Interelectron effects and the conductivity of disordered two-dimensional electron systems

B. L. Al'tshuler, A. A. Varlamov, and M. Yu. Reĭzer

Moscow Institute of Steel and Alloys

(Submitted 30 December 1982)
Zh. Eksp. Teor. Fiz. **84**, 2280–2289 (June 1983)

The effect of interelectron interaction on the conductivity of disordered electron systems is investigated in the case of a weak coupling constant. Summation of the ladder diagrams in the Cooper interaction channel in the case of a two-dimensional conductor leads to the appearance of corrections to the conductivity which are proportional to $\ln\ln(T_0/T)$. In the three-dimensional case the corrections are of the form $T^{1/2}/\ln(T_0/T)$. The relation between the various temperature-dependent contributions to the conductivity of disordered electron systems is discussed.

PACS numbers: 72.10.Di

1. Interest has increased lately in the singularities of kinetic phenomena in two-dimensional disordered metals and semiconductors. The unusual properties of two-dimensional conductors, particularly the temperature dependence of the conductivity, are the result of the quantum character of the interaction of the electrons with the impurities and of the associated localization phenomenon,[1–3] as well as the results of the electron-electron interaction.[4–6] The electron-electron interaction in these studies was taken to be the screened Coulomb interaction. In addition, in Refs. 7 and 8, in an investigation of the magnetoresistance of disordered electronic systems, the electron-electron interaction in the Cooper channel (particle-hole channel) was described with the aid of an effective coupling constant. This approach, in the case of a negative effective coupling constant, corresponds to allowance for the influence of superconducting fluctuations on the conductivity.[9–11] The experimental investigation of the magnetoresistance of disordered conductors is the subject of many studies, see, e.g., Refs. 12 and 13.

The present paper is devoted to the study of the influence of electron-electron interaction in a Cooper channel on the temperature dependence of the conductivity. The analysis is carried out, with a two-dimensional disordered system as the example, for the case of a positive coupling constant. It turns out that the contribution of a definite class of diagrams leads to temperature-dependent corrections $\propto \ln\ln(T_0/T)$ to the conductivity (at $|\ln(T_0/T)| \gg 1$), where T_0 is a temperature of the order of the Fermi energy.

At the end of the article are presented the results of a similar calculation for a three-dimensional disordered electron system, and it is also shown that all the results can be used verbatim for the case of a negative coupling constant by formally replacing T_0 by the corresponding superconducting-transition temperature T_c.

2. Consider the influence of the electron-electron interaction on the conductivity of the disordered electron system. We simulate the disorder by scattering of the electrons from impurities with short-range and isotropic interaction potential. It is convenient to describe the electron-electron interaction in a Cooper channel with the aid of an effective propagator $L_{\alpha\beta\gamma\delta}(\mathbf{q},\Omega_k)$, a graphic solution for which is shown in Fig. 1. The propagator is denoted by a wavy line, and the thick circle corresponds to the effective coupling constant g. A solid line denotes the Green's function of the normal metal, averaged over the impurity positions:

$$G_{\alpha,\beta}(p,\varepsilon_n)=\delta_{\alpha,\beta}[i\tilde{\varepsilon}_n-\xi(\mathbf{p})]^{-1},\qquad \tilde{\varepsilon}_n=\varepsilon_n+\frac{1}{2\tau}\operatorname{sign}\varepsilon_n. \qquad (1)$$

Here α and β are the spin indices and $\varepsilon_n=(2n+1)T$. The shaded three-point vertices $C(\mathbf{q},\omega_1,\omega_2)$ (copperons) denote the sum of ladder diagrams that are produced when account is taken of the interaction of electrons with impurities. It is convenient to write the vertex $C(\mathbf{q},\omega_1,\omega_2)$ in the form[14]

$$C(\mathbf{q},\omega_1,\omega_2)=\frac{|\tilde{\omega}_1-\tilde{\omega}_2|}{|\omega_1-\omega_2|+\tau\langle(\mathbf{qv})^2\rangle\theta(-\omega_1\omega_2)}; \qquad (2)$$

$\mathbf{v}$ is the electron-velocity vector, and the angle brackets denote averaging over the Fermi surface; θ is the Heaviside theta function; $\tilde{\omega}$ is related to ω as $\tilde{\varepsilon}$ is to ε in (1).

For $g>0$ the propagator $L_{\alpha\beta\gamma\delta}(\mathbf{q},\Omega_k)$ takes the form[15]

$$L_{\alpha\beta\gamma\delta}(\mathbf{q},\Omega_k)=-\frac{1}{\nu}\left[\ln\frac{T}{T_0}+\psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\alpha_\mathbf{q}\right)-\psi\left(\frac{1}{2}\right)\right]^{-1}\times(\delta_{\alpha\beta}\delta_{\gamma\delta}-\delta_{\alpha\delta}\delta_{\gamma\beta}), \qquad (3)$$

where ν is the single-spin density of states on the Fermi surface ($\nu=m/2\pi$ in the two-dimensional case); T_0 is defined by the equation $T_0=\varepsilon_F\exp\{1/\nu g\}$; ε_F is the Fermi energy; $\psi(x)$ is the logarithmic derivative of the gamma function;

$$\alpha_\mathbf{q}=\tau\langle(\mathbf{qv})^2\rangle/4\pi T=Dq^2/4\pi T,$$

where D is the diffusion coefficient of the conduction electrons.

