Electromagnetic generation of sound in a metal plate in a perpendicular magnetic field

F. L. Fal'ko

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences
(Submitted 25 December 1982)
Zh. Eksp. Teor. Fiz. **85**, 300–310 (July 1983)

Electromagnetic generation of sound (EGS) is considered in a metal plate in a perpendicular magnetic field **H** for a complicated electron dispersion law. The electron reflection from the boundaries is assumed to be specular. A model is used in which the EGS is due to mechanisms of conduction-electron interaction with the lattice. It is shown that it is best to observe the resonant generation effects in plates having a thickness d such that the inequalities $L_{ac} > d > l, L_b, \delta$ are satisfied (L_{ac} is the sound damping length, l is the electron mean free path, L_b is the damping length of the natural electromagnetic mode of the metal, and δ is the depth of the skin layer). If the Fermi surfaces of the metal have inflection sections, the amplitude $\mathbf{u}(H)$ of the excited sound undergoes periodic drastic changes of the Doppler-shifted cyclotron resonance. The features of this resonance in EGS are investigated. It is shown that under certain conditions the EGS is accompanied by a Doppler-phonon resonance. In strong magnetic fields, when $\mathbf{q}R \ll 1$ ($\mathbf{q}$ is the wave vector of the sound and R is the electron Larmor radius), the character of the $\mathbf{u}(H)$ dependence is determined not by the shape of the Fermi surface, but by whether the metal is compensated or not. In an uncompensated metal in the classical region of strong fields H, the function $|u(H)|$ varies linearly. In a compensated metal the function $|u(H)|$ reaches at $H = H_{ext}$ a maximum value. The position of the maximum of $|u(H)|$ depends on the electron-collision frequency and varies with temperature.

PACS numbers: 43.35.Rw, 72.55. + s

1. Electromagnetic generation of sound in metals in a magnetic field is due to two interaction mechanisms between the conduction electrons and the lattice: introduction and deformation. The first is the averaged Lorentz force, and the second is determined by direct deformational interaction. In a magnetic field **H** perpendicular to the sample surface, sound generation was theoretically investigated in a number of papers.[1–7] It was shown that allowance for only the induction mechanism leads to a linear dependence of the amplitude **u** of the excited sound on H.[1] Such a dependence takes place in strong fields H, where $qR \ll 1$ (q is the wave vector of the sound and R is the Larmor radius of the electron). In weak fields, $qR > 1$, the contribution of the deformation mechanism to the generation is the principal one and causes a substantial deviation of $\mathbf{u}(H)$ from linearity,[3] as observed in potassium.[8–10] Electromagnetic generation of sound was recently investigated experimentally in metals with complex Fermi surface, such as tungsten,[11–13] tin,[14] and others. In weak fields H, corresponding to the region of existence of Doppler-shifted cyclotron resonance (DSCR) and Doppler-phonon resonance (DPR), sharply peaked singularities were observed in $\mathbf{u}(H)$ of tungsten.[11–13] Substantial deviation of the sound amplitude from linearity were observed in strong fields in tungsten[13] and in tin.[14] These effects cannot be obtained by results[1–5] obtained for alkali metals, and were not considered in the theory[6] for aluminum. The present communication is devoted to a theoretical study of electromagnetic generation of sound in a perpendicular magnetic field in a metal plate with a complex dispersion law $\varepsilon(\mathbf{p})$. The singularities of the manifestation of DSCR in electromagnetic generation of sound are investigated. It is shown that resonant interaction with weakly damped waves (dopplerons, DPR)[15] should be observed also in electromagnetic generation of sound in metals.

In strong fields ($qR \ll 1$) the character of the $\mathbf{u}(R)$ dependence is determined not by the shape of the Fermi surface, but by whether the metal is compensated or not. For uncompensated metals $\mathbf{u}(H)$ is linear in a wide field interval. For compensated metals in the same H interval, the amplitude first increases linearly with increasing H, reaches a maximum value at $H = H_{ext}$, and then decreases. The position of the maximum of the amplitude H_{ext} depends on the electron-collision frequency and consequently on the temperature.

