Radiative transitions between highly excited atomic states in the presence of a strong microwave field

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The wave functions for highly excited states of atoms located in a strong radiation field whose frequency ω is lower than or of the order of the level spacing is found in a quasiclassical approximation. The wave functions obtained are used to calculate the probabilities W_k of radiative transitions that result in absorption of a photon of frequency Ω ($\Omega \gg \omega$) and of k photons of frequency ω. The calculated W_k agree satisfactorily with the measured dependences of W_k on k, on the principal quantum number n of the final state, and on the intensity and frequency of the microwave field.

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§1. INTRODUCTION

The excitation of hydrogen atoms from a level with principal quantum number $n_i = 10$ to a level with n_f of the order of 50 by an infrared laser in the presence of a strong microwave field was recently[1] investigated. Besides the principal resonance corresponding to a transition from n_i to n_f there was observed a large number of satellite resonances corresponding to absorption of a photon from the infrared field simultaneously with the absorption (emission) of k photons of the microwave field. The probability of excitation at various k was investigated as a function of nf the intensity and frequency of the microwave field.

To calculate the probabilities of such radiation transitions it is necessary to find the wave function of the highly excited states of a hydrogen atom in a strong low-frequency field. The search for such a wave function was first undertaken by Blokhintsev.[2] He found that the basic action of the low-frequency field on the degenerate hydrogen states reduces to a change of the time-dependent part of the wave function, and that the function takes the form

$$\Psi(\mathbf{r}, t) = \varphi_{nn_1n_2}(\mathbf{r}) \exp\left\{it/2n^2 - i\frac{3F_0 n(n_1-n_2)}{2\omega}\sin\omega t\right\}, \quad (1)$$

where $\varphi_{nn_1n_2}(\mathbf{r})$ is the coordinate part of the wave function of the hydrogen atom in parabolic coordinates[3]; n, n_1, and n_2 are the parabolic quantum numbers; F_0 and ω are the amplitude and frequency of the alternating field. It can be seen that the only difference from the unperturbed wave function lies in the appearance of a sine term in the exponential, preceded by a factor comprising the ratio of the Stark energy to the photon energy. The wave function (1) was obtained in the basis of the wave function of one shell, and can therefore be used at frequencies ω much lower than the distances between the levels. Allowance for the influence exerted on the wave function by states of other shells is based in this method on perturbation theory.

In Ref. 4 were calculated the probabilities of the radiative transitions between states described by the wave function (1) under the influence of a second field of frequency Ω. It is typical that the probability of such radiative transitions with absorption of one photon of frequency Ω and of k photons of frequency ω is proportional to the square of a Bessel function whose argument is equal to the factor preceding the sine function in (1). In Ref. 4, however, was considered only a transition from the ground state to the first excited level. In a recent study[5] undertaken to explain the experiment of Ref. 1, an expression differing somewhat from that obtained in Refs. 2 and 4 was obtained for the probability of exciting the k th satellite. The excitation probability on the basis of this expression oscillates strongly with change of field intensity, whereas in experiment there is observed only one peak, followed by a plateau. In Ref. 5 only one parabolic state of the final level was taken into account. We note that a wave function of the form (1) was considered also in Refs. 6 and 7 to find the spectrum of atoms and in Ref. 8 to find the probabilities of their tunnel ionization in a low-frequency field.

In the present paper the wave function of highly excited states of atoms in a strong low-frequency field is obtained in the basis of a quasiclassical approximation. In the case of multiphoton ionization, quasiclassical perturbation theory[9] turned out to be accurate enough even at small n. The wave function obtained is valid in a larger interval of the frequencies ω of the alternating field than the function (1). The wave function obtained was used in §3 to calculate the probabilities of the radiative transitions of frequency Ω as applied to the experimental conditions of Ref. 1.

