$\langle\theta(t)\rangle=\langle\theta^+(t)\rangle=0, \qquad \langle\theta^+(t)\theta(t')\rangle=\gamma n_T\delta(t-t'),$$
$$\langle\theta(t)\theta^+(t')\rangle=\gamma(n_T+1)\delta(t-t').$$

Here n_T is the average number of quanta in the thermostat at the frequency ω:

$$n_T=[e^{\hbar\omega/kT}-1]^{-1}.$$

The binary terms containing θ^2 and θ^{+2} yield zero on averaging.

When calculating the variance of the quantum-number operator $n=a^+a$ in the oscillator, use is made of fourth-order moments. Assuming that θ and θ^+ have Gaussian distributions, they reduce to second-order moments in accordance with the expansion[9]:

$$\langle\theta(t_1)\theta^+(t_2)\theta(t_3)\theta^+(t_4)\rangle=\langle\theta(t_1)\theta^+(t_2)\rangle\langle\theta(t_3)\theta^+(t_4)\rangle+\langle\theta(t_1)\theta^+(t_4)\rangle\langle\theta^+(t_2)\theta(t_3)\rangle.$$

We present expressions for the mean values of the operators X_1, X_2, and n and their variances, but do not specify concretely for the time being the initial value of the oscillator density matrix:

$$\langle X_1(t)\rangle=e^{-\gamma t/2}[(\operatorname{ch} rt-\cos\psi\operatorname{sh} rt)\langle X_1(0)\rangle-\sin\psi\operatorname{sh} rt\langle X_2(0)\rangle+l(\Phi(t)+\Phi^*(t)],\qquad(5)$$

$$\langle X_2(t)\rangle=e^{-\gamma t/2}[(\operatorname{ch} rt+\cos\psi\operatorname{sh} rt)\langle X_2(0)\rangle-\sin\psi\operatorname{sh} rt\langle X_1(0)\rangle+il(\Phi^*(t)-\Phi(t))],\qquad(6)$$

$$\langle n(t)\rangle=e^{-\gamma t}[\langle m^+(t)m(t)\rangle+\Phi(t)\langle m^+(t)\rangle+\Phi^*(t)\langle m(t)\rangle+|\Phi(t)|^2+\langle T^+T\rangle].\qquad(7)$$

We note that at $\sin\psi=0$ the mean values of X_1 and X_2 do not depend on the initial mean values X_2 and X_1, respectively.

We write down the variances of X_1 and X_2 by means of a single formula, in which the upper sign pertains to δX_1^2 and the lower to δX_2^2:

$$\delta X_{1,2}^2=\langle X_{1,2}^2\rangle-\langle X_{1,2}\rangle^2=e^{-\gamma t}[(\operatorname{ch} rt\mp\cos\psi\operatorname{sh} rt)^2\times\delta X_{1,2}^2(0)+\sin^2\psi\operatorname{sh}^2 rt\,\delta X_{2,1}^2(0)-\sin\psi\operatorname{sh} rt(\operatorname{ch} rt\mp\cos\psi\operatorname{sh} rt)K(0)+l^2(\pm\langle T^+T^+\rangle\pm\langle TT\rangle+\langle T^+T\rangle+\langle TT^+\rangle)],\qquad(8)$$

$$K(0)=\langle X_1(0)X_2(0)\rangle+\langle X_2(0)X_1(0)\rangle-2\langle X_1(0)\rangle\langle X_2(0)\rangle.$$

It can be seen that the variances δX_1^2 and δX_2^2 do not depend on the acting force. For the variance of the number of quanta we have the general formula

$$\begin{aligned}e^{2\gamma t}\delta n^2(t)&=[\langle(m^+m)^2\rangle-\langle m^+m\rangle^2]+\Phi[\langle m^+mm^++m^{+2}m\rangle-2\langle m^+\rangle\langle m^+m\rangle]+\Phi^*[\langle mm^+m+m^+m^2\rangle-2\langle m^+m\rangle\langle m\rangle]\\&+\Phi^2[\langle m^{+2}\rangle-\langle m^+\rangle^2]+\Phi^{*2}[\langle m^2\rangle-\langle m\rangle^2]\\&+|\Phi|^2[\langle m^+m+mm^+\rangle-2\langle m\rangle\langle m^+\rangle]+\langle T^2\rangle[\langle m^{+2}\rangle+2\Phi^*\langle m^+\rangle+\Phi^{*2}]+\langle T^{+2}\rangle[\langle m^2\rangle+2\Phi\langle m\rangle+\Phi^2]\\&+\langle TT^+\rangle[\langle m^+m\rangle+\Phi\langle m^+\rangle+\Phi^*\langle m\rangle+|\Phi|^2]\\&+\langle T^+T\rangle[\langle mm^+\rangle+\Phi\langle m^+\rangle+\Phi^*\langle m\rangle+|\Phi|^2]\\&+\langle(T^+T)^2\rangle-\langle T^+T\rangle^2.\end{aligned}\qquad(9)$$

Expressions for the mean values containing products of T and T^+ are given in the Appendix.

