Expulsion of a magnetic field by a hot-electron cloud

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The expulsion of a magnetic field by a hot-electron gas whose pressure is significantly higher than the pressure of the magnetic field is considered. It is shown that such an expulsion process in a highly conducting medium is accompanied by a reconstruction of the magnetic field over periods of time significantly shorter than the skin-penetration time. Nonlinear wave solutions are constructed for the case of a constant propagation velocity u that can be significantly lower than the velocity of the hot electrons.

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In the present paper we solve the problem of the expulsion, by a hot-electron gas, of a magnetic field H frozen in a conducting medium. We shall assume that the hot electrons are electrically neutralized because of the high conductivity of the medium. The freely moving electrons of a such a cloud then fly into the surrounding magnetic field and revolve in it along orbits with the Larmor radius. The diamagnetic current j' of the electrons of the cloud induces in the conducting medium, over a time τ significantly shorter than the skin time

$$\tau_s = 4\pi\sigma r^2/c^2$$

(σ is the conductivity of the medium and r is the characteristic dimension of the current region) a reverse current j, which cancels out the diamagnetic current of the hot electrons.[1,2] Since the reverse current $\mathbf{j} = \sigma\mathbf{E}$ is proportional to the induced electric field, this results in the lowering of the magnetic field strength H over a characteristic time

$$\tau \sim \tau_s/\beta \ll \tau_s, \qquad \beta = 8\pi p'/H^2 \gg 1$$

(p' is the pressure of the electron cloud). This means that the magnetic field is expelled by the hot-electron cloud.

The above-formulated problem differs essentially from the traditional formulation for magnetohydrodynamic shock waves and tangential discontinuities within the framework of magnetohydrodynamics[3] by the presence of an additional hot component.

In magnetohydrodynamics the pressure p of the medium can be counterbalanced by the magnetic-field pressure $H^2/8\pi$. In the problem under consideration this is impossible, since the cloud pressure $p' = n'\gamma m v'^2$ (n' is the hot-electron density, v' is the electron velocity, and γ is the relativistic factor) is significantly higher than the magnetic field presure: $p' \gg H^2/8\pi$. The possibility of the pressure of the hot-electron cloud being counterbalanced by the inertia of the medium then arises. The electromagnetic force $c^{-1}\mathbf{j}\times\mathbf{H}$ that arises in the conducting medium in the course of the expulsion of the magnetic field by the hot electrons as a result of the Hall effect produces an electric field, which transfers momentum from the cloud of hot electrons to the ions of the medium.

The problem under consideration turns out to be close in its formulation to the problem of the propagation of heat in a longitudinal magnetic field,[4] where the hot electron component, moving along the strong magnetic field, displaces the cold electron component, which leads to heat propagation. The front of such a wave is determined in this case by electric potential fields. The present formulation of the problem differs essentially from the formulation for the displacement wave in that in it the hot-electron cloud is restrained by the field.

The problem as formulated here, in which a hot-electron gas and a conducting medium are present, is, to use a more remote anlogy, somewhat similar to the problem of the interpenetration of a superconducting and a normal fluid in the theory of superconductivity.[5] The hot-electron gas, carrying current and transporting heat without hindrance, is analogous to the superconducting component, while the medium with a finite conductivity is analogous to the normal component. An important difference here lies in the character of the inertia: the superconducting component is characterized by the inertia of the electrons, while the motion of the cloud is inertialess.

Let us now proceed to derive the basic equations describing the wave of expulsion of a magnetic field by a hot-electron cloud.

The evolution of the magnetic field is described by the equation

$$\operatorname{rot}\mathbf{H} = 4\pi c^{-1}(\mathbf{j}'+\mathbf{j}),$$

where $\mathbf{j}'$ is the diamagnetic current of the hot electrons and $\mathbf{j}$ is the current carried by the conducting medium. The conducting medium is described by the hydrodynamic equations for the ions

$$M d\mathbf{V}/dt = Ze\mathbf{E} \tag{2}$$

and the inertialess equation for the electrons

$$0 = -e\mathbf{E} - \frac{e}{c}[\mathbf{v}\times\mathbf{H}] - \frac{\nabla p}{n} - m\nu_{ei}(\mathbf{v}-\mathbf{V}), \tag{3}$$

which has the meaning of Ohm's law. Here $\mathbf{V}$ and $\mathbf{v}$ are respectively the ion and electron velocities, M and m are the ion and electron masses respectively, ν_{ei} is the electron–ion

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collision rate; p is the electron pressure, and n is the electron density.

