Resistive state in broad superconducting films

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(Submitted 16 November 1982)
Zh. Eksp. Teor. Fiz. **84**, 2216–2227 (June 1983)

The critical current and the current-voltage characteristic of a superconducting film whose width is large compared with the effective penetration depth of the magnetic field is obtained. It is shown that at the critical value of the current the nonvortical superconducting state becomes unstable and that vortex changes are generated at the edges of the films. The current-voltage characteristic of the film shows a voltage jump at a total current much higher than the critical value, and the film resistance ahead of the jump is small compared with the normal resistance. Pinning can lead to the appearance of voltage jumps also at small currents.

PACS numbers: 74.40. + k, 73.60.Ka

1. INTRODUCTION

Destruction of superconductivity by current in thin films is attracting much interest of late. The transition from the norma state is "smeared" this case, so that there exists a large range of currents, higher than the film critical current I_c, at which the film resistance is already different from zero, but is still lower than the normal resistance (the resistive state).

The nature of the resistive state depends substantially on the relation between the width b of the film (the films usually are long strips of given width) and the coherence length ξ in the superconductor. In narrow films (superconducting channels: $b<\xi$) there arise the so-called phase-slip centers, at which a voltage is generated.[1-2] In broad films the magnetic film of the current generates on the edges of the film chains of Abrikosov vortices whose motion across the film leads to the appearance of resistance.

In the present paper, this picture is used as the basis for finding the critical current and the current-voltage characteristic (CVC) of a broad superconducting film without an external magnetic field at temperatures close to critical. The width is assumed large both compared with ξ and relative to the effective depth λ_{eff} of the penetration of the magnetic film into the film. The critical current is determined from the condition that the nonvortical current superconducting state (the Meissner state) be stable to small perturbations of the order parameter. Instability sets in at a certain value of the vector potential at the edge of the film, and the critical perturbation is found to depend periodically on the longitudinal coordinate. Such an analysis permits also an estimate of the time of the instability development that leads to generation of the vortex chain at the film edge.

To find the CVC we investigated the viscous motion of the vortices in the film. We used the hydrodynamic approximation that becomes valid already at current closae to critical (if the degree of supercriticality is low). We have introduced the densities of the vortices and of the average current, which depend on the transverse coordinate, and write down the macroscopic equations that connect these quantities with the average electric field E in the film. We obtain thus the dependence of the voltage drop along the film on the total current I.

When the current is increased the vortex density increases and the current distribution becomes more and more uniform. At a certain current value (considerably exceeding the film critical current I_c) the current density becomes equal to the critical not only at the edges, where the vortices are generated, but also in the middle of the film. Although the distance between vortices is in this case still large compared with the coherence length (the vortex cores do not overlap), the vortical superconducting state becomes unstable and the film goes over jumpwise into the normal state. We obtain the corresponding values of the current and of the voltage jump.

We investigated also the effect of pinning on the critical current in the CVC of a broad superconducting film. To this end, a phenomenological pinning force was introduce into the equations, of viscous motion. This force has little effect on the vortex generation at the film edges, where the current density is high (equal to the critical density), but slows down the vortices and decreases thus the differential resistance of the film. At a certain value of the pinning force it becomes capable of stopping the chain of vortices at the center of the film, where the current density is lowest. At larger pinning the critical current increases, and a voltage appears jumpwise across the film, since stationary motion of a definite vortex structure sets in right away. The effect of the pinning on the transition of the film into the normal state is weak. The current density is everywhere large in this case and the principal role is played by the interaction of the transport current.

2. CRITICAL CURRENT OF FILM

So long as the value of the total current does not exceed the critical value of I_c, the film is in the Meissner state. The order parameter, the current density **j**, and the vector potential **A** depend then in a definite manner on the transverse coordinate x, and the modulus of the order parameter differs from zero everywhere in the film. These relations are obtained from the Ginzburg-Landau equations, which are best written in dimensionless units:

$$\varkappa_{\text{eff}}^{-2}\nabla^2F+F(1-F^2-\mathbf{Q}^2)=0, \tag{1}$$

$$\operatorname{rot}\operatorname{rot}\mathbf{Q}=-F^2\mathbf{Q}\delta(z)\theta(x)\theta(\tilde{b}-x). \tag{2}$$

