Investigation of antiferromagnet elastic oscillations generated by parametric excitation of spin waves

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Generation of elastic oscillations in an antiferromagnetic $MnCO_3$ specimen by parametric excitation of spin waves in it is observed. The elastic oscillation frequencies ω_e of the various modes are in the 2 to 10 MHz range. The frequency of the parametrically excited spin waves is $\omega_k = 9$ GHz. The spectral interval of the elastic oscillations generated is of the order of 100 Hz, which is significantly narrower than the contours of the elastic-mode and of the spin-wave resonances. By means of a number of control experiments it is shown that the generation is due to stimulated emission of a spin-wave phonon with formation of a secondary spin wave. Concurrently, generation of spin waves with frequencies $\omega_k + \omega_e$ and $\omega_k - \omega_e$ occurs. A simplified theoretical model of the phenomenon is developed. By using supplementary microwave pumping and a coherent sound wave it is shown experimentally that the initial parametrically excited spin wave system is incoherent with respect to the individual wave phases. Collective parametric excitation of spin-wave oscillations by sound waves is observed.

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1. INTRODUCTION

In the study of parametric excitation of spin waves in antiferromagnetic $MnCO_3$ by the parallel pumping method, we observed that at large microwave pump amplitudes the power absorbed by the sample oscillates about a stationary level. The frequency of these oscillations does not depend on the frequency and power of the microwave pumping, or on the magnetic field, but depends on the size of the sample and lands frequency in the range of its natural elastic oscillations, i.e., of the order of 1 MHz for a sample of size ~ 1 mm. The frequency of the parametrically excited spin waves (PSW) is of the order of 10 GHz. We assumed that in the presence of a definite number of PSW, natural elastic oscillations of the sample are produced and these in turn cause oscillations of the absorbed power. A similar observation and assumption are described in Ref. 1, where a study was made of parametric excitation of spin waves in antiferromagnetic $FeBO_3$.

The present paper is devoted to an experimental confirmation of this assumption, to the study of the mechanism of generation of elastic oscillations, and also to an investigation of the nonlinear interaction of acoustic waves with an excited spin system of an antiferromagnet.

Instability of a sample excited by a microwave field to the onset of elastic oscillations was observed also in spherical ferromagnetic yttrium iron garnet cyrstals.[2,3] The phenomenon investigated in Refs. 2 and 3, apart from the fact that the investigated objects were different (a ferromagnet and an antiferromagnet), differs from that described also by us in Ref. 1 by two circumstances. First, the elastic oscillations of the ferromagnet were observed when only long-wave oscillations of the spin system were excited, with a wavelength of the order of the sample size. In our case, the elastic oscillations are observed upon excitation of spin waves of length of the order 10^{-3} of the sample size. Second, the long-wave oscillations excited parametrically are coherent because of the strong influence of the sample boundaries, while the resonantly excited oscillations are coherent to the same degree as the pump. At the same time, individual phases of the parametrically excited short-wave spin waves can be randomized by even weak interaction with the thermal excitations. This follows from the fact that in thermal excitation the pump fixes only the sum of the phases of the pair of excited waves (see, e.g., Ref. 4). The question of what is decisive for the coherence or non-coherence of PSW, the influence of the boundaries or the randomizing action of the thermal excitations, has remained open to this day.

The methodological aspect of the study also differs from the preceding investigations of the related phenomena. We use here direct detection of acoustic oscillations and measure their amplitude. To identify the mechanism of the phenomenon by experiment, we use a second probing pump.

2. EXPERIMENT

The object of our investigation was the antiferromagnet $MnCO_3$ with easy-plane anisotropy and with a Néel temperature 32 K. The properties of microwave spin waves in it and the characteristics of their parametric excitation were already investigated earlier.[5,6]

The experimental setup is shown in Fig. 1. A cylindrical $MnCO_3$ single crystal of diameter and height 1 mm is placed in an aperture in the wall of a strip microwave resonator in such a way that it protrudes 0.2 mm both into the interior of the resonator and outside the external surface. An ultrasound sensor of the longitudinal type in the form of a plate made of radiotechnical piezoceramic measuring $2\times2\times0.4$ mm is glued to the end face of the sample protruding to the outside. The natural frequency of the longitudinal oscillations of the piezo-ceramic plate is approximately 7 MHz.

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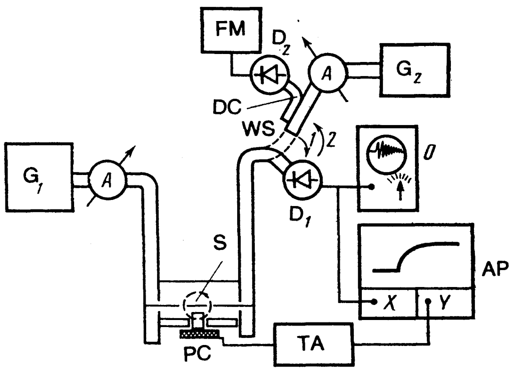

FIG. 1. Experimental setup: G_1,G_2) microwave generators; A) attenuators; DC) directional coupler; WS) waveguide switch; PC) piezoceramic; S) Strip of strip resonator (the dashed lines show the direction of the microwave magnetic field); D_1,D_2) microwave detectors; FM) frequency meter; O) oscilloscope; AP) automatic plotter; TA) tuned amplifier.

This sensor was used as a detector and as a source of elastic vibrations in the range from 1 to 10 MHz, and also to excite acoustic ringing in the system. The sensor was clamped between a dielectric liner and two springs that grounded one of its surfaces. The springs are strips of copper foil 0.1 mm thick and 0.3 mm wide. Thus, the sample and the sensor constituted a single vibrating system with weak acoustic coupling to the surrounding parts of the instrument. There is no significant loss of elastic-oscillation energy in the region where the sample and sensor were glued. This was determined by an independent experiment on the passage of sound through the sample. Therefore, in the case of elastic oscillations that correspond to the fundamental modes of the sample piezo-sensor system, the amplitude of the deformation should be of the same order in the sensor and in the sample. The signal of the piezoceramic sensor was amplified with a tuned amplifier and fed to the Y coordinate of an automatic plotter.

The external constant field **H** and the microwave magnetic field at the location of the sample were parallel and located in the easy plane of the antiferromagnetism, which coincided with the plane of the end faces of the sample. The microwave power from the generator G_1, which operated in the cw mode at a frequency $\omega_{p1}/2\pi = 18$ GHz, was fed through the waveguide channel with the aid of a coupling antenna into a resonator with $Q \approx 1000$. The power of the generator G_1 sufficed to produce at the location of the sample a microwave magnetic field with amplitude h exceeding by more than 15 times the parametric-excitation threshold h_c threshold. We recall that parametric excitation (i.e., an exponential growth of the spin-wave amplitude in time until any one of the limitation mechanisms comes into play) takes placed only at $h > h_c$, and the threshold field h_c is determined in the simplest case by the frequency $\Delta\nu_{\mathbf{k}}$ of the relaxation of the spin waves and by the coefficient $V_{\mathbf{k}}$ of the coupling of the microwave field with the pairs of spin waves:

$$h_c = \pi \Delta\nu_{\mathbf{k}} / V_{\mathbf{k}}. \tag{1}$$

An expression for the coefficient $V_{\mathbf{k}}$ was obtained in Ref. 7. The amplitude increases in two spin waves with wave vectors **k** and $-$**k**, which make up the so-called parametric pair. The natural frequency $\omega_{\mathbf{k}}$ of these waves satisfies the parametric-resonance condition

