Effective conductivity of an inhomogeneous medium in a strong magnetic field

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The effective conductivity problem for an isotropic medium with three-dimensional random concentration inhomogeneities in a strong magnetic field is considered. It is shown that for an infinite medium the field dependence of the transverse tensor components $\hat{\sigma}^e$ in maximally strong fields is universal, i.e., the field dependence is independent of the properties of the medium and obeys a 4/3 law $\sigma_\perp^e = \sigma_{xx}^e \propto H^{-4/3}$. It is also shown that the finiteness of the medium (specimen) in the longitudinal direction (along **H**) markedly influences the field dependence of $\sigma_\perp^e$ even when the thickness of the specimen L_z is much greater than the dimension a of the inhomogeneities; the $\sigma_\perp^e(H, L_z)$ dependences are also determined. It is found that $\sigma_\perp^e \propto \Delta\,(a/L_z)^{1/2} H^{-1}$, and $(\Delta^2 = \langle \delta\sigma^2 \rangle / \langle \sigma \rangle^2)$ in extremely strong fields, and that in intermediate fields the behavior of $\sigma_\perp^e(H)$ depends on L_z and varies over a thickness on the order of $L_0 = a/\Delta^2$.

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§1. INTRODUCTION

Steady flow of current in an inhomogeneous medium with inhomogeneities greater than the mean free path of the carriers l is described by a local conductivity tensor $\hat{\sigma}(r)$, which relates the current and electrical field at a given point of the medium[1)]

$$\mathbf{j}(\mathbf{r}) = \hat{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r}). \tag{1}$$

Besides a regular component which varies slowly at the system limits, the inhomogeneities may include a random, small-scale component with characteristic dimension (correlation radius) a greater than l, but much less than the dimensions of the medium. The influence of the random inhomogeneities on the conductivity of medium may be taken into account by averaging the current and electrical field over each region with dimensions greater than a. We end up with the problem of determining the effective conductivity tensor $\hat{\sigma}^e$, which relates the smoothened values of the current and the field:

$$\langle \mathbf{j} \rangle = \hat{\sigma}^e \langle \mathbf{E} \rangle. \tag{2}$$

The tensor $\hat{\sigma}^e$ does not, in general, coincide with $\langle \hat{\sigma} \rangle$ and may greatly differ from the latter. In addition to the obvious case of strong inhomogeneities, such a situation may also arise in a weakly inhomogeneous medium in a strong magnetic field ($\lambda = (\mu H/c)^{-1} < 1$, with μ the mobility of the carriers) for the transverse components $\hat{\sigma}^e$ (Ref. 1); this is due to the markedly anisotropic nature of the conductivity in a strong magnetic field.

In the case of an isotropic medium, a case typical of many semiconductors[1] and plasma,[2] the local-conductivity tensor has the form (the Z axis is directed along **H**)

$$\hat{\sigma}(\mathbf{r}) = \begin{pmatrix} \lambda^2 & \lambda & 0 \\ -\lambda & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sigma_0(\mathbf{r}) \equiv \hat{\zeta}(\lambda)\sigma_0(\mathbf{r}), \tag{3}$$

where $\sigma_0(\mathbf{r}) = e\mu n$, n being the carrier density.

Below we will consider a medium with three-dimensional inhomogenieties [$\hat{\sigma} = \hat{\sigma}(x, y, z)$]; moreover, it will be assumed that the fluctuations of $\hat{\sigma}$ are small. In this case, the influence of the inhomogeneities on the Hall and longitudinal components is insignificant,[3] i.e., $\sigma_{xy}^e \cong \langle \sigma_{xy} \rangle$ and $\sigma_{zz}^e \cong \langle \sigma_{zz} \rangle = \sigma_0$. However, for the transverse components $\sigma_\perp^e = \sigma_{xx}^e = \sigma_{yy}^3$, since $\langle \sigma_\perp \rangle = \lambda^2 \sigma_0$ is small, the presence of relatively strongly fluctuating Hall components $\sigma_{xy} \sim \lambda$ leads to anomalously high values of the correction $\delta\sigma_\perp = \sigma_\perp^e - \langle \sigma_\perp \rangle$ for the inhomogeneity in a strong enough magnetic field; here the contribution of the diagonal components of $\hat{\sigma}(r)$ to $\delta\sigma_\perp$ is slight by comparison with the Hall contribution and may be ignored.[3] Thus, though fluctuations of both the carriers density and mobility may occur in an inhomogeneous medium, the mobility fluctuations are negligible, since the Hall components are independent of the mobility.

The computation of $\delta\sigma_\perp$ has been considered in many studies (a history of the problem may be found in Ref. 3). In a first approximation with respect to the fluctuation parameter $\Delta^2 = \langle \delta n^2 \rangle / \langle n \rangle^2$, the influence of the inhomogeneities is described by the Herring correction[1]

$$\delta\sigma_\perp \approx \lambda\Delta^2\sigma_0, \quad \sigma_0 = \langle \sigma_0(\mathbf{r}) \rangle. \tag{4}$$

The correction (4) depends anomalously on the magnetic field ($\propto H^{-1}$) and, in a strong enough field, exceeds the zeroth-order approximation: $\langle \sigma_\perp \rangle \propto H^{-2}$. As a result, the applicability of ordinary perturbation theory in the parameter Δ^2 is restricted (even when $\Delta < 1$) to the region of "weak" fields in which $\delta\sigma_\perp < \langle \sigma_\perp \rangle$. Study of the higher orders of the expansion of $\delta\sigma_\perp$ in a series in Δ^2 shows that $\gamma = \lambda^{-1}\Delta^2$ is the true parameter of the expansion.[3]

Methods of solving the problem that make it possible to go beyond the ordinary perturbation theory in the region of "strong" fields ($\gamma > 1$) have been developed by Dreĭzin and Dykhne.[3] Using a renormalized perturbation theory, Dreĭzin and Dykhne showed that, under definite assumptions to be discussed below, the solution in the region of strong magnetic fields for the quantity $\overline{\delta\sigma_\perp}$ averaged over the ensemble of random fields $n(r)$ has the form

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$$\overline{\delta\sigma_{\perp}}=A(\lambda\Delta)^{\nu}\sigma_0, \tag{5}$$

where $\nu = 4/3$ and A is a numerical coefficient whose value remains undefined within this method.

The diffusion analogy method, which was also applied in Ref. 3, does not offer any advantages over the renormalized perturbation theory in the study of the analytic properties of the exact solution (this may be seen by constructing the exact influence function for Eq. (8) of Ref. 3, see Eq. (26) of the present article). Besides giving a qualitative, physically graphic picture of the phenomenon,[3] the diffusion analogy also makes it possible to use direct (numerical) methods to solve the problem.[4] By means of the Monte Carlo method, a numerical solution of the problem has been obtained[4] for a particular model of the medium. It was found that $\sigma_{\perp}^{e}$ is well described by (5), but with $1.1 < \nu < 1.27$ and a most probable value $\nu = 1.19$.

There are two possible fundamental reasons for the divergence between the results of Refs. 3 and 4. First, the exponent ν in (5) depends on the properties of the medium, i.e., the field dependence of $\sigma_{\perp}^{e}$ in the region of strong fields is not universal. In fact, the 4/3 law was derived in Ref. 3 under definite assumptions imposed on the quantities that depend on the properties of the medium (see below), and this could affect the choice of the ensemble over which $\delta\sigma_{\perp}$ was averaged. By contrast, the computation in Ref. 4 was performed for a specific inhomogeneity model in which these assumptions may also not be satisfied if there is no universality.

The second reason can be the possibility that $\sigma_{\perp}^{e}$ may depend on the dimensions of the system, since the 4/3 law was obtained for an infinite medium, whereas Ref. 4 discussed the motion of a particle in a bounded region with periodic boundary conditions.

Our goal in the present article is to consider precisely these two questions.

We prove that for an infinite randomly inhomogeneous medium the field dependence of $\sigma_{\perp}^{e}$ in the region of strong fields is universal and that the 4/3 law is satisfied. We simultaneously show that the inhomogeneity correction $\delta\sigma_{\perp}$ is positive, thus ensuring that the stability condition holds for steady flow of currents[5] $\sigma_{\perp}^{e} > 0$ in the case of anomalous conductivity ($|\delta\sigma_{\perp}| > \langle\sigma_{\perp}\rangle$).

We also show that the transverse conductivity of a randomly inhomogeneous medium with three-dimensional inhomogeneities depends (in a strong enough magnetic field) on the longitudinal dimension (thickness of the specimen) L_z even if

$$L_x,\ L_y,\ L_z \gg a, \tag{6}$$

where L_α are the dimensions of the system. Because $\hat{\sigma}^{e}$ is self-averaged,[6] the approximation of an infinite medium can be used for random inhomogeneities by virtue of conditions (6) if there is no magnetic field. The size effect considered below is thus attributable to non-uniform flow of current in a medium with strongly anisotropic conductivity in a strong magnetic field. The qualitative meaning of this effect may be explained by the following example. A dependence of $\sigma_{\perp}^{e}$ on L_z means that if the specimen is cut, say in half, in the direction transverse to the magnetic field, when its resistance to a direct current $\langle \mathbf{j}\rangle = (\langle j_x\rangle,0,0)$ is measured in a strong enough magnetic field, the resistance will change. This is because the current lines in an inhomogeneous specimen immersed in a strong magnetic field will be stretched along $\mathbf{H}$,[1] while the characteristic dimension $\mathscr{L}$ of the trajectories in the direction of the magnetic field increases with increasing H. Therefore, when $\mathscr{L}(H)$ becomes of the order of the thickness L_z of the specimen when the specimen is cut most of the current lines will also be cut, which leads to a rearrangement of the pattern of flow of current, i.e., to a change in the resistance.