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Here $\varkappa_{\text{eff}} = \lambda_{\text{eff}}/\xi$ is the effective Ginzburg-Landau parameter renormalized for a film of thickness d smaller than the magnetic-field penetration depth λ ($\lambda_{\text{eff}} = \lambda^2/d$); the distances are measured in units of λ_{eff} ($\tilde{b}/b/\lambda_{\text{eff}}$); F is the modulus of the order parameter and is measured in units

$$\Delta_{GL}=[8\pi^2 T(T_c-T)/7\zeta(3)]^{1/2};$$

$$\mathbf{Q}=\mathbf{A}-\varkappa_{\text{eff}}^{-1}\nabla\chi$$

is the gauge-invariant vector potential (χ is the phase of the order parameter) and is measured in units of $\varphi_0/2\pi\xi$ (φ_0 is the magnetic-field quantum). The film is a strip of width b, the transverse coordinate is measured from the edge of the film, and z is the coordinate perpendicular to the plane of the film (the film is assumed to be infinitely thin in this direction). The right-hand side of Eq. (2) is the expression for the density of the superconductng current.

The boundary conditions for the system (1) and (2) are the conditions for the vanishing of the derivative of the modulus of the order parameter and of the transverse component of the current density at the edges of the film:

$$\partial F/\partial x|_{x=0,\tilde{b}}=0, \qquad Q_x|_{x=0,\tilde{b}}=0. \tag{3}$$

Usually in the film $\xi_{\text{eff}} \gg \xi$ and the Ginzburg-Landau parameter $\varkappa_{\text{eff}} \gg 1$. In this case, at distances large compared with ξ from the edges of the film, the gradient term in (1) can be neglected, so that the modulus of the order parameter and the vector potential are connected by the relation

$$F^2=1-\mathbf{Q}^2. \tag{4}$$

The distribution of the vector potential is obtained from Eq. (2), which must be solved at a given value of the integral

$$\int_0^{\tilde{b}} F^2\mathbf{Q}\,dx$$

(of the total current) and under condition that the magnetic field vanish far from the film (curl $\mathbf{Q} = 0$ as $x, z\to\infty$). The vector potential is directed along the y axis and decreases monotonically along x from the edges towards the center of the film. Its value $Q = Q_y$ ($z = 0$) in the film is obtained from the equation

$$\frac{dQ}{dx}=-\frac{1}{2\pi}\int_0^{\tilde{b}}\frac{Q(x')[1-Q^2(x')]}{x'-x}\,dx'. \tag{5}$$

When finding the critical current we shall be interested in the distribution of the potential at distances small compared with λ_{eff} ($x\ll 1$) from the film edges. In this region, the asymptotic solution of (5) is of the form

$$Q=Q_0\left[1-\frac{1-Q_0^2}{2\pi}x\ln\frac{1}{x}\right]. \tag{6}$$

The vector potential Q_0 at the edge of the film is determined the total current in the film I.

We investigate now the stability of the Meissner state to infinitely small perturbations of the order parameter f and of the vector potential $\mathbf{q}$. Such a question was first considered in Refs. 3 and 4. We use for our problem a method developed for the determination of the maximum magnetic field for superheating the Meissner state of a bulky superconductor.[5] Linearizing the system (1), (2) we have

$$\varkappa_{\text{eff}}^{-2}\nabla^2 f+f(1-3F^2-Q^2)-2F\mathbf{Q}\mathbf{q}=0, \tag{7}$$

$$\text{rot rot}\,\mathbf{q}=-(F^2\mathbf{q}+2fF\mathbf{Q})\,\delta(z)\,\theta(x)\,\theta(\tilde{b}-x). \tag{8}$$

Equations (7) and (8) correspond to the condition that the second variation of the free energy vanish at the extremal point determined by the Ginzburg–Landau equations (1) and (2). The existence of nontrivial solutions of Eqs. (7) and (8) means that the extremum point is a saddle point and that the Meissner state is unstable.

The critical perturbation turns out to be localized at distances from the film edge that are small compared with λ_{eff}. We can therefore substitute Eq. (6) and the corresponding expression (4) for the modulus of the order parameter in (7) and (8) to obtain the unperturbed potential. For the same reason, we can neglect the magnetic self-field produced by the perturbation of the potential $\mathbf{q}$. This gives the connection between the components q_x and q_y of the potential in the film:

$$\partial q_y/\partial x-\partial q_x/\partial y=0 \tag{9}$$

and makes it possible to neglect the component q_z of the potential. The critical potential is found to depend on the coordinate y direction along the film.

For each component of the potential of the critical perturbation this dependence turns out to be stronger than the dependence on the transverse coordinate x, so that the corresponding derivatives can be omitted.