By analogy with superconductivity, the interaction described by the propagator $L_{\alpha\beta\gamma\delta}(\mathbf{q},\Omega_k)$ at $g>0$ will be called fluctuation electron-electron interaction. We must empha-

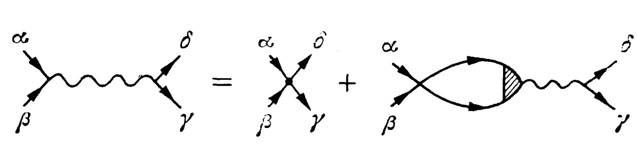

FIG. 1. Diagramatic equation for the effective propagator.

 0038-5646/83/061329-06$04.00

size that it differs from the Coulomb interaction. The fluctuation interaction takes in fact account of the Coulomb interaction in Cooper-channel diagrams in all orders of perturbation theory in the ladder approximation. Then, however, the exact dynamic screened Coulomb interaction considered in Ref. 4 and 5 in first-order perturbation theory is replaced by a model interaction described by a positive coupling constant, so that both Coulomb and electron-phonon interactions can be taken into account.

We proceed to calculate the static conductivity, which is expressed in standard fashion in terms of the operator of the electromagnetic response $Q(\omega_\nu)$. Calculating $Q(\omega_\nu)$ and carrying out the analytic continuation into the upper half-plane of the complex frequency $\omega_\nu = -i\omega$, we obtain the conductivity $\sigma = Q^R(\omega)/(-i\omega)$, $\omega \to 0$.

In first order in the fluctuation interaction, and with account taken of the averaging over the impurities, $Q(\omega_\nu)$ is determined by the ten diagrams shown in Fig. 2. At the vertices of the diagrams are the factors $e\mathbf{v}$, where e is the charge of the electron, and the shaded rectangles are ladders made up of impurity strokes:

$$\Gamma(\mathbf{q}, \omega_1, \omega_2) = \frac{1}{2\pi\nu\tau} C(\mathbf{q}, \omega_1, \omega_2). \tag{4}$$

We consider the calculation of the diagrams for the electromagnetic response, using as the example the first diagram of Fig. 2. For the diagonal component of the tensor $Q_{ii}(\omega_\nu) \equiv Q(\omega_\nu)$ we have

$$Q_1(\omega_\nu) = e^2 T \sum_{\Omega_k} \int \frac{d\mathbf{q}}{(2\pi)^2} L_{\alpha\gamma\alpha\gamma}(\mathbf{q}, \Omega_k) T \times \sum_{\varepsilon_n} C^2(\mathbf{q}, \varepsilon_n, \Omega_k - \varepsilon_n) I(\varepsilon_n, \Omega_k, \omega_\nu, \mathbf{q}); \tag{5}$$

where summation over repeated indices is implied. The block of Green's functions $I(\varepsilon_n, \Omega_k, \omega_\nu, \mathbf{q})$ is defined by the expression

$$I(\varepsilon_n, \Omega_k, \omega_\nu, \mathbf{q}) = \frac{1}{16} \int \frac{d\mathbf{p}}{(2\pi)^2} (v_i)^2 G_{\alpha\alpha}^2(\mathbf{p}, \varepsilon_n) G_{\beta\beta}(\mathbf{q}-\mathbf{p}, \Omega_k - \varepsilon_n) G_{\gamma\gamma}(\mathbf{p}, \varepsilon_{n+\nu}) \tag{6}$$

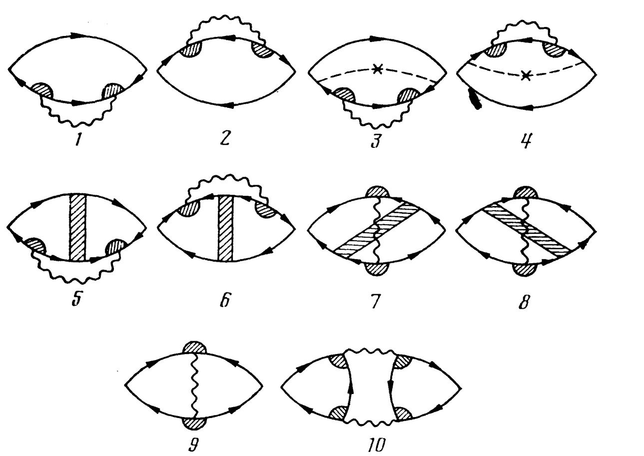

FIG. 2. Diagrams that determine the contribution made to the conductivity by the fluctuation electron-electron interaction.