In Eq. (2) we have neglected the term with the magnetic field, assuming that the characteristic dimensions in the problem are significantly smaller than the ion Larmor radius. We assume the conducting medium to be quasineutral, and, in view of the fact that the density n' of the electron cloud is low, i.e., that $n' \ll n$, we set

$$Zn_i = n_e = n.$$

The variation of the temperature $T = p/n$ of the electron component in the course of the expulsion of the magnetic field is determined by the ohmic dissipation:

$$\frac{n}{\alpha-1}\frac{dT}{dt} = \frac{j^2}{\sigma(T)}, \tag{4}$$

where α is the effective adiabatic exponent. We have neglected in Eq. (4) the heat transfer because of the presence of the electronic thermal conduction. As will be shown below, this condition can be fulfilled for typical parameters of the problem.

We shall, in considering the wave of expulsion of a magnetic field by a hot-electron cloud, investigate the one-dimensional formulation in which at the initial instant of time the hot electrons occupy the half-space $x < 0$, while the magnetic field H_z occupies the half-space $x > 0$. We shall in this case seek the wave solutions propagating in the region of positive x, and depending only on the variable $s = x - ut$. Equation (1) allows us to estimate the characteristic magnitude of the expulsion rate u. Setting $j_y = \sigma E_y$, and using for the determination of E_y the induction equation

$$-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = \operatorname{rot}\mathbf{E}, \tag{5}$$

we obtain in the high-conductivity approximation, in which the reverse current of the medium cancels out the diamagnetic current $j_y' \approx \sigma E_y$, the following estimate for the expulsion rate:

$$u/c \sim j_y'/\sigma H_z. \tag{6}$$

Below we shall assume the following apoproximation for the conductivity σ:

$$\sigma = \sigma_0 T^k, \quad k > 0. \tag{7}$$

If the plasma electrons are not magnetized, the condition $\omega_{He} \ll \nu_{ei}$ ($\omega_{He} = eH/mc$ is the electron cyclotron frequency), together with (6), leads to $v_y \ll u$, where v_y is the current velocity of the electrons of the medium. This, with allowance for the fact that $nv_y \sim n'v'$, yields

$$u \gg v'n'/n. \tag{8}$$

Below we shall, in investigating the expulsion wave, neglect the motion of the ions, assuming their velocity V_x to be significantly lower than the wave velocity u. Then, equating the ion velocity to the sound velocity calculated from the pressure of the hot electrons, we obtain

$$u^2 \gg \frac{p'}{\rho} = Z\frac{n'}{n}\frac{\gamma m}{M}v'^2. \tag{9}$$

We shall, with the above assumptions taken into account, investigate the expulsion wave within the framework of the equations

$$j + j' = -\frac{c}{4\pi}\frac{d^2A}{ds^2}, \tag{10}$$

$$\frac{n}{\alpha-1}\frac{dT}{ds} = -\frac{1}{c}j\frac{dA}{ds}, \tag{11}$$

where

$$H_z = \frac{dA}{ds}, \quad j = \sigma\frac{u}{c}\frac{dA}{ds},$$

and A is the y component of the vector potential.

The diamagnetic current j' generated by the hot-electron cloud is equal to

$$j' = -e\int v_y' f\, d^3p, \tag{12}$$

where the distribution function f is found from the Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\nabla f - e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)\frac{\partial f}{\partial \mathbf{p}} = 0.$$

This equation has a solution that depends on $s = x - ut$,

$$f = f\left(\varepsilon - p_x u,\, p_y - \frac{e}{c}A,\, p_z\right). \tag{13}$$

In deriving (13) we neglected the small Hall electric field E_x.