If at the initial instant of time $t=0$ the oscillator was at equilibrium with the thermostat, it is described by the density matrix

$$\rho=[1-\exp(-\hbar\omega/kT)]\sum_{n=0}^{\infty}e^{-n\hbar\omega/kT}|n\rangle\langle n|,$$

and then

$$\langle X_1(0)\rangle=\langle X_2(0)\rangle=K(0)=0,\quad n(0)=n_T,$$
$$\delta X_1^2(0)=\delta X_2^2(0)=l^2(2n_T+1),\quad \delta n^2(0)=n_T(n_T+1).$$

Expressions for the mean values and the variances that arise under the action of f and p on an equilibrium oscillator are given in the Appendix.

Different aspects of the behavior of an oscillator in a thermostat under the action of a force and (or) parametric pumping were analyzed in Refs. 10 and 11.

§2. PRODUCTION OF SQUEEZED STATES

A decrease (squeezing) of the variance of the operator X_1 or X_2 is reached by a special choice of the phase ψ of the parametric generator. Assume that at $t=0$ the oscillator was in thermodynamic equilibrium with the thermostat. We put $\psi=0$. It follows then from (A3) that

$$\begin{aligned}\delta X_1^2(t)&=l^2(2n_T+1)(\gamma+2r)^{-1}(\gamma+2re^{-(\gamma+2r)t}),\\\delta X_2^2(t)&=l^2(2n_T+1)(\gamma-2r)^{-1}(\gamma-2re^{-(\gamma-2r)t}),\end{aligned}\qquad(10)$$

i.e., in the variance of X_1 decreases with time, and the variance of X_2 increases, compared with the equilibrium value (we assume for the sake that $r>0$). The maximum squeezing of the variance of X_1 as $t\to\infty$ is always finite, and the equilibrium value $\delta X_1^2(0)$ decreases by a factor $\gamma/(\gamma+2r)$:

$$\delta X_1^2(t\to\infty)=l^2(2n_T+1)[\gamma/(\gamma+2r)].\qquad(11)$$

The increase of the variance of X_2 is finite if $2r<\gamma$, and increases exponentially with time if $2r>\gamma$. If the phase $\psi=\pi$ is chosen, the variances of X_1 and X_2 interchange roles (in (10) this is equivalent to replacing r by $-r$). At arbitrary ψ, the operators Y_1 and Y_2, which are certain linear combinations of X_1 and X_2, are subject to squeezing and to dilatation.

At arbitrary $2r\gg\gamma$, squeezing of the variance of X_1 to a value that practically reaches the limit takes place within a characteristic time

$$t_c=(2r)^{-1}\ln(2r/\gamma)\ll\gamma^{-1}.$$

Considerable squeezing is possible only at $2r$ comparable with γ or exceeding γ, and we shall therefore consider hereafter the cases $2r=\gamma$ and $2r\gg\gamma$.

The upper bound of the parameter $2r$ is 2ω. At this value $|p|=1$ and the Hamiltonian (2) would no longer be positive-definite. Consequently the oscillator would become unstable and the region of permissible values of the oscillator energy would range from $-\infty$ to $+\infty$.

As already noted, the variances of the operators X_1 and X_2 depend on the parameter r but not on f. The force enters only into the law that governs the mean values of X_1 and X_2. At $\psi=0$, as follows from (A.10), we obtain

$$\begin{aligned}\langle X_1(t)\rangle&=4l|f|\frac{\omega}{\gamma+2r}\sin\varphi[1-e^{-(r+\gamma/2)t}],\\\langle X_2(t)\rangle&=-4l|f|\frac{\omega}{\gamma-2r}\cos\varphi[1-e^{(r-\gamma/2)t}].\end{aligned}\qquad(12)$$

We now turn to the mean value and to the variance of the number of quanta. Under a force generator and a parametric generator act jointly on an oscillator placed in a thermostat, $n(t)$ and $\delta n^2(t)$ vary in accordance with Eqs. (A2) and (A4). In contrast to δX_1^2 and δX_2^2, the variance δn^2 depends on f, so that the force increases not only n but also δn^2.

To observe a weak force (e.g., a gravitational wave) it is desirable to create states with low variance and with large number of the quanta. Large $\langle n(t)\rangle$ is desirable because in a squeezed state, just as in a coherent one, the response of the oscillator, i.e., the change Δn of the number of quanta under the action of the signal, increases in proportion to the square root of the number of quanta contained in the oscillator. Relegating the proof of this statement and a detailed analysis of the detection criteria to §3, we discuss the conditions under which the squeezing, i.e., the use of a parametric generator with $r\neq 0$, makes it possible to decrease the ratio $\delta n/n^{1/2}$ in the oscillator.