and is already averaged over the spin states. As a result of simple calculations we obtain for the block $I(\varepsilon_n, \Omega_k, \omega_\nu, q)$ a rather cumbersome expression, which in the case of small mean free paths $ql \ll 1$ of interest to us, where $l = v_F\tau$, takes at Ω_k, ω, $\varepsilon_n \ll \tau^{-1}$ the form

$$I(\varepsilon_n, \Omega_k, \omega_\nu, 0) = -\frac{v_F^2}{2} 2\pi\nu\tau^3 [\theta(\varepsilon_n \varepsilon_{n+\nu}) \theta(\varepsilon_n(\varepsilon_n - \Omega_k)) + \theta(-\varepsilon_n \varepsilon_{n+\nu}) \theta(-\varepsilon_n(\varepsilon_n - \Omega_k)) - 2\theta(-\varepsilon_n \varepsilon_{n+\nu}) \theta(\varepsilon_n(\varepsilon_n - \Omega_k))]. \tag{7}$$

Owing to the presence of θ functions in this expression, we can rearrange its terms in such a way that the sum over ε_n is represented by two sums with finite and infinite limits

$$D(\Omega_k, \omega_\nu, \mathbf{q}) = T \sum_{\varepsilon_n} C^2(q, \varepsilon_n, \Omega_k - \varepsilon_n) I(\varepsilon_n, \Omega_k, \omega_\nu, 0)$$

$$= -\pi\nu\tau v_F^2 T \left(\sum_{n=-\infty}^{\infty} - 3 \sum_{n=-\nu}^{-1} \right) \frac{\theta(\varepsilon_n(\varepsilon_n - \Omega_k))}{(|2\varepsilon_n - \Omega_k| + 4\pi T \alpha_\mathbf{q})^2}. \tag{8}$$

The first sum is independent of the external frequency ω_ν and is consequently cancelled out by analogous contributions from the remaining diagrams. Calculating the second sum, we obtain

$$D(\Omega_k, \omega_\nu, \mathbf{q}) = \frac{3\nu v_F^2 \tau}{16\pi T} \{D_1(\Omega_k, \omega_\nu, \mathbf{q}) + D_2(\Omega_k, \omega_\nu, \mathbf{q})\};$$

$$D_1(\Omega_k, \omega_\nu, \mathbf{q}) = \theta(\Omega_k) \left[\psi'\left(\frac{1}{2} + \frac{|\Omega_k|}{4\pi T} + \frac{\omega_\nu}{2\pi T} + \alpha_\mathbf{q}\right) - \psi'\left(\frac{1}{2} + \frac{\omega_\nu}{2\pi T} + \alpha_\mathbf{q}\right) \right];$$

$$D_2(\Omega_k, \omega_\nu, \mathbf{q}) = \theta(-\Omega_k)\theta(\Omega_k + \omega_{\nu-1}) \times \left[\psi'\left(\frac{1}{2} + \frac{\omega_\nu}{2\pi T} - \frac{|\Omega_k|}{4\pi T} + \alpha_\mathbf{q}\right) - \psi'\left(\frac{1}{2} + \frac{|\Omega_k|}{4\pi T} + \alpha_\mathbf{q}\right) \right]. \tag{9}$$

The function $D(\Omega_k, \omega_\nu, q)$ must now be continued analytically in the frequency ω_ν into the upper half-plane. The contributions to the electromagnetic response functions, analytic in the upper frequency complex half-plane and corresponding to $D_1(\Omega_k, \omega_\nu, \mathbf{q})$ and $D_2(\Omega_k, \omega_\nu, \mathbf{q})$, will be denoted by $Q_1'^R(\omega)$ and $Q_1''^R(\omega)$.

Since the function $\theta(\Omega_k)$ in the expression for $D_1(\Omega_k, \omega_\nu, q)$ does not contain ω_ν, the analytic continuation is effected by the simple substitution $\omega_\nu \to -i\omega_\nu$. Putting next $\omega \ll T$, we obtain

$$Q_1'^R(\omega) = -\frac{3i\omega\sigma}{16\pi^2 \nu T} \times \sum_{k=0}^{\infty} \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{\psi''((1+k)/2 + \alpha_\mathbf{q})}{\ln(T_0/T) - \psi((1+k)/2 + \alpha_\mathbf{q}) + \psi(1/2)}, \tag{10}$$

where $\sigma = 2e^2\nu D$ is the residual conductivity.