$$\omega_{\mathbf{k}} = \omega_p/2, \tag{2}$$

where ω_p is the microwave-pump frequency and, according to Ref. 8,

$$\omega_{\mathbf{k}} = \gamma [H(H+H_D) + H_\Delta^2/T + \alpha^2 k^2]^{1/2} \tag{3}$$

is the dispersion law of the spin waves in antiferromagnets with easy-plane anisotropy for the orientation **H** realized in our case. The parameters of Eq. (3) are the following: $H_D = 4.4$ kOe is the Dzyaloshinskii field, $H_\Delta^2 = 5.8\ \mathrm{kOe}^2 \cdot \mathrm{K}$ is the hyperfine interaction constant, $\alpha = 0.78 \cdot 10^{-5}$ kOe · cm is the exchange constant, $\gamma = 2\pi \cdot 2.8$ GHz/kOe is the gyromagnetic ratio, and T is the temperature. At the conditions of our experiment, when the magnetic field H is varied from zero to $H_c = 1.2$ kOe, the PSW vector changes, according to (2) and (3), from a value $10^{-5}\ \mathrm{cm}^{-1}$ to zero. Usually in the course of the experiment the presence of parametric excitation is evidenced by absorption of microwave power by the sample. This absorption vanishes at $H > H_c$, where H_c is the magnetic field above which simultaneous satisfaction of Eqs. (2) and (3) is impossible at any values of **k**.

The process of establishment of the absorbed power is investigated by observing the waveform of the envelope of the pulse of the microwave power passing through the resonator when the pumping is by rectangular pulses. The absorption of the power by the sample becomes noticeable not immediately after the start of the pulse, since the thermal level of the spin-wave excitation is very low. As a result, the envelope of the pulse of the microwave power passing through the resonator has at $1.2h_c > h > h_c$ a rather prolonged horizontal section, and only after a certain time (of the order of 100 μsec) a characteristic distortion, in the form of a notch, appears on the envelope after the start of the pulse does followed by a new stationary state of the transmitted power, lower than at the start of the pulse. A similar distortion occurs also on the envelope of the pulse of the microwave power reflected from the resonator. The notch sets in earlier the higher the initial excitation level and the larger $(h/h_c - 1)\Delta\nu_{\mathbf{k}}$ (for details see Refs. 5 and 9). The time variation of the absorbed power can be assessed by observing the signal of a detector D_1, which generates a voltage proportional to h^2, while the change of this voltage upon excitation of PSW is proportional to the power absorbed in the sample.

The measurements were made at a temperature 1.65 K. In this case, as follows from the results of Refs. 5 and 6, the lifetimes of the magnons with frequency 10 GHz is of the order of 1 μsec. The lifetime $\tau_{\mathbf{k}}$ is connected with the relaxation frequency $\Delta\nu_{\mathbf{k}}$ by the sample relation $\tau_{\mathbf{k}} = (2\pi\Delta\nu_{\mathbf{k}})^{-1}$.

Measurement of the absorbed power and of $\tau_{\mathbf{k}}$ make it possible to determine the total number N of the PSW in the steady state. At $h/h_c \approx 2$ we have $N \sim 5 \cdot 10^{16}\ \mathrm{cm}^{-3}$.

To estimate the PSW occupation numbers we shall assume that the excitation is uniform over the entire equal-energy surface $\omega_{\mathbf{k}} = \omega_p/2$ of **k**-space. The basis for this assumption is that the coefficient $V_{\mathbf{k}}$ is the same for all waves of this surface. We shall also assume that the PSW are excited near the surface in a layer whose thickness Δk is determined by the lifetime of the spin wave: $(\Delta k)^{-1} \approx \tau_k (d\omega_{\mathbf{k}}/dk)$. This assumption is confirmed by experiment.[10] After making these assumptions, we find that the value of N cited above corresponds to occupation numbers $n_{\mathbf{k}} \sim 10^7$.

From the theory of the above-threshold state of a PSW (Ref. 4) and from experiments that confirm this theory (e.g., Ref. 6) it follows than then h is changed the number of PSW changes like

$$N \propto \Delta \nu_{\mathbf{k}} [(h/h_c)^2 - 1]^{1/2}. \quad (4)$$

In Sec. 4 we shall describe experiments performed with simultaneous action of two microwave fields of different frequency on the sample. The pump frequencies differed by 2–15 MHz. In place of the detector D_1, a second microwave generator G_2 was connected to the resonator (see Fig. 1, waveguide switch in position 2). Generator G_2 operated in a pulsed regime with pulse duration 1 msec and repetition frequency 50 Hz. The detector D_2 receives simultaneously signals from two generators: the signal from generator G_1 which passed through the resonator, and the signal of G_2 reflected from the resonator. The oscillogram of the voltage of the detector D_2 shows the beats of the microwave signals of these two generators, modulated by the envelope of the reflected microwave power. Thus, the envelope of the beats reveals the instant of the notch for the pump produced by G_2. The power of the generator G_2 made it possible to reach an excess $h/h_c \approx 3$ above threshold field. The difference between the frequencies of G_1 and G_2 was determined from the beam frequency with the aid of a digital frequency meter.

3. BASIC RESULTS

When the microwave pump power is gradually increased, 2.4-MHz elastic oscillations of a sample are produced once the microwave field h exceeds a certain threshold value h_e. The dependence of the amplitude A of the elastic oscillations on the square of the pump field h^2 can be recorded with an automatic plotter by gradually increasing the damping introduced into the waveguide channel of the generator G_1 and by using the signals of the piezo-sensor and the detector D_1. A typical plot is shown in Fig. 2.

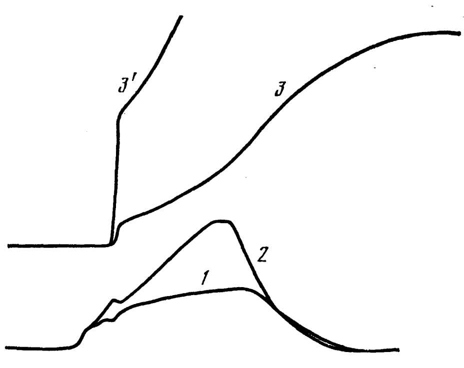

FIG. 2. Plot of piezo-sensor signal as a function of the microwave pump power (signal of detector D_1). The tuned amplifier with passband 0.01 MHz is tuned to the frequency (in MHz); 1) 2.38; 2) 2.40; 3) 3.32; $H = 0.4$ kOe. The plot of line 3′ was obtained at a higher sensitivity of the amplifier.

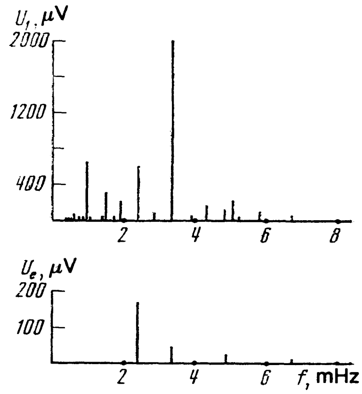

FIG. 3. Spectrum of the oscillations of the microwave power passing through the resonator (U_I) and of the piezoceramic signal U_e at $h/h_c \approx 15$ and $H = 0.4$ kOe.

The quantity h_e exceeds the threshold field of the parametric excitation h_c by 7–10 times. Changes of h_e within these limits take place when the sample is reattached. At $h \approx h_e$, the integral density of the parametrically excited magnons is approximately $5 \cdot 10^{17}$ cm^{-3}, and their occupation numbers are of the order of 10^8.