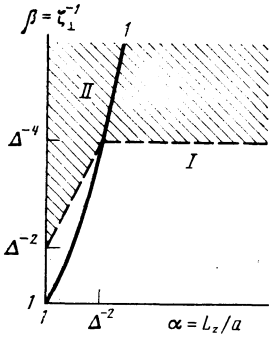

FIG. 1. "Phase" diagram in the α-β plane, where $\alpha = L_z/a$ and $\beta = \zeta_{\perp}^{-1} = \sigma_0/\sigma_{\perp}^{e}$. Curve 1 which depicts the relation $\beta = \alpha^2$ is the boundary between phase I (infinite medium) and phase II (region of size effect). Anomalous conductivity corresponds to the shaded region, the 4/3 law ($\sigma_{\perp}^{e} \sim (\lambda\Delta)^{4/3}\sigma_0$) holds below curve 1, and the law $\sigma_{\perp}^{e} \sim \lambda\Delta\,(a/L_z)^{1/2}\sigma_0$, where $\lambda = (\mu H/c)^{-1}$ holds above it.

For a given thickness of the specimen, the condition $\mathscr{L}(H) = L_z$ defines the characteristic field in which there occurs a transition from the infinite-medium approximation ($\mathscr{L} < L_z$) to the region of the size effect (Fig. 1). The most important consequence of the size effect is that the dependence of $\delta\sigma_{\perp}$ on λ in sufficiently strong magnetic fields again becomes linear ($\delta\sigma_{\perp} \sim \lambda\Delta\,(a/L_z)^{1/2}$), as in (4), but with a coefficient that depends on L_z. It is also of interest that the nature of the $\delta\sigma_{\perp}(H)$ dependence in the transitional region differs for thick ($L_z > L_0$) and thin ($L_z < L_0$) specimens (Fig. 2) with $L_0 = a/\Delta^2$. The size effect is considered in Sec. 4.

Before passing on to the main part of the article, let us refine our formulationof the problem. In proving the univer-

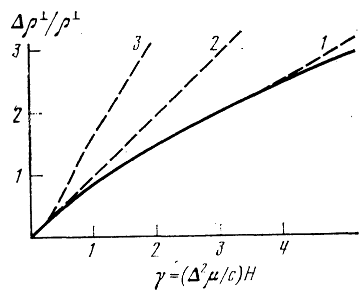

FIG. 2. Field dependences of inhomogeneity correction to the transverse magnetoresistance at different thicknesses of the specimen for $\Delta = 0.2$. The solid curve corresponds to the case of an unbounded specimen ($L_z = \infty$). The variation in the field dependence of $\Delta\rho^{\perp}/\rho^{\perp}$ with L_z/L_0 equal to 2 (curve 1), 1 (curve 2) and 1/3 (curve 3) is shown by dashed lines ($L_0 = a/\Delta^2$).

sality of the 4/3 law, we will make essential use of results obtained for $\sigma_\perp^e$ by Dreĭzin and Dykhne by the renormalized perturbation theory method. The quantity $\sigma_\perp^e(\mathbf{k}) = \langle\sigma_\perp\rangle + \delta\sigma_\perp(\mathbf{k})$ (**k** is the wave vector) was introduced in Ref. 3 and defined by an expansion of $\sigma_\perp^e$ in a perturbation-theory series theory and by the relation

$$\delta\sigma_\perp = \lim_{\mathbf{k}\to 0} \delta\sigma_\perp(\mathbf{k}) \tag{7}$$

associated with $\sigma_\perp^e$. By averaging the expansion for $\delta\sigma_\perp(\mathbf{k})$ over the ensemble, Dreĭzin and Dykhne obtained, by means of a diagram technique, an equation in the form of a renormalized perturbation-theory series for the averaged quantity $\overline{\delta\sigma_\perp(\mathbf{k})}$. Solution of this equation in the case of three-dimensional inhomogeneities encounters a difficulty connected with the presence of singular integrations with respect to k_z. The result of such an integration determines the asymptotic behavior of $\overline{\delta\sigma_\perp(\mathbf{k})}$, meaning also of $\delta\sigma_\perp$, as $\lambda\to 0$, but depends in turn on the sign of $\overline{\delta\sigma_\perp(\mathbf{k})}$. Assuming that

$$\overline{\delta\sigma_\perp(\mathbf{k})} > 0, \tag{8}$$

Dreĭzin and Dykhne proved that in a strong field there exists a solution

$$\overline{\delta\sigma_\perp(\mathbf{k})} = A^{\mathbf{k}}(\lambda\Delta)^{4/3}\sigma_0,$$

where A^k is determined by the properties of the medium. Thus, in view of (7), to prove that the 4/3 law is universal it is sufficient to show that condition (8) is satisfied.

It is difficult to obtain any information about the behavior of $\overline{\delta\sigma_\perp(\mathbf{k})}$ as a function of **k** directly from the expansion in a renormalized-perturbation-theory series, since the series is asymptotic and the strong-coupling case is realized.[3] Therefore, we develop below a different approach to the problem, based on Dyson-type equations for an arbitrary inhomogeneous medium. In these equations the exact (unaveraged) quantity[2)] $\delta\sigma_\perp(\mathbf{k})$ plays the role of the self-energy part. The Dyson equations are derived in §3, where it is proved that if the fluctuations of all the components of $\hat{\sigma}(\mathbf{r})$ other than the Hall components are inessential the following inequality holds for an arbitrary inhomogeneous medium

$$\delta\sigma_\perp(\mathbf{k}) > 0, \tag{9}$$

and leads to the condition (8).

In Appendix 2 we will derive the Dreĭzin and Dykhne expansion for $\overline{\delta\sigma_\perp(\mathbf{k})}$ from the Dyson equations for a randomly inhomogeneous medium by taking the limit as $V\to\infty$ (V is the volume of the system). Besides proving that our methods are equivalent to those of Ref. 3, the derivation will also show that the Dreĭzin and Dykhne expansion is applicable for $\overline{\delta\sigma_\perp(\mathbf{k})}$ in the case of a bounded medium, particularly for describing the size effect and for obtaining a sufficient condition under which the fluctuations $\delta\sigma_\perp(\mathbf{k})$ due to the finite dimensions of the system are small.

§2. BASIC EQUATIONS

The steady flow of the current is described by the equations

$$\operatorname{div}\mathbf{j}=0, \qquad \operatorname{rot}\mathbf{E}=0. \tag{10}$$

Together with (1)–(3), these equations form a complete system of initial equations.[3)] Following Herring,[1] we separate the mean values (over the volume) of $\langle\mathbf{j}\rangle$, $\langle\mathbf{E}\rangle$, and $\langle\hat{\sigma}\rangle$, and rewrite (1)–(3) and (10) as equations for the Fourier components of the fluctuations $\delta\mathbf{j}$, $\delta\mathbf{E}$, $\delta\hat{\sigma}$, which are defined as follows:

$$\delta\mathbf{j}(\mathbf{r}) = \mathbf{j}(\mathbf{r}) - \langle\mathbf{j}\rangle = \sum_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r}}\delta\mathbf{j}^{\mathbf{k}}, \quad \delta\mathbf{j}^{\mathbf{k}} = \frac{1}{V}\int d\mathbf{r}e^{-i\mathbf{k}\mathbf{r}}\delta\mathbf{j}(\mathbf{r}),$$

and analogously for $\delta\mathbf{E}$ and $\delta\hat{\sigma}$. Using the expressions for $\langle\mathbf{j}\rangle$ and $\delta\mathbf{j}^{\mathbf{k}}$, as well as the conditions of transversality of $\delta\mathbf{j}_{\mathbf{k}}$ and longitudinality of $\delta\mathbf{E}_{\mathbf{k}}$, which follow from (10), and also bearing in mind (2), the following expression of $\hat{\sigma}^e$ is obtained:

$$\hat{\sigma}^e = \langle\hat{\sigma}\rangle + \delta\hat{\sigma}, \qquad \delta\hat{\sigma} = \delta\hat{\zeta}(\lambda)\sigma_0,$$

where

$$\delta\zeta_{\alpha\beta} = \sum_{\mathbf{k}_1,\mathbf{k}_2} (\mathbf{e}_\alpha\hat{\xi}\mathbf{n}_1)\xi^{-\mathbf{k}_1}G^{\mathbf{k}_1\mathbf{k}_2}\xi^{\mathbf{k}_2}(\mathbf{n}_2\hat{\zeta}\mathbf{e}_\beta), \tag{11}$$

$\mathbf{e}_\alpha$ are unit vectors along the coordinate axes, $\mathbf{n} = \mathbf{k}/k$,

$$G^{\mathbf{k}\mathbf{k}'} = g^{\mathbf{k}}\Delta(\mathbf{k}-\mathbf{k}') + g^{\mathbf{k}}W^{\mathbf{k}\mathbf{k}'}g^{\mathbf{k}'}$$

and

$$W^{\mathbf{k}\mathbf{k}'} = \gamma^{\mathbf{k}\mathbf{k}'} + \sum_{n\geqslant 2} W_n^{\mathbf{k}\mathbf{k}'},$$

$$W_n^{\mathbf{k}\mathbf{k}'} = \sum_{\mathbf{k}_1,\ldots,\mathbf{k}_{n-1}} \gamma^{\mathbf{k}\mathbf{k}_1}g^{\mathbf{k}_1}\gamma^{\mathbf{k}_1\mathbf{k}_2}\ldots g^{\mathbf{k}_{n-1}}\gamma^{\mathbf{k}_{n-1}\mathbf{k}'}. \tag{12}$$

The quantities $g^{\mathbf{k}}$, $\gamma^{\mathbf{k}\mathbf{k}'}$, and $\xi^{\mathbf{k}}$ are defined by the expressions

$$g^{\mathbf{k}} = -\frac{\bar{\Delta}(\mathbf{k})}{n_z^2+\lambda^2}, \quad \bar{\Delta}(\mathbf{k}) = 1-\Delta(\mathbf{k}), \quad \Delta(\mathbf{k}) = \begin{cases}1, & \mathbf{k}=0\\ 0, & \mathbf{k}\neq 0\end{cases},$$
$$\gamma^{\mathbf{k}\mathbf{k}'} = \xi^{\mathbf{k}-\mathbf{k}'}(\mathbf{n}\hat{\zeta}\mathbf{n}') \equiv \xi^{\mathbf{k}-\mathbf{k}'}\zeta_{\alpha\beta}n_\alpha n_\beta, \qquad \xi^{\mathbf{k}} = \delta n^{\mathbf{k}}/\langle n\rangle. \tag{13}$$