In the considered temperature region we have $\ln(T_0/T) \gg 1$; consequently, when integrating over the momentum, only the dependence of the numerator of the integrand in (10) on $\mathbf{q}$ is of significance. The remaining sum over k will be replaced in accordance with the condition $\Omega_k \tau \ll 1$ by an integral with upper limit $(\tau T)^{-1}$, and we obtain ultimately with logarithmic accuracy

$$Q_1'^R(\omega)=\frac{3i\omega\sigma}{4\pi(\varepsilon_F\tau)}\ln\left[\frac{\ln(T_0/T)}{\ln(T_0\tau)}\right]. \tag{11}$$

The analytic continuation of $D_2(\Omega_k,\omega_\nu,\mathbf{q})$ turns out to be more complicated because the θ functions in (9) limit the region of summation over k to finite limits that depend on the frequency ω_ν. When calculating the sum over k in $Q_1''(\omega_\nu)$ and in the subsequent analytic continuation of this function in ω_ν, we follow Ref. 14. To this end we represent $Q_1''(\omega_\nu)$ in the form

$$Q_1''(\omega_\nu)=\frac{3\sigma}{8\pi\nu}\left[\int\frac{d\mathbf{q}}{(2\pi)^2}F(\omega_\nu,\mathbf{q}) -\frac{1}{2}\int\frac{d\mathbf{q}}{(2\pi)^2}\frac{\psi'(1/2+\omega_\nu/2\pi T+\alpha_\mathbf{q})-\psi'(1/2+\alpha_\mathbf{q})}{\ln(T_0/T)-\psi(1/2+\alpha_\mathbf{q})+\psi(1/2)}\right], \tag{12}$$

where

$$F(\omega_\nu,\mathbf{q})={\sum_{k=0}^{\nu-1}}'\frac{\psi'((1+k)/2+\alpha_\mathbf{q})-\psi'((1-k)/2+\alpha_\mathbf{q})}{\ln(T_0/T)-\psi((1+k)/2+\alpha_\mathbf{q})+\psi(1/2)}. \tag{13}$$

The prime on the summation sign means that the term with $k=0$ is taken with a factor 1/2.

The analytic continuation of the last term in (12) is carried out just as above ($\omega_\nu\rightarrow -i\omega$), after which we can expand in terms of ω/T. To calculate $F(\omega_\nu,\mathbf{q})$ we transform the sum into a contour integral

$$F(\omega_\nu,\mathbf{q})={\sum_{k=0}^{\nu-1}}'f(k,\omega_\nu,\mathbf{q}) =\frac{1}{2}f(0,\omega_\nu,\mathbf{q})+\frac{1}{2i}\int_C \operatorname{cth}\pi z f(-iz,\omega_\nu,\mathbf{q})\,dz, \tag{14}$$

where the contour C is shown in Fig. 3. The integral along this contour reduces, by change of variable, to an integral along the real axis, as a result of which the dependence on ω_ν goes over from the integration limits into the argument of the function $f(x)$, after which the analytic continuation is carried out directly by the substitution $\omega_\nu - i\omega$, and we get

$$F^R(\omega,q)=\frac{1}{2i}\int_{-\infty}^{\infty}\operatorname{cth}\pi z\left[f(-iz)-f\left(-iz+\frac{i\omega}{2\pi T}\right)\right]dz+\frac{1}{2}f(0). \tag{15}$$

Expanding the obtained expression in terms of the parameter ω/T and integrating by parts, we obtain

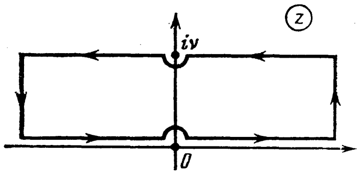

FIG. 3. Integration contour used in the analytic continuation of $Q_1''(\omega_\nu)$.

$$F^R(\omega,q)=-\frac{i\omega}{4T}\times\int_{-\infty}^{\infty}\frac{dz}{\operatorname{sh}^2\pi z}\frac{\psi'((1+iz)/2+\alpha_\mathbf{q})-\psi'((1-iz)/2+\alpha_\mathbf{q})}{\ln(T_0/T)-\psi((1-iz)/2+\alpha_\mathbf{q})+\psi(1/2)}. \tag{16}$$

Finally, calculating the remaining integral with respect to $\mathbf{q}$ in (12), we get

$$Q_1''^R(\omega)=-\frac{3\pi^2}{32}\frac{i\omega\sigma}{(\varepsilon_F\tau)}\left[\frac{1}{\ln(T_0/T)}+\frac{8K(\mathbf{q}=0)}{\ln^2(T_0/T)}\right], \tag{17}$$

where

$$K(\mathbf{q})=\int_{-\infty}^{\infty}\frac{dt}{\operatorname{sh}^2\pi t}\left[\psi\left(\frac{1+it}{2}+\alpha\right)-\psi\left(\frac{1-it}{2}+\alpha_\mathbf{q}\right)\right] \times\left[\psi\left(\frac{1-it}{2}+\alpha_\mathbf{q}\right)-\psi\left(\frac{1}{2}\right)\right];\quad K(0)\approx\frac{\pi^3}{24}. \tag{18}$$

From a comparison of expressions (11) and (17) it can be seen that the contribution of the considered diagram to the conductivity stems from the term $Q_1'^R(\omega)$, and allowance for the term $Q_1''^R(\omega)$ together with $Q_1'^R(\omega)$ is an exaggeration of the accuracy.