In the one-dimensional formulation under consideration, the hot-electron cloud will be modeled by a flux of relativistic electrons that travel from $x = -\infty$ in the positive direction, turn around in the magnetic field H_z, and return. Let us, assuming A is equal to zero at $x = -\infty$, choose f in the form

$$f = F(\varepsilon - p_x u)\,\delta\left(p_y - \frac{e}{c}A\right)\delta(p_z). \tag{14}$$

Let the distribution function for the relativistic electrons injected at $x = -\infty$ in the positive x direction be equal to

$$f_0 = F_0(\varepsilon)\,\delta(p_y)\,\delta(p_z). \tag{15}$$

Then, as follows from (14), the resulting distribution function for the electron cloud in the region $x > -\infty$ will depend on u, which is due to the reflection of the initial function f_0 from the magnetic field moving with velocity u.

We can, bearing in mind the fact that the distribution function f is constant along each trajectory of the individual particles, derive the following expression for $F(\tilde{\varepsilon})$ from (14):

$$F(\tilde{\varepsilon}) = 2F_0[\gamma_u^2\tilde{\varepsilon} + (\gamma_u^2-1)^{1/2}(\tilde{\varepsilon}^2\gamma_u^2 - m^2c^4)^{1/2}], \tag{16}$$

where

$$\gamma_u = (1-u^2/c^2)^{-1/2}, \quad \tilde{\varepsilon} = \varepsilon - p_x u.$$

Below we shall consider the most interesting case, in which $u \ll c$ and the expulsion wave velocity is significantly lower than the characteristic velocity of the electrons of the cloud. In this case $F \cong 2F_0$.

Notice that the expulsion wave velocity should, when the foregoing limitations are taken into consideration, satisfy the following inequalities:

$$1 \gg \frac{u}{c} \gg \left(Z \frac{\gamma m}{M} \frac{n'}{n}\right)^{1/2}. \tag{17}$$

If we normalize the distribution function (14) by the condition

$$\int f\, d^3p|_{x=-\infty} = n_0',$$

then we can derive the following expression for the diamagnetic current of the relativistic-electron cloud:

$$j' = -2en_0'c \frac{(e/c)A[P_0^2-(eA/c)^2]^{1/2}}{m^2c^2\{\gamma_{\max}(\gamma_{\max}^2-1)^{1/2}+\ln[\gamma_{\max}+(\gamma_{\max}^2-1)^{1/2}]\}}, \tag{18}$$

where

$$P_0 = (\varepsilon_{\max}^2/c^2 - m^2c^2)^{1/2} \equiv mc(\gamma_{\max}^2-1)^{1/2}.$$

Here we have chosen the function $F_0(\varepsilon)$ in the form

$$F_0(\varepsilon) \sim \varepsilon[\theta(\varepsilon - mc^2) - \theta(\varepsilon - \varepsilon_{\max})], \quad \theta(x) = \begin{cases} 1 & x>0 \\ 0 & x<0 \end{cases},$$

which corresponds to electron energy in the interval $mc^2 < \varepsilon < \varepsilon_{\max}$. Such a choice of $F_0(\varepsilon)$ allows us to obtain a fairly simple final equation for the magnetic-field profile in the wave.

Let us now use the expression obtained above for the diamagnetic current of the relativistic-electron cloud, and substitute j from Eq. (10) into Eq. (11). Performing the integration over s, we obtain as a result an expression for the plasma temperature as a function of A and dA/ds:

$$T = T_0 + \frac{\alpha-1}{8\pi n}\left[\left(\frac{dA}{ds}\right)^2 - H_0^2\right] + \frac{2n_0'(\alpha-1)}{3m^2cn} \cdot \frac{[P_0^2-(eA/c)^2]^{3/2}}{\gamma_{\max}(\gamma_{\max}^2-1)^{1/2}+\ln[\gamma_{\max}+(\gamma_{\max}^2-1)^{1/2}]}. \tag{19}$$

Here T_0 and H_0 are the plasma temperature and the magnetic field at the boundary of the hot-electron cloud. To this boundary corresponds a magnetic-field flux per unit length along the y axis, $A_0 = cP_0/e$, that can cause the hot electrons to turn around.