It can be seen from (A2) and (A4) that were we to have $r=0$ i.e., if a coherent state (plus thermal noise) were produced in the oscillator under the influence of a laboratory force, we would have by the instant of time $t=t_1$

$$\langle n(t_1)\rangle = n_T + |\alpha|^2, \tag{13}$$

$$\delta n^2(t_1) = n_T(n_T+1) + (2n_T+1)|\alpha|^2, \tag{14}$$

where

$$\alpha = (2\omega|f|/\gamma)(1-e^{-\gamma t/2})e^{i(\varphi-\pi/2)}.$$

The parameter α defines also the mean values of X_1 and X_2:

$$\langle X_1\rangle + i\langle X_2\rangle = 2l\alpha.$$

It can be seen therefore that even within a time exceeding the relaxation time $\tau^* = \gamma^{-1}$ it is impossible to obtain a better result than

$$\delta n/n^{1/2} \approx (2n_T+1)^{1/2}.$$

This quantity does not differ in practice from the equilibrium value

$$\delta n/n^{1/2} = (n_T+1)^{1/2}.$$

If, however, $r\neq 0$, the choice of the phase shift $\varphi - \psi/2$ becomes important. For the choice $\cos(\varphi-\psi/2)=0$ and under the assumption $2r \gg \gamma$, the joint action of the force and parametric generators during the time

$$t \geqslant t_c = (2r)^{-1}\ln(2r/\gamma)$$

leads to

$$n(t_c) \approx \frac{|f|^2}{|p|^2}, \quad \delta n^2(t_c) \approx (2n_T+1)\frac{|f|^2}{|p|^2}\frac{\gamma}{2r}.$$

It was assumed here in the calculation that the deviation from the equilibrium state is appreciable, i.e., the terms containing $|f|$ predominate, and, concretely,

$$|f|^2/|p|^2 \gg (2n_T+1)(2r/\gamma)^3.$$

As a result we obtain a squeezed state with a value

$$\delta n/n^{1/2} \approx (2n_T+1)^{1/2}(\gamma/2r)^{1/2}, \tag{15}$$

which is better by a factor $(\gamma/2r)^{1/2}$ than in the coherent state. The obtained squeezed state can be used to record a weak force within a time interval that is short compared with τ^*, i.e., small compared with the decay time of this state.

Large deviations in the choice of the parameters used in the derivation of (15) make this estimate worse. The choice of the phase $\cos(\varphi-\psi/2)=0$ when considering $\langle n(t)\rangle$ and δn^2 follows the same purpose as the choice of the phases $\psi=0$ and $\cos\varphi=0$ when considering the most effective decrease of $\delta X_1^2(t)$ and the most effective increase of $\langle X_1(t)\rangle$.

In the derivation of (15) it was assumed that the force and parametric generators act jointly during the characteristic squeezing time t_c. A longer action of the generators is not advantageous, since it leads to dominance of terms that do not contain $|f|$, and then

$$\delta n/n^{1/2} \approx (2n_T+1)^{1/2}\exp[(r-\gamma/2)t].$$

At the same time, after a short time t_c the average number of the quanta does not increase strongly enough to permit it to grow under the influence of only the force generator in a time $\tau^* \gtrsim t \gg t_c$. Indeed, at $t\approx\tau^*$ we have

$$|\alpha|^2 \approx \frac{4\omega^2|f|^2}{\gamma^2} = \frac{|f|^2}{|p|^2}\left(\frac{2r}{\gamma}\right)^2 \gg \frac{|f|^2}{|p|^2}.$$

Under the experimental conditions it may be desirable to increase the oscillator response Δn that is proportional to the initial $\sqrt{n}$, with the preservation of the condition (15).

It is possible to obtain the same estimate (15), but with larger n, if the force and parametric generators act separately and in succession. The idea is to produce first with the aid of the force generator a large $\langle n(t)\rangle$, and then turn off the force action and turn on the parametric action. Then, at a suitable choice of the phases, after a short squeezing time t_c the variance δn^2 decreases strongly, and $\langle n(t)\rangle$ does not change so much within the same time.

Let us consider the proposed method in greater detail. Assume that an oscillator in equilibrium with the thermostat is acted upon by a force during a time t_1. Then by the instant t_1 we have

$$a(t_1) = e^{-i\omega t_1}e^{-\gamma t_1/2}[a_0 + \Phi(t_1) + T(t_1)]$$

and the Hermitian-adjoint expression for $a^+(t_1)$. The values of $\langle n(t_1)\rangle$ and $\delta n^2(t_1)$ are determined by Eqs. (13) and (14). At the instant t_1 the force is turned off and r is turned on. The subsequent evolution of the oscillator can be calculated from the general formulas (7) and (9) with the already known initial data. In this case $\Phi\equiv 0$ and it is convenient to reckon the time from the instant $t=t_1$. We then obtain for $\langle n(t)\rangle$