Transforming to the Fourier representation with respect to the coordinate y (with a corresponding wave number k), we obtain from Eqs. (7)–(9) for the component $q_y(x, k)$ of the perturbation potential

$$\frac{\partial^2 q_y}{\partial x^2}-(1-Q_0^2)^{-1}k^2 \times\left[1-3Q_0^2+\frac{3}{\pi}Q_0^2(1-Q_0^2)\,x\ln\frac{1}{x}+\frac{k^2}{2\varkappa_{\text{eff}}^2}\right]q_y=0. \tag{10}$$

The condition that the total current be invariant yields the boundary condition for this equation

$$\partial q_y/\partial x|_{x=0}=0, \tag{11}$$

and we are interested in solutions that decrease towards the interior of the film.

The significant values are $k\gg 1$, and with logarithmic accuracy the solution of Eq. (10) is an Airy function Φ:

$$q_y=\Phi\left\{\left(\frac{2}{3\pi}k^2\ln k\right)^{1/3}\left[x-\frac{9\pi}{4}\ln^{-1}k\left(3Q_0^2-1-\frac{1}{2}k^2\varkappa_{\text{eff}}^{-2}\right)\right]\right\}, \tag{12}$$

and the boundary condition (11) gives the connection between k and Q_0

$$3Q_0^2-1={}^1/_2k^2\varkappa_{\text{eff}}^{-2}-\zeta_0\left(\frac{2}{3}\right)^{5/3}\pi^{-2/3}k^{-2/3}\ln^{2/3}k, \tag{13}$$

where $\zeta_0 = -1.02$ is the point of the first maximum of the Airy function.

Equation (13) allows us to find the minimum value of Q_0 at which the instability sets in

$$Q_{0c} = \frac{1}{\sqrt{3}}\left[1 + \frac{2}{3}(3\pi)^{-1/4}(-\zeta_0)^{3/4}\left(\frac{\ln \varkappa_{\text{eff}}}{\varkappa_{\text{eff}}}\right)^{1/2}\right] \tag{14}$$

and the corresponding value of k

$$k_c = (-\zeta_0)^{3/8} 2^{1/2} 3^{-3/4} \pi^{-1/4} \varkappa_{\text{eff}}^{3/4} \ln^{1/4} \varkappa_{\text{eff}} . \tag{15}$$

It follows from (7) and (8) that a similar instability sets in at $Q_0 = W_{0c}$ for the transverse component of the potential and for the modulus of the order parameter.

The critical value of the potential Q_{0c} at the edge of the film turns out to be close to $1/\sqrt{3}$. The corresponding current density $j = Q(1 - Q^2)$ at the edge of the film is close, at the onset of the instability, to the density of the pair-breaking current of a one-dimensional superconduction channel (j is measured in units of $3\sqrt{3}jc/2$, where $j_c = \varphi_0/12\sqrt{3}\pi^2\lambda^2\xi$). The critical perturbation is localized in the region

$$x_0 \sim k_c^{-2/3} \ln^{-1/3} k_c \sim \varkappa_{\text{eff}}^{-1/2} \ln^{-1/2} \varkappa_{\text{eff}}$$

at the edge of the film, and oscillates along the film with a period

$$y_0 \sim k_c^{-1} \sim \varkappa_{\text{eff}}^{-3/4} \ln^{-1/4} \varkappa_{\text{eff}} .$$

(We recall that the length is measured here in units of λ_{eff}.)

To determine the total current I_c at which the potential at the edge of the film reaches the critical value we must know the distributions of the current and of the potential in the entire film. At distances larger than λ_{eff} from the film edges, the current density is already much lower than critical, and $Q \ll 1$. On the other hand, this is precisely the region that makes the principal contribution to the total current. The corresponding solutions of (5) are of the form

$$Q = (I/I_0)\, \tilde{b}^{1/2} x^{-1/2} (\tilde{b} - x)^{-1/2}.$$

Using the exact solution[6] of the linearized equation (5), we can match this solution to the function $Q(x)$ given by (6) near the edge. As a result we obtain for the critical current the expression

$$I_c = (2/3\pi)^{1/2} I_0, \quad I_0 = \frac{3\sqrt{3}}{2}\pi j_c d (\lambda_{\text{eff}}\, b)^{1/2}. \tag{16}$$

Estimates show that allowance for the nonlinear terms in (5) can change the coefficient in (16) by not more than 30%. A similar expression for the critical current was obtained in Ref. 7 from an estimate of the condition for the vanishing of the Bean–Livingston barrier to the entry of the vortex into the film.

So long as the total current in the film is less than the value of I_c (and accordingly $Q_0 < Q_{0c}$), the film is in the Meissner state. Only widely separated vortices penetrate in this case through the barrier and, moved by the transport current, produce an exponentially small fluctuating resistance of the film. At a current I_c (and correspondingly at a potential Q_{0c}) instability sets in. The periodic perturbation of the potential and of the order parameter begin to increase, and the Meissner state is destroyed.