Substituting the expression obtained for T into $\sigma(T)$, and using the expression $j = \sigma uc^{-1}dA/ds$ for the current, we obtain from (10) the following final equation for $a = eA/cP_0$:

$$\frac{2}{3\beta} h \frac{dh}{da} = a(1-a^2)^{1/2} - \lambda h\left[\tau_0 + \frac{1}{3\beta}(h^2 - h_0^2) + \frac{1}{3}(1-a^2)^{3/2}\right]^k, \tag{20}$$

where $h = H/H_*$, $h_0 = H_0/H_*$, $\tau_0 = T_0/T_*$,

$$\lambda = \frac{u}{c}\frac{H_*\sigma_*}{j_*}, \qquad \sigma_* = \sigma_0 T_*^k,$$

$$\gamma_0^2 - 1 \equiv \frac{(\gamma_{\max}^2-1)^{3/2}}{\gamma_{\max}(\gamma_{\max}^2-1)^{1/2}+\ln[\gamma_{\max}+(\gamma_{\max}^2-1)^{1/2}]}.$$

Here $\beta = 8\pi p_0'/H_*^2$, where $p_0' = \frac{2}{3}n_0'(\gamma_0^2-1)mc^2$ is the pressure of the relativistic electron cloud at $x = -\infty$.

The characteristic magnetic-field value H_* is uniquely determined by the parameter $\beta > 1$, which is equal to the ratio of the pressure of the relativistic electrons to the magnetic-field pressure. The temperature

$$T_* = 2(\alpha-1)\frac{n_0'}{n}(\gamma_0^2-1)mc^2$$

characterizes the heating of the conducting medium by the reverse current and T_0 is the temperature of the conductive medium at the boundary of the diamagnetic current. The quantity j_* gives the order of magnitude of the diamagnetic current (18). The expulsion wave velocity $u/c = \lambda j_*/\sigma_* H_*$ is determined by the eigenvalue λ of Eq. (20), and depends on the boundary conditions of this equations.

Let us now show that the system of equations (1),(3)-(5) has an integral that allows us to unamnbiguously determine the expulsion wave velocity. Using the y component of Eq. (3), we represent Eq. (4) in the form

$$\frac{n}{\alpha-1}\frac{\partial T}{\partial t} = -\frac{1}{c}j\frac{\partial A}{\partial t},$$

where we have neglected the small velocity along x due to the motion of the ions.

Now, substituting the expression for j from (1) into this equation, we have

$$\frac{\partial}{\partial t}\left\{\frac{nT}{\alpha-1} - \frac{1}{c}\int_{A_0}^{A} j'(A)\,dA + \frac{H^2}{8\pi}\right\} = \frac{1}{4\pi}\frac{\partial}{\partial x}\left(\frac{\partial A}{\partial t}\frac{\partial A}{\partial x}\right). \tag{21}$$

If the Poynting vector $cE_yH_z/4\pi$ vanishes as $x \to +\infty$, then, integrating (21), we find that the quantity that has the physical meaning of the plasma free energy is conserved during the heating of the plasma by the extraneous current. This is valid if $H_z \to 0$ as $x \to +\infty$.

Let us discuss in greater detail the case in which $H_z \to H_\infty$ as $x \to +\infty$. Since the plasma current $j_y = \sigma E_y$ vanishes at $x \to +\infty$, $E_y = 0$ when σ is nonzero at $x \to +\infty$. Thus, the conservation of the energy (and of the magnetic flux) is possible only when $\sigma_\infty \neq 0$. But, strictly speaking, a stationary magnetic-field-expulsion wave exists only if by chance the conductivity σ_∞ in front of the wave is equal to zero and the magnetic field is expelled unimpeded to infinity. It is natural to assume that $\sigma_\infty \neq 0$, but small compared to σ_*. Then an expulsion wave with constant velocity u will exist if the characteristic skin dimension

$$\delta_s = (c^2\tau/4\pi\sigma_\infty)^{1/2} \sim r(\sigma_*/\beta\sigma_\infty)^{1/2}$$

is significantly greater than the characteristic dimension r of the wave front, a condition fulfilled when $\beta \ll \sigma_*/\sigma_\infty$. The propagation of the wave leads to the expulsion of the magnetic field from the region $x \sim r$ into the region $x \sim \delta_s$, where the magnetic energy is dissipated. In the process, the expelled magnetic-field flux gets distributed over a region of characteristic dimension $\sim \delta_s$, which leads over a time period $\sim \tau$ to the increase of the magnetic field strength in comparison with H_∞ by an amount