Examination of the second diagram of Fig. 2 shows that its contribution $\sigma^{(2)}$ to the conductivity coincides with the contribution $\sigma^{(1)}$ of the first. It can be shown that the contributions of the third and fourth diagrams are also equal to each other. They are smaller than $\sigma^{(1)}$ by a factor of three and are of opposite sign, i.e., $\sigma^{(3)}=\sigma^{(4)}=-(1/3)\sigma^{(1)}$.

We proceed to calculate diagrams 5–8 (see Fig. 2). In view of the vector character of the current vertices $e\mathbf{v}$, to obtain an nonzero result the Green's functions of both sides of the four-point vertex $\Gamma(\mathbf{q},\omega_1,\omega_2)$ must be expanded in powers of $\mathbf{q}\cdot\mathbf{v}$. Leaving out the intermediate calculations, which are similar to the preceding ones, we present only the expression for the conductivity due to the fifth diagram:

$$\sigma^{(5)}=\frac{D\sigma}{(4\pi)^3T^2\nu}\times{\sum_{k=-\infty}^{\infty}}'\int\frac{d\mathbf{q}}{(2\pi)^2}\frac{q^2\psi'''((1+|k|)/2+\alpha_\mathbf{q})}{\ln(T_0/T)-\psi((1+|k|)/2+\alpha_\mathbf{q})+\psi(1/2)}. \tag{19}$$

The sixth diagram reduces to a similar expression, while the seventh and eighth make twice as large a contribution to the conductivity, i.e., $-\sigma^{(7)}=-\sigma^{(8)}=2\sigma^{(5)}=2\sigma^{(6)}$. Gathering them all together, we find that at temperatures $T\ll T_0$ [$\ln(T_0/T)\gg 1$] we obtain, accurate to terms of order $[\ln(T_0/T)]^{-1}$ that the contribution made to the conductivity from diagrams 1–8 and corresponding to the change of the density of states as a result of the electron-electron interaction, is of the form[1)]

$$\sigma_2^{DS}=\sigma^{(1-8)}=\frac{e^2}{2\pi^2\hbar}\ln\left[\ln\frac{T_0}{T}\Big/\ln(T_0\tau)\right]. \tag{20}$$

The ninth diagram, when calculating fluctuation conductivity of a film above the temperature of the superconducting transition, contains an anomalous Maki–Thompson contribution[10,11] and is the principal one in a wide range of temperatures $T \gg T_c$.[7,14] In the case of a nonsuperconducting amorphous film, in analogy with the preceding, it is possible to obtain for the contribution of the ninth diagram to the conductivity the following expression:

$$\sigma_2^{MT} = -\frac{e^2}{2\pi T}\frac{1}{\ln^2(T_0/T)}\int\frac{d\mathbf{q}}{(2\pi)^2}\frac{K(\mathbf{q})}{2\alpha_q+\Delta}; \tag{21}$$

here Δ is the cutoff parameter of the integral at the lower limit, which is introduced in analogy with the theory of superconducting fluctuations.[11] For a two-dimensional electron system, the important role in the integration with respect to $\mathbf{q}$ in (21) is assumed by the region of small momenta. The logarithmic divergence at the lower limit is eliminated by cutoff at $\Delta \sim (T\tau_\varphi)^{-1}$, where τ_φ is the electron phase-relaxation time.[8] In this case, the integral builds up in expression (18) for the function $K(q)$ in the region of small $t \ll 1$ and the ψ function can be expanded in powers of α_q and it, after which integration in (21) leads to the result

$$\sigma_2^{MT} = \frac{\pi^2 e^2}{24\hbar}\frac{\ln(T\tau_\varphi)}{\ln^2(T_0/T)}. \tag{22}$$

We note the following circumstance. The contribution (20) to the conductivity was obtained accurate to terms of order $\ln^{-1}(T_0/T)$, therefore at first glance the retention of $\sigma_2^{MT} \propto \ln^{-2}(T_0/T)$ with it is not valid. In the numerator of (22), however, there is a large logarithm $\ln(T\tau_\varphi)$, which can make σ_2^{MT} comparable with σ_2^{DS}. In addition, we recall that the result (22) was obtained in the approximation with $\ln(T_0/T) \gg 1$. However, in analogy with Ref. 7, the region of its applicability can be expanded by replacing $(\pi^3/24)/\ln^{-2}(T_0/T)$ by the function $\beta(T_0/T)$, which is tabulated in Ref. 7.