Note that, because the medium is isotropic,

$$\operatorname{Im}\xi^{\mathbf{k}} = \langle\xi(\mathbf{r})\sin\mathbf{k}\mathbf{r}\rangle = 0.$$

In view of (3) and the fact that $\xi^{\mathbf{k}}$ is real, $\gamma^{\mathbf{k}\mathbf{k}'}$ may be represented in the form of a symmetric ($\gamma_s^{\mathbf{k}\mathbf{k}'}$) and an antisymmetric ($\gamma_a^{\mathbf{k}\mathbf{k}'}$) part: $\gamma = \gamma_s + \gamma_a$, where

$$\gamma_a^{\mathbf{k}\mathbf{k}'} = -\gamma_a^{\mathbf{k}'\mathbf{k}} = \xi^{\mathbf{k}-\mathbf{k}'}\lambda(n_x n_y' - n_y n_x'),$$

$$\gamma_s^{\mathbf{k}\mathbf{k}'} = \gamma_s^{\mathbf{k}'\mathbf{k}} = \xi^{\mathbf{k}-\mathbf{k}'}(n_z n_z' + \lambda^2(\mathbf{n}_\perp\mathbf{n}_\perp')). \tag{14}$$

Expressions (11)–(13) yield a representation of $\sigma_\perp^e$ in the form of an expansion in a series of the ordinary perturbation theory, in which g and γ play the role of the bare Green's functions and vertices. The Herring correction (4) is obtained if we limit ourselves, in the case of $G^{\mathbf{k}\mathbf{k}'}$, to the first term of the expansion $g^{\mathbf{k}}\Delta(\mathbf{k}-\mathbf{k}')$ and ignore the diagonal (symmetric) part $\hat{\zeta}$ in (11).

Equations (11)–(13) are not suitable for studying the behavior of $\sigma_\perp^e$ as $\lambda\to 0$, since the expansion parameter is $\lambda^{-1}\Delta^2$ (Ref. 3). In an attempt to get around this difficulty, Dreĭzin and Dykhne reconstructed the expansion (11)–(13) by means of a diagram technique, thus eliminating the divergences as $\lambda\to 0$. However, difficulties remain even when the renormalized perturbation theory is used, since a strong coupling case is then realized. Therefore, we will develop below a renormalization technique that does not make use of perturbation theory, but can yield Dyson-type equations for the renormalized quantities.

§3. DYSON EQUATIONS FOR AN ARBITRARY INHOMOGENEOUS MEDIUM. PROOF OF INEQUALITY (9)

Let us determine the quantity $S^{\mathbf{k}}_{\alpha\beta}$ related to $\delta\zeta_{\alpha\beta}$ by an equation similar to (7). Following Dreizin and Dykhne,[3] we may introduce $S^{\mathbf{k}}_{\alpha\beta}$ by replacing $\xi^{-\mathbf{k}_1}$ and $\xi^{\mathbf{k}_2}$ in (11) by $\xi^{\mathbf{k}-\mathbf{k}_2}$ and $\xi^{\mathbf{k}_2-\mathbf{k}}$; as can be easily seen, $S^{\mathbf{k}}_{\alpha\beta}$ then coincides with $W^{\mathbf{kk}}_{\alpha\beta}$, which is determined in Appendix 1 [see (A1)]. This definition of $S^{\mathbf{k}}_{\alpha\beta}$ is not, however, entirely exact, since $W^{\mathbf{kk}}_{\alpha\beta}$ is not an analytic function of $\mathbf{k}$. To prove this we consider the structure of $W^{\mathbf{kk}'}$. From (12) it is clear that every $W^{\mathbf{kk}'}_n$ contains non-connective terms of the form $W^{\mathbf{kk}}_m g^{\mathbf{k}} W^{\mathbf{kk}'}_{n-m}$, or $W^{\mathbf{kk}'}_m g^{\mathbf{k}'} W^{\mathbf{k}'\mathbf{k}'}_{n-m}$, which correspond to the cases in which one of the inner momenta $\mathbf{k}_m$ is equal to $\mathbf{k}$ or $\mathbf{k}'$. Since all the $W^{\mathbf{kk}'}_n$ have a structure analogous to (A1), the presence of non-connective terms makes $W^{\mathbf{kk}}_{\alpha\beta}$ non-analytic.

As for $\delta\hat{\zeta}$, because of our definition (13) of $g^{\mathbf{k}}$ the constraint $\mathbf{k}_i\neq 0$ is imposed on the inner momenta in (11) and (12), as a result of which there are no non-connective contributions to $\delta\hat{\zeta}$.

Thus, it is more correct to define $S^{\mathbf{k}}_{\alpha\beta}$ as a quantity which is obtained from $W^{\mathbf{kk}}_{\alpha\beta}$ after eliminating the non-connective terms.

Let us consider the quantity $\Gamma^{\mathbf{kk}'}$, which is obtained from $W^{\mathbf{kk}'}$ as a result of eliminating the non-connective contributions. From the foregoing it follows that $\Gamma^{\mathbf{kk}'}$ is determined by the expansion (12) in which we have introduced additional constraints on the innter momenta ($\mathbf{k}_i\neq\mathbf{k},\mathbf{k}'$). We introduce also the quantities $L^{\mathbf{kk}'}$ and $R^{\mathbf{kk}'}$ obtained from $W^{\mathbf{kk}'}$ by means of the constraints $\mathbf{k}_i\neq\mathbf{k}$ and $\mathbf{k}_i\neq\mathbf{k}'$, respectively. It will be convenient below to introduce common notation for the diagonal and nondiagonal parts of the arbitrary tensor $X^{\mathbf{kk}'}$ for use throughout the present article:

$$X^{\mathbf{kk}'}=X^{\mathbf{k}}\Delta(\mathbf{k}-\mathbf{k}')+\tilde{X}^{\mathbf{kk}'},\quad X^{\mathbf{k}}\equiv X^{\mathbf{kk}}.$$

The diagonal parts of Γ, L, and R coincide; we denote them $S^{\mathbf{k}}$. Thus, $\tilde{\Gamma}^{\mathbf{kk}'}$ and $S^{\mathbf{k}}$ are fully irreducible quantities, while $\tilde{L}^{\mathbf{kk}'}$ and $\tilde{R}^{\mathbf{kk}'}$ are irreducible with respect to $\mathbf{k}$ and $\mathbf{k}'$, respectively.

It is clear from our definition of the quantities $\tilde{\Gamma}$, $\tilde{L}$, $\tilde{R}$, and S that the expansion (12) plays the role of a generating series for these quantities; consequently, we introduce the notation

$$\begin{aligned}&\tilde{L}^{\mathbf{kk}'}=\tilde{W}^{\mathbf{kk}'}(\mathbf{k}),\quad \tilde{R}^{\mathbf{kk}'}=\tilde{W}^{\mathbf{kk}'}(\mathbf{k}'),\\ &\tilde{\Gamma}^{\mathbf{kk}'}\tilde{W}^{\mathbf{kk}'}(\mathbf{k},\mathbf{k}'),\quad S^{\mathbf{k}}=W^{\mathbf{k}}(\mathbf{k}),\end{aligned}\tag{15}$$

which will be used also in other cases, with the excluded inner moments indicated in parentheses. For example, $\tilde{\Gamma}^{\mathbf{kk}'}$ can be written as $\tilde{L}^{\mathbf{kk}'}(\mathbf{k}')$ or $\tilde{R}^{\mathbf{kk}'}(\mathbf{k})$, while $\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})$ and $S^{\mathbf{k}}(\mathbf{p})$ should be understood as $\tilde{W}^{\mathbf{kk}'}(\mathbf{k},\mathbf{k}',\mathbf{p})$ and $W^{\mathbf{k}}(\mathbf{k},\mathbf{p})$, respectively.

Let us consider the quantity $S^{\mathbf{k}}_{\alpha\beta}$, which is related to $S^{\mathbf{k}}$ by an equation analogous to (A1). Since $S^{\mathbf{k}}$ is irreducible, $S^{\mathbf{k}}_{\alpha\beta}$ is a function of $\mathbf{k}$ analytic at zero and we have[4)]

$$\delta\zeta_{\alpha\beta}=\lim_{\mathbf{k}\to 0}S_{\alpha\beta}{}^{\mathbf{k}}.\tag{16}$$

Let us now derive the Dyson equations, in which $S^{\mathbf{k}}$ and $\tilde{\Gamma}^{\mathbf{kk}'}$ will play the role of the self-energy part and the exact vertex while $G^{\mathbf{k}}$ will serve as the exact Green's function. From the definition of $W^{\mathbf{kk}'}$ in (12) and the definitions of $\tilde{R}$ and $\tilde{L}$ in (15) it follows that the nondiagonal parts of $\tilde{R}$ and $\tilde{L}$ satisfy the equations

$$\tilde{R}^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}g^{\mathbf{k}_1}\tilde{R}^{\mathbf{k}_1\mathbf{k}'},\quad \tilde{L}^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\tilde{L}^{\mathbf{kk}_1}g^{\mathbf{k}_1}\gamma^{\mathbf{k}_1\mathbf{k}'},\tag{17}$$

and that $S^{\mathbf{k}}$ is related to $\tilde{R}$ and $\tilde{L}$ by the equations

$$S^{\mathbf{k}}=\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}g^{\mathbf{k}_1}\tilde{R}^{\mathbf{k}_1\mathbf{k}}=\sum_{\mathbf{k}_1}\tilde{L}^{\mathbf{kk}_1}g^{\mathbf{k}_1}\gamma^{\mathbf{k}_1\mathbf{k}}.\tag{18}$$