The elastic oscillations are accompanied by oscillations of the absorbed power at the same frequency. With further increase of the microwave field amplitude, oscillations of the absorbed power set in and signals are produced by the piezo-sensor at higher frequencies. When $h/h_c \approx 15$, the spectrum of the oscillations of the absorbed power contains approximately 10 lines of different intensity (see Fig. 3). The spectrum of the piezoceramic signal has five reliably distinguishable lines whose frequencies are contained in the oscillation spectrum of the absorbed power.

To determine the natural frequencies of the piezo-sensor + sample system, we observed acoustic ringing of the system following a mechanical shock. The shock was produced with a short voltage pulse applied to the piezoceramic, and the ringing was revealed by the response of the PSW to the shock. The oscillogram of the microwave power passing through the resonator after the shock is a superposition of several damped oscillations, from among which we can separate oscillations with frequency $\omega_e^{1,2} = 4.2$ and 3.3 MHz. It is precisely at these frequencies that the most intense elastic oscillations, produced by the microwave pump, are observed. The characteristic damping time of the 2.4-MHz oscillation on the oscillogram amounts approximately 30 μsec (see Fig. 4a). The amplitude of the oscillations of the absorbed power is proportional to the amplitude of the acoustic oscillations when the amplitude of the oscillations of the relative strain does not exceed 10^{-8}. We have verified this by using a piezo-sensor as the ultrasound source. Thus, the damping time of the energy of the elastic oscillations τ_e is approximately 15 μsec, which exceeds by more than one order of magnitude the lifetime of the spin waves $\tau_{\mathbf{k}}$. As can be seen from the $A(h_2)$ plot in Fig. 2, the amplitude of the elastic oscillation of frequency 2.4 MHz reaches a maximum value when the microwave power is increased, after which it drops to zero. The amplitude of the other elastic modes either satu-

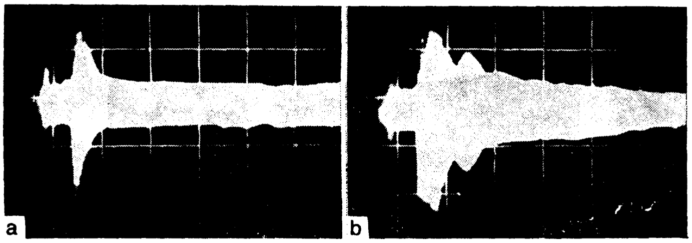

FIG. 4. Oscillogram of microwave power passing through the resonator close to the instant of time when a mechanical shock is produced in the sample ($H = 0.4$ kOe, sweep 100 μsec/div): a) $h/h_e = 0.15$; b) $h/h_e = 0.9$.

rates after a rapid increase in the vicinity of the h_e threshold value or changes little. The maximum amplitude of the elastic oscillations was produced by a signal of approximately 300 μV from the piezo-sensor, corresponding to a relative strain 10^{-9} of the material of the sensor in elastic oscillations, to an elastic energy stored in the sample of the order of $4 \cdot 10^{-17}$ J, and to an acoustic power $4 \cdot 10^{-12}$ W.

Elastic oscillations are generated in the entire magnetic-field interval that admits of parametric excitation of spin waves. The dependence of the microwave power threshold for the excitation of the sound on H for one of the elastic modes is shown in Fig. 5.

The frequency of the generated oscillations turned out to depend little on the magnetic field; when H was varied in the entire interval from zero to H_c, the frequency changed within the limits of the width of the resonance curve of the elastic mode. For example, for the mode with frequency 2.4 MHz this change takes place in the range from 2.41 to 2.47 MHz. The amplitudes of the output voltage of a resonant amplifier to whose input the piezo-sensor was connected, as functions of the magnetic field at different tuning of the amplifier, are plotted in Fig. 6.

The instability of the frequency of the generated acoustic oscillation turned out to be much less than the value $(2\pi\tau_e)^{-1}$. The frequency instability of the 2.4-MHz mode does not exceed 100 Hz.

The antiferromagnetic crystal with the PSW excited in it can operate as an elastic-oscillation amplifier when the microwave pump power level is lower than the sound-generation threshold. With the aid of our device it was possible to demonstrate the transfer of energy from the PSW to the elastic oscillations by observing the increase of the lifetime of the elastic oscillations with increasing number of PSW. An oscillogram of the acoustic ringing (Fig. 4b), obtained at a sufficiently high power level of the microwave pump, shows that this increase does indeed take place. As $h \to h_e$, the value of τ_e increased without limit. A plot of $\tau_e(h^2)$ is shown in Fig. 7.

4. BASIC ASSUMPTION. TEST EXPERIMENTS

We consider the interactions of quasiparticles with participation of spin waves, as a result of which elastic oscillations are generated. The number of oscillation modes is smallest when a spin wave decays into an oscillation of an elastic mode and a secondary spin wave or into an elastic oscillation and phonon. Recognizing that $\omega_e \ll \omega_{\mathbf{k}}$ for the elastic oscillations observed by us, we find that the second variant of the decay is allowed by the conservation laws only in a narrow region of **k**-space, where the frequency and the wave vector of the spin waves are approximately equal to these values for the phonons. We shall therefore dwell on the first spin-wave decay scheme:

$$\mathbf{k}_1 \to \begin{matrix} \nearrow \mathbf{k}_2 \\ \\ \searrow \mathbf{q} \end{matrix} . \qquad (5)$$

We note that, when analyzing the interactions of the quasiparticles, it is necessary to stipulate satisfaction of the momentum conservation law only accurate to a value of the order of $\hbar/d$, where d is the size of the sample. Therefore, even though magnon decay into a magnon and a phonon in $MnCO_3$ is forbidden by the conservation laws, excitation of

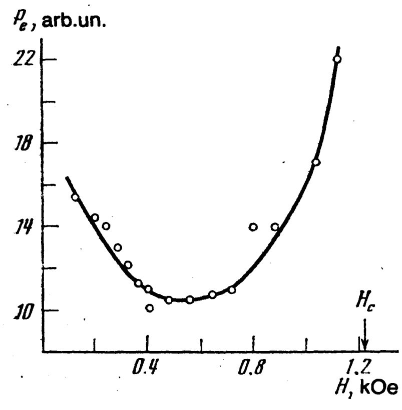

FIG. 5. Dependence of the threshold generation power of 2.4-MHz elastic oscillations on the magnetic field.

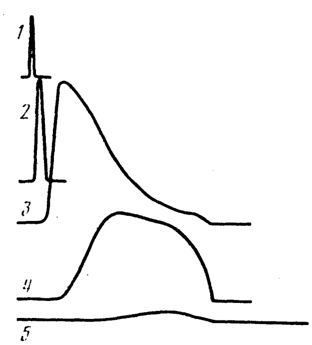

FIG. 6. Plot of signal of the piezo-sensor as a function of the magnetic field. Width of passband of the tuned amplifier 0.01 MHz. The tuned amplifier is tuned to the frequency (in MHz): 1) 2.42; 2) 2.43; 3) 2.45; 4) 2.46; 5) 2.47; ($\pm$ 0.01).