§2. WAVE FUNCTION OF HIGHLY EXCITED STATES OF ATOMS IN A STRONG HIGH-FREQUENCY FIELD

We use as the basis of the quasiclassical theory developed in Refs. 9 and 10 for the description of an electron moving simultaneously in a Coulomb field and in a periodic radiation field. The wave function for the electron is sought in the form

$$\Psi = \sum_{N=-\infty}^{\infty} \sum_{l=|M|}^{\infty} \frac{f_{Nl}(r)}{r} Y_{lM}(\vartheta, \varphi) \exp\{-it(E+N\omega)\}, \quad (2)$$

$$f_{Nl}(r) = \frac{1}{2ik_{Nl}^{1/2}(r)}\{a_{Nl}^{+}(r)\exp[i(S_{Nl}(r)+\pi/4)] - a_{Nl}^{-}(r)\exp[-i(S_{Nl}(r)+\pi/4)]\}, \quad (3)$$

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$$S_{Nl}(r)=\int_{r_1}^{r} dr\, k_{Nl}(r), \tag{4}$$

$$k_{Nl}^2(r)=2\left[E+N\omega+\frac{1}{r}-\frac{(l+1/2)^2}{2r^2}\right], \tag{5}$$

where E is the electron energy, Y_{lM} are spherical functions; M is the magnetic quantum number which is a conserved quantity for the linear field polarization case considered here.

The quasiclassical approximation is valid when the action S_{Nl} is large. It becomes greatly simplified if it is possible to separate from among the quantities S_{Nl} in all the channels that make a noticeable contribution to the considered process a large term S_0 that is common at all N and l. Physically, separation of such a term means that it is possible to separate in the electron motion the classical motion along a certain trajectory in the Coulomb field (this motion predominates). In this case one can neglect the influence of the alternating field on the classical motion of the electron, but account must be taken of the transitions, due to the alternating field, between the states with different N and l. It follows from (5) that separation of such a general motion is possible if two conditions are satisfied. First, the orbital momenta l in all the channels must differ insignificantly from a certain mean value $L \gg 1$. Second, either $E + N\omega$ must be considerably less than the last two terms in (5), meaning motion along a parabola, or $|N|\omega$ is considered smaller than $|E|$, meaning motion along an ellipse. In both cases

$$S_{Nl}\approx S_0+N\omega\tau-m\varphi(\tau), \tag{6}$$

$$m=l-L\ll L, \tag{7}$$

and for the slowly varying amplitudes we obtain the following simple equations[9]:

$$i\frac{da_{Nm}(\tau)}{d\tau}=-\frac{F_0 r(\tau)}{4}(1-M^2/L^2)^{1/2}\sum \exp[i(N'-N)\omega\tau + i(m-m')\varphi(\tau)]a_{N'm'}(\tau). \tag{8}$$

Here τ and $\varphi(\tau)$ are the classical time and angle, while $a_{Nm}(\tau)$ is equal to $a_{Nm}{}^{+}(\tau)$ and $a_{Nm}{}^{-}(\tau)$ at $\tau>0$ and $\tau<0$, respectively. The summation in (8) is over all $N'=N\pm1$ and $m'=m\pm1$. In the case of multiphoton ionization it was shown in Ref. 9 that under the condition $\omega \gg n^{-3}$ it suffices to consider motion along a parabola. At ω of the order of or less than n^{-3} the energy in all the channels differs insignificantly from E, and we must consider motion along an ellipse, defined by the following parametric equations[11]:

$$r=\nu_0^2(1-\varepsilon\cos u),\quad \tau=\nu_0^3(u-\varepsilon\sin u),$$
$$\varphi=2\operatorname{arctg}\left[\left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{1/2}\operatorname{tg}\frac{u}{2}\right],\quad \varepsilon=\left(1-\frac{L^2}{\nu_0^2}\right)^{1/2}, \tag{9}$$
$$\nu_0=(-2E)^{-1/2},\quad -\pi\leqslant u\leqslant\pi.$$