The instability-evolution time can be stimated by using the nonstationary Ginzburg-Landau equations. It suffices for this purpose to add to the left-hand side of (7) the term $-12\partial f/\partial t$ and to their right hand side of (8) the term

$$(-\partial \mathbf{q}/\partial t)\,\delta(z)\,\theta(x)\,\theta(\tilde{b} - x),$$

where the time is measured in units of the relaxation time τ_{GL} of the order parameter.[8] Taking the Laplace transform with respect to time and proceeding in accord with the analysis described above we find that at $Q_0 > Q_{0c}$ Eqs. (7) and (8) have solutions that now satisfy the boundary conditions in a definite range of values of k. The fastest to grow, however, is the mode with $k = k_c$, in proportion to $\exp(p_0 t)$, where $p_0 = 2\varepsilon/7$ and $\varepsilon = (Q_0 - Q_{0c})/Q_{0c}$ is assumed to be small. With logarithmic accuracy, the time of instability evolution is given by

$$\tau = (7/2)\,\tau_{GL}\,\varepsilon^{-1} \ln(\Delta_{GL}/\Delta_{\text{fl}}), \tag{17}$$

where Δ_{fl} is the characteristic scale of the initial fluctuation.

The instability development leads to formation of vortex chains at one edge of the film. The transport current moves the vortices to the center of the film, where they are annihilated by oppositely directed vortices produced on the other edge of the film. At low supercriticality, the repulsion forces of the vortices with like direction prevent generation of a next chain at the edge before the first chain is annihilated at the center. When the current in the film is increased, a second chain of vortices can be generated, etc. Unfortunately, the linear equations (7), and (8) do not make it possible to track the formation of the vortex chain at the edge of the film, and it is impossible to analyze the nonlinear equations. It is relatively easy, however, to investigate the vortex motion (and to obtain the CVC), for even at a low degree of supercriticality the vortex density is high enough and the hydrodynamic approximation is valid.

3. CURRENT-VOLTAGE CHARACTERISTIC OF FILM

The quasistationary picture of the viscous flow of a vortex "fluid" can be described with the aid of hydrodynamic equations. Such an equation is obtained from the condition that connects the average magnetic field induction $n\varphi_0$ (where $n(x)$ is the vortex density, to which we ascribe a definite sign that depends on their polarity) with the averaged linear current density $i(x)$. This connection is given by the generalized London equation[9]:

$$8\pi \frac{\lambda_{\text{eff}}}{b}\frac{di}{dx} + 2\int_{-1}^{1} \frac{i(x')\,dx'}{x' - x} = -n\varphi_0. \tag{18}$$

We note that in this and succeeding sections we reckon the coordinate x from the center of the film and measure it in units of the film half-width $b/2$.

The right hand side of (18) can be obtained by using the continuity equation for the vortex fluid:

$$\frac{d}{dx} nv = 0, \tag{19}$$

where v is the velocity of the vortex. It means that the number nv of the vortices passing through a unit length of film per unit time is constant. Recognizing that the passage of each vortex changes the phase χ of the order parameter by 2π, and taking into account the Josephson relation $\chi = 2$ eV, we obtain $-nv = E/\varphi_0$, where E is the average electric field strength in the film (the voltage is $V = EL$, where L is the film length).

On the other hand, the velocity of the viscous motion of vortices is determined by the current density[10]

$$v=-\eta^{-1}\varphi_0 i \operatorname{sign} x, \tag{20}$$

where η is the viscosity coefficient, and the expression sign x indicates that the vortices of opposite polarity move opposite to one another in different halves of the film. For the right-hand side of (18) we obtain ultimately

$$n\varphi_0=(\eta E/i\varphi_0)\operatorname{sign} x. \tag{21}$$

Since the width b of the film is assumed to be large compared with λ_{eff}, the gradient term of (18) is small in almost the entire region of the film. Neglecting this term, we can easily invert the integral equation (18), and it is convenient to separate the current density $i_0(x)$ produced by the transport current at the critical value I_c. As a result we obtain for $i_1 = i - i_0$

$$i_1=\frac{1}{2\pi^2[1-x^2]^{1/2}}\left[\frac{\eta E}{\varphi_0}\int_{-1}^{1}\frac{[1-(x')^2]^{1/2}\operatorname{sign} x'}{i(x')\,(x'-x)}dx'+\frac{4\pi}{b}(I-I_c)\right], \tag{22}$$

where I is the specified value of the total current.

The current-voltage characteristic of the film is obtained from the following boundary condition: at the edges of the film the current density $i(\pm 1)$ must equal the critical value $i_0(\pm 1)$ at which the instability sets in. Substituting accordingly in Eq. (22) the value $i_1(\pm 1) = 0$, we can find the connection between I and E if the current distribution $i(x)$ is known.