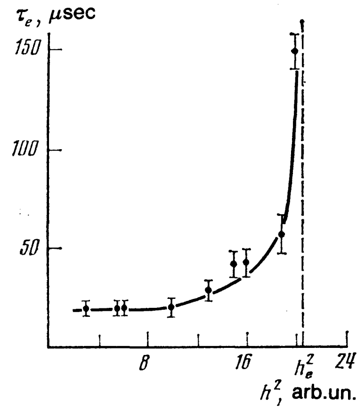

FIG. 7. Dependence of the lifetime of the elastic oscillations of the 2.4-MHz mode on the microwave pump power.

elastic oscillations on account of the interaction (5) is possible in principle if the difference between the wave vectors of the magnons whose frequency difference equals the frequency ω_e of the elastic mode is not too large compared with $1/d$. In this case the spin-wave Hamiltonian terms corresponding to this type of interaction will contain in place of $\Delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q})$, where $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{q}$ are the wave vectors of the magnons and the phonon, the factor

$$\eta_{12}{}^{e} = \frac{1}{V} \int \cos(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q})\, d^3 r. \tag{6}$$

For the PSW and the elastic oscillations in our experiment $\eta_{12}^e \approx 0.2$ in the magnetic-field interval from 0 to 1 kOe. Thus, as a working hypothesis we can propose that the oscillations of the elastic mode are generated in the situation described above on account of the interaction (5). As $H \to H_c = 1.2$ kOe we have $\eta_{12}^e \to 0$, since $d\omega_{\mathbf{k}}/d\mathbf{k} \to 0$, as follows from the form of the spectrum (3). Therefore such an excitation of the elastic modes of the sample should stop as $H \to H_c$. In our experiment one observes an increase of the sound generation threshold h_e as $H \to H_c$ (see Fig. 5), thus offering evidence in favor of the hypothesis advanced concerning the sound-generation mechanism.

If the sound is generated in the manner described above, spin waves with frequency $\omega_p/2 - \omega_e$ (we abbreviate them SW $-$) and spin waves with frequency $\omega_p/2 + \omega_e$ (SW $+$) should be generated in the sample besides the elastic oscillations. The waves SW $+$ will be excited in the course of coalescence of an elastic oscillation with the initial PSW. This process has the same probability as the direct process (5). We note, however, that the presence of SW $+$ and SW $-$ waves is not unequivocal evidence in favor of this particular proposed sound generation mechanism since, if the sound is formed by some other method, SW $+$ and SW $-$ can be excited as waves of the next generation. Quantitative estimates of the amplitude $\Phi^e_{1,2}$ of the interaction (5) can be obtained by exciting in the sample, in addition to the initial PSW, also PSW with frequency $\omega_p/2 + \omega_e$ or $\omega_p/2 - \omega_e$ (i.e., SW $+$ or SW $-$ waves). In this case there generation of elastic oscillations should be observed not in the threshold regime, but with an amplitude that varies smoothly when the number of the PSW changes. By determining with such an experiment the value of $\Phi^e_{1,2}$ averaged over the PSW distribution, we can estimate (see Sec. 5 below) the number of PSW necessary to reach the sound-generation threshold. Thus, it is possible to verify in principle whether the interaction (5) is capable of leading to generation of sound upon excitation of that number of PSW which is reached in the given experiment. It turns out that experiments with artificial excitation of SW $+$ and SW $-$ provide also a more direct confirmation of the fact that the sound generation is due to the interaction (5).

To excite and observe the SW $+$ and SW $-$ waves we connected to the resonator one more microwave generator, G_2 (Fig. 1, waveguide switch in position 2). We denote by ω_{p2} the frequency of the second generator and by h_2 the amplitude of the microwave magnetic field produced by it at the location of the sample. The value of h_2 corresponding to the threshold of the parametric excitation of PSW with frequency $\omega_{p2}/2$ will be denoted h_{2c}.

Turning on both pumps, we followed the shape of the envelope of the pulse of the second pump at $h_2 > h_{2c}$. It turned out that when the G_1 power was sufficient to generate elastic oscillations (i.e., at $h > h_e$), and the frequencies of the generators satisfied the relation

$$\omega_{p2}/2 - \omega_{p1}/2 = \omega_e \tag{7a}$$

with accuracy not worse than 0.1 MHz, a qualitative change was observed in the shape of this envelope. The change consists in the fact that the notch on the envelope of the pulse appears immediately after the second pump is turned on, and not after the usually observed horizontal segment. The envelopes of the signal of the detector D_2, which illustrate the change of the shape of the envelope of the second-pump pulse, are shown in Fig. 8. No horizontal section of the oscillogram in the initial stage of the process was observed even at the smallest excesses of the threshold of the second pump: $0 < h_2 - h_{2c} \lesssim 0.1 h_{2c}$. In accordance with the statements made in Sec. 2 concerning the origin of the horizontal section of the oscillogram, the presence of power absorption even during the first microseconds after G_2 is turned on is evidence that beyond the generation threshold of the elastic oscillations the occupation numbers of SW $+$ become of the order of 10^6.

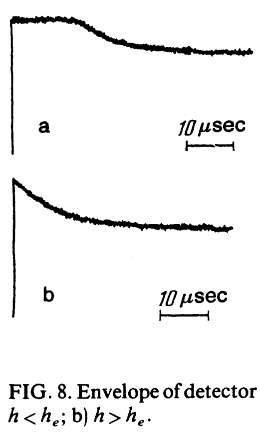

FIG. 8. Envelope of detector signal D_2 when SW $+$ waves are observed: a) $h < h_e$; b) $h > h_e$.

The oscillogram shown in Fig. 8b is also evidence that the aggregate of the PSW is incoherent in the sense of the individual phases. Indeed, assume that the initial group of the PSW is coherent. Then the group of the SW + waves will also be coherent, since it is produced by connecting a coherent elastic oscillation. Thus, in the SW + group, the sum of the phases of the waves with wave vectors **k** and $-$ **k** as well as individual phases will be determined by the phase of the microwave pump of the generator G_1. In this case the absorption of the microwave power from the second pump at the instant when it is turned on will be determined by the phase shift of the second pump relative to the sum of the phases of the pair of waves from the group SW +. Namely, the absorbed power W is determined by the relation (see, e.g., Ref. 4 and the experiment of Ref. 6)

$$W=\hbar\omega_{\mathbf{k}}N_{\mathbf{k}}hV_{\mathbf{k}}\sin\psi_{\mathbf{k}}, \qquad (8)$$

where $N_{\mathbf{k}}$ is the number of the PSW, and $\psi_{\mathbf{k}}=\varphi_{\mathbf{k}}+\varphi_{-\mathbf{k}}$ is the aforementioned phase shift. Obviously, at any new instant when the second pump is turned on the quantity $\sin\varphi_{\mathbf{k}}$ changes randomly in the interval from -1 to 1, and then the oscillogram obtained by repeating the switching of the second pump with frequency 50 Hz should reveal a scatter of the envelopes of the pulse of the second pump in the vicinity of the instant of its observation.[1)] The scatter of the envelopes should be of the order of the deviation of the oscilloscope beam from the zero-absorption line. Experiment shows, however, that the oscilloscope beam always traces the same path and reveals no scatter whatever.

The SW + waves can be excited also at $h<h_e$ if the elastic oscillations are induced artificially with a piezoceramic plate. The onset of microwave-power absorption after the start of the pulse of generator G_2 was observed also in this case. Such a procedure makes it possible to vary the amplitude and frequency of the elastic oscillations continuously, to use elastic oscillations of larger amplitude, and consequently obtain a larger number of SW +. We used elastic oscillations of frequency ω_s in the range 4–8 MHz. By tuning the generator G_2 to the frequency $\omega_{p2}=\omega_{p1}+2\omega_s\pm0.1$ MHz, we observed, at all sound amplitudes, the start of the absorption at the frequency ω_{p2} immediately after turning-on the second pump at a zero value of the absorbed power, at the very instant when G_2 was turned on. There was likewise no scatter of the envelopes in the case of periodic repetition of the pulses of generator G_2. The ability of the apparatus to reveal this scatter was monitored in the following manner. When the sound source is turned off, a scatter of the oscillogram is observed after turning off the generator G_2 for a short time (0.2 μsec) at the midpoint of each pulse. In this case the quantity $\sin\varphi_{\mathbf{k}}$ at the instant of the G_2 is turned on again is determined by the sum of the PSW phases by the pump prior to the instant of turning off and by the random phase of the pump when the generator is turned-on.