$$\langle n(t)\rangle = e^{-\gamma t}|\alpha|^2[\operatorname{ch} 2rt - \operatorname{sh} 2rt\cos(\psi-2\varphi_1)] + n_0,$$

where $\varphi_1 = \varphi - \omega t_1 - \pi/2$ and the term n_0 stands for expression (A2) in which we must put $|\Phi|^2=0$. For $\delta n^2(t)$, using the notation of the Appendix, we can obtain

$$\begin{aligned} e^{2\gamma t}\delta n^2(t) &= n_T(n_T+1)\operatorname{ch} 4rt \\ &+ |\alpha|^2(2n_T+1)[\operatorname{ch} 4rt - \operatorname{sh} 4rt\cos(\psi-2\varphi_1)] \\ &+ {}^1/_2\gamma|\alpha|^2(2n_T+1)\{e^{2rt}[1-\cos(2\varphi_1-\psi)](I_c-I_s) \\ &+ e^{-2rt}[1+\cos(\psi-2\varphi_1)](I_c+I_s)\} \\ &+ {}^1/_2[\operatorname{sh}^2 2rt - e^{\gamma t} + 1 + \gamma(2n_T+1)^2(I_c\operatorname{ch} 2rt - I_s\operatorname{sh} 2rt)]. \end{aligned}$$

The principal growing terms in $\langle n(t)\rangle$ and $\delta n^2(t)$ are eliminated by choosing the phase $\cos(2\varphi_1 - \psi) = 1$. We assume also that $2r \gg \gamma$ and

$$|\alpha|^2 \gg [n_T(n_T+1)/(2n_T+1)](2r/\gamma)^4.$$

By the instant of time t_c we then have

$$n(t_c) \approx |\alpha|^2 \frac{\gamma}{2r}, \quad \delta n^2(n_1, t_c) \approx |\alpha|^2 (2n_T+1) \left(\frac{\gamma}{2r}\right)^2, \quad (16)$$

i.e., $\delta n/(n)^{-1/2}$ we return to Eq. (15) for, but now we have $n(t_c)$ larger by $2r/\gamma$ times than the value of n reached in the variant with joint action of f and r. We note that by that instant of time the average values of the operators a and a^+ are equal, apart from a phase factor, to the quantity $|\alpha|(\gamma/2r)^{-1/2}$, i.e., to $(n)^{-1/2}$.

Worthy of special consideration is the case $2r = \gamma$. In Eqs. (12) and (10) for $\langle X_1(t)\rangle$ and $\delta X_1^{\,2}(t)$ this case is not at all remarkable, but in the expressions for $\langle X_2(t)\rangle$ and $\delta X_2^2(t)$ it leads to a singularity:

$$\langle X_2(t)\rangle = -2l|f|\omega t \cos\varphi \quad \delta X_2^{\,2}(t) = l^2(2n_T+1)(1+\gamma t). \quad (17)$$

If the experimenter can use time intervals that are long compared with the relaxation time, then at $t \gg \gamma^{-1}$ we can obtain (at $\cos\varphi = -1$)

$$\delta X_2/X_2 \approx [(2n_T+1)^{1/2}/2|f|\omega](\gamma/t)^{1/2},$$

i.e., $\delta X_2/X_2 \to 0$ at $t \to \infty$. We note that at $2r \gg \gamma$ and $t \to \infty$ we would obtain a nonvanishing and rather large quantity

$$\delta X_2/X_2 = (2n_T+1)^{1/2}(r/2|f|\omega).$$

For $\langle n(t)\rangle$ and $\delta n^2(t)$ at $2r = \gamma$, $\cos(\varphi - \psi/2) = \pm 1$ and $\gamma t \gg 1$ we have from (A2) and (A4)

$$\langle n(t)\rangle \approx |f|^2\omega^2 t^2, \quad \delta n^2 \approx (2n_T+1)|f|^2\omega^2\gamma t^3. \quad (18)$$

As will be shown in §3, the case $2r = \gamma$ can be used to improve the sensitivity if the signal can be accumulated during many relaxation times.

The variance δn of the number of quanta must be set in correspondence with the variance $\delta\hat{\varphi}$ of the phase operator, even though this operator is known to be defined subject to some stipulations. We present nevertheless illustrative arguments from which it can be seen that in squeezed states a decrease of δn is accompanied by an increase of $\delta\hat{\varphi}$ and an increase of δn is accompanied by a decrease of $\delta\hat{\varphi}$. We consider first coherent states. On a plane with coordinates $\langle X_1\rangle$ and $\langle X_2\rangle$ the coherent state $|\alpha\rangle$ is mapped by a circle with center at the points $2l\alpha$ in accordance with the equation $\langle X_1\rangle + i\langle X_2\rangle = 2l\alpha$. The diameter of the circle is determined by the variance $\delta X_1 = \delta X_2 = l(2n_T+1)^{1/2}$. The uncertainty of the number of quanta at