$$\Delta H \sim H_\infty r/\delta_s \ll H_\infty.$$

The use of the above-indicated energy integral in this case is inconvenient, since it leads to integration over an x region of dimension $\sim\delta_s$, which is arbitrary with respect to the problem under consideration and, moreover, increases without restriction as σ_∞ decreases, which leads to the divergence of the energy integral. Let us show that in this case there an integral over the region $x \leqslant L$ ($<\delta_s$) is conserved, up to terms $\sim r/\delta_s$. It is in fact equal to the difference between the energy and the magnetic flux multiplied by $H_\infty/4\pi$. Let us introduce the notation $H_L \equiv H_z(x=L)$, where $r \ll L < \delta_s$. Integrating (21) over x in the region $-\infty < x \leqslant L$, we obtain

$$\frac{\partial}{\partial t}\int_{-\infty}^{L} dx\left\{\frac{nT}{\alpha-1}-\frac{1}{c}\int_{A_0}^{A} j'(A)\,dA+\frac{(H_z-H_L)^2}{8\pi}\right\}$$

$$=\frac{1}{4\pi}\frac{\partial H_L}{\partial t}\int_{-\infty}^{L} dx\,(H_L-H_z).$$

We can, on the basis of the arguments set forth above, estimate the right-hand side of this integral to be

$$\frac{1}{4\pi}\frac{(\Delta H)^2}{\tau}L \sim \frac{1}{4\pi}\frac{H_\infty^2 L}{\tau}\frac{r^2}{\delta_s^2}.$$

Since $L<\delta_s$, we find that the integral

$$W=\int_{-\infty}^{+\infty} dx\left\{\frac{nT}{\alpha-1}-\frac{1}{c}\int_{A_0}^{A} j'(A)\,dA+\frac{(H-H_\infty)^2}{8\pi}\right\}, \quad (22)$$

in which we have set $L=+\infty$ in view of the convergence of the integral, exists at least accurate to $\sim r/\delta_s$. Formally, this corresponds to the limiting case $\sigma_\infty=0$.

For the solutions that depend on $s=x-ut$ and satisfy Eq. (20), the integral assumes the form

$$W=\frac{1}{4\pi}\int_{-\infty}^{+\infty} dx\,H(H-H_\infty). \quad (23)$$

The integral is defined such that it is equal to zero for the magnetic field jump: $H_z=0$ for $x<x_0$ and $H_z=H_\infty$ for $x>x_0$, where x_0 is an arbitrary coordinate. Thus, the infinite magnetic-field energy for $x>0$ is not included in the integral W. This integral will be used below to determine unambiguously the wave velocity u.

The wave velocity u will now be determined for the various cases depicted in Fig. 1. Figure 1a corresponds to the situation in which the magnetic field for $x\to+\infty$ has c onstant value $H_z=H_\infty$. In this case the form of the solution depends essentially on the exponent k in the formula (7).

Let us analyze the character of the solutions to Eq. (20) as a function of the value of the exponent k. If we introduce the function

$$g(a,h)=\tau_0+\frac{1}{3\beta}(h^2-h_0^2)+\frac{1}{3}(1-a^2)^{3/2},$$

then Eq. (20) is reduced to the form

$$dg/da=-\lambda h g^k. \quad (24)$$

Integrating, we obtain the relation

$$\frac{1}{1-k}(g^{1-k}-\tau_0^{1-k})=-\lambda\int_1^a h\,da, \quad (25)$$

which turns out to be useful for the determination of the form of the function $h(a)$ in the region $0<a<1$.

Let us represent the magnetic field near the point $h=h_0$ in the form

$$h=h_0+h_1(1-a^2)^{\delta_1}+h_2(1-a^2)^{\delta_2}+\dots. \quad (26)$$

In the case when $\tau_0=0, h_0=1$. Representing $h-1$ in the form of a series in powers of $1-a^2$, and substituting this expansion into Eq. (25), we find that the magnetic-field profile should have a peak in the $k<\frac{1}{3}$, and that it is monotonic when $<\frac{1}{3}<k<1$. The wave velocity for $\tau_0=0$ can then be found from the solution to Eq. (20) for the boundary conditions

$$h=0 \text{ for } a=0 \text{ and } h=1 \text{ for } a=1. \quad (27)$$

The intetral W can be represented in the dimensionless form

$$w=\frac{1}{4\pi}\int_0^{a_{\max}} da\,(h-h_\infty), \quad (28)$$

where $a_{\max}$ corresponds to the total magnetic flux in the wave. From this it follows that $w<0$ for the monotonic profile in the region $<\frac{1}{3}<k<1$, since the integrand in (28) is nonzero in the region $0\leqslant a\leqslant 1$, where $h\leqslant h_0=h_\infty$.