In the remaining diagram 10 of Fig. 2, which corresponds to the Aslamozov–Larkin process,[2] we obtain after calculating the blocks of the Green's functions and after analytic continuation in the frequency[14]

$$\sigma_2^{AL} = \frac{\sigma}{2(\varepsilon_F\tau)}\int_0^\infty dx x\int_{-\infty}^{\infty}\frac{dt}{\mathrm{sh}^2\pi t}\left[\psi'\left(\frac{1+it}{2}+x\right)\right]^2 \times\left[\ln\frac{T_0}{T}-\psi\left(\frac{1+it}{2}+x\right)+\psi\left(\frac{1}{2}\right)\right]^{-1}\left[\ln\frac{T_0}{T} -\psi\left(\frac{1-it}{2}+x\right)+\psi\left(\frac{1}{2}\right)\right]^{-1}. \tag{23}$$

The region $t \lesssim 1$, $x \gg 1$ turns out to be significant in the evaluation of the integrals in (23); therefore, neglecting the dependence of the ψ functions on t, we obtain

$$\sigma_2^{AL} = \frac{e^2}{\pi^2\hbar}\left[\frac{1}{\ln(T_0\tau)}-\frac{1}{\ln(T_0/T)}\right]. \tag{24}$$

In the calculation of σ_2^{DS} we have already neglected the contributions of this order, therefore in the considered temperature region allowance for the term σ_2^{AL} is an exaggeration of the accuracy.

Thus, the influence of the fluctuation interelectron interaction on the conductivity of a two-dimensional disordered electronic system is determined by expressions (20) and (22), which correspond to a change, due to this interaction, of the density of states and to the fluctuation process of Maki and Thompson.

3. We discuss now the influence of the fluctuation interaction on the conductivity in three-dimensional disordered electronic systems. The corresponding corrections are determined by the same diagrams of Fig. 2, but the integrations in (10), (19), (20), and (23) must be carried out with allowance for the fact that the electronic spectrum is three-dimensional. In the integration with respect to $\mathbf{q}$ the expression corresponding to σ_3^{DS} diverges formally at large momenta. This divergence is due to the fact that the expressions employed for the propagator and the vertices were obtained for momenta $a < l^{-1}$, and by taking this circumstance into account it is easy to eliminate this divergence by subtracting the corresponding quantity taken at $T = 0$. This yields

$$\Delta\sigma_3^{DS}(T) = \sigma_3^{DS}(T) - \sigma_3^{DS}(0) = \frac{0.915e^2}{2\pi^2\hbar}\left(\frac{T}{D}\right)^{1/2}\frac{1}{\ln(T_0/T)}. \tag{25}$$

The Maki–Thompson contribution, which corresponds to the ninth diagram, has no singularities in the three-dimensional case and can be obtained from expressions (12) and (16) with account taken of the integration over the three-dimensional electron momentum $\mathbf{q}$. Since it cannot be assumed beforehand that t is small, we expand the numerator and denominator of the integrand in (16) in Taylor series in powers of it. Integrating the obtained series with respect to t and confining ourselves to the first nonvanishing term in the expansion in reciprocal powers of the large $\ln(T_0/T)$, we obtain

$$\sigma_3^{MT} = -\frac{3^{5/2}\sigma(T\tau)^{1/2}}{16\pi^{1/2}(\varepsilon_F\tau)^2}\int_0^\infty\frac{dx}{\sqrt{x}}\left[\ln\frac{T_0}{T}-\psi\left(\frac{1}{2}+x\right) +\psi\left(\frac{1}{2}\right)\right]^{-2}\sum_{k,n=0}^{\infty}(-1)^{n+k}\frac{\psi^{(2n+1)}(1/2+x)\,\psi^{(2k+1)}(1/2+x)}{(2k+1)!(2n+1)!4^{n+k}}|B_{2(n+k+1)}|, \tag{26}$$

where B_{2m} are Barnoulli numbers.

An analysis of the convergence of this series shows that it suffices, with good accuracy, to retain its first term, after which we obtain

$$\sigma_3^{MT} = -\frac{6.8e^2}{2\pi^2\hbar}\left(\frac{T}{D}\right)^{1/2}\ln^{-2}\frac{T_0}{T}. \tag{27}$$

Calculation of the Aslamazov–Larkin contribution, which corresponds to the tenth diagram, is carried out in analogy with that for the first diagrams and leads to the result

$$\sigma_3^{AL} = -\frac{2.3e^2}{2\pi^2\hbar}\left(\frac{T}{D}\right)^{1/2}\ln^{-2}\frac{T_0}{T}. \tag{28}$$

Thus, for a three-dimensional disordered electronic system the contributions from the Maki–Thompson and Aslamazov–Larkin processes, in the temperature region $T \ll T_0$, turn out to be of the same order of magnitude. They are, however, small compared with the conductivity (25) corrections due to the influence of the fluctuating interaction on the density of states.