$$|\alpha|^2(2n_T+1) \gg n_T(n_T+1)$$

is $\delta n \approx (2n_T+1)^{1/2}|\alpha|$, i.e., $\delta n = S(m\omega/2\pi\hbar)$, where S is the area of a ring made up by rotating the circle around the origin (see Fig. 1). The phase uncertainty $\delta\hat{\varphi}$ is expressed in terms of the (doubled) angle from which the diameter is seen from the origin, so that

$$\delta\hat{\varphi} \approx (2n_T+1)^{1/2}/|\alpha|.$$

As a result we obtain

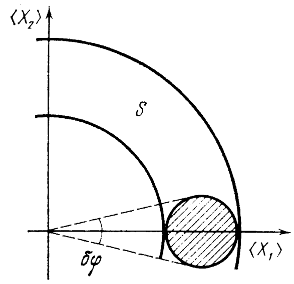

FIG. 1

$$\delta n\delta\hat{\varphi} \approx (2n_T+1).$$

In the squeezed state, the variance circle is transformed into an ellipse compressed along the axis for which the variance is a minimum. If the phases $\psi = 0$ and $\cos\varphi = 0$ are chosen, the ellipse is compressed along the X_1 axis, and its center is located on the X_1 axis; at $\psi = 1$ and $\cos\varphi = \pm 1$, the ellipse is compressed along the same axis, but its center is located on the X_2 axis (see Fig. 2). Accordingly, in the former case the area of the ring decreases, and the angle increases, so that [cf. (16)]

$$\delta n \approx (m\omega/\hbar)\langle X_1\rangle\delta X_1 \approx (2n_T+1)^{1/2}|\alpha|(\gamma/2r),$$

$$\delta\hat{\varphi} \approx 2\delta X_2/\langle X_2\rangle \approx (2n_T+1)^{1/2}|\alpha|^{-1}(2r/\gamma)$$

and

$$\delta n\delta\hat{\varphi} \approx (2n_T+1).$$

In the latter case δn increases and $\delta\hat{\varphi}$ decreases, so that again

$$\delta n\delta\hat{\varphi} \approx 2n_T+1.$$

§3. USE OF SQUEEZED STATES TO DETECT A WEAK FORCE

The possibilities of using squeezed states can be naturally separated into two versions, depending on whether the duration of the signal and other conditions of the experiment will permit accumulation of a signal during several relaxation times, or whether the accumulation time is much shorter than γ^{-1}. If $\gamma\tau \ll 1$, it is reasonable to first "prepare" the squeezed state, and then use it to observe the force before the variance of the measured quantity manages to increase substantially. The action of the signal itself in the course of preparation of the state is immaterial. We consider this technique for the measurement of $X_1(t)$ and of the number of quanta.

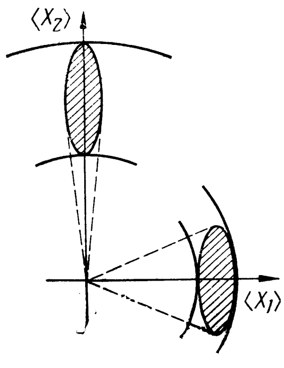

FIG. 2

Assume that at the instant when the laboratory generators are turned off (we choose this instant of time to be zero time) the squeezing reached (at $2r \gg \gamma$) is

$$\delta X_1^2(0) \approx l^2(2n_T+1)(\gamma/2r). \tag{19}$$

Subsequently $\delta X_1^2(t)$ increases in accordance with (8):

$$\delta X_1^2(t) = l^2(2n_T+1)\left[\frac{\gamma}{2r}e^{-\gamma t}+1-e^{-\gamma t}\right]$$
$$\approx l^2(2n_T+1)\left(\frac{\gamma}{2r}+\gamma t\right). \tag{20}$$

The expression (20), which is approximate at $\gamma t \ll 1$, makes it possible to refine the permissible observation time τ. It is desirable for this time to be such that the growing term in δX_1^2, i.e., the term proportional to γt, still has not exceeded the initial value $\delta X_1^2(0)$. In other words, if $\tau \lesssim (2r)^{-1}$, the variance did increase by not more than a factor of 2, but if $(2r)^{-1} \ll \tau < \gamma^{-1}$, the variance is still smaller than the equilibrium value, but comes close to it. The change of $\langle X_1(t)\rangle$ under the action of the gravitational force f_g to be observed proceeds in accordance with (5), where we must put $r = 0$, $f = f_g$, and $\varphi = \varphi_g$. This yields

$$\Delta X_1(t) = 4l|f_g|\frac{\omega}{\gamma}\sin\varphi_g(1-e^{-\gamma t/2}) \approx 2l|f_g|\omega t\sin\varphi_g \tag{21}$$