To ensure the correct behavior of the functions $f_{Nl}(r)$ in the classically forbidden regions, the following condition must be imposed on $a_{Nm}(u)$:

$$a_{Nm}(-\pi)\exp(-2i\pi\nu_N)=a_{Nm}(\pi), \tag{10}$$

$$\nu_N=(-2E-2N\omega)^{-1/2}. \tag{11}$$

The solution of such first-order differential equations as (8), in which the matrix elements depend only on the quantum-number differences $N-N'$ and $m-m'$, can be obtained with the aid of a generating function.[12,13] The general solution of the system (8) takes the form

$$a_{Ns}(u)=R\sum_{N's'=-\infty}^{\infty} b_{N's'}J_{N-N'+s-s'}(h_-(u))J_{s'-s}(h_+(u)) \times\exp[i\gamma_-(u)(N-N'+s-s')+i\gamma_+(u)(s'-s)], \tag{12}$$

$$h_\pm(u)=[c_\pm^2(u)+d_\pm^2(u)]^{1/2},\quad \operatorname{tg}\gamma_\pm(u)=c_\pm(u)/d_\pm(u), \tag{13}$$

$$c_\pm(u)=\frac{F_0\nu_0^3}{2}\left(1-\frac{M^2}{L^2}\right)^{1/2} \times\int_{-\pi}^{u} du'r(u')(1-\varepsilon\cos u')\cos[\omega\tau(u')\pm\varphi(u')], \tag{14}$$

$$d_\pm(u)=\frac{F_0\nu_0^3}{2}\left(1-\frac{M^2}{L^2}\right)^{1/2} \times\int_{-\pi}^{u} du'r(u')(1-\varepsilon\cos u')\sin[\omega\tau(u')\pm\varphi(u')],$$

$$m=N+2s. \tag{15}$$

Here R is a normalization factor, and the arbitrary constants b_{Ns} must be determined from the boundary condition (10). Since $h_\pm(-\pi)=0$ and Bessel functions of zero argument differ from zero only if the index is zero, it follows from (12) that

$$a_{Ns}(-\pi)=Rb_{Ns}. \tag{16}$$

Substituting (12) and (16) in (10) and recognizing that $d_\pm(\Pi)=0$, we obtain the following infinite system of algebraic equations:

$$b_{Ns}\exp[-2i\pi(\nu_0+N\omega\nu_0^3)] = \sum_{N',s'=-\infty}^{\infty} i^{N-N'}J_{N-N'+s-s'}(c_-(\pi))J_{s'-s}(c_+(\pi))b_{N's'}. \tag{17}$$

Since $N\omega$ is assumed small compared with $|E|$, the quantities ν_N in (17) are replaced by the first terms of the series in $N\omega$.

Assume that the system (17) can be satisfied if b_{Ns} is chosen in the form

$$b_{Ns}=J_{N+s}(g_-)J_s(g_+)\exp[i(N\varphi_1+s\varphi_2)]. \tag{18}$$

The constants g^-, g^+, φ_1, φ_2, and ν_0 should be so defined that the system (17) turns into an identity. Substituting (18) in (17) and using the addition theorem for Bessel functions,[14] we find that the system is indeed satisfied if

$$\nu_0=n,\quad \varphi_1=\pi+\pi\omega n^3,\quad \varphi_2=\pi,$$
$$g_\pm=c_\pm(\pi)/2\sin\pi\omega n^3. \tag{19}$$

Here n is an integer and according to the definition (9) it corresponds to the principal quantum number.

Substituting (18) in (12) and using again the addition theorem for Bessel functions, we obtain ultimately

$$a_{Ns}(u)=RJ_{N+s}(w_-(u))J_s(w_+(u))$$
$$\times\exp\{iN[\gamma_-(u)+\psi_-(u)]$$
$$+is[\gamma_-(u)-\gamma_+(u)+\psi_-(u)-\psi_+(u)+\pi]\}, \quad (20)$$

$$w_\pm(u)=\{h_\pm^2(u)+g_\pm^2-2h_\pm(u)g_\pm\cos[\gamma_\pm(u)-\pi\omega n^3]\}^{1/2},$$

$$g_\pm\sin[\gamma_\pm(u)-\pi\omega n^3]=w_\pm\sin\psi_\pm. \quad (21)$$

Expressions (2), (3), and (20) determine the sought quasiclassical wave function of a highly excited state with large quantum numbers n, L, and M in a radiation field. Although neither the energy nor the orbital quantum number is preserved in the presence of a field, they remain on the average the same as without a field. It follows from (19) that in our approximation the level with the quantum number n neither shifts nor splits in the presence of a field, and only quasienergy states are produced, separated from the given level by $N\omega$. In the same manner, the field mixes in states with other orbital quantum number into the given state. The admixture of the quasienergy states and of states with other orbital quantum numbers depends substantially on the radiation-field intensity. The fact that in our treatment the level neither shifts nor splits is a consequence of the approximation of equidistant levels.