In the $\tau_0\neq 0$ case, using the expansion (26), we find that the magnetic field has a maximum, and that it falls off when $a\geqslant 1$. Because of the heating of the plasma by the ohmic dissipation, $\tau_0=(hh_0^2-h_\infty^2)/3\beta$. The magnetic flux is then not conserved. The velocity u of this maximum can be determined from the condition that the spreading of the magnetic field on account of the finite conductivity should be canceled by its steepening as a result of the expulsion by the hot-electron cloud. Here it is convenient to assume that $H_*=H_0$ and $h_\infty=1$. The expulsion-wave velocity for this case can be determined from the solution to Eq. (20) for the boundary conditions

$$h=0 \text{ for } a=0, \quad h=h_0(w) \text{ for } a=1. \quad (29)$$

The function $h_0(w)$ entering into these conditions can be determined from (28). Notice that, for $\tau_0\neq 0$, the equation possesses solutions for both $k>1$ and $k<1$. Integrating (24) over

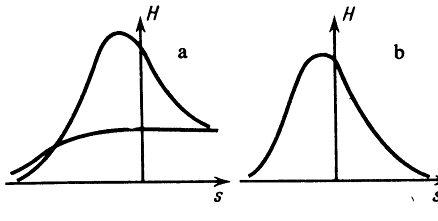

FIG. 1 Schematic profile of the magnetic field of the expulsion wave: a) two types of solutions for the case of a finite magnetic field at $x\to+\infty$; b) the case of zero magnetic field at $x\to+\infty$.

a in the region $(0,a)$, we obtain

$$\frac{1}{1-k}\left[\left(\frac{h^2-1}{3\beta}\right)^{1-k}-\left(\frac{\beta-1}{3\beta}\right)^{1-k}\right]=-\lambda w-\lambda a, \qquad a>1, \tag{30}$$

It follows from (30) that, if for $k<1$ the magnetic-field flux at the wave front at the wave front is finite ($h \to 1$ for $a<\infty$), for $k>1$ it is infinite ($h\to 1$ for $a\to\infty$).

Figure 1b corresponds to the case in which the magnetic field has a maximum and decreases to zero in the region outside the diamagnetic current. In this case the finite magnetic flux $A_\infty = A\,(x=\infty)$, which is conserved, is frozen in at the expulsion-wave front. The boundary conditions for (20) in this case will be

$$h=0 \text{ for } a=0, \quad h=h_0(a_\infty) \text{ for } a=1, \tag{31}$$

where $h_0(a_\infty)$ is given by the relation

$$\frac{1}{\beta}\int_0^{h_0}\frac{dh}{(\tau_\infty+h^2/3\beta)^k}=\frac{3\lambda}{2}(a_\infty-1).$$

Here $a_\infty = eA_\infty/cP_0$ and $\tau_\infty = T_\infty/T_*$ is the dimensionless temperature corresponding to the preheating at $x\to\infty$. As follows from the formula given, for $\tau_\infty=0$, a finite magnetic flux exists at the wave front only when $2k<1$. Equation (20) allows us to establish a relation between a_∞ and w.

We shall now show that Eq. (20) always has an eigenvalue λ, the proof being given for the case of the boundary conditions (29). For the proof, consider Eq. (24). To the eigenvalue $\lambda=0$ corresponds $g=\text{const}$. Figure 2 shows those curves passing through the points $a=0$, $h=0$ and $a=1, h=h_0$ which correspond to the boundaries of the hot-electron cloud. Let us draw through the intercept ($a=a_*$) of the second curve on the a axis a straight line parallel to the h axis up to the point ($h=h_*$) where it intersects the first curve. Let us integrate Eq. (20) over a in the intervals $(0,a_*)$ and $(a_*,1)$:

$$\left(\tau_0+\frac{h_1^2}{3\beta}\right)^{1-k}-\left(\frac{\beta-1}{3\beta}\right)^{1-k}=-\lambda(1-k)\int_0^{a_*}h\,da, \tag{32}$$

$$\left(\frac{h_0^2-1}{3\beta}\right)^{1-k}-\left(\tau_0+\frac{h_2^2}{3\beta}\right)^{1-k}=-\lambda(1-k)\int_{a_*}^{1}h\,da. \tag{33}$$

Here h_1 is the point of intersection of the integral curve emanating from the point $a=0, h=0$ with the straight line $a=a_*$, while h_2 is the point of intersection of the integral curve emanating from the point $a=1, h=h_0$ with the straight line $a=a_*$.