These observations indicate that the wave systems PSW and SW + are incoherent in the sense of individual phases; consequently, excitation of elastic modes of the sample is produced by the action of incoherent waves.

It should be noted that the action of a relatively strong pump, produced by the generator G_1, causes a substantial change in the threshold h_{2c} compared with the threshold field in the unperturbed crystal. At $\omega_{p2}>\omega_{p1}$ the threshold field h_{2c} increases by 30–40%. At $\omega_{p1}-2\pi\cdot10$ MHz $<\omega_{p2}<\omega_{p1}$ the critical field h_{2c} decreases and for $\omega_{p1}-\omega_{p2}\approx2\omega_e$ this decrease is by approximately a factor of 3, so that it is impossible to observe the shape of the second-pump pulse envelope at threshold power levels, owing to the interfering influence of the beats with the high-power pump. This circumstance makes impossible also direct indication of the SW $-$ in the manner described above for SW +. The described change of the threshold field h_{2c} has no singularities of resonant type at $\omega_{p1}-\omega_{p2}=\pm2\omega_e$ and consequently cannot be explained within the framework of the considered interaction. The change of h_{2c} retains the same character also at $h<h_e$.

We turn now to an analysis of the piezo-sensor signal under conditions of simultaneous action of two pumps on the sample. Plots of $A(h^2)$ at two different values of $\omega_{p2}-\omega_{p1}$ are shown in Fig. 9. Excitation of the additional group of PSW when the condition (7a) is not satisfied and the condition

$$\omega_{p2}/2-\omega_{p1}/2=-\omega_e \qquad (7b)$$

is not satisfied decreases the threshold generation field of the elastic oscillation h_e to a value h_e'. Here h_e is the microwave field produced by the generator G_1. A plot of $A(h^2)$ for this case is shown in Fig. 9a (curve 2). When the condition (7a) is satisfied and $h_2>h_{2c}$, nonthreshold generation of elastic oscillations is observed. The amplitude of the sound has then a noticeable value even in the region of values $h<h_e'$ (see Fig. 9a, curve 3). When the condition (7b) is satisfied, nonthreshold generation of elastic oscillations upon excitation of the second PSW group is also observed (Fig. 9b, curve 3). In a narrow region of h near h_e, namely $0.95h_e<h<h_e$,

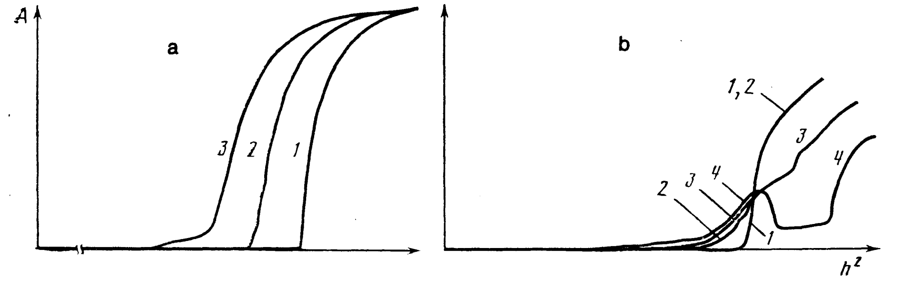

FIG. 9. Dependence of the piezo-sensor signal amplitude at 2.4 MHz on the power of the first pump under the action of the second pump. a) $\Delta f=(\omega_{p2}-\omega_{p1})/2\pi>0$: 1) generator G_2 turned off; 2) $h_2/h_{2c}\approx2$ and $\Delta f=4.5$ MHz; 3) $h_2/h_{2c}\approx2$ and $\Delta f=4.8$ MHz; b) $\Delta f=-4.8$ MHz; 1) generator G_2 turned off; 2) $h_2=h_{2c}/4$; 3) $h_2=1.2h_{2c}$; 4) $h_2\approx2h_{2c}$.

nonthreshold generation of sound is observed also at below-threshold level of the second pump, i.e., a $h_2 < h_{2c}$ (Fig. 9b, curve 2). The last phenomenon is apparently due to the appreciable growth of the number of SW $-$ as $h \to h_e$ (below-threshold heating) and the below-threshold absorption of the energy of the second pump by the SW $-$ waves. (See Refs. 11 and 12 concerning below-threshold absorption.) There exists thus indirect evidence of the presence of SW $-$ waves.

The instability of the frequency of the sound generated in the nonthreshold regime amounts to approximately 10 kHz, i.e., it approximately coincides with the width of the resonance curve of the elastic mode.

As can be seen from Fig. 9b (curves 3,4), excitation of SW $-$ waves increases the amplitude of the sound at small values of h, followed by a region of small values of h where the amplitude of the sound decreases upon excitation of SW $-$, or can be completely suppressed. This characteristic behavior of the function $A(h^2)$ is observed only if the condition (7b) is satisfied. It is explained on the basis of a simple analysis of the probabilities of the direct and indirect transitions for the process (5) in the incoherent wave system (see, e.g., Ref. 13). In this case the change of the occupation numbers of different quasi-particles is described by the kinetic equations. From this analysis it follows that the energy fraction going into oscillations of the elastic mode, a fraction that depends on the number of SW $-$ waves, is proportional to $n_{\mathbf{k}-}(n_{\mathbf{k}0} - n_e)$, where $n_{\mathbf{k}-}$, $n_{\mathbf{k}0}$, and n_e are the occupation numbers of the SW $-$, PSW, and the elastic modes [see also Eq. (10c)]. This expression reverses sign at $n_e = n_{\mathbf{k}0}$, i.e., at $n_e \sim 10^8$, which corresponds to an elastic-oscillation amplitude amounting to approximately 10% of the observed maximum amplitude value. Obviously, the reversal of the sign of this contribution to the energy of the elastic oscillation causes the aforementioned characteristic behavior of the $A(h^2)$ dependence when the SW $-$ are artificially excited. If the generation of the elastic oscillations were to occur without a considerable participation of the process (5), the satisfaction of non-satisfaction of the condition (7b) would not lead to such a change in the form of this dependence.

Thus, we have shown experimentally that generation of elastic oscillations is due mainly three-wave interaction of the form (5). The initial PSW system is in this case incoherent. The narrow spectral width of the generated sound is an indication of the stimulated character of the emission of phonons by spin waves.