At low supercriticality (when $\Delta I = I - I_c$) is small compared with I_c), Eq. (22) can be solved by perturbation theory. In this case the current density $i(x)$ under the integral sign in the right-hand side is assumed equal to the current density $i_0(x)$ in the Meissner state. The edges of the film make a small contribution to the inegral with respect to x', and in the region $1 - x^2 \gg \lambda_{\text{eff}}/b$ the following asymptotic expression holds for the current density:

$$i_0(x)=(2I_c/\pi b)\,(1-x^2)^{-1/2}. \tag{23}$$

Substituting this expression in (22) we obtain

$$i_1=\frac{2I_c}{\pi b}\frac{E}{E_0}(1-x^2)^{-1/2}\left[(1-x^2)\ln\left(\frac{1-x^2}{x^2}\right)-1+\frac{E_0}{E}\frac{I-I_c}{I_c}\right],$$

$$E_0=\frac{8\varphi_0 I_c^2}{\eta b^2}. \tag{24}$$

This expression has singularities at the edge of the film at $x = \pm 1$ as well as halfway at $x = 0$. They are eliminated when account is taken of the gradient term in (18). (For example, the singularity near an edge is "cut off" but substituting the value $1 - x^2 \sim \lambda_{\text{eff}}/b$.) The condition $i_1(\pm 1) = 0$ can then be satisfied by relating I and E so that the coefficient of the singular term vanishes, and allowance for the gradient term leads only to corrections small in the parameter λ_{eff}/b.

As a result we have for the initial section of the CVC of the film

$$E=E_0(I-I_c)/I_c, \quad I-I_c\ll I_c. \tag{25}$$

Thus, the initial section of the CVC turns out to be linear, and its slope E_0 is smaller the closer the temperature to T_c and the larger the film width b.

With increasing current, its density can differ noticeably already in almost over the entire film from the Meissner distribution $i_0(x)$. To obtain the CVC it is then necessary to solve the nonlinear integral equation (22). At $I \gg I_c$, however, this equation can be easity written in dimensionless form in such a way that we get for the CVC from the condition $i_1(\pm 1) = 0$, accurate to a numerical coefficient C,

$$E=CE_0(I/I_c)^2, \quad I\gg I_c. \tag{26}$$

Thus, at a high degree of supercriticality the CVC of the film becomes a parabola.

Formula (26) for the CVC of the film remains valid so long as current density inside the film is everywhere less than the density of the pair-breaking current. With increasing current, however, this condition is violated. To find the current limit I_m we investigate the current distribution at $I > I_c$. The current density has a maximum at $x = 0$ (midway in the film), since the current density produce by the vortices is added to the Meissner distribution (23). Each vortex decreases the current density in the film by an amount $\propto r^{-2}$, where r is the distance from the vortex. For a chain of vortices the decrease of the current density is now hyperbolic, $\propto r^{-1}$, and the integral over a large number of chains diverges loarithmically midway in the film. The asymptotic current distribution at $I \gg I_c$ near the middle of the film can be obtained from Eq. (22):

$$i(x)=\left(\frac{\eta E}{\pi^2\varphi_0}\ln x^{-2}\right)^{1/2}, \quad \frac{\lambda_{\text{eff}}}{b}\ll x\ll 1, \tag{27}$$

and the current density midway in the film (the maximum of the current density) is obtained with logarithmic accuracy by substituting $x \sim \lambda_{\text{eff}}/b$. Equating the value of $i(0)$ to the critical current density $i_c = j_c d$, we can obtain from (27) the maximum value of the electric field in the film, and then calculate with the aid of (26) the corresponding maximum current. As a result we obtain

$$I_m=Cbdj_c\ln^{-1/2}(b/\lambda_{\text{eff}}). \tag{28}$$

When the total current in the film reaches the value I_m, we get on entirely different picture of the stationary viscous flow of the liquid of the vortices from the edge of the film (where they are generated) to its middle (where vortex annihilation takes place). Although the distance between the vortices is still large

$$n^{-1/2}\gtrsim(\lambda_{\text{eff}}\,\xi)^{1/2}\ln^{1/4}(b/\lambda_{\text{eff}}),$$

the current density produced by them midway in the film becomes equal to the pair-breaking-current density. An instability similar to that produced at the edge of the film at the critical current is then produced midway in the film.