The derivation of the wave function obtained above is based on the assumption that the field frequency ω is lower than or of the order of the distance between levels. As can be seen from (19), at frequencies ω that are multiples of n^{-3}, i.e., at resonance, it is apparently impossible to use the obtained wave function, since rapid excitation of the atom takes place. As a result it suffices to confine ourself to the equidistant-level approximation used in the derivation of Eqs. (17), and it is necessary to take into account the succeeding terms of the expansion of ν_N in terms of $N\omega$. Actually there is a certain upper bound on the field, but this bound is difficult to determine, since it depends on the proximity to resonance. In practice it is convenient to formulate this bound in the form of the following requirement: it is necessary that those effective N_{eff} that make a substantial contribution to the wave function (2) be such that ωN_{eff} remain in the region where the levels are equidistant. The closer the frequency ω to resonance, the weaker the fields at which this requirement is violated.

If we use the expansion (6) and the asymptotic representation for the Legendre functions at large l, the sums over l and N in (2) can be convoluted with the aid of the addition theorem for the Bessel functions, and ψ_{nLM} can be expressed in terms of elementary functions. But since the arguments of these functions have a very complicate form and the variables in them are entangled, it is convenient not to do so in the calculations that follow. It is easy to find the normalization function: it coincides with the unperturbed value $(2\pi n^3)^{1/2}$. We note that the wave function obtained does not go over into the function (1) at $\omega n^3 \ll 1$, for in contrast to (1) our wave function was obtained in spherical coordinates and in a quasiclassical approximation.

§3. PROBABILITIES OF RADIATIVE TRANSITIONS IN THE PRESENCE OF A STRONG MICROWAVE FIELD

We calculate now the probabilities of radiative transitions between the states obtained in the preceding section, as a result of the action of a field with frequency Ω and unit polarization vector $\mathbf{e}$ whose direction relative to the z axis (the direction of the polarization of the field of frequency ω) is given by the polar angles ϑ_Ω and φ_Ω. The probability of this transition is determined by the matrix element

$$T=-i\int_0^t dt\langle\Psi^{M_f}_{n_fL_f}|(\mathbf{er})e^{-i\Omega t}|\Psi^{M_i}_{n_iL_i}\rangle, \quad (22)$$

where the indices i and f pertain respectively to the initial and final states. Substituting (2) in (22), carrying out a simple integration with respect to the angle variables,[3] and using time-dependent perturbation theory, we find that the transition probability per unit time, summed over M_f, is

$$dW=\sum_{k=-\infty}^{\infty}\delta(1/2n_i^2-1/2n_f^2-\Omega+k\omega)W_k d\Omega, \quad (23)$$

$$W_k=2\pi^2\alpha I_\Omega\{(3\cos^2\vartheta_\Omega-1)(1-M_i^2/L_i^2)|R_k^+ + R_k^-|^2$$
$$+2\sin^2\vartheta_\Omega(|R_k^+|^2+|R_k^-|^2)\}, \quad (24)$$

$$R_k^\pm=\sum_{N_il_i}\int drrf^*_{N_i+k,l_i\pm1}f_{N_il_i}, \quad (25)$$

where I_Ω is the intensity of the field of frequency Ω, and α is the fine-structure constant. Since M_f and L_f differ little from the corresponding M_i and L_i, whereas these quantities themselves are large, the factor, $1-M_f^2/L_f^2$ was replaced everywhere in (24) and (25) by $1-M_i^2/L_i^2$. Expression (23) has a structure typical of multiphoton processes. The presence of a δ function in (23) means that the transitions occur at frequencies equal to the energy difference between the final and initial states of the atom plus $k\omega$.