The quantities $\tilde{R}^{\mathbf{kk}'}$ and $\tilde{L}^{\mathbf{kk}'}$ are reducible with respect to $\mathbf{k}$ and $\mathbf{k}'$, respectively. In Appendix 1 it is shown that, by separating the non-connective contributions from $\tilde{R}$ and $\tilde{L}$ we find that [see (A9) and (A10)]

$$g^{\mathbf{k}}\tilde{R}^{\mathbf{kk}'}=G^{\mathbf{k}}(\mathbf{k}')\tilde{\Gamma}^{\mathbf{kk}'},\quad \tilde{L}^{\mathbf{kk}'}g^{\mathbf{k}'}=\tilde{\Gamma}^{\mathbf{kk}'}G^{\mathbf{k}'}(\mathbf{k}),\tag{19}$$

where

$$G^{\mathbf{k}}=\frac{g^{\mathbf{k}}}{1-g^{\mathbf{k}}S^{\mathbf{k}}}=-\frac{\hat{\Delta}(\mathbf{k})}{n_z{}^2+\lambda^2+S^{\mathbf{k}}},\quad G^{\mathbf{k}}(\mathbf{p})=\frac{g^{\mathbf{k}}}{1-g^{\mathbf{k}}S^{\mathbf{k}}(\mathbf{p})}.\tag{20}$$

In view of (19), we obtain from (17) and (18)

$$\begin{aligned}&S^{\mathbf{k}}=\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}(\mathbf{k})\tilde{\Gamma}^{\mathbf{k}_1\mathbf{k}},\\ &\frac{G^{\mathbf{k}}(\mathbf{k}')}{g^{\mathbf{k}}}\tilde{\Gamma}^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}(\mathbf{k}')\tilde{\Gamma}^{\mathbf{k}_1\mathbf{k}'},\end{aligned}\tag{21}$$

as well as the equations

$$\begin{aligned}&S^{\mathbf{k}}=\sum_{\mathbf{k}_1}\tilde{\Gamma}^{\mathbf{kk}_1}G^{\mathbf{k}_1}(\mathbf{k})\gamma^{\mathbf{k}_1\mathbf{k}},\\ &\tilde{\Gamma}^{\mathbf{kk}'}\frac{G^{\mathbf{k}'}(\mathbf{k})}{g^{\mathbf{k}}}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\tilde{\Gamma}^{\mathbf{kk}_1}G^{\mathbf{k}_1}(\mathbf{k})\gamma^{\mathbf{k}_1\mathbf{k}'}\end{aligned}\tag{22}$$

which are conjugate to (21). Together with (A12), which relate $G^{\mathbf{k}}(\mathbf{p})$ to G and $\tilde{\Gamma}$, Eqs. (20)–(22) form a closed system and are the sought Dyson equations for S, $\tilde{\Gamma}$, and G.

To prove inequality (9), we rewrite Eqs. (21) and (22) for $\tilde{\Gamma}^{\mathbf{kk}'}$ in the form of equations for the symmetric and antisymmetric (relative to the permutation of $\mathbf{k}$ and $\mathbf{k}'$) vertices $\tilde{\Gamma}^{\mathbf{kk}'}_s$ and $\tilde{\Gamma}^{\mathbf{kk}'}_a$. We consider the case in which the fluctuations of all the components of $\hat{\sigma}(\mathbf{r})$, other than the Hall components, may be ignored; it follows then from (13) and (14) that $\gamma_s=0$ and $\gamma=\gamma_a$. As a result, since (21) and (22) are real, we obtain

$$S^{\mathbf{k}}=\sum_{\mathbf{k}_1}\gamma_a^{\mathbf{k}\mathbf{k}_1}G^{\mathbf{k}_1}(\mathbf{k})\tilde{\Gamma}_a^{\mathbf{k}_1\mathbf{k}},$$

$$\tilde{\Gamma}_a^{\mathbf{k}\mathbf{k}'}\frac{G^{\mathbf{k}'}(\mathbf{k})}{g^{\mathbf{k}'}}=\gamma_a^{\mathbf{k}\mathbf{k}'}+\sum_{\mathbf{k}_1}\tilde{\Gamma}_s^{\mathbf{k}\mathbf{k}_1}G^{\mathbf{k}_1}(\mathbf{k})\gamma_a^{\mathbf{k}_1\mathbf{k}'},\tag{23}$$

$$\frac{G^{\mathbf{k}}(\mathbf{k}')}{g^{\mathbf{k}}}\tilde{\Gamma}_s^{\mathbf{k}\mathbf{k}'}=\sum_{\mathbf{k}_1}\gamma_a^{\mathbf{k}\mathbf{k}_1}G^{\mathbf{k}_1}(\mathbf{k}')\tilde{\Gamma}_a^{\mathbf{k}_1\mathbf{k}'}.\tag{24}$$

In view of (23), $S^{\mathbf{k}}$ may be represented in the form

$$S^{\mathbf{k}}=-\sum_{\mathbf{k}_1}|\tilde{\Gamma}_a^{\mathbf{k}\mathbf{k}_1}|^2\frac{[G^{\mathbf{k}_1}(\mathbf{k})]^2}{g^{\mathbf{k}_1}}$$

$$-\sum_{\mathbf{k}_1}\tilde{\Gamma}_s^{\mathbf{k}\mathbf{k}_1}G^{\mathbf{k}_1}(\mathbf{k})\sum_{\mathbf{k}_2}\gamma_a^{\mathbf{k}_1\mathbf{k}_2}G^{\mathbf{k}_2}(\mathbf{k})\tilde{\Gamma}_a^{\mathbf{k}_2\mathbf{k}},$$

whence, in view of (24) and the definition of g in (13), we obtain

$$S^{\mathbf{k}}=\sum_{\mathbf{k}_1}(n_{1z}^2+\lambda^2)[G^{\mathbf{k}_1}(\mathbf{k})]^2\{|\tilde{\Gamma}_a^{\mathbf{k}\mathbf{k}_1}|^2+|\tilde{\Gamma}_s^{\mathbf{k}\mathbf{k}_1}|^2\},\tag{25}$$

which proves in fact inequality (9) and that $S^{\mathbf{k}}$ is positive, since $S^{\mathbf{k}}=S_\perp^{\mathbf{k}}(n_x^2+n_y^2)$ at $\gamma=\gamma_a$. We emphasize that $S^{\mathbf{k}}$ can be represented in the form (25) as a consequence of the absence of any bare vertex γ_s in the equations, i.e., ultimately, as a consequence of the fact that the fluctuations of all the components of $\hat{\sigma}(\mathbf{r})$, other than the Hall components, are inessential.

Since $S^{\mathbf{k}}\leqslant S_\perp^{\mathbf{k}}$ if $\gamma=\gamma_a$, we then find [bearing in mind (16)], that the inhomogeneity correction $\delta\sigma_\perp$ is positive also as a consequence of (25).

§4. SIZE EFFECT

The fact that the sequence in which the limits as $V\to\infty$ and $H\to\infty$ are taken is essential, which is noted in the derivation of the Dyson equations for a randomly inhomogeneous medium (Appendix 2), shows that $\sigma_\perp^e$ may depend on the dimensions of the specimen in a sufficiently strong magnetic field.

It is best to begin an explanation of the nature of this dependence with the diffusion analogy,[3] which provides a qualitative picture of the phenomenon. Dreĭzin and Dykhne noted that Eqs. (1) and (10), which are written for the potential Ψ, describe the process of stationary strongly anisotropic diffusion of particles of density $\Psi(\mathbf{r})$ in a field of random velocities $\mathbf{v}(\mathbf{r})=(-\nabla_y\sigma_{xy},\nabla_x\sigma_{xy},0)$. The longitudinal-diffusion coefficient $\langle\sigma_{zz}\rangle=\sigma_0$, and the transverse-diffusion coefficient $\langle\sigma_\perp\rangle=\lambda^2\sigma_0$.

Because of the random convection across the magnetic field, the transverse diffusion coefficient must be renormalized, i.e., $\langle\sigma_\perp\rangle$ is replaced by $\sigma_\perp^e=\langle\sigma_\perp\rangle+\delta\sigma_\perp$. It has been proposed[3] to obtain $\delta\sigma_\perp$ by considering for the particle a random walk described by the equation

$$\frac{\partial}{\partial t}\Psi=\hat{\mathscr{L}}\Psi,\quad \hat{\mathscr{L}}=\sigma_0\frac{\partial^2}{\partial z^2}+\lambda^2\sigma_0\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-(\mathbf{v}\nabla),\tag{26}$$

with initial condition $\Psi(\mathbf{r},t=0)=\delta(\mathbf{r}-\mathbf{r}_0)$. Then $\sigma_\perp^e$ is determined from the well-known expression

$$\sigma_\perp^e=\lim_{t\to\infty}\frac{\overline{\langle\mathbf{r}_\perp^2(t)\rangle}}{4t},\tag{27}$$

where $\langle\ldots\rangle$ denotes the average over all possible particle trajectories.