This instability can also be analyzed with the aid of Eqs. (7) and (8). The instability appears at a critical value of the vector potential Q, close to $1/\sqrt{3}$ (the correction to this value are small relative to the parameter $\varkappa_{\rm eff}^{-1}$, just as in Eq. (14); it leads to formation of infinitely small fluctuations of the order parameter. The development of the instability, however, unlike near the edge of the film at a current I_c, does not permit formation of a chain of vortices. The appearance of such vortices would lead to a decrease of the value of the potential Q midway in the film, but would increase its value at the edge. Thus the steady-state picture of the viscous motion of the vortices, at which Q exceeds nowhere the critical value, becomes impossible, and the film goes over into the normal state. A jump should be observed on the CVC of the film at $I = I_m$.

It can be seen from Eqs. (16) and (28) that I_m exceeds I_c considerably. At the same time, prior to its transition into the normal state, the film differential resistance at the current I_m is still small compared with the normal resistance R_N of the film. Using the known expressions for the viscosity coefficient,[10] we have the estimate

$$\eta \gtrsim \sigma\varphi_0^2 d/\xi^2,$$

where σ is the conductivity in the normal state. This yields

$$\frac{R_m}{R_N} \propto \frac{\sigma d\varphi_0 I_m}{\eta b} \lesssim \varkappa_{\rm eff}^{-1} \ln^{-1/2}\left(\frac{b}{\lambda_{\rm eff}}\right). \tag{29}$$

We note also that in the present paper no account was taken of the nonlinear effects that occur when the vortices move,[11] for the viscosity coefficient η was assumed to be independent of the vortex velocity. This is justified for a zero-gap superconductor or near the transition temperature:

$$1-T/T_c \ll (T\tau_\varepsilon)^{-2/3},$$

where τ_ε is the energy relaxation time; in this case the effects are small even at current densities of the order of critical.

4. EFFECT OF PINNING ON THE FILM CVC

The inhomogeneities of the electron-phonon interaction constant, of the mean free path, of the film thickness, and others hinder the viscous motion of the vortices under the action of the transport current.[9,12] As a result, the motion of the entire vortex structure in the film occurs only when the current density at each point exceeds a certain critical value i_p. Usually i_p is small compared with the critical current density i_c. Nonetheless, even weak pinning can lead to substantial changes in the film CVC.

As already noted, near the critical value I_c the current is not uniformly distributed in the film. Although at the film edges the current density is of the order of critical, midway in the film, as follows from (23), the current density $i_0(0) = 2I_c/\pi b$ is relatively small. Therefore even at value of i_p on the order of $i_0(0)$ the pinning influences the film CVC.

When the pinning is taken into account, the equation of the viscous motion of the vortices takes the form

$$v = -\eta^{-1}\varphi_0(i-i_p)\,\mathrm{sign}\,x, \tag{30}$$

which differs from (20). This causes also a change in the integral equation (22) for the increment $i_1(x)$ to the Meissner current distribution:

$$i_1 = \frac{1}{2\pi^2[1-x^2]^{1/2}} \times \left[\frac{\eta E}{\varphi_0}\int_{-1}^{1} \frac{[1-(x')^2]^{1/2}\,\mathrm{sign}\,x'}{i_0(x')+i_1(x')-i_p}\frac{dx'}{x'-x} + \frac{4\pi}{b}(I-I_c)\right]. \tag{31}$$

The current-voltage characteristic of the film, with allowance for pinning, is determined as before from the boundary condition $i_1(\pm 1) = 0$ for Eq. (31).

We investigate now Eq. (31) at values of i_p close to $i(0)$. This suffices to understand qualitatively the form of the CVC also at $|i_p - i_0(0)| \sim i_0(0)$. We consider first the case when the small dimensionless parameter

$$\alpha = [i_0(0)-i_p]/i_p$$

is positive (the pinning forces are not yet able to stop the vortices). At low supercriticality $\Delta I = I - I_c \ll I_c$ there are then two characteristic current regions. When the current I is very close to the critical I_c, we have $i_1 \ll i_0 - i_p$ in the entire film region and the quantity i_1 in the denominator of the integrand of (31) can be neglected. The integral is then easily calculated and we have for the initial section of the CVC the expression

$$E = E_0 \ln^{-1}\left(\frac{1}{\alpha^2}\right)\frac{I-I_c}{I_c}, \tag{32}$$

$$\Delta I/I_c \ll \alpha^2 \ln \alpha^{-1}. \tag{33}$$

Comparing this expression with (25) we find that at positive α allowance for the pinning leads to a decrease of the differential resistance of the film on the initial section of the CVC by a factor $\ln(1/\alpha^2)$, while the critical current of the film remains unchanged.