5. THEORETICAL ESTIMATES

Theoretical estimates of the threshold level of excitation of spin oscillations of a ferrite for generation of elastic waves have been described in the literature.[14,15] They were obtained for experimental situations that take place in studies of the type in Refs. 2 and 3. These theoretical models presuppose coherence of initial spin oscillations (this, of course, is valid for magnetostatic waves). They do not take into account, however, the formation of oscillations with frequency $\omega_{\mathbf{k}} + \omega_e$ (in our case this is SW $+$). We construct a model of the phenomenon, following Refs. 16 and 17. In Ref. 16 a theoretical and experimental study was made of the instability of PSW to generation of spin waves of the "bottom of the spectrum" on account of four-wave interactions. In Ref. 17, the instability of PSW to phonon generation was theoretically investigated in the case of a decay spin-wave spectrum. The structural difference between Ref. 17 on the one hand, and Refs. 14 and 15, on the other, consists both in the assumption that the individual phases are random and in consideration of the SW $+$ waves. According to Refs. 16 and 17, owing to the randomness of the individual phases of the PSW, the time variation of the occupation numbers of the SW $+$, SW $-$, and elastic modes ($n_{\mathbf{k}+}$, $n_{\mathbf{k}-}$, and n_e, respectively) can be described with the aid of kinetic equations made up in accordance with standard procedure (see, e.g., Ref. 13). The occupation number of the initial PSW will be designated $n_{\mathbf{k}0}$. The decay of the spin waves into a spin wave and an elastic oscillation, and also the inverse processes, contribute to the Hamiltonian

$$H^{(3)} = \sum_{1,2,e} \Phi_{1,2}(b_1 b_2^+ c_e^+ + \text{c.c.})\eta_{1,2}. \tag{9}$$

The factor $\eta^e_{1,2}$ is determined here by Eq. (6), while b_1^+, b_2^+, b_1, and b_2 are the magnon creation and annihilation operators, while c_e^+ and c_e are the elastic-oscillation quantum creation and annihilation operators. The equations for the occupation numbers are similar to those given in Ref. 17 and take for our case the form

$$\frac{d}{dt}n_{\mathbf{k}+} = \frac{8\pi}{\hbar}\sum_{\mathbf{k}_0}|\Phi^e_{\mathbf{k}0,\mathbf{k}+}\eta^e_{\mathbf{k}0,\mathbf{k}+}|^2(n_{\mathbf{k}0}n_e - n_{\mathbf{k}+}n_e - n_{\mathbf{k}+}n_{\mathbf{k}0} - n_{\mathbf{k}+}) \times\delta(\hbar\omega_{\mathbf{k}+} - \hbar\omega_{\mathbf{k}0} - \hbar\omega_e) - \gamma_{\mathbf{k}+}n_{\mathbf{k}+}. \tag{10a}$$

$$\frac{d}{dt}n_{\mathbf{k}-} = \frac{8\pi}{\hbar}\sum_{\mathbf{k}_0}|\Phi^e_{\mathbf{k}-,\mathbf{k}0}\eta^e_{\mathbf{k}-,\mathbf{k}0}|^2(n_{\mathbf{k}0}n_e + n_{\mathbf{k}0}n_{\mathbf{k}-} + n_{\mathbf{k}0} - n_{\mathbf{k}-}n_e) \times\delta(\hbar\omega_{\mathbf{k}0} - \hbar\omega_{\mathbf{k}-} - \hbar\omega_e) - \gamma_{\mathbf{k}-}n_{\mathbf{k}-}, \tag{10b}$$

$$\frac{d}{dt}n_e = \frac{8\pi}{\hbar}\sum_{\mathbf{k}_-,\mathbf{k}_0}|\Phi^e_{k-,\mathbf{k}0}\eta^e_{\mathbf{k}-,\mathbf{k}0}|^2(n_{\mathbf{k}0}n_{\mathbf{k}-} + n_e n_{\mathbf{k}0} + n_{\mathbf{k}0} - n_e n_{\mathbf{k}-}) \times\delta(\hbar\omega_{\mathbf{k}0} - \hbar\omega_{\mathbf{k}-} - \hbar\omega_e) + \frac{8\pi}{\hbar}\sum_{\mathbf{k}_0,\mathbf{k}_+}|\Phi^e_{\mathbf{k}0,\mathbf{k}+}\eta^e_{\mathbf{k}0,\mathbf{k}+}|^2(n_{\mathbf{k}+}n_{\mathbf{k}0} + n_e n_{\mathbf{k}+} + n_{\mathbf{k}+} - n_e n_{\mathbf{k}0})\delta(\hbar\omega_{\mathbf{k}+} - \hbar\omega_{\mathbf{k}0} - \hbar\omega_e) - \gamma_e n_e. \tag{10c}$$

Here $\gamma_i = 1/\tau_i$; the sums over $\mathbf{k}_0$, $\mathbf{k}_+$, and $\mathbf{k}_-$ correspond to changes of the occupation numbers on account of processes of type (5), and the terms of the type $\gamma_i n_i$ correspond to natural relaxation of the considered oscillations on account of the interaction with the remaining excitations of the crystal, which we shall assume to be at the thermal level.

Spin waves with wave vectors $\mathbf{k}_0$, $\mathbf{k}_+$, and $\mathbf{k}_-$, connected by the relations

$$\omega_{\mathbf{k}0} - \omega_{\mathbf{k}-} = \omega_e, \qquad |\mathbf{k}_0 - \mathbf{k}_-| \approx 1/d,$$

$$\omega_{\mathbf{k}+} - \omega_{\mathbf{k}0} = \omega_e, \qquad |\mathbf{k}_+ - \mathbf{k}_0| \approx 1/d,$$

have close values of the frequencies and of the wave vectors, since $\omega_e \ll \omega_{\mathbf{k}0}$, and the differences between waves with wave vectors $\mathbf{k}_+$ and $\mathbf{k}_0$ or $\mathbf{k}_0$ and $\mathbf{k}_-$ with respect to frequency and wave vector are identical; we shall therefore assume that $\Phi^e_{\mathbf{k}0,\mathbf{k}+} = \Phi^e_{\mathbf{k}-,\mathbf{k}0}$ and are the same for all $\mathbf{k}_0$, i.e., $\Phi^e_{\mathbf{k}0,\mathbf{k}+} = \Phi^e_{\mathbf{k}-,\mathbf{k}0} \equiv \Phi$. We shall also assume that the PSW are excited uniformly over a sphere of constant frequency $\omega_{\mathbf{k}0}=\omega_{p1}/2$, i.e., $n_{\mathbf{k}0}\equiv n_0$, and that the waves SW + and SW − are also excited uniformly over the corresponding spheres in k space: $n_{\mathbf{k}+}\equiv n_+$ and $n_{\mathbf{k}-}\equiv n_-$. The thickness Δk of the spherical layer in which the PSW are distributed, will be assumed, as above, to be determined by the lifetime $\tau_{\mathbf{k}}$. The factors $\eta^e_{\mathbf{k}-,\mathbf{k}0}$ and $\eta^e_{\mathbf{k}0,\mathbf{k}+}$ differ noticeably from zero only when $|\mathbf{k}_0-\mathbf{k}_+|$, $|\mathbf{k}_0-\mathbf{k}_-|\sim\omega_e/v_{\mathbf{k}}\approx 2/d$, where $v_{\mathbf{k}}$ is the group velocity of the spin waves. Thus, at fixed $\mathbf{k}_0$, the contribution to the sums that enter in (10) will be taken into account only from small sections of the aforementioned equal-energy spheres; the areas of these sections are of the order of $(\omega_e/v_{\mathbf{k}})^2$. We replace the quantities $\eta^e_{\mathbf{k}\mp,\mathbf{k}0}$ by certain values η averaged over these sections. The damping factor $\gamma_{\mathbf{k}\mp}$ will be assumed to be independent of the direction of $\mathbf{k}_\mp$, i.e., $\gamma_{\mathbf{k}_+}\equiv\gamma_+$ and $\gamma_{\mathbf{k}-}\equiv\gamma_-$. In addition, to find the threshold value of n_0 we linearize (10) with respect to n_+, n_- and n_e. Then Eq. (10) take the form

$$\frac{d}{dt}n_+=\beta(n_0n_e-n_+n_0)-\gamma_+n_0, \tag{11a}$$

$$\frac{d}{dt}n_-=\beta(n_0n_e-n_-n_0)-\gamma_-n_-, \tag{11b}$$

$$\frac{d}{dt}n_e=\beta\rho(n_+n_0+n_-n_0)-\gamma_en_e. \tag{11c}$$

Here

$$\beta=\frac{V\Phi^2\eta^2\omega_e^2}{\pi^2\hbar^2v_{\mathbf{k}}^3},\quad \rho=\frac{k_0^2}{2\pi^2}\frac{V}{v_{\mathbf{k}}\tau_{\mathbf{k}}},$$

with ρ the number of wave states near the surface $\omega_{\mathbf{k}}=\omega_{p1}/2$ in a frequency interval of width $1/\tau_{\mathbf{k}}$, and V is the volume of the sample.