We calculate now the radial integrals (25). Inasmuch as the experiment[1] dealt with radiative transitions between states widely spaced in energy ($n_i = 10$ and n_f of the order of 50) we continue the analysis subject to the following restriction on the frequency Ω:

$$\Omega \gg n_i^{-3}. \quad (26)$$

When this condition is satisfied, the calculation of the radial integrals is greatly simplified, since a noticeable contribution to the transition probability between states widely separated in energy is made only by states with $L \ll n$ ($\varepsilon \sim 1$), and by the region of variation of r near the turning point closest to the nucleus.[9,16] If ε is replaced by $1-L_i^2/2n_i^2$, a new variable $v = un_i/L_i$ is introduced, and account is taken of (3), (6), and (9), we have in this case

$$R_k^\pm=\frac{1}{4}\sum_{N_il_i}\int d\tau' r(\tau')a_{N_il_i}(v)a_{N_i+k,l_i\pm1}(v)$$
$$\times\exp[-i\Omega\tau'\pm i\varphi(\tau')], \quad (27)$$

$$r=L_i^2(1+v^2)/2, \quad \tau'=L_i^3(v+v^3/3)/2, \quad \varphi=2\operatorname{arctg} v. \quad (28)$$

The amplitudes $a_{Nl}(\nu)$ in the integral (27) also depend on the variable ν, but at small u and at $L \ll n$ this dependence can be

neglected. Indeed, in this case the functions (14) take at $\omega = 0$ the form

$$c_{\pm}(v, \omega=0) = -\frac{3\pi F_0 n^5}{4}\left(1-\frac{M^2}{L^2}\right)^{1/2}\left[1-\left(\frac{L^2}{2n^2}+\frac{L^5}{6\pi n^5}v\right)\right],$$
$$d_{\pm}(v, \omega=0) = \mp F_0 n^4 L\left(1-\frac{M^2}{L^2}\right)^{1/2}\left[1-\frac{L^4}{8n^4}v^2\right]. \quad (29)$$

It follows therefore that the second terms in the square brackets of these functions can be neglected both at small and at large values of the parameter ΩL^3 ($\Omega \gg n^{-3}$) which determines the value of the integral (27). At nonzero ω it is more difficult to obtain such an estimate, but it can be shown that if ωn^3 is less than or of the order of unity it is still possible to neglect in the integral of (27) the dependence of the amplitudes a_{Nl} on ν. As a result the amplitude $a_{N_i s_i}(0)$ takes the form

$$a_{N_i s_i}(0) = \left(\frac{2}{\pi n_i^3}\right)^{1/2} J_{N_i+s_i}\left(\frac{w_i}{2}\cos\beta\right) \times J_{s_i}\left(\frac{w_i}{2}\cos\beta\right)\exp\{i(2N_i\gamma_i+s_i\pi)\}, \quad (30)$$

$$w_i = c_i \operatorname{ctg} \pi\omega n_i^3 - d_i, \quad (31)$$

$$c_i = -F_0 n_i^5 \int_0^{\pi} du\,(1-\cos u)^2 \cos \omega n_i^3 (u-\sin u), \quad (32)$$

$$d_i = F_0 n_i^5 \int_0^{\pi} du\,(1-\cos u)^2 \sin \omega n_i^3 (u-\sin u),$$
$$\operatorname{tg}\gamma_i = c_i/d_i, \quad \sin\beta = M_i/L_i. \quad (33)$$

A similar expression holds for the amplitude $a_{n_f s_f}(0)$.