With increasing current, the condition (33) is violated, and when the inverse inequality is satisfied we have $i_0(0) - i_p \ll i_1(0)$. Near the middle of the film the principal role is assumed in the denominator of the integrand in (31) by the function $i_1(x')$, which can be seen to be of the order of $(\eta E/\varphi_0)^{1/2}$ and to decrease from the middle of the film towards the edges. An asymptotic expression for i_1 can be obtained at small values of x'

$$i_1(x') = \left(\frac{\eta E}{\pi^2\varphi_0}\right)^{1/2} \ln^{1/2}\frac{x_0^2}{x'^2},$$
$$x_0 = \left(\frac{\eta E}{\varphi_0 i_p^2}\right)^{1/4}, \qquad \frac{\lambda_{\rm eff}}{b} \ll x' \ll x_0 \ll 1. \tag{34}$$

On the other hand, as follows from (23), the quantity $i_0(x') - i_p$ increases quadratically on going from the middle of the film to the edges, $\propto i_0(0)(x')^2$ at $x' \ll 1$. In the region $x' \sim x_0$ the quantities $i_0(x') - i_p$ and $i_1(x')$ become of the same order. This is precisely the region that makes, with logarithmic accuracy, the principal contribution to the integral of (31). As a result we obtain for the CVC of the film

$$(E/E_0)\ln(E_0/E) = (I-I_c)/I_c, \tag{35}$$

$$\alpha^2 \ln \alpha^{-1} \ll \Delta I/I_c \ll 1. \quad (36)$$

It can be seen from (35) that the initial section of the CVC of the film becomes nonlinear as a result of the pinning. At $I \gg I_c$ the influence of the pinning is insignificant, since the current density is everywhere large compared with i_p. In this case the CVC of the film is given by (26).

An interesting situation arises when i_p exceeds the value $i_0(0)$ (the parameter α is negative). In this case the flow of the liquid of the vortices can no longer start at arbitrary small supercritically. The integrand in (31) has at small values of $i_1(x')$ a pole and the equation has no solutions. The condition $i_0 + i_1 > i_p$ is violated and the motion of the entire vortex structure becomes impossible.

With increasing current in the film, the pole of the integrand in (31) vanishes. This takes place at $i_1(0) \sim i_p - i_0(0)$. The quantity

$$\Delta I_c/I_c \sim \alpha^2 \ln|\alpha^{-1}| \quad (37)$$

gives the pinning-caused change ΔI_c of the critical current (it assumed that $|\alpha| \ll 1$). At the critical current the film voltage appears jumpwise, and its value ΔE_c can be obtained from (32):

$$\Delta E_c \sim E_0 \alpha^2. \quad (38)$$

At $\Delta I \gtrsim \Delta I_c$ the CVC of the film takes the form (35), and at $I \gg I_c$ it is determined by (26). Equations (37) and (38) remain qualitatively valid also at $|\alpha| \sim 1$.

Thus, when the pinning is strong enough and i_p exceeds the current density $i_0(0)$ midway in the film, the critical current of the film increases and a jump appears on the CVC of the film. We note that in this case a static vortical structure can appear in the film even at currents lower than critical (the instability at the edges of the film sets in at the old value of the critical current I_c). A decrease of the current leads to rearrangement of this structure, accompanied by voltage pulses. The steady-state motion of the vortex structure becomes possible, however, only at currents higher than $I_c + \Delta I_c$.

5. DISCUSSION OF RESULTS

The transition of a wide superconducting film into the resistive state is thus the result of loss of stability of a vortex-free superconducting state, a loss that occurs at the critical current value determined by Eq. (16). The order parameter at the edges of the film is suppressed at the edges of the film under the influence of the magnetic self-field to such an extent that a critical perturbation is generated there. It is found to be oscillating along the film with a period $\sim \lambda_{\text{eff}}/k_c$, where k_c is defined by Eq. (15). The instability develops within a time τ [Eq. (17)] and leads to formation of a chain of vortices at the edge of the film. The critical value of the vector potential is determined by Eq. (14), and the corresponding density of the current at the edges of the film, however, two, three, and more vortex chains can move in the film. It turns out to be close to the density of the pair-breaking current.

The density of the vortices in the film depends on the supercriticality $\Delta I = I - I_c$. After the generation of the vortex chain, the order parameter at the edge of the film increases. At very low supercriticality the next chain of vortices can be generated at the edge of the film only when the preceding reaches the middle of the film, where it is annihilated by a chain of oppositely directed vortices generated on the other edge of the film. With increasing current in the film. It turns out that even at a low degree of supercritically

$$\Delta I/I_c \gtrsim \varkappa_{\text{eff}}^{-1} (\lambda_{\text{eff}}/b)^{1/2} \quad (39)$$

the average distance between the vortices becomes small compared with the film width b. The time of vortex creation then also becomes small compared with the time it takes to negotiate the distance between the vortices.