We seek the solution of (11) in the form $n_+, n_-, n_e\propto e^{\nu t}$. The threshold value $n_0=n_0^*$ is determined from the condition $\nu=0$. Under the conditions of our experiment, when $\gamma_+\approx\gamma_-\gg\gamma_e$ and $\rho\gg 1$ we obtain

$$(n_0^*)^2=\gamma_{\mathbf{k}}\gamma_e/2\beta^2\rho. \tag{12}$$

The amplitude of the three-wave interaction with participation of two magnons and a phonon, for antiferromagnets with easy-plane anisotropy was calculated in Ref. 18:

$$\Phi^e_{1,2}=i\Theta\left(\frac{\hbar\omega_e}{2Mc^2}\right)^{1/2}\frac{J_0}{(\hbar\omega_1\hbar\omega_2)^{1/2}}\left(\frac{v_0}{V}\right)^{1/2}; \tag{13}$$

here Θ is the characteristic value of the magnetoelastic energy (for $MnCO_3$, according to the data of Ref. 19, $\Theta\approx 5$ K); $J_0\approx 30$ K is the exchange energy, M and v_0 are the mass and volume of the unit cell, and c is the speed of sound ($Mc^2=2\cdot 10^5$ K). Using the presented numerical values, we obtain for $V=v_0$ an amplitude $\Phi\approx 6\cdot 10^{-3}$ K. Then, according to (11) we have for our sample $n_0^*\approx 10^9$, which is larger by one order of magnitude than the value of n_0^* given in Sec. 3 and based on the experimental data. Taking into account the qualitative character of the calculation presented here, the agreement between these quantities can be regarded as good.

We have also calculated the threshold value of n_0^* for a situation with coherent waves under the same assumptions as above. To this end, following Ref. 4, we wrote down equations for the amplitudes of the SW +, SW −, and the elastic oscillation. Just as in the preceding calculation, the answer is determined by the amplitude $\Phi^e_{1,2}$. In this case we obtain for the integral number of PSW at the sound-excitation threshold

$$N^*=\frac{k_0^2V}{2\pi^2\tau_{\mathbf{k}}v_{\mathbf{k}}}n_0^*=\frac{\hbar^2}{\tau_e\Phi^2}\left(\frac{1}{\tau_+}-\frac{1}{\tau_-}\right). \tag{14}$$

Under the conditions of our experiment, calculation by formula (14) yields a value $n_0^*\approx 10^5$, i.e., in the case of coherent waves the phonon generation should have started immediately past the PSW excitation threshold.

This circumstance can explain why the generation of elastic oscillations in ferrites was not observed at large k. In parametric excitation of spin waves with small k, the initial PSW system is coherent and, consequently, the threshold number of PSW needed for sound generation is small. If the PSW were to remain coherent at large values of k, n_0^* would grow by approximately one order of magnitude on account on the decrease of η, but generation of sound could still be observed under the conditions of the same experiments. However, the transition to the incoherent wave system increases the value of n_0^* by several orders of magnitude.

The presented calculation makes it possible to estimate the amplitude of the acoustic oscillations produced without a threshold under the action of two pumps. Indeed, in a stationary state in the presence of artificially maintained nonzero occupation numbers n_+ and n_0 we obtain from (11c)

$$n_e=n_+n_0\beta\rho/\gamma_e. \tag{15}$$

A similar formula holds also for excitation of SW −. We estimate now n_e at $n_0\approx n_0^*$. Then, using (12), we obtain

$$n_e\approx n_+[(\gamma_{\mathbf{k}}/\gamma_e)\rho]^{1/2}. \tag{15a}$$

This yields for the energy of the elastic oscillation in the nonthreshold regime a value $5\cdot 10^{17}$ J, which agrees in order of magnitude with the observed value of the energy of the elastic oscillations generated in the nonthreshold regime at $h\approx h_e$. We note that Eqs. (15) and (15a) do not contain the amplitude $\Phi^e_{1,2}$. These arguments can also be reversed: by measuring the elastic oscillation energy in the nonthreshold generation regime and by using relation (15) we can estimate the value β and then, using (12), approximately predict the threshold occupation numbers n_0^*. Obvious, in this case we obtain a qualitative agreement with the experimentally observed value $n_0^*\approx 10^8$, since we have demonstrated above that the experimental data are in aggrement if the inverse chain of reasoning is followed. Thus, we have performed the calculations with the aid of which we planned to verify that the amplitude of the interaction (5) is sufficient to generate sound when a given number of PSW is excited.

6. ANALOGY WITH MASER

In the experiments described above we generated coherent elastic oscillations in transitions between crystal states that differ in the number of the spin waves with wave vectors $\mathbf{k}_0$, $\mathbf{k}_+$, and $\mathbf{k}_-$. These transitions are stimulated, as is attested by the small spectral width of the generated oscillations. This width is less than 1% of the width of the resonance curves of the elastic oscillation and of the spin wave, as well as of the width of the spectral interval of the microwave pump. The stimulated character of the transitions is due to the fact that the number of transitions per unit time depends essentially on n_e [see Eqs. (10) and (11)].

For the onset of coherent generation it is necessary to produce in the sample an essentially nonequilibrium distribution of the occupation numbers of the spin waves, namely, inversion of the occupation numbers of the spin waves with spin vectors $\mathbf{k}_0$ and $\mathbf{k}_-$.

These circumstances indicate that the observed effect is similar to maser generation of electromagnetic and sound waves. The difference between the effect observed by us and an ordinary maser is that transitions are used between ground and excited states not of individual atoms or ions, but of the entire crystal. In addition, the spin system has a quasicontinuous excitation spectrum, in contrast to the discrete spectra of the atoms. The maser described by us, besides generation of sound, produces spin waves SW + and SW −. Just as an ordinary maser, the crystal of an antiferromagnet in which PSW are excited can operate as a sound amplifier (see Sec. 3).

The possibility of realizing one of the variants of the maser effect on collective excitations in an antiferromagnet was discussed in Ref. 20.

7. INTERACTION OF SOUND WITH COLLECTIVE PSW OSCILLATIONS

Investigating the PSW reaction to excitation of sound waves in a sample with the aid of a piezoceramic plate, we have observed that at sufficiently large sound amplitude, in a certain interval of the pump fields h (in the subthreshold region for sound generation) the absorbed microwave power executes oscillations not only with the frequency of the sound ω_s, but also with half the frequency, i.e., $\omega_s/2$. Oscillations with half the frequency have a threshold with respect to the sound amplitude. Consequently, we observe here one more parametric process.

It turned out that the threshold amplitude of the sound is a minimum at that microwave power when the frequency of the collective oscillations of the PSW is approximately equal to $\omega_s/2$. The collective oscillations of the PSW in the above-threshold state are oscillations of the number and phase $\psi_\mathbf{k}$ of the PSW relative to their equilibrium values at the given pump level. The spectrum of such oscillations is described in Ref. 4 and they were observed experimentally and investigated in antiferromagnets in Ref. 6. These oscillations were excited by impact via rapid change of the pump phase and manifested themselves in the form of damped oscillations of the absorbed power. Starting from the indicated agreement, we have assumed that the sound wave excites parametrically collective oscillations of PSW, which manifest themselves as subharmonics in the oscillations of the absorbed power.