Taking ΔX_1 to be the signal S and δX_1 the noise N and assuming also $\sin\varphi_g = 1$, we obtain within an observation time $\tau \lesssim 1/2r$

$$S/N \approx [2|f_g|\omega\tau/(2n_T+1)^{1/2}]\left(\frac{2r}{\gamma}\right)^{1/2} \tag{22}$$

and after a time $1/2r \ll \tau \lesssim 1/\gamma$

$$S/N \approx [2|f_g|\omega/(2n_T+1)^{1/2}]\left(\frac{\tau}{\gamma}\right)^{1/2}. \tag{23}$$

If the oscillator were to be in a coherent state, then after an observation time $0 < \tau \ll \gamma^{-1}$ we would obtain the estimate

$$S/N \approx 2|f_g|\omega\tau/(2n_T+1)^{1/2}, \tag{24}$$

which is worse by $(2r/\gamma)^{1/2}$ times than (22), and worse by $(\gamma\tau)^{-1}$ times than (23). Of course, if it is possible to accumulate a signal within a time on the order of γ^{-1}, the minimum observable force during that time is, according to (24),

$$f_{min} \approx (2n_T+1)^{1/2}\gamma/2\omega,$$

much better than the estimate

$$f_{min} \approx \frac{(2n_T+1)^{1/2}}{2\omega}(2r\gamma)^{1/2}$$

that follows from Eq. (22) at $\tau \approx 1/2r$, and somewhat better than the estimate that follows from (23). This is so because the time required to reach the maximum ΔX_1, as follows from (21), is γ^{-1} while the decay time of the squeezed state is also γ^{-1}. Thus, it is advantageous to use squeezed states that are prepared beforehand if the time of observation is strongly limited for some reason or another.

Perfectly analogous conclusions are obtained also by analyzing measurements of the number of quanta. The action of the force changes $\langle n(t)\rangle$ in accordance with (7):

$$\langle n(t)\rangle = n_0e^{-\gamma t}+n_T(1-e^{-\gamma t})+\frac{2i\omega}{\gamma}e^{-\gamma t}(e^{\gamma t/2}-1)$$
$$\times[\langle a_0\rangle f_g^* - \langle a_0^+\rangle f_g] + \frac{4\omega^2}{\gamma^2}|f_g|^2(1-e^{-\gamma t/2})^2, \tag{25}$$

where n_0 is the initial average number of quanta.

Inasmuch as in the squeezed state $\langle a_0\rangle \neq 0$ and $\langle a_0^+\rangle \neq 0$, the term linear in the small quantity f_g is preserved, just as in the case of the coherent state (but not in the n-quantum or equilibrium state). We are interested in such squeezed states in which at a given average number of quanta the variance δn^2 is much less than in the coherent state with the same n. In such states, as can be seen from a discussion of Eqs. (16), $\langle a_0\rangle$ and $\langle a_0^+\rangle$ are equal, apart from phase factors, to $\sqrt{n_0}$. Thus, according to (25) the change of the number of quanta under the action of f_g and at $\gamma t \ll 1$ is

$$\Delta n(t) \approx 2|f_g|\sqrt{n_0}\omega t.$$

The variance δn^2 increases from an initial value δn_0^2, which we choose in accordance with (16) in the form

$$\delta n_0^2 = n_0(2n_T+1)\gamma/2r.$$

The law of variation of the variance is determined by the general formula (9) at $r = 0$, but we assume f_g to be so small that the increase of δn^2 is determined by the interaction with the thermostat and not with f_g. Then

$$\delta n^2(t) \approx n_0(2n_T+1)(\gamma/2r+\gamma t)$$

and for $\tau \lesssim (2r)^{-1}$ we have

$$\frac{S}{N} = \frac{\Delta n}{\delta n} \approx \frac{2|f_g|\omega\tau}{(2n_T+1)^{1/2}}\left(\frac{2r}{\gamma}\right)^{1/2}$$

which coincides with (22). At $1/2r \ll \tau \lesssim \gamma^{-1}$ we obtain a formula that coincides with (23).

We consider now the observation of prolonged signals with the aid of the variant $2r = \gamma$. As seen from (17) and (18), the mean values of X_2 and n and of their variances increase in the course of time, but the growth of the mean values is faster. The oscillator is acted upon jointly by the laboratory force f_l and by the gravitational f_g; i.e.,

$$f \equiv |f|e^{i\varphi} = |f_l|e^{i\varphi_l}+|f_g|e^{i\varphi_g}.$$

The increment of $\langle X_2(t)\rangle$ due to the force f_g is, if the phases are suitably chosen

$$\Delta X_2 \approx 2l|f_g|\omega t,$$

and the variance is

$$\delta X_2^2 \approx l^2(2n_T+1)\gamma t,$$

from which we obtain at an observation time $\tau \approx \gamma^{-1}$ or $\tau \gg \gamma^{-1}$

$$S/N = [2|f_g|\omega/(2n_T+1)^{1/2}](t/\gamma)^{1/2}.$$

Exactly the same formula is obtained for $S/N = \Delta n/\delta n$. Thus, the minimum observable force decreases without limit with increasing observation time. The actual sensitivity limits will be determined by the accuracy with which the necessary relations between the phases and the parameters are satisfied.