The integral (27) can now be expressed in terms of an Airy function and its derivative.[17] Using also the addition theorem, we obtain ultimately a representation for the radial integrals:

$$R_k^{+} = P_+ J_{k+\varkappa}\left(\frac{x}{2}\cos\beta\right) J_{\varkappa}\left(\frac{x}{2}\cos\beta\right)\exp[ik(\psi+2\gamma_f)+i\varkappa\pi], \quad (34)$$

$$x = [w_i^2 + w_f^2 - 2w_i w_f \cos 2(\gamma_i+\gamma_f)]^{1/2}, \quad (35)$$
$$w_i \sin 2(\gamma_i+\gamma_f) = x \sin\psi,$$

$$P_{\pm} = (n_i n_f)^{-3/2}(2^2/\Omega^5)^{1/3}[-\mathrm{Ai}'(y) \pm y^{1/2}\mathrm{Ai}(y)], \quad (36)$$

$$\varkappa = (L_i - L_f - k + 1)/2, \quad y = (\Omega L_i^3/2)^{2/3}. \quad (37)$$

The radial integral R_k^- is obtained from (34) by replacing $\varkappa$ by $\varkappa - 1$ and P_+ by P_-. $\varkappa$ takes on only integer values.

After substituting $R_k^{\pm}$ in (24) we must carry out a number of averagings and summations. First, summation over L_f, which reduces to summation over all the integer $\varkappa$. We assume next an equiprobable population of the initial states. The averaging of the obtained expression over all M_i can be replaced by integration with respect to β between $-\pi/2$ and $+\pi/2$, and averaging over L_i by integration with respect to y (Ref. 9). Integrations of the squares of the Airy function and of its derivative entail no difficulty, so that we obtain as a result the following expression for the probability of the radiative transition per unit time with absorption of k microwave photons:

$$\overline{W}_k = \frac{4\pi\alpha I_{\Omega}}{3^{1/2} n_i^5 n_f^3 \Omega^4}[(3\cos^2\vartheta_{\Omega}-1)F_k(x) + 2\sin^2\vartheta_{\Omega} G_k(x)], \quad (38)$$

$$F_k(x) = \sum_{\varkappa}\int_{-\pi/2}^{\pi/2} d\beta \cos^3\beta U_{\varkappa}^k (U_{\varkappa}^k - U_{\varkappa-1}^k/3), \quad (39)$$

$$G_k(x) = \sum_{\varkappa}\int_{-\pi/2}^{\pi/2} d\beta \cos\beta (U_{\varkappa}^k)^2, \quad (40)$$

$$U_{\varkappa}^k = J_{k+\varkappa}\left(\frac{x}{2}\cos\beta\right) J_{\varkappa}\left(\frac{x}{2}\cos\beta\right). \quad (41)$$

We shall consider next only the case $\vartheta_{\Omega} = 0$, i.e., the case when the polarizations of both fields coincide. The probability of radiative transition with absorption of k photons is then proportional to the function $F_k(x)$. If we use for the squares of Bessel functions the representation[14]

$$\pi J_k^2(z) = \int_0^{\pi} d\varphi J_0(2z\sin\varphi)\cos 2k\varphi \quad (42)$$

as well as the addition theorem, we can obtain after some transformations a relatively simple representation for the function $F_k(x)$:

$$F_k(x) = 2\int_0^1 dz\,(1/3 + z^2) J_k^2(xz). \quad (43)$$

This function has also the following asymptotic representation at large x ($x \gg |\mathbf{k}|$):

$$F_k(x) = [2\ln 2x - 2\psi(|k|+1/2) + 3 - (-1)^k\cdot 4\cos 2x/x + O(x^{-2})]/3\pi x, \quad (44)$$

where $\psi(z)$ is a psi function.[17]

As seen from (43), $F_k(x)$ are symmetric with respect to a change of the sign of k and x. The behavior of the functions $F_k(x)$ is the following. At small x and $|k| > 0$ they increase in proportion to $x^{2|k|}$, reach a maximum at a certain $x_{\max}(k)$, and then decrease. This decrease becomes very slow at large x. The larger $|k|$ the farther from zero the maximum of the corresponding $F_k(x)$. For $k = 1,2,7$ the functions $F_k(x)$ reach their maximum at $x_{\max}(k)$ equal respectively to 2.38, 3.9, 5.1, 6.3, 7.5, 8.7, and 9.9. Although the functions $F_k(x)$ oscillate past the maximum, as can be seen also from (44), these oscillations are relatively small.