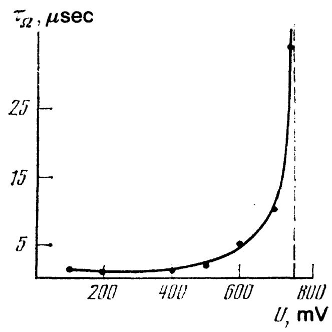

FIG. 10. Lifetime of collective oscillations of frequency $\Omega = 1.23$ MHz vs amplitude of ac voltage of frequency 2Ω on the piezosensor.

For an exact clarification of this assumption, we investigated the dependence of the lifetime of the collective oscillations on the amplitude of the sound wave. Collective oscillations were excited, as in Ref. 6, by impact. It turned out that their characteristic damping time τ_Ω increases substantially with increasing sound amplitude and becomes infinite at the threshold of the onset of the absorbed-power oscillation with frequency $\omega_s/2$. This increase takes place only at a definite value of the phase of the elastic oscillation at the instant of the impact excitation. Consequently, if the pulses that excite the collective oscillations are periodic, a substantial scatter in the envelopes of the collective oscillations is observed. The values of τ_Ω were measured at the extreme positions of the envelopes corresponding to the favorable phase of the sound oscillations. The dependence of τ_Ω on the amplitude of the alternating voltage on the piezo-sensor is shown in Fig. 10.

We have thus shown that a sound wave in which the amplitude of the relative deformation is 10^{-8} excites parametrically collective oscillations of PSW. The parametric excitation of the collective oscillation by sound can be explained in the following manner. It is known that, because of the magnetoelastic interaction, the mechanical stresses change the spectrum of the spin waves (see, e.g., Ref. 21). Then, in the field of the sound wave, the natural frequency of the PSW will oscillate with a frequency ω_s, thus causing oscillations in the number and phase of the PSW with the same frequency. This can be traced with the aid of the equations that describe the evolution of $N_\mathbf{k}$ and of $\psi_\mathbf{k}$ [Eqs. (3.6) in Ref. 4]. In addition, the oscillations in the PSW system, with frequency ω_s, are seen directly on the oscillogram of the transmitted microwave power. Since the frequency Ω of the collective oscillations is proportional to $N_\mathbf{k}$, Ω will oscillate at a frequency ω_s about a certain mean value. Thus, the standard situation in parametric excitation is produced—the natural frequency of the oscillatory system is modulated harmonically. When the modulation frequency equals double the natural frequency, parametric resonance sets in.

Parametric excitation of collective oscillations was observed earlier[22] under the action of an alternating magnetic field on the sample.

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Note added in proof (27 April 1983). The proof that generation of elastic oscillations is due to interaction of the type (5), given previously, can be given also in a more lucid form. As seen from Figs. 9a and 9b (curve 3), excitation of SW + or SW − (and only of these waves) leads to sound generation when $h < h_e'$ for the main pump, and its amplitude is of the same order as in the basic experiment, i.e., under the action of one pump with $h > h_e$. This generation is explained in the analysis of processes of type (5) with participation of PSW and SW +, or PSW and SW −, in the following manner. According to Eqs. (10) and (11), part of the energy flux into oscillations of the elastic mode is proportional to $n_{\mathbf{k}+} n_{\mathbf{k}0} + n_{\mathbf{k}-} n_{\mathbf{k}0}$. This quantity is increased by several orders of magnitude upon excitation of SW + or SW −. It was shown in Sec. 4 that under conditions of the basic experiment, at $h > h_e$, the SW + waves, and apparently also SW −, occur spontaneously, and approximately in the same amount when they are excited by the additional pump. Since simultaneous presence of PSW and SW + or of PSW and SW − in the sample leads to generation of sound, it follows that in the basic experiment also the generation of elastic oscillations is due primarily to interaction of PSW with SW + and PSW with SW −, in accordance with the scheme (5).

[1)]Oscillograms of the transmitted microwave power near the instant of turning on the pump with different values of sin $\varphi_{\mathbf{k}}$ are shown in Figs. 4 and 5 of Ref. 6.

[1]B. Ya. Kotyuzhanskiĭ and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **81**, 1913 (1981) [Sov. Phys. JETP **54**, 1013 (1981)].
[2]R. Ya. Ikola, J. Appl. Phys. **36**, 3260 (1965).
[3]Ya. A. Monosov, V. V. Surin, and V. I. Shcheglov, Pis'ma Zh. Eksp. Teor. Fiz. **7**, 315 (1968) [JETP Lett. **7**, 247 (1968)].
[4]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].
[5]V. V. Kveder, B. Ya. Kotyuzhanskiĭ, and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **63**, 2205 (1972) [Sov. Phys. JETP **47**, 1165 (1973)].
[6]L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teor. Fiz. **67**, 1952 (1974) [Sov. Phys. JETP **40**, 970 (1975)].
[7]A. S. Borovik-Romanov, Zh. Eksp. Teor. Fiz. **36**, 76 (1959) [Sov. Phys. JETP **9**, 54 (1959)].
[8]V. I. Ozhogin, Zh. Eksp. Teor. Fiz. **58**, 2079 (1970) [Sov. Phys. JETP **31**, 1121 (1970)].
[9]A. I. Smirnov, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 126 (1978) [JETP Lett. **27**, 117 (1978)].
[10]G. A. Melkov and V. L. Grankin, Zh. Eksp. Teor. Fiz. **69**, 1415 (1975) [Sov. Phys. JETP **42**, 721 (1975)].
[11]H. Yamazaki, J. Phys. Soc. Jpn. **37**, 667 (1974).
[12]L. A. Prozorova and A. I. Smirnov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 148 (1976) [JETP Lett. **23**, 130 (1976)].
[13]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, Spin Waves, Wiley, 1968 [Russ. Orig. Nauka, 1967, p. 254).
[14]G. Tomas and S. Wang, J. Appl. Lett. **38**, 2198 (1967).
[15]V. P. Lukomskiĭ and A. V. Kuz'ko, Fiz. Tverd. Tela (Leningrad) **11**, 1295 (1969) [*sic*].
[16]A. V. Lavrinenko, V. S. L'vov, G. A. Melkov, and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **81**, 1022 (1981) [Sov. Phys. JETP **54**, 542 (1981)].
[17]G. E. Fal'kovich, Preprint, Inst. Atomation and Electrometry, Siberian Div. USSR Acad. Sci., No. 172, 1983.
[18]V. S. Lutovinov, Fiz. Tverd. Tela (Leningrad) **20**, 1807 (1978) [Sov. Phys. Solid State **20**, 1044 (1978)].
[19]V. R. Gakel', Zh. Eksp. Teor. Fiz. **67**, 1827 (1974) [Sov. Phys. JETP **40**, 908 (1975)].
[20]O. A. Ol'khov and V. S. Lutovinov, Fiz. Tverd. Tela (Leningrad) **19**, 2282 (1977) [Sov. Phys. Solid State **19**, 1336 (1977)].
[21]A. S. Borovik-Romanov and E. G. Rudashevskiĭ, Zh. Eksp. Teor. Fiz. **47**, 2095 (1964) [Sov. Phys. JETP **20**, 1407 (1965)].
[22]V. V. Zautkin, V. S. L'vov, B. I. Orel', and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **72**, 272 (1977) [Sov. Phys. JETP **45**, 143 (1977)].

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