4. In conclusion, we compare the results with other temperature-dependent corrections to the conductivity and discuss the situation as a whole. The foregoing analysis of the influence of the fluctuation interaction on the conductivity of two-dimensional and three-dimensional disordered electronic systems has shown that the decisive corrections are those connected with the change of the density of states under the influence of the electron-electron interaction in the Cooper channel, σ^{DS}. In the two-dimensional case, at large τ_φ, the Maki–Thompson contribution σ_2^{MT} may also turn out to be significant.

It must be noted that summation of all the ladder diagrams of the interaction in the Cooper channel is most significant in the considered problem. In Ref. 16 were calculated the analogous corrections to the conductivity in first-order perturbation theory in the coupling constant. As seen from (20) and (22), we can confine ourselves to first-order perturbation theory only in the case of extremely weak coupling constant $g \ll [\nu \ln(T_0/T)]^{-1}$. In the opposite case, as shown in the present paper, allowance for all the diagrams in the Cooper channel leads to a considerable change in the form of the singularity of the temperature-dependent corrections to the conductivity: in place of $\ln(T\tau)$, as in Ref. 16, the contribution σ_2^{DS} turns out according to (20) to be proportional to $\ln\ln(T_0/T)$. In addition, in first order in g it is impossible in principle to calculate the Maki–Thompson contribution.

It is of interest to compare the conductivity corrections due to the fluctuation interaction with the contribution made to the conductivity by the interaction in the diffusion channel, $\delta\sigma^D$.[4,5] As shown in Refs. 17 and 18, it is convenient to represent the expression for $\delta\sigma_2^D$ as a sum of contributions from the interaction with a total spin of the electron and hole j equal to 0 and 1:

$$\delta\sigma_2^D=\delta\sigma_2^{j=0}+\delta\sigma_2^{j=1},\quad \delta\sigma_2^{j=0}=(e^2/2\pi^2\hbar)\ln(T\tau),$$
$$\delta\sigma_2^{j=1}=(3\lambda_\sigma^{j=1}e^2/4\pi^2\hbar)\ln(T\tau), \qquad (29)$$

where $\lambda_\sigma^{j=1}$ is a constant that depends on the magnitude and the character of the electron-electron interaction. It can be seen from (29) that the correction $\delta\sigma_2^{j=0}$ is larger and increases more rapidly with temperature than σ_2^{DS} (at $\ln(T_0/T)\gg 1$). We note, however, the following circumstances: 1. The contributions $\delta\sigma_2^{j=0}$ and $\delta\sigma_2^{j=1}$ can cancel one another to a considerable degree. 2. The value of $\delta\sigma^D$ in the presence of strong spin-orbit scattering is universal: $\delta\sigma^D=\delta\sigma^{j=0}$. 3. The quantity $\lambda_\sigma^{j=1}$ can sometimes be determined independently. 4. The corrections to the conductivity on account of the interaction in the Cooper channel and in the diffusion channel depend differently on the magnetic field H: the contribution σ^{DS} ceases to depend on the temperature at

$$H \gtrsim H_{int}=\pi cT/2De. \qquad (30)$$

The corresponding contribution to the magnetoresistance is considered in Ref. 8, and the corrections to the state density, proportional to $\ln\ln(HT_0)$, were obtained in Ref. 19. At the same time, $\delta\sigma^{j=0}$ does not depend at all on H in the region of classically weak magnetic fields, while in $\delta\sigma^{j=1}$ the temperature dependence is suppressed[20] at $\gamma\mu_B H \sim T$ (γ is the gyromagnetic ratio and μ_B is the Bohr magneton). Since usually $H_{int} \ll T/\gamma\mu_B$, a study of the temperature dependences of the conductivity in different magnetic fields makes it possible to separate the contributions $\delta\sigma^{j=0}$, $\delta\sigma^{j=1}$, and σ^{DS}.

For all the reasons above, the contribution $\sigma_2^{DS}+\sigma_2^{MT}$ of the interaction in the Cooper channel should be taken into account even at $\ln(T_0/T)\gg 1$, which is a sufficient condition for the validity of expressions (20), (22), and (25). The same arguments remain in force also in the three-dimensional case, where the temperature dependence of σ_3^D differs from σ_3^{DS} (25) in that the former does not contain the factor $\ln^{-1}(T_0/T)$ (Ref. 4).

The fluctuation propagator of a Cooper pair in a superconductor coincides with (3) at $T_c<T$ if T_0 is replaced by T_c. Therefore, in a superconductor, likewise, at temperatures $T_c \ll T \ll \tau^{-1}$ the contribution made to the conductivity by the electron-electron interaction in the Cooper channel and corresponding to the usual superconducting fluctuations[14] is described by the same expressions (20), (22), and (25) with $\ln(T_0/T)$ replaces by $\ln(T_c/T)$. We point out that in this case the signs of the corrections σ^{DS} and σ^{AL} are reversed, while the sign of σ^{MT} remains unchanged.