The functions $F_k(x)$ depend on the intensity and frequency of the microwave field and on the principal quantum numbers n_i and n_f only via one dimensionless quantity x. This quantity is proportional to the field intensity F_0, but it follows from (35), (31), and (32) that its dependence on ω, n_i, and n_f is quite complicated. If $n_f^2 \gg n_i^2$, we have approximately

$$x = c_f \operatorname{ctg}\pi\omega n_f^3 - d_f, \quad (45)$$

where c_f and d_f are defined by the integrals in (32). We recall that our analysis becomes incorrect in the case of resonance, when ωn_f^3 is equal to an integer.

If $\omega n_f^3 \ll 1$, then

$$x = \frac{3F_0(n_i^2 - n_i^2)}{2\omega}\left[1 + \frac{35}{72}\omega^2 n_f^6\right]. \quad (46)$$

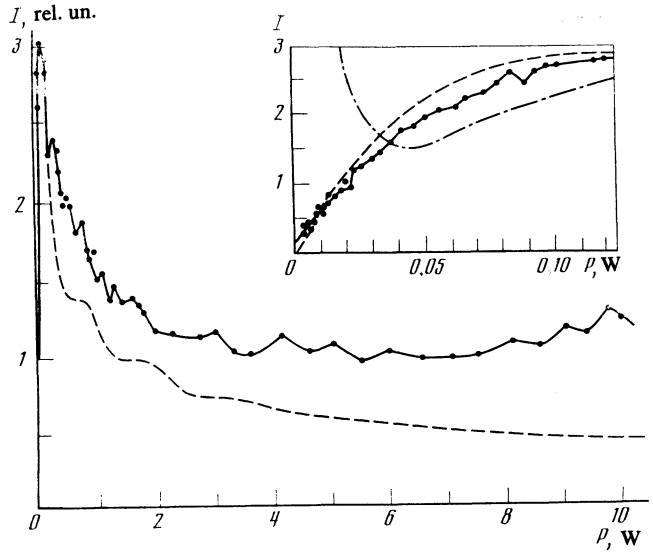

FIG. 1. Relative intensity I of satellite at $k = -1$ and $n = 44$ vs the power of a microwave source of frequency $\omega = 7.829$ GHz. The inset shows this dependence at low powers. Points and continuous curve—experiment[1]; dashed curve—the function (43) normalized the experimental curve at the maximum; dash-dot curve—asymptotic form (44) for the function (43) at high microwave-source powers.

It can be seen that the quantity in front of the square bracket in this expression is the difference between the ratios of the energies of the outermost Stark components for the final and initial states to the photon energy. The second term in the square brackets is the first correction in ωn_f^3.

In this limiting case of small ωn_f^3 it is possible to establish a connection with the Blokhintesev theory. As noted above, calculations with the aid of the function (1) make the intensity of the k th satellite proportional to the square of a Bessel function whose argument is the difference between the ratios of the Stark energies of the initial and final states to the photon energy. Taking the sum over all values of $n_1 - n_1$, which at large n can be replaced by an integral with respect to $z = |n_1 - n_2|/n$ from zero to unity, we arrive at an expression of the type (42). To find the factor preceding the square of the Bessel function it is necessary, however, to calculate the dipole matrix elements of the parabolic wave functions in a quasiclassical approximations for states greatly separated in energy, as was done above and in Refs. 9 and 16 in the case of spherical coordinates. No such quasi-classical estimate of the dipole matrix elements in parabolic coordinates has been obtained so far. Actually, at $\omega n_f^3 \ll 1$ there is no need for using the quasiclassical theory expounded above, and it is better to perform all the calculations with the aid of the function (1) which is more accurate in this case.