Let us form the difference

$$\Delta=\left(\tau_0+\frac{h_2^2}{3\beta}\right)^{1-k}-\left(\tau_0+\frac{h_1^2}{3\beta}\right)^{1-k}=\left(\frac{h_0^2-1}{3\beta}\right)^{1-k}-\left(\frac{\beta-1}{3\beta}\right)^{1-k}+\lambda(1-k)\int_0^1 h\,da.$$

Since $h_0^2<\beta$ we have $\Delta<0$ when $k<1$ and $\Delta>0$ when $k>1$ as $\lambda\to 0$. Let us now choose the eigenvalue $\lambda=\lambda_{\max}$ corresponding to $h_2=h_*$ in (33). Substituting $\lambda=\lambda_{\max}$ in (33), and using the expression $h_*^2=\beta-h_0^2$, we obtain

$$\Delta=\lambda_{\max}(1-k)\int_0^{a_*}h\,da.$$

From this it follows that $\Delta>0$ when $k<1$ and $\Delta<0$ when $k>1$ at $\lambda=\lambda_{\max}$. Thus, there exists a value of λ in the continuous interval $0<\lambda<\lambda_{\max}$ such that $\Delta(\lambda)=0$. This indicates that the trial integral curves emanating from the points $a=0$, $h=0$ and $a=1, h=h_0$ "join" at some value of λ. In this case the derivatives also turn out, on account of Eq. (20), to be continuous at $a=a_*$.

In the present investigation we carried out a numerical computation of the rate of expulsion of a magnetic field by a hot-electron cloud for the case of a constant magnetic field H_∞ at $x\to\infty$ and for the case of a finite frozen-in flux A_∞ with allowance for the preheating T_∞. In both cases the exponent in the formula (7) was equal to $k=\frac{3}{2}$. In the case of a constant magnetic field at infinity, the conservatin of the quantity w has to be used in order to obtain a unique velocity u. Using (24), we find from (28) that

$$w+1=\frac{(3\beta)^{k-1}}{\lambda}\left\{\int_1^{h_0}\frac{2dh}{(h+1)^k(h-1)^{k-1}}+\frac{1}{1-k}\left[(\beta-1)^{1-k}-(h_0^2-1)^{1-k}\right]\right\}, \qquad k<2. \tag{34}$$

This expression gives the above-indicated function $h_0(w)$, which uniquely determines the boundary conditions (29) for Eq. (20). Let us note that a similar problem of the unambiguous determination of a wave velocity arises in the self-similar problems considered in Refs. 6 and 7.

In the case when the magnetic field $H_\infty=0$ the quantity w has the meaning of a magnetic energy. Then it can be shown that the eigenvalue λ determining the expulsion-wave velocity can simply be expressed in terms of w. Using Eq. (20), we obtain

$$\lambda w=\frac{1}{k-1}\left[\tau_\infty^{1-k}-(\tau_\infty+1/3)^{1-k}\right]. \tag{35}$$

Thus, the expulsion-wave velocity in the case when the magnetic flux is conserved is inversely proportional to the magnetic energy at the wave front. In this case the wave velocity increases significantly as the preheating τ_∞ decreases. This can be explained in the following manner. Although the total

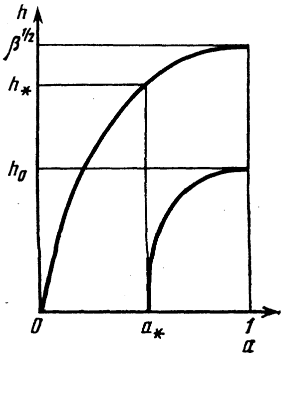

FIG. 2.