Let us consider a simple model in which the medium consists of regions with dimensions on the order of the correlation radius a. In each correlation cell, the velocity $\mathbf{v}(\mathbf{r})$ has a definite value

$$v\sim(\langle\delta\sigma_{xy}^2\rangle)^{1/2}/a\sim\lambda\Delta\sigma_0/a$$

and a definite direction. The directions of the velocities in the different cells are specified randomly and are not correlated. Since the longitudinal diffusion is rapid, and the transverse motion is slow ($\langle\sigma_\perp\rangle\sim\lambda^2$, $v\sim\lambda$), a particle is able to travel through a significant number of correlations cells in the direction of the magnetic field before shifting noticeably in the transverse direction. Let τ be the time it takes for a particle to cross a cell in the transverse direction. If $t\leqslant\tau$, the particle will often return to its initial cell, since the one-dimensional diffusion is recurrent. This leads to a unique time correlation between the transverse displacements caused by convective transport.[3] We assume that if $t>\tau$, the probability that a particle will return to its initial cell is small (which means that the motion described by Eq. (26) is not recurrent, as is three-dimensional diffusion[7]); then there is no correlation between $\mathbf{r}_\perp(t>\tau)$ and $\mathbf{r}_\perp(t<\tau)$, and consequently $\sigma_\perp^e$ may be estimated at[3]

$$\sigma_\perp^e\sim\overline{\langle\mathbf{r}_\perp^2(\tau)}\rangle/\tau,\tag{28}$$

whence, recalling the definition of τ, we find that

$$\tau\sim a^2/\sigma_\perp^e.\tag{29}$$

Let us evaluate the convective contribution to the transverse displacement of a particle when $t\lesssim\tau$. Suppose that $N(t)$ is the number of distinct cells through which the particle succeeds in traveling in time t; then the average time that the particle spends in an individual cell is about $t/N(t)$. Since the displacements in the different cells do not correlate, we have

$$\mathbf{r}_\perp^2(t)\sim(vt/N(t))^2N(t)\sim(vt)^2/N(t),$$

whence, in view of (28), we find that

$$\delta\sigma_\perp\sim v^2\frac{\tau}{N(\tau)}.\tag{30}$$

Here $N(\tau)\sim\mathscr{L}/a$, where $\mathscr{L}$ is the distance the particle succeeds in traveling in the direction of the magnetic field in the time τ. In an infinite medium, $\mathscr{L}\sim(\sigma_0\tau)^{1/2}$, which, in view of (29) and (30), leads to the following equation[3] for $\delta\sigma_\perp$:

$$\delta\sigma_\perp\sim\lambda^2\Delta^2\left(\frac{\sigma_0}{\lambda^2\sigma_0+\delta\sigma_\perp}\right)^{1/2}\sigma_0,$$

the solutions of which in weak ($\delta\sigma_\perp<\lambda^2\sigma_0$) and strong ($\delta\sigma_\perp>\lambda^2\sigma_0$) fields coincide with (4) and (5).

Using (29) for τ, we can estimate $\mathscr{L}$ for an infinite medium[5]:

$$\mathscr{L}\sim(\sigma_0\tau)^{1/2}\sim(\sigma_0/\sigma_\perp^e)^{1/2}a\sim a/\zeta_\perp^{1/2}\gg a.\tag{31}$$

Thus, even if conditions (6) are satisfied in a sufficiently strong magnetic field, the longitudinal particle trajectory $\mathscr{L}$ can become comparable with the thickness of the specimen L_z. What happens in this case? Clearly, the answer depends on the boundary conditions imposed on Eq. (26). These conditions may be obtained from the boundary conditions imposed on the potential Ψ in the stationary problem, which are determined from (1) and (3) by specifying the current component j_n normal to the surface of the specimen. Since $\sigma_\perp^e$ is independent of the current, we may set $j_n = 0$. For longitudinal diffusion this leads as $\lambda \to 0$ to the following homogeneous boundary conditions:

$$\frac{\partial}{\partial z}\Psi|_{z=0} = \frac{\partial}{\partial z}\Psi|_{z=L_z} = 0,$$

which correspond to reflection of the particle from the surface of the specimen.[7] Consequently, the fact that the specimen is bounded in the direction of the magnetic field leads simply to finite the longitudinal diffusion of the particle.

Thus, in sufficiently strong magnetic fields we have $N(\tau) \sim L_z/a$. In light of (30) this leads to the following equation for $\delta\sigma_\perp$ in the region of the size effect:

$$\delta\sigma_\perp \approx \lambda^2\Delta^2 \frac{a}{L_z}\sigma_0 \frac{\sigma_0}{\lambda^2\sigma_0 + \delta\sigma_\perp}. \tag{32}$$

Let us consider the solutions of Eq. (32). For a weak field ($\delta\sigma_\perp < \lambda^2\sigma_0$):

$$\delta\sigma_\perp \sim \Delta^2 \frac{a}{L_z}\sigma_0. \tag{33}$$

In the region of anomalous conductivity ($\delta\sigma_\perp > \lambda^2\sigma_0$), i.e., in a strong field,

$$\delta\sigma_\perp \sim \lambda\Delta \left(\frac{a}{L_z}\right)^{1/2} \sigma_0. \tag{34}$$

Note that at $L_z \sim a$ Eqs. (33) and (34) turn into the corresponding solutions for the case of two-dimensional inhomogeneities $n = n(x, y)$ (Ref. 3).

Let us find the ranges of the magnetic field (λ) and of the longitudinal dimensions of the specimen L_z in which (4), (5), (33), and (34) hold. We note first that the solutions (4) and (5) were obtained for an infinite-medium approximation, which is valid so long as $\mathscr{L} \ll L_z$, i.e., under the condition

$$\zeta_\perp = \sigma_\perp^e/\sigma_0 > (a/L_z)^2. \tag{35}$$

In the language of time, this means that $\tau < \tau_L = L_z^2/\sigma_0$ is the time it takes for a particle to pass through the specimen along the magnetic field. Accordingly, the region of the fields in which the size effect exists is determined from the condition $\tau \gtrsim \tau_L$. this leads to the inequality

$$\zeta_\perp = \sigma_\perp^e/\sigma_0 \lesssim (a/L_z)^2. \tag{36}$$

Bearing in mind conditions (35) and (36), as well as the constraints imposed by the weak-field and strong-fields conditions, our results may be represented as follows. We associate with weak and maximally strong fields the solutions (4) and (34):

$$\delta\sigma_\perp \sim \begin{cases} \lambda\Delta^2\sigma_0, & \max(\Delta^2, a/L_z) \ll \lambda < 1, \\ \lambda\Delta(a/L_z)^{1/2}\sigma_0, & \lambda \ll \min(\lambda_1, \lambda_2), \end{cases}$$
$$\lambda_1 = \Delta^{-1}(a/L_z)^{3/2}, \quad \lambda_2 = \Delta(a/L_z)^{1/2}. \tag{37}$$

In the region of intermediate fields, either solution (5) or solution (33) is realized, depending upon the thickness of the specimen:

$$\delta\sigma_\perp \sim \begin{cases} (\lambda\Delta)^{4/3}\sigma_0, & L_z \gg L_0, \quad \lambda_1 \ll \lambda \ll \Delta^2, \\ \Delta^2 \dfrac{a}{L_z}\sigma_0, & L_z \ll L_0, \quad \lambda_2 \ll \lambda \ll a/L_z, \end{cases} \tag{38}$$

where $L_0 = a/\Delta^2$. Regions within which $\sigma_\perp^e$ behaves differently may be conveniently represented graphically in the form of a phase diagram in the $(L_z/a, \zeta_\perp^{-1})$ plane (Fig. 1).

The relations (37) and (38) seem more natural for the transverse magnetoresistance $\Delta\rho^\perp/\rho^\perp$, which is related to $\hat{\sigma}^e$ as[8]

$$\frac{\Delta\rho^\perp}{\rho^\perp} = \frac{\sigma_0\sigma_\perp^e}{(\sigma_\perp^e)^2 + (\sigma_{xy}^e)^2} - 1 \cong \frac{\delta\sigma_\perp}{\lambda^2\sigma_0}.$$

Taking into account (37) and (38) and treating $\Delta\rho^\perp/\rho^\perp$ as a function of the variable $\gamma = \lambda^{-1}\Delta^2 = (\Delta^2\mu/c)H$, we find that in thick specimens, i.e., at $L_z \gg L_0$,

$$\frac{\Delta\rho^\perp}{\rho^\perp} \sim \begin{cases} \gamma, & 1 \gg \gamma > \Delta^2, \\ \gamma^{2/3}, & \varkappa^3 \gg \gamma \gg 1, \\ \varkappa^{-1}\gamma, & \gamma \gg \varkappa^3, \end{cases} \tag{39}$$

$$\varkappa = (L_z/L_0)^{1/2} = \Delta(L_z/a)^{1/2},$$

in thin specimens at $L_z \ll L_0$,

$$\frac{\Delta\rho^\perp}{\rho^\perp} \sim \begin{cases} \gamma, & \varkappa^2 \gg \gamma > \Delta^2, \\ \varkappa^{-2}\gamma^2, & \varkappa \gg \gamma \gg \varkappa^2 \\ \varkappa^{-1}\gamma, & \gamma \gg \varkappa, \end{cases} \tag{40}$$

and at $L_z \cong L_0$, regardless of the field strength,

$$\Delta\rho^\perp/\rho^\perp \sim \gamma. \tag{41}$$

These relations are depicted in Fig. 2.

It is clear from (39)–(41) that, from a study of the dependence of $\Delta\rho^\perp/\rho^\perp$ on H and L_z, it is possible in principle to assess not only the size Δ of the inhomogeneities but also their dimension a (and of the correlation radius when there are several inhomogeneity scales). For this purpose, we require measurements in both weak fields (this yields Δ) and in strong fields (which makes it possible to determine a/L_z if Δ is known). Thin specimens are preferable here, since in this case weaker fields are required.

The solution obtained for the problem by means of the diffusion analogy is in fact not rigorous (we will return to this question later). Let us show that the exact solution in the region of the size effect differs nevertheless from (34) only by a numerical coefficient (we limit ourselves to the case of strong fields, since ordinary perturbation theory is applicable to weak fields). To explain the properties of the exact solution, we will use an expansion of $\delta\sigma_\perp$ in the series of the renormalized perturbation theory [Eqs. (A21) and (A22)].

Let us first consider the solution of the truncated equation for $\delta\sigma_\perp$, which is obtained if we limit ourselves to the first graph in the diagram expansion (A22). This leads to the following equation for S^k:

$$S^{\mathbf{k}}=\frac{\lambda^2}{2}\sum_{\mathbf{k}_1}\frac{k_{1\perp}{}^2}{k_{1z}{}^2+(\lambda^2+S^{\mathbf{k}})k_{1\perp}{}^2}|\xi^{\mathbf{k}-\mathbf{k}_1}|^2.$$

The correlation between the longitudinal and transverse motions manifests itself analytically in the presence of singular integrations with respect to k_{1z}. At first glance, therefore, we can take into account the face that the specimen is bounded by cutting off the integration with respect to k_{1z} at small $|k_{1z}|\sim L_z{}^{-1}$. However, before passing on to the integration, it is necessary to separate from the sum the term with $k_{1z}=0$, which also makes the main contribution to the effect. Since the main contribution to the sum is produced by the momentum domain $k\lesssim k_0\sim a^{-1}$ and since $S^{\mathbf{k}}\sim\delta\zeta_\perp$ when $k\lesssim k_0$, and in view of (A25), we obtain the following equation for $\delta\zeta_\perp$:

$$\delta\zeta_\perp\sim\lambda^2\Delta^2\frac{a}{L_z}\frac{1}{\lambda^2+\delta\zeta_\perp}+\lambda^2\Delta^2\int\limits_{a/L_z}^{\infty}dx\frac{1}{x^2+\lambda^2+\delta\zeta_\perp}. \quad (42)$$

Condition (35) corresponds to the approximation of an infinite medium. In this case, the first term in (42) is not essential and the solution coincides with previous results.[1,3] The region of the size effect is determined by inequality (36). In this case, the second term $\sim\lambda^2\Delta^2 L_z/a$ in (42) is small by comparison with the first term. Neglecting it we obtain (32).