§4. COMPARISON WITH EXPERIMENT

In the experiment of Ref. 1, $n_i = 10$ and n_f is approximately 50, i.e., the initial and final states are far from each other. This justifies the approximations on which the calculations of the radial matrix elements in the preceding sections are based. Since ωn_f^3 does not exceed one-fifth in the experiment, the correction in the square brackets in (46) can be neglected. The expression for $F_k(x)$ was obtained under the assumption that the initial-state populations are equally probable. As noted by the authors of Ref. 1, this assumption is apparently not valid in their experiment. But since the true distribution is unknown, it makes sense nevertheless to compare the $F_k(x)$ obtained under this assumption with the measured transition intensities.

In Ref. 1 the curves are plotted not as functions of F_0 but as function of the microwave power P. It is noted there that a power 0.12 W corresponds to a peak field intensity close to 6 V/cm. Although satisfactory agreement with experiment is observed also in this respect, the agreement is somewhat better if the last number is chosen somewhat smaller. We use hereafter the value 5.23 V/cm.

In Fig. 1 are compared the resonance intensity measured at $k = -1$ and $n = 44$ with the function $F_1(x)$ normalized to the experimental curve at the maximum. Here and elsewhere n_f will be designated by n. It must be noted first of all that the function $F_1(x)$ has its maximum approximately at the same values of the power P as in the experiment. The falloff of the resonance intensity on both sides of the maximum is described with sufficient accuracy by the function $F_1(x)$. At high powers the experimental and theoretical curves diverge, inasmuch as in the experiment the intensity falloff slows down with increasing P, and above 7.5 W the intensity even increases. The cause of this discrepancy is still unclear. Nor does the theoretical curve show the experimentally ob-

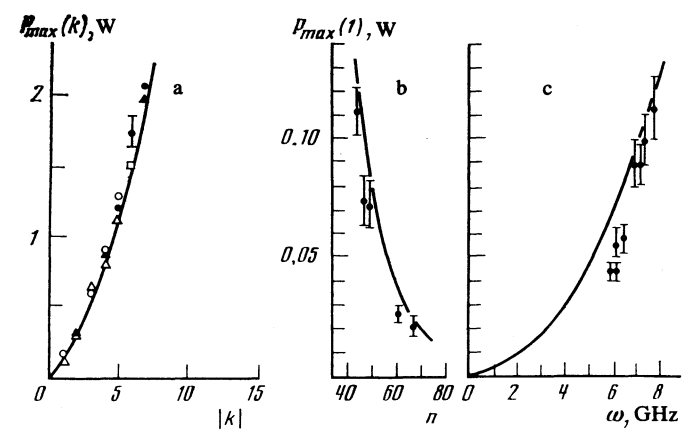

FIG. 2. a) Microwave-source power $P_{max}(k)$ corresponding to the maximum excitation of the k th satellite: ● (n = 43) and ○ ($n = 44$) – $k < 0$; △ ($n = 44$) and ▲ ($n = 45$) – $k > 0$; b) dependence of $P_{max}(1)$ on the principal quantum number n of the final state; c) dependence of $P_{max}(1)$ on the frequency ω of the microwave source. Points—experiment,[1] solid curves—Eq. (47).

tained small oscillations of the resonance intensities with change of P.

Knowing $x_{max}(k)$ we can find the power $P_{max}(k)$ corresponding to the maximum probability of excitation of the k th satellite:

$$P_{max}(k)=1.2\cdot 10^3\omega^2 x_{max}^2(k)/(n^2-n_i^2)^2. \quad (47)$$

Here P is in watts and ω in GHz. The plots in Fig. 2 are based on (47) and on the values of $x_{max}(k)$ given at the end of the last section. It can be seen that they duplicate well the experimental dependence of $P_{max}(k)$ on k, n, and ω. A possible exception is the dependence of $P_{max}(1)$ on ω, since experiment indicates that $P_{max}(1)$ is proportional to $\omega^{3\pm 0.5}$ and not to ω^2 as follows from (47). Although the correction in ωn^3 obtained above can raise the power of ω in the dependence of $P_{max}(1)$ on ω, in this experiment the correction is too small to require consideration. When the frequency is increased the dependence of $P_{max}(k)$ on the frequency is no longer quadratic and becomes more complicated, as follows from (45) and (32). It would be of undoubted interest to carry out experiments at frequencies ω that approach resonance.

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