§4. COMPARISON OF THE SQUEEZED-STATES TECHNIQUE AND QND MEASUREMENTS IN THE PROBLEM OF OBSERVING A WEAK FORCE

The squeezed-states technique and the use of QND measurements[1–3,12–14] have common features. In both cases one of the tasks of the method is to decrease the variance of the measured quantity. In QND measurements the decrease of the variance is reached by the very act of the first measurement, as a result of which a reduction of the wave function takes place, while in the squeezed-states technique the variance is decreased by specially chosen force and parametric action on the oscillator. Although the postulates of quantum mechanics permit the variance to be decreased after the measurement to zero, its actual value is determined by the coupling constants and by the measuring instrument, by the measurement time, etc. In addition, as already noted, the variance will inevitably increase with time as a result of the interaction of the system with the thermostat. In this sense, we lose the fundamental difference between the final variance obtained by squeezing, and that (final) dispersion which can be obtained after the first measurement performed in a realistic situation.

The initial requirement and theory of QND measurements is to choose quantities that can be measured with arbitrary accuracy without demolishing the state of the system. The system, by assumption is, after the first measurement in one of the eigenstates of the QNDF operator or of the QNDP operator, and it is precisely this quantity which is measured subsequently. The squeezed-states technique does not presuppose an obligatory measurement of the QND-variable. Prior to the arrival of the gravitational signal and during the time of its action, the oscillator is in a squeezed state and not, say, in an eigenstate $X_1(t)$ of the QNDF-operator. In this case the force can be detected, e.g., by the changes of the energy, i.e., of a quantity that is not a QNDF-variable. However, the experimenter must not claim too high an accuracy of the measurements and, consequently, too large a perturbation of the system. The accuracy of the measurements must not be better than the (*a priori*) variance of the measured quantity.

To compare the two considered methods, we assume that we measure the QNDF-variable $X_1(t)$. In accordance with the QND-measurement principle, we assume that at the initial instant of time an exact measurement of $X_1(t)$ was performed on the equilibrium oscillator, as a result of which the oscillator is brought to an eigenstate of the operator X_1 with a value $X_1(0)=0$; then $\delta X_1^2(0)=0$. Subsequently, the variance will increase in accordance with the general formula (8), and the average value will change under the influence of f_g in accordance with (5). The S/N ratio after a time τ of the action of the force will be

$$S/N=[2|f_g|\omega/(2n_T+1)^{1/2}](\tau/\gamma)^{1/2}. \tag{26}$$

We assume now that at the initial instant of time the oscillator is in a squeezed state. We measure again the quantity X_1, but not too accurately, within the limits of the variance. Then the S/N ratio is determined by Eqs. (22) and (23). We know that the measurement of the energy leads to similar equations. The difference from formula (26) takes place only at $0<\tau\lesssim 1/2r$. In this time interval the sensitivity of (26) is generally speaking better than that of (22). However, when $2r$ tends to a definite limit equal to 2ω, the difference vanishes, since the duration of the action of f_g is assumed at any rate to be not shorter than $2\pi/\omega$. Of course, as $r\to\omega$ the conclusions concerning the sensitivity are only qualitatively valid.

Summarizing, we can state that when account is taken of the inevitable damping in the oscillator ($\gamma\neq 0$, $n_T\neq 0$) the QND-measurement method loses its advantages to a considerable degree. At large squeezing parameters the state-squeezing technique is not inferior to that of QND measurements, having the advantage that the quantity measured must not necessarily be a QND variable.

The use of squeezed states can improve the realizability of laboratory experiments on the study and detection of gravitational waves. In Ref. 15 was considered a concrete scheme, in which the radiation and detection were proposed to be effected with electromagnetic fields. The estimate of the sensitivity started out with the fact that it is possible to observe the change of the number of quanta $\sqrt{n}$ against the background of the n quanta present in the cavity-detector. In other words, estimates were used for an oscillator in a coherent state. In this case the technical requirements on the facility are very stringent[15]: the total volume of the system $V=25\times 10^9$ cm^3, characteristic field intensity $E\sim H\sim 3\times 10^5$ G, signal frequency $\omega=2\times 10^{18}$ sec^{-1}, Q-factor of the cavity-detector $Q=\omega\tau^*=7\times 10^{13}$, time of signal accumulation (equal to the relaxation time) $\hat{\tau}=4\times 10^5$ sec. Were it possible to improve the Q of the resonator-detector, e.g., to $Q\approx 8\times 10^{15}$ and produced in it a squeezed state with value $2r/\gamma\approx 10^2$, the sensitivity could be increased during the same accumulation time by a factor of 10, and consequently relax the requirements on the other parameters of the system. These values of Q are apparently perfectly attainable.[16]