It is easily verified that the main contribution to the region of the size effect in the remainder of expansion (A22) is made by terms with zero values of the longitudinal inner momenta, $k_{iz}=0$. Here it is sufficient to take into account expansion terms that contain only paired correlators, since graphs with irreducible correlators of higher order contain extra powers of the small parameter a/L_z. As a result, we obtain the following equation for $\delta\zeta_\perp$:

$$\sum_{n\geqslant 1}(-1)^{n-1}f_n\left(\frac{\lambda\Delta(a/L_z)^{1/2}}{\delta\zeta_\perp}\right)^{2n}=1,$$

where $f_n>0$ are numerical coefficients that are determined by integration with respect to the angular variables and by the topological properties of the diagrams (as in the case of an infinite medium, the strong coupling case is realized and $f_n\sim n!$ at $n\gg 1$). The substitution $\delta\zeta_\perp=A\lambda\Delta(a/L_z)^{1/2}$ yields for the coefficient A an equation that does not contain the essential variables λ, Δ, and a/L_z. Thus, for random fields, the exact solution in the region of the size effect differs from (34) only by a numerical factor.

Let us trace in more detail the relation between the methods of the diffusion analogy and the methods of renormalized perturbation theory. For this purpose, we use for $\delta\sigma_\perp$ the exact expression which follows from (27) (Ref. 3):

$$\delta\sigma_\perp=\int\limits_0^\infty dt\mathscr{K}(t),\quad \mathscr{K}(t_1-t_2)=\frac{1}{2}\overline{\langle\mathbf{v}(\mathbf{r}(t_1)\mathbf{v}(\mathbf{r}(t_2))\rangle}. \quad (43)$$

The role of the distribution function of particle trajectories is assumed by the Green's function (26), which may be represented in the form of a perturbation-theory series in the operator $-(\mathbf{v}\cdot\nabla)$. Substituting this series in (43) we obtain after averaging over the ensemble the expansion (A22) for $\delta\sigma_\perp$. Note that in the general case the averaging over the ensemble average must follow path averaging over the trajectories.

As already noted, the reasoning used in deriving Eq. (32) is inexact, since it does not take into account effects corresponding to higher terms of the expansion (A22). This is due to our assumption that the motion of the particle described by equation (26) is not recurrent. In fact, if the particle's return to the initial cell may be ignored when $t>\tau$, when $\mathscr{K}(t>\tau)=0$, which, in view of (29), leads to equation (32). The motion of the particle is actually recurrent and higher terms of the expansion (A22) correspond precisely to allowance for the random walk of the particle and its return to the initial cell. The fact that this leads only to a numerical renormalization of $\delta\sigma_\perp$ indicates that the effective transverse diffusion coefficient is formed over times $t\lesssim\tau$.

§5. CONCLUSION

The results above pertain to the case $n=n(x,y,z)$. The results for two- and one-dimensional inhomogeneities differ from (37)–(41) (see footnote 4). In a medium with two-dimensional inhomogeneities stretched along the magnetic field $[n=n(x,y)]$, the influence of inhomogeneities in weak fields ($\Delta\ll\lambda=(\mu H/c)^{-1}<1$) is described by the Herring correction for the two-dimensional case $\delta\sigma_\perp\sim\Delta^2\sigma_0$ (Ref. 1), and in strong fields ($\lambda\ll\Delta$), anomalous conductivity ($\delta\sigma_\perp\gg\langle\sigma_\perp\rangle$) takes place, with $\delta\sigma_\perp\sim\lambda\Delta\sigma_0$ (Refs. 3 and 9). In this case there is no size effect.

For a medium with $n=n(x,z)$ or with $n(x)$, the inhomogeneities affect only the conductivity along the Y-axis, and $\sigma^e_{xx}=\langle\sigma_\perp\rangle$. At $n=n(x)$ Herring's[1] correction $\delta\sigma_{yy}$ for the inhomogeneity is then exact,[10,11] while at $n=n(x,z)$, the applicability of (4) is limited by the size effect, i.e., by the condition $a/L_z\ll\lambda<1$. If $\lambda\ll a/L_z$ we have in this case

$$\delta\sigma_{yy}\sim\Delta^2(a/L_z)\sigma_0 \text{ and } \Delta\rho_{yy}/\rho^\perp\sim\Delta^2(a/L_z)(\mu H/c)^2.$$

The case of a quantizing magnetic field, in which $\omega_H=|e|\cdot H/mc>\max(T,\varepsilon_f)$, also requires special treatment, since the quantum effects influence the field dependences of the diagonal components of the local conductivity tensor $\hat{\sigma}(\mathbf{r})$,[12] as a consequence of which the $\sigma^e_\perp(H,L_z)$ dependences may differ from the dependence obtained above.

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APPENDIX I

1. From the definitions (12) of $W^{\mathbf{kk}'}$ and the definition of (13) of $\gamma^{\mathbf{kk}'}$ it follows that

$$W^{\mathbf{kk}'}=W^{\mathbf{kk}'}_{\alpha\beta}n_\alpha n_\beta{}',\quad \mathbf{n}=\mathbf{k}/k, \quad (A1)$$

and similarly for $W^{\mathbf{kk}'}_n$.

2. To derive a number of relations used in the paper, we first obtain equations that relate $W(\mathbf{p})$ to W, and $W(\mathbf{p},\mathbf{p}')$ to $W(\mathbf{p})$ and $W(\mathbf{p}')$. Let us consider $W^{\mathbf{kk}'}_n$ [see (12)]. We separate in W_n the term with $\mathbf{k}_1=\mathbf{p}$:

$$W_n^{\mathbf{kk}'}=\gamma^{\mathbf{kp}}g^{\mathbf{p}}W_{n-1}^{\mathbf{pk}'}+\sum_{\mathbf{k}_1\neq\mathbf{p}}\gamma^{\mathbf{kk}_1}g^{\mathbf{k}_1}W_{n-1}^{\mathbf{k}_1\mathbf{k}'},$$

and then separate from the remaining terms with $\mathbf{k}_1\neq\mathbf{p}$ the term with $\mathbf{k}_2=\mathbf{p}$. Continuing this procedure until the term with $\mathbf{k}_{n-1}=\mathbf{p}$ is separated, we obtain as a result

$$W_n^{\mathbf{kk}'}=W_n^{\mathbf{kk}'}(\mathbf{p})+\sum_{i=1}^{n-1}W_i^{\mathbf{kp}}(\mathbf{p})g^{\mathbf{p}}W_{n-i}^{\mathbf{pk}'}. \tag{A2}$$

Summing (A1) with respect to n and bearing in mind that

$$\sum_{n=1}^{\infty}\sum_{i=1}^{n-1}W_i(\mathbf{p})W_{n-i}=\sum_{i=1}^{\infty}W_i(\mathbf{p})\sum_{n\geqslant i+1}W_{n-i},$$

we find [see (15)]

$$W^{\mathbf{kk}'}=W^{\mathbf{kk}'}(\mathbf{p})+R^{\mathbf{kp}}g^{\mathbf{p}}W^{\mathbf{pk}'}. \tag{A3}$$

Repeating this process from right to left (starting with $\mathbf{k}_{n-1}$), we obtain

$$W^{\mathbf{kk}'}=W^{\mathbf{kk}'}(\mathbf{p})+W^{\mathbf{kp}}g^{\mathbf{p}}L^{\mathbf{pk}'}. \tag{A4}$$

Analogously, we derive relations connecting $W(\mathbf{p},\mathbf{p}')$ with $W(\mathbf{p})$ and $W(\mathbf{p}')$. If $\mathbf{p}'\neq\mathbf{p}$, we find that

$$W^{\mathbf{kk}'}(\mathbf{p}')=W^{\mathbf{kk}'}(\mathbf{p},\mathbf{p}')+R^{\mathbf{kp}}(\mathbf{p}')g^{\mathbf{p}}W^{\mathbf{pk}'}(\mathbf{p}'), \tag{A5}$$

$$W^{\mathbf{kk}'}(\mathbf{p})=W^{\mathbf{kk}'}(\mathbf{p},\mathbf{p}')+W^{\mathbf{kp}'}(\mathbf{p})g^{\mathbf{p}'}L^{\mathbf{p}'\mathbf{k}'}(\mathbf{p}). \tag{A6}$$

3. Let us consider the corollaries of (A3) and (A4). Setting $\mathbf{p}=\mathbf{k}$ and $\mathbf{p}=\mathbf{k}'$ successively and bearing in mind the definitions of R, L, and S [see (15) and above], as well as the definition of $G^{\mathbf{k}}$, we obtain

$$W^{\mathbf{k}}=\frac{S^{\mathbf{k}}}{1-g^{\mathbf{k}}S^{\mathbf{k}}},\quad G^{\mathbf{k}}\equiv(1+g^{\mathbf{k}}W^{\mathbf{k}})g^{\mathbf{k}}=\frac{g^{\mathbf{k}}}{1-g^{\mathbf{k}}S^{\mathbf{k}}},$$

$$g^{\mathbf{k}}W^{\mathbf{k}}=G^{\mathbf{k}}S^{\mathbf{k}}, \tag{A7}$$

$$g^{\mathbf{k}}\tilde{W}^{\mathbf{kk}'}=G^{\mathbf{k}}\tilde{L}^{\mathbf{kk}'},\quad \tilde{W}^{\mathbf{kk}'}g^{\mathbf{k}'}=\tilde{R}^{\mathbf{kk}'}G^{\mathbf{k}'}. \tag{A8}$$