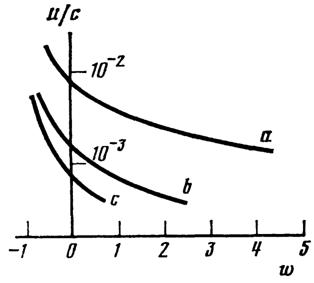

FIG. 3. Dependence of the expulsion rate u/c on the magnitude of the integral w for different values of β: a) 10^3; b) 2×10^2; c) 10^2.

magnetic-field flux is conserved, as the temperature decreases, the conductivity at the wave front decreases, and the characteristic dimension of the front increases. As a result, the magnetic field at the boundary with the cloud decreases, and the rate of its expulsion increases significantly. To find the dependence of the velocity u/c on the magnitude a_∞ of the frozen-in flux, we must find the relatin between w and a_∞ from Eq. (20).

In Figs. 3 and 4 we present the expulsion-wave-velocity values obtained as a result of the integration of Eq. (20) with the aid of Euler's implicit method.[8]

Figure 3 shows the dependence of the expulsion-wave velocity on the integral w for three different values of the parameter β in the case when the magnetic field assumes a constant value as $x \to +\infty$. The magnetic field profile then corresponds to the curve with the peak in Fig. 1a. The computations were carried out for the characteristic values of the hot-electron energy $\varepsilon = 1$ MV, the hot-electron density $n_0' = 10^{15}$ cm^{-3}, and the plasma density $n = 10^{19}$ cm^{-3}.

Figure 4 shows the dependence of the expulsion-wave velocity u/c on the dimensionless total magnetic field flux a_∞ frozen in at the wave front. The typical behavior of the magnetic field corresponds to Fig. 1b. It can be seen that the expulsion-wave velocity depends essentially on the temperature T_∞ characterizing the preheating. The wave velocity decreases with increasing T_∞.

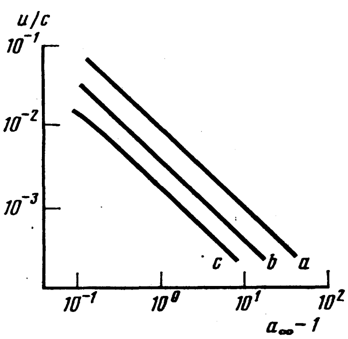

FIG. 4. Dependence of the expulsion rate u/c on the magnitude a_∞ of the magnetic flux at the wave front for different values of the preheating temperature T_∞: a) 0.2 eV; b) 0.5 eV; c) 1.0 eV.

In conclusion, let us analyze the equation for the electron temperature with allowance for the heat conduction and the collisional transfer of energy to the ions:

$$\frac{n}{\alpha-1}\frac{\partial T}{\partial t}=\frac{j^2}{\sigma}+\operatorname{div}(\varkappa\nabla T)-\frac{3m}{M}n\nu_{ei}(T-T_i), \tag{36}$$

where T_i is the ion temperature. Comparing the left-hand side with the second term on the right-hand side, we find that the effect of the heat conduction can be neglected when

$$n/\tau > \varkappa/r^2, \tag{37}$$

where $\varkappa$ is the thermal conductivity coefficient.

Let us, to begin with, assume that the thermal conduction is not magnetized[9]:

$$\varkappa = 3.16nT/m\nu_{ei}.$$

To estimate the temperaure T, let us use its characteristic value $T_* \sim T'n'/n$ [see the formula (20)], and take as the characteristic time $\tau \sim \tau_s/\beta$. Substituting these estimates in (37), we find that the inequality (37) is equivalent to

$$\beta > 5\cdot10^{-26}\left(\frac{n'}{n}\right)^4\frac{\sigma_0^2}{n}.$$

From this it follows that, when $\beta \gtrsim 10^2$ and $n = 10^{19}$ cm^{-3}, this inequality is satisfied for $n'/n = 10^{-4}$. Under this condition the term with the thermal conductivity coefficient in Eq. (36) can be neglected, and Eq. (4) is valid. In the case $\omega_{He} \gg \nu_{ei}$, when the thermal conduction is magnetized, the term corresponding to heat transport in Eq. (36) is still smaller. The last term in Eq. (36) can be neglected when $n'/n > m/M$, which is fulfilled in the case of the n'- and n-parameter values chosen here for heavy ions.

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