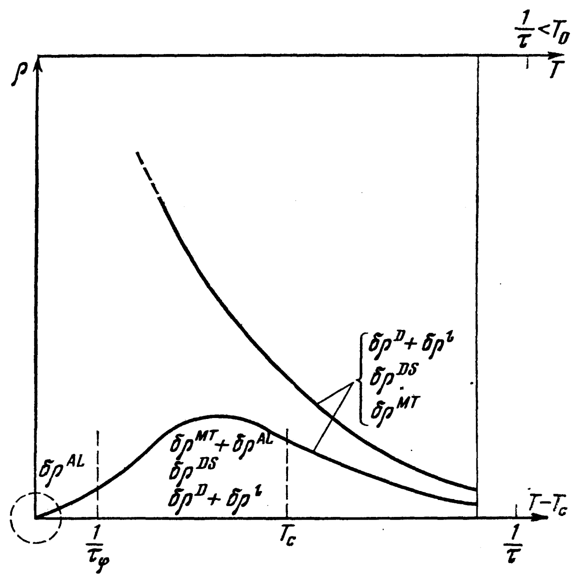

FIG. 4. Qualitative form of the temperature dependence of the resistance of a disordered superconductor (lower curve) and of a normal metal (upper curve). $\delta\rho^D$—contribution made to the resistance by the interaction in the diffusion channel[4,5]; $\delta\rho^L$—correction to the resistance from the localization effect[2]; $\delta\rho^{DS}=\delta\sigma^{DS}\sigma^{-2}$—resistance contribution obtained above and due to the change of the density of the single-electron states as a result of the electron-electron interaction in the Cooper channel; $\delta\rho^{MT}$ and $\delta\rho^{AL}$—Maki-Thompson[10,11,14] and Aslamazov-Larkin[9,14] contributions. The dashed circle separates the critical region. We note that in the two-dimensional case $\delta\rho^D\sim\delta\rho^l$, and in the three-dimensional case $\delta\rho^D\gg\delta\rho^l$. For a superconductor in the temperature region $\tau_\varphi^{-1}\ll T-T_c\ll T_c$ in the two-dimensional case, the predominant contribution to the resistance is $\delta\rho^{MT}$ ($|\delta\rho^{MT}|\gg|\delta\rho^{AL}|$), whereas in the three-dimensional case these contributions turn out to be of the same order of magnitude.

The qualitative form of the dependence of the resistance on the temperature for normal and superconducting disordered conductors is shown in Fig. 4. The downward sequence of the designations of the corrections to the resistance corresponds to the hierarchy of these quantities in this temperature region. In recent experiments on thin aluminum films[21] it was observed that the resistance depends on temperature in just this manner, and there was also a quantitative agreement with the results cited above.

The authors are deeply grateful to A. A. Abrikosov, A. G. Aronov, A. I. Larkin, and D. E. Khmel'nitskiĭ for valuable discussions and to M. E. Gershenzon and T. A. Polyanskaya for a discussion of the experimental situation.

[1)]Starting with Eq. (20), the subscript of σ indicates the dimensionality of the space.

[1]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[2]P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).
[3]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[4]B. L. Al'tshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)].
[5]B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).
[6]B. L. Altshuler, D. E. Khmel'nitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
[7]A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 239 (1980) [JETP Lett. **31**, 219 (1980)].
[8]B. L. Al'tshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 41 (1981)].
[9]L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela (Leningrad) **10**, 1104 (1968) [Sov. Phys. Solid State **10**, 875 (1968)].
[10]K. Maki, Progr. Theor. Phys. **39**, 892 (1968).
[11]R. S. Thompson, Phys. Rev. B **1**, 327 (1970).
[12]T. A. Polyanskaya and I. I. Saĭdashev, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 378 (1981) [JETP Lett. **34**, 361 (1981)].
[13]M. E. Gershenson and V. N. Gubankov, Sol. St. Commun. **49**, 33 (1982).
[14]L. G. Aslamazov and A. A. Varlamov, J. Low Temp. Phys. **38**, 223 (1980).
[15]L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. Zh. Eksp. Teor. Fiz. **67**, 647 (1974) [Sov. Phys. JETP **40**, 321 (1975)].
[16]H. J. Fukuyama, J. Phys. Soc. Jpn. **50**, 3407 (1981).
[17]A. M. Finkel'shteĭn, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)].
[18]B. L. Altshuler and A. G. Aronov, Sol. St. Commun. **45**, (1983).
[19]B. L. Altshuler and A. G. Aronov, Sol. St. Commun. **38**, 11 (1981).
[20]P. A. Lee and T. V. Ramakrishnan, Phys. Rev. B **26**, 4009 (1982).
[21]M. E. Gershenzon, V. N. Gubanov, and Yu. E. Zhuravlev, Zh. Eksp. Teor. Fiz. **85**, 287 (1983) [Sov. Phys. JETP **58**, 1983, in press].

Translated by J. G. Adashko