Let us consider the corollaries of (A5) and (A6). From (A5) with $\mathbf{p}'=\mathbf{k}=\mathbf{k}'\neq\mathbf{p}$ it follows that

$$W^{\mathbf{k}}(\mathbf{p})=\frac{S^{\mathbf{k}}(\mathbf{p})}{1-g^{\mathbf{k}}S^{\mathbf{k}}(\mathbf{p})},$$

$$G^{\mathbf{k}}(\mathbf{p})\equiv(1+g^{\mathbf{k}}W^{\mathbf{k}}(\mathbf{p}))g^{\mathbf{k}}=\frac{g^{\mathbf{k}}}{1-g^{\mathbf{k}}S^{\mathbf{k}}(\mathbf{p})}. \tag{A9}$$

Setting $\mathbf{p}=\mathbf{k}$ and $\mathbf{p}'=\mathbf{k}'$ in (A5) and (A6) and bearing in mind the definition (15) of $\tilde{\Gamma}$, we find that

$$g^{\mathbf{k}}\tilde{R}^{\mathbf{kk}'}=G^{\mathbf{k}}(\mathbf{k}')\Gamma^{\mathbf{kk}'},\quad \tilde{L}^{\mathbf{kk}'}g^{\mathbf{k}'}=\Gamma^{\mathbf{kk}'}G^{\mathbf{k}'}(\mathbf{k}). \tag{A10}$$

4. Let us find $G^{\mathbf{k}}(\mathbf{p})$ when $\mathbf{p}\neq\mathbf{k}$. Setting $\mathbf{k}=\mathbf{k}'\neq\mathbf{p}$ in (A3), we have

$$W^{\mathbf{k}}=W^{\mathbf{k}}(\mathbf{p})+\tilde{R}^{\mathbf{kp}}g^{\mathbf{p}}\tilde{W}^{\mathbf{pk}}. \tag{A11}$$

Using (A7), (A8), and (A9), we may transform (A11) to the form

$$G^{\mathbf{k}}=G^{\mathbf{k}}(\mathbf{p})+g^{\mathbf{k}}\tilde{R}^{\mathbf{kp}}\tilde{L}^{\mathbf{pk}}g^{\mathbf{k}}G^{\mathbf{p}}=G^{\mathbf{k}}(\mathbf{p})+\Gamma^{\mathbf{kp}}\Gamma^{\mathbf{pk}}G^{\mathbf{p}}[G^{\mathbf{k}}(\mathbf{p})]^2,$$

whence we find that

$$G^{\mathbf{k}}(\mathbf{p})=\frac{2}{1+(1+4\tilde{N}^{\mathbf{kp}})^{1/2}}G^{\mathbf{k}},\quad \tilde{N}^{\mathbf{kp}}=G^{\mathbf{k}}G^{\mathbf{p}}\tilde{\Gamma}^{\mathbf{kp}}\tilde{\Gamma}^{\mathbf{pk}}. \tag{A12}$$

5. Let us derive an expression for $G^{\mathbf{k}}(\mathbf{k}',\mathbf{p})\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})$. For this purpose, we rewrite (A3) and (A4) in the form

$$\tilde{W}^{\mathbf{kk}'}(\mathbf{p})=\tilde{W}^{\mathbf{kk}'}-\tilde{W}^{\mathbf{kp}}g^{\mathbf{p}}\tilde{L}^{\mathbf{pk}'},\quad \tilde{W}^{\mathbf{kk}'}(\mathbf{p})=\tilde{W}^{\mathbf{kk}'}-\tilde{R}^{\mathbf{kp}}g^{\mathbf{p}}\tilde{W}^{\mathbf{pk}'}.$$

We multiply both equations by $g^{\mathbf{k}}g^{\mathbf{k}'}$ and take into account the fact that

$$g^{\mathbf{k}}\tilde{W}^{\mathbf{kk}'}g^{\mathbf{k}'}=G^{\mathbf{k}}(\mathbf{k}')\Gamma^{\mathbf{kk}'}G^{\mathbf{k}'}=G^{\mathbf{k}}\Gamma^{\mathbf{kk}'}G^{\mathbf{k}'}(\mathbf{k})$$

follows from (A8) and (A10). As a result, we find

$$G^{\mathbf{k}}(\mathbf{k}',\mathbf{p})\Gamma^{\mathbf{kk}'}(\mathbf{p})=\frac{G^{\mathbf{k}'}(\mathbf{k})}{G^{\mathbf{k}'}(\mathbf{p})}G^{\mathbf{k}}\Gamma^{\mathbf{kk}'}-G^{\mathbf{k}}(\mathbf{p})\Gamma^{\mathbf{kp}}G^{\mathbf{p}}\Gamma^{\mathbf{pk}'} \tag{A13}$$

and the conjugate equation

$$\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})G^{\mathbf{k}'}(\mathbf{k},\mathbf{p})=\Gamma^{\mathbf{kk}'}G^{\mathbf{k}'}\frac{G^{\mathbf{k}}(\mathbf{k}')}{G^{\mathbf{k}}(\mathbf{p})}-\Gamma^{\mathbf{kp}}G^{\mathbf{p}}\Gamma^{\mathbf{pk}'}G^{\mathbf{k}'}(\mathbf{p}). \tag{A14}$$

APPENDIX II

1. Dyson equation for randomly inhomogeneous medium

In passing to a randomly inhomogeneous medium, Eqs. (21) and (22) are replaced by another form of the equation for the exact vertex $\tilde{\Gamma}$. Taking into account the definitions of $\tilde{R}$ and $\tilde{\Gamma}$ from (15) as well as (17), we find that

$$\Gamma^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}g^{\mathbf{k}_1}\tilde{R}^{\mathbf{k}_1\mathbf{k}'}(\mathbf{k}),$$

whence, repeating the derivation of (19) for $\tilde{R}(\mathbf{p})$, we obtain

$$\Gamma^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}(\mathbf{k},\mathbf{k}')\Gamma^{\mathbf{k}_1\mathbf{k}'}(\mathbf{k}) \tag{A15}$$

and similarly for the conjugate equation. In view of (A13) for $G^{\mathbf{k}}(\mathbf{k}',\mathbf{p})\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})$, equation (A15) is entirely equivalent to Eq. (21) for $\tilde{\Gamma}$.

For a randomly inhomogeneous medium the Fourier components of the density fluctuations are statistically small:

$$|\delta n^{\mathbf{k}}|^2\underset{V\to\infty}{\sim}O\left(\frac{1}{V}\right),$$

and consequently the bare vertex γ is statistically small. Therefore, it is natural to impose the asymptotic conditions

$$\lim_{V\to\infty}S^{\mathbf{k}}\sim O(1),\quad \lim_{V\to\infty}\Gamma^{\mathbf{kk}'}=0 \tag{A16}$$

on the solution of the Dyson equations for a randomly inhomogeneous medium. It can be shown that S and $\tilde{\Gamma}$ satisfy Eqs. (A16) for a set of realizations of the random field $n(\mathbf{r})$ such that the Fourier components of the quantities $\eta_n(\mathbf{r})=\xi^n(\mathbf{r})-\langle\xi^n\rangle$ are statistically small, i.e.,

$$|\eta_n^{\mathbf{k}}|^2\underset{V\to\infty}{\sim}O(1/V),$$

where $\xi(\mathbf{r})=\delta n(\mathbf{r})/\langle n\rangle$.

By means of conditions (A16) it is possible to markedly simplify the Dyson equations. Let us consider the quantities $G^{\mathbf{k}}(\mathbf{p})$ and $G^{\mathbf{k}}(\mathbf{k}',\mathbf{p})\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})$ defined in (A12) and (A13). From

(A16) it follows that, if the system is sufficiently large, $\tilde{N}$ in (A12) may be ignored. As a result, we obtain $G^{\mathbf{k}}(p)=G^{\mathbf{k}}, G^{\mathbf{k}}(\mathbf{k}',\mathbf{p})\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})=G^{\mathbf{k}}\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})=G^{\mathbf{k}}[\tilde{\Gamma}^{\mathbf{kk}'}-\tilde{\Gamma}^{\mathbf{kp}}G^{\mathbf{p}}\tilde{\Gamma}^{\mathbf{pk}'}]$, as a consequence of which the Dyson equations (20), (21), and (A15) assume the form

$$S^{\mathbf{k}}=\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}\tilde{\Gamma}^{\mathbf{k}_1\mathbf{k}},\quad G^{\mathbf{k}}=-\frac{\bar{\Delta}(\mathbf{k})}{n_z{}^2+\lambda^2+S^{\mathbf{k}}},\tag{A17}$$

$$\tilde{\Gamma}^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{\mathbf{k}_1}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}\tilde{\Gamma}^{\mathbf{k}_1\mathbf{k}'}(\mathbf{k}),\tag{A18}$$

$$\tilde{\Gamma}^{\mathbf{kk}'}(\mathbf{p})=\tilde{\Gamma}^{\mathbf{kk}'}-\tilde{\Gamma}^{\mathbf{kp}}G^{\mathbf{p}}\tilde{\Gamma}^{\mathbf{pk}'}.\tag{A19}$$

Equations (A17)–(A19) cannot be used for determining directly the asymptotic behavior of $S^{\mathbf{k}}$ as $\lambda\to 0$, since they contain the strongly fluctuating exact vertex $\tilde{\Gamma}$. Therefore, it is necessary to first eliminate $\tilde{\Gamma}$ from (A17) and obtain a closed equation for $S^{\mathbf{k}}$.