In this case viscous motion of a vortex fluid, which leads to the appearance of a voltage on the film, can be treated in the hydrodynamic approximation. The vortex density n determines the average magnetic field connected with the average current density i by the London equation. At the same time, the current density determines the vortex velocity v, and the continuity equation sets the vortex flux nv. By solving these equations we obtained the distributions of the current, of the density, and of the velocity of the vortices in the film and calculated the film CVC.

The electric field intensity in the film is determined by the number of vortices passing through a unit length of the film per unit time: $E \sim (\pi\hbar)/e)nv$. At a low supercriticality, $\Delta I \ll I_c$, the vortex density n, as follows from (18), is proportional to the supercriticality: $n \sim \varphi_0^{-1} \Delta I/bc$ (c is the speed of light, which we set equal to unity). The vortex velocity is proportional to the current density: $v \sim \hbar i/\eta e$, where i is of the order of I_c/b in almost the entire film (except the edges). As a result, the electric field intensity in the film is found to proportional to the supercriticality: $E \sim E_0 \Delta I/I_c$, where the coefficient $E_0 \sim \hbar I_c^2/c^2 e\eta b^2$ [which is proportional to $(1 - T/Tc)$] gives the order of magnitude of the intensity at currents $I \sim I_c$ [Eq. (25)]. At large supercriticality $I \gg Ic$ the vortex density n, as well as their velocity v, is proportional to the current density $i \sim I/b$. Therefore $E \sim E_0 (I/I_c)^2$ [Eq. (26)].

The current distribution in the film also depends on the supercriticality. At the critical current I_c its density reaches the critical value i_c only at the edges of the film, and elsewhere it is smaller than i_c by a factor b/λ_{eff}. The appearance of a vortex structure at $I > I_c$ leads to the onset of a local maximum of the current density midway in the film (the current density produced by each vortex decreases relatively slowly with increasing distance from the vortex). With increasing current, its density midway in the film increases, while at the edges it remains equal to the critical value. At the current value I_m defined in (28) the current density midway in the film also reaches the critical value I_c. In this case the current is almost uniformly distributed over the film (in the remaining region of the film its density is less than critical only by a factor $\ln^{1/2}(b/\lambda_{\text{eff}})$.

The stationary motion of the vortex structure at $I > I_m$ becomes unstable. Although the cores of the vortices still do not overlap in this case, the entire film goes over into the normal state. The film resistance (and hence the voltage

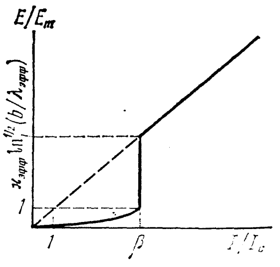

FIG. 1. Current-voltage characteristic of film, drawn in enlarged scale ($\beta = (b/\lambda_{\text{eff}})^{1/2} \ln^{-1/2}(b/\lambda_{\text{eff}})$; $E_m = \beta^2 E_0$).

across it) increases jumpwise at the current value I_m determined by Eq. (29); the corresponding CVC of the film is shown in Fig. 1.

The inhomogeneities randomly distributed in the film hinder the vortex motion and decrease the film resistance. Qualitative changes occur in the CVC also in the case of weak pinning, when the pinning forces are capable of stopping the vortex chain at the middle of the film, where the initial current density is small compared with the critical. As a result, the initial section of the film becomes nonlinear [Eq. (35)]. The critical current of the film can increase, and a jump of the film voltage takes place [Eqs. (37) and (38)]. The corresponding CVC of the film is shown in Fig. 2. We note that if the average pinning forces on different sections of the film differ from one another, several voltage jumps can appear on the CVC.

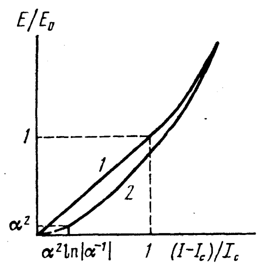

FIG. 2. Current voltage characteristic of film at currents of the order of critical: 1—without pinning, 2—with allowance for pinning.

The performed theoretical investigation of the resistive state of a broad superconducting film agrees with the experimental results of Ref. 13. The transition of the film into the normal state takes place jumpwise at a large (compared with the critical) current. Also observed are a much weaker inhomogeneity of the current distribution at the instant of the transition, and a relatively small voltage on the film prior to the jump. In a number of studies,[14] however, the nature of the voltage jumps on the CVC is apparently different and is possibly connected with nonlinear effects connected with the vortex motion.

The authors are deeply grateful to A. A. Abrikosov and A. I. Larkin for valuable discussions of the results.

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Translated by J. G. Adashko