It is advantageous to use squeezed states also when detecting monochromatic cosmic radiation, e.g., from a pulsar in the Crab nebula. Experiment is being presently planned on detection of gravitational radiation from this pulsar.[17] The parameters of the resonant antenna are the following: $\nu=60$ Hz, $Q=2\times 10^8$ $M=1400$ kg, antenna length $L=1.65$ m, and the temperature at which the antenna will operate is $T=3\times 10^{-3}$ K. The signal accumulation time τ should amount to $\approx 5\times 10^5$ sec. At a frequency $\nu=60$ Hz, however, the Q of the antenna can possibly be raised to $Q\approx 10^{11}$ (Ref. 16). The relaxation time in such a system will be approximately 9 years. If the technique of squeezed states is used, the time of signal accumulation up to satisfaction of the observability criterion $S/N=1$ could be decreased to approximately 10^3 sec.

APPENDIX

We present first an expression for the quantities containing the operators T and T^+, which do not depend on the initial state of the oscillator. We introduce the notation

$$I_c=\int_0^t e^{\gamma\tau}\,\mathrm{ch}\,2r(\tau-t)\,d\tau,\quad I_s=\int_0^t e^{\gamma\tau}\,\mathrm{sh}\,2r(\tau-t)\,d\tau,$$

$$E=\int_0^t e^{\gamma\tau}\,d\tau.$$

Then

$$\langle T^+T\rangle=\gamma(n_T+{}^1/_2)I_c-{}^1/_2\gamma E,\quad \langle TT^+\rangle=\gamma(n_T+{}^1/_2)I_c+{}^1/_2\gamma E,$$
$$\langle T^+T^+\rangle=\langle TT\rangle^*=\gamma(n_T+{}^1/_2)e^{-i\psi}I_s,$$
$$\langle(T^+T)^2\rangle-\langle T^+T\rangle^2={}^1/_4\gamma^2[(2n_T+1)^2(I_c^2+I_s^2)-E^2].$$

We write out also the expression, which is used in the test for Φ:

$$\Phi(t)$$
$$=2\omega|f|e^{i\psi/2}e^{\gamma t/2}\left\{-i\cos\left(\varphi-\frac{\psi}{2}\right)\frac{1-e^{(r-\gamma/2)t}}{\gamma-2r}\sin\left(\varphi-\frac{\psi}{2}\right)\right.$$
$$\left.\times\frac{1-e^{-(r+\gamma/2)t}}{\gamma+2r}\right\}.$$

The general formulas (5)–(9) become simpler if at the initial instant of time the oscillator was at equilibrium with the thermostat. Then

$$\langle X_1(t)\rangle=le^{-\gamma t/2}[\Phi(t)+\Phi^*(t)],$$
$$\langle X_2(t)\rangle=le^{-\gamma t/2}i[\Phi^*(t)-\Phi(t)],\qquad\text{(A1)}$$

$$\langle n(t)\rangle=e^{-\gamma t}\left[|\Phi(t)|^2+\frac{r}{2}(2n_T+1)\left(\frac{e^{-2rt}}{\gamma+2r}-\frac{e^{2rt}}{\gamma-2r}\right)\right]$$
$$+\frac{n_T\gamma^2+2r^2}{\gamma^2-4r^2}.\qquad\text{(A2)}$$

For the variances we have

$$\delta X_{1,2}(t)=l^2(2n_T+1)e^{-\gamma t}[\operatorname{ch}2rt\mp\cos\psi\operatorname{sh}2rt+\gamma(I_c\pm\cos\psi I_s)],\qquad\text{(A3)}$$

$$\delta n^2(t)=e^{-2\gamma t}\left\{\frac{1}{4}(2n_T+1)^2\operatorname{ct}4rt-\frac{1}{4}e^{2\gamma t}+\frac{1}{4}\gamma(2n_T+1)^2[\gamma(I_c^2+I_s^2)\right.$$
$$+e^{2rt}(I_c-I_s)+e^{-2rt}(I_c+I_s)]+4(2n_T+1)|f|^2\omega^2\left[\cos^2\left(\varphi-\frac{\psi}{2}\right)\left(\frac{e^{\gamma t/2}-e^{rt}}{\gamma-2r}\right)^2\right.$$
$$\left.\left.\times(e^{2rt}+\gamma I_c-\gamma I_s)+\sin^2\left(\varphi-\frac{\psi}{2}\right)\left(\frac{e^{\gamma t/2}-e^{-rt}}{\gamma+2r}\right)^2(e^{-2rt}+\gamma I_c+\gamma I_s)\right]\right\}.\qquad\text{(A4)}$$

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