2. Derivation of equation for $S^{\mathbf{k}}$. Relation between the Dyson equations and the Dreizin and Dykhne expansion for $\delta\sigma(\mathbf{k})$

Let us find an explicit expression for $\tilde{\Gamma}$ by means of Eqs. (A18) and (A19). For this purpose we solve these equations by successive approximations at fixed G. In each approximation, two types of terms will appear: those which make a contribution to $S^{\mathbf{k}}$ that does not vanish as $V\to\infty$, and also free terms which depend anomalously on the magnetic field (λ) and on the volume and which do not contribute to $S_{\mathbf{k}}$ as $V\to\infty$. Ignoring the anomalous terms, we obtain

$$\tilde{\Gamma}^{\mathbf{kk}'}=\gamma^{\mathbf{kk}'}+\sum_{n\geqslant 2}\tilde{\mathscr{F}}_n^{\mathbf{kk}'},$$

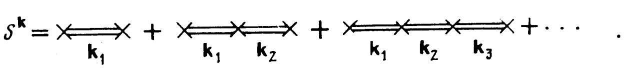
$$\tilde{\mathscr{F}}_n^{\mathbf{kk}'}=\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_{n-1}\\(\mathbf{k}_i\neq\mathbf{k}_j,\mathbf{k},\mathbf{k}')}}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}\gamma^{\mathbf{k}_1\mathbf{k}_2}\ldots G^{\mathbf{k}_{n-1}}\gamma^{\mathbf{k}_{n-1}\mathbf{k}'}.\tag{A20}$$

Substituting next (A20) in expression (A17) for $S^{\mathbf{k}}$ we find

$$S^{\mathbf{k}}=\sum_{n\geqslant 2}\mathscr{F}_n{}^{\mathbf{k}},\quad \mathscr{F}_n{}^{\mathbf{k}}=\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_{n-1}\\(\mathbf{k}_i\neq\mathbf{k}_j,\mathbf{k})}}\gamma^{\mathbf{kk}_1}G^{\mathbf{k}_1}\gamma^{\mathbf{k}_1\mathbf{k}_2}\ldots G^{\mathbf{k}_{n-1}}\gamma^{\mathbf{k}_{n-1}\mathbf{k}}.\tag{A21}$$

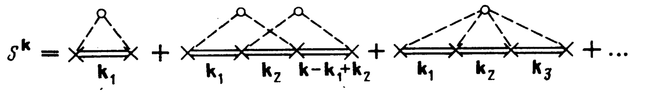
Comparing (A20) and (A21) to the initial expressions (15) and (12) for $\tilde{\Gamma}$ and S we see that the new expressions have the same structure, though with $g^{\mathbf{k}_i}$ replaced by $G^{\mathbf{k}_i}$ and with the additional constraints $\mathbf{k}_i\neq\mathbf{k}_j$ imposed on the internal momenta. The latter appear upon iteration of Eqs. (A18) and (A19), first because $\tilde{\Gamma}(\mathbf{p})$ and $\tilde{\Gamma}$ are not diagonal, and second because of the second term in $\tilde{\Gamma}(\mathbf{p})$ in (A19), which cancels some of the terms with $\mathbf{k}_i=\mathbf{k}_j$.

Graphically, the expansion (A21) for $S^{\mathbf{k}}$ may be represented as follows:



We associate γ with the crosses on the graphs, and $G^{\mathbf{k}_i}$ with the double lines; summation (under the constraint $k_i\neq k_j,k$) is performed over all internal momenta $\mathbf{k}_i$. Whenever $\gamma=\gamma_a$, there are no graphs with an odd number of crosses.

In passing from summation with respect to $\mathbf{k}_i$ to integration, it is first necessary to determine the contributions $\sim O(1)$ of the values of $\mathbf{k}_i$ corresponding to the conditions $\mathbf{q}_{\alpha^1}+\ldots+\mathbf{q}_{\alpha_m}=0$, where $\mathbf{q}_i=\mathbf{k}_i-\mathbf{k}_{i-1}$, which is equivalent to the expansion of $\delta\sigma(\mathbf{k})$ in terms of irreducible correlators in Ref. 3. The result may be represented graphically in the form



(A22)

Allowance for only compact graphs in (A22) corresponds to the conditions $\mathbf{k}_i\neq\mathbf{k}_j$ in (A21). Taking into account (A17), the expansion (A22) is the sought equation for $S^{\mathbf{k}}$.

Both (A22) and the expansion for $\delta\bar{\sigma}(\mathbf{k})$ given in Ref. 3 are series exapnsions in the irreducible momenta $\xi(\mathbf{r})$ and differ only in the averaging method. In (A22) the series is $\langle\xi^2\rangle,\langle\xi^4\rangle-\langle\xi^2\rangle^2$, etc., and in Ref. 3 $\overline{\xi^2}$, $\overline{\xi^4}-\overline{\xi^2}^2$, etc. Thus, under the assumption the averagings over the volume and over the ensemble are equivalent, the expansion (A22) for $S^{\mathbf{k}}$ turns into the expansion for $\overline{S}^{\mathbf{k}}=\overline{\delta\sigma(\mathbf{k})}/\sigma_0$ as $V\to\infty$, which means that $S^{\mathbf{k}}$ is self-averaged.

Let us now consider the validity of (A22) or, what is the same thing, of the Dreĭzin and Dykhne expansion for the description of a system with finite dimensions. For this purpose, it is necessary to evaluate the contribution of the anomalous terms discarded in the course of deriving (A20) and (A21). These terms appear in the course of the iterations, starting with terms in γ. The anomalous part of $\tilde{\Gamma}^{(3)}$ has the form (at $\gamma=\gamma_a$)

$$|\gamma_a^{\mathbf{kk}'}|^2\gamma_a^{\mathbf{kk}'}G^{\mathbf{k}}G^{\mathbf{k}'},$$

which leads to a correction biquadratic in γ (quadratic in Δ^2) to $S^{\mathbf{k}}$:

$$\sum_{\mathbf{k}_1}|\gamma_a^{\mathbf{kk}_1}|^4(G^{\mathbf{k}_1})^2G^{\mathbf{k}}.\tag{A23}$$

Taking into account the inequlity (A26) and comparing (A23) with the term quadratic in γ in (A21), we find that (A23) differs from it by a factor of the order of

$$\delta=\left(\frac{\lambda\Delta\sigma_0}{\sigma_\perp{}^e}\right)^2\frac{a^3}{V}.\tag{A24}$$

It can be shown that the anomalous contributions of higher order are corrections $\propto\delta^m$ (with $m\geqslant 1$) to the terms of the expansion (A21).

From (A24) it follows that δ depends on the magnetic field, and that the nature of this dependence is determined by the behavior of $\sigma_\perp^e(H)$. Therefore, strictly speaking, the expansions (A21) and (A22) are suitable for studying the asymptotic behavior of $\sigma_\perp^e(H)$ as $H\to\infty\,(\lambda\to 0)$ only in the limit as $V\to\infty$. Let us show, however, that the solutions obtained by means of the expansion (A22) are valid also for systems with finite volume. To estimate the value of δ, we will use the solution for $\sigma_\perp^e(H)$ found by neglecting the anomalous contributions. Using (37) and (38), we find that in "weak" fields,

$$\delta\sim\lambda^{-1}\Delta^2a^3/V<a^3/V.$$

Let us consider the case of "strong" fields. Within the range of applicability of the 4/3 law, taking into account (5) and (31) we find that

$$\delta\sim\frac{a^2}{V_\perp}\frac{\mathscr{L}\sigma_0}{L_z\sigma_\perp{}^e}\sim\frac{a^2}{V_\perp}\frac{\mathscr{L}(\mathrm{H})}{L_z}<\frac{a^2}{V_\perp},$$

where $V_\perp=L_xL_y$. In the region of the dimensional effect ($\mathscr{L}\gtrsim L_z$), taking (34) into account, we obtain $\delta\sim a^2/V_\perp$, i.e., δ no longer is a function of H. Thus, δ will be small if conditions (6) are satisfied, which justifies our use of the expansion (A22) for systems of finite dimensions.

APPENDIX 3

Let us obtain estimates for certain quantities used in the body of the article. Consider the correlator of the density fluctuations δn (more precisely, of the quantities $\xi=\delta n/\langle n\rangle$)

$$C(\mathbf{r})=\int\frac{d\mathbf{R}}{V}\xi\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right)\xi\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right)=\sum_{\mathbf{k}}e^{i\mathbf{k}\mathbf{r}}|\xi^{\mathbf{k}}|^2.$$

$C(\mathbf{r})$ possesses the obvious properties

$$C(0)=\Delta^2,\quad \int d\mathbf{r}C(\mathbf{r})=0.$$

We will assume that $|C(\mathbf{r})|$ at $r>a$ and $|\xi^{\mathbf{k}}|^2$ at $k>k_0=a^{-1}$ decrease quite rapidly to zero; then the following estimates hold:

$$\sum_{\mathbf{k}_\perp}k_\perp|\xi^{\mathbf{k}_\perp}|^2\approx\Delta^2,\quad \sum_{\mathbf{k}_\perp}|\xi^{\mathbf{k}_\perp}|^2\approx\Delta^2\frac{a}{L_z},\tag{A25}$$

$$|\xi^{\mathbf{k}}|^2\leqslant\Delta^2\frac{a^3}{V}.\tag{A26}$$

[1]For the sake of brevity, we define the electric field as $E=-\nabla\Psi$, where Ψ is the electrochemical potential.

[2]More precisely, the quantity $S^{\mathbf{k}}=\delta\sigma(\mathbf{k})/\sigma_0$.

[3]Since $n(\mathbf{r})$ occurs in (1)–(3), it may seem that the Poisson equation $\Delta\varphi=-4\pi en$ should be added. In the steady-state, however, this is unnecessary, since the equations are linear in the electric field. In fact, allowance for the change of the volume charge density when an external field is applied leads in (1)–(3) to nonlinear corrections to the current.

[4]Note that in a laminar medium (one-dimensional inhomogeneities), $\gamma_a^{hh'}$, and likewise $S_{\mathbf{k}}$, are identically zero and $\delta\sigma_\perp$ is described by the Herring correction, and similarly for $n=n(x,z)$ and $n(y,z)$.

[5]It can be shown that in the conductivity problem, $\mathscr{L}$ has the meaning of the longitudinal correlation length for the current fluctuations $\delta\mathbf{j}(\mathbf{r})$, whereas the transverse correlation length is on the order of a.

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