2. Propagation of electromagnetic and acoustic waves in a metal and their mutual transformation are described by a system of equations comprising Maxwell's equations, the linearized kinetic equation, and the lattice-vibration equations (see, e.g., Ref. 3). We choose a coordinate frame in which the z axis is normal to the surface of a plate occupying the space $0 \leqslant z \leqslant d$; the wave propagation direction and the vector of the external constant and uniform-magnetic-field vector **H** are parallel to the z axis; the x axis coincides with the electric-field vector of the electromagnetic wave in vacuum. The boundary conditions for the system of equations are the following: continuity of the tangential components of the alternating electric and magnetic fields on the surfaces $z = 0$ and $z = d$; equality of the voltages on these surfaces to zero; specular reflection of the electrons from the boundaries. If electromagnetic generation of sound is considered outside the region of the sound coupling with the natural electromagnetic modes of the metal, the wave transformation coefficient T is small in terms of the parameter m/M (m and M

 0038-5646/83/070175-06$04.00 © 1984 American Institute of Physics

are the masses of the electron and ion). The system is solved by successive approximations in T. The solution so obtained is valid also in the case of weak coupling of the waves, but is not valid for strong coupling.

If the wave propagates along a many-fold symmetry axis of the crystal, the longitudinal and two transverse modes of the sound separate. In a perpendicular magnetic field the equations and their solution can be made simpler by introducing the circular polarization of the vectors: $A_\pm = A_x \pm iA_y$. Solving the system of equations for a plate of thickness d, we obtain the following expression for the amplitude $u_\pm$ of circularly polarized sound excited by the electromagnetic wave:

$$u_s(z,H)$$

$$=-\frac{iHq_s^2 \operatorname{sgn} s}{2\pi c\rho\omega^2}\int_{-\infty}^{\infty} dk\mathscr{E}_s(k)\frac{\sigma_s(k)}{k^2-q_s^2}\left[e^{ikz}+i\frac{k}{q_s}\frac{\cos q_s z}{\sin q_s d}e^{ikd}\right]$$

$$-\frac{iq_s^2}{2\pi\rho\omega^2}\int_{-\infty}^{\infty} dk\mathscr{E}_s(k)\frac{k\eta_s(k)}{k^2-q_s^2}\left[e^{ikz}+i\frac{q_s}{k}\frac{\cos q_s z}{\sin q_s d}e^{ikd}\right]. \quad (1)$$

In this equation the first term stems from the induction force in the equation for the lattice vibrations, and the second from the deformation source. The second terms in the square brackets of (1) take into account the finite thickness of the plate and the sound reflection from the faces $z=0$ and $z=d$. The following notation was introduced: the subscript s denotes "+" polarization, for which $\operatorname{sgn} s = 1$, or "−" polarization, for which $\operatorname{sgn} s = -1$; ρ is the density of the metal; ω is the frequency of the external wave; q_s is the value of the wave vector of the sound wave and includes the damping and renormalization of the velocity as the wave propagates in the metal. In the region of existence of resonances, the wave vector can differ substantially for the two polarizations.[16] The Fourier component of the conductivity $\sigma_s(k)$ of the metal is defined by the expressions

$$\sigma_s(k)=\sigma_{xx}(k)+i\operatorname{sgn} s\sigma_{yx}(k),$$

$$\sigma_{\alpha\beta}=\sum_n \sigma_{\alpha\beta}^{(n)}, \quad \sigma_{\alpha\beta}(k)=\sigma_{\alpha\beta}(-k),$$

$$\sigma_{\alpha\beta}^{(n)}(k,H)=\frac{2e^2}{h^3}\left|\frac{eH}{c}\right|\int_{\varepsilon_{Fn}} dp_z\oint dt\, v_\alpha(t)\int_0^\infty d\xi v_\beta(t-\xi)$$

$$\times\exp\left[-(\nu-i\omega)\xi+ik\int_{t-\xi}^{t} v_z\, dt_z\right]. \quad (2)$$

The Fourier component of the "deformation conductivity" $\eta_s(k)$ is of the form

$$\eta_s(k)=\eta_{xx}(k)+i\operatorname{sgn} s\eta_{yx}(k),$$

$$\eta_{\alpha\beta}=\sum_n \eta_{\alpha\beta}^{(n)}, \quad \eta_{\alpha\beta}(k)=-\eta_{\alpha\beta}(-k),$$

$$\eta_{\alpha\beta}^{(n)}(k,H)=\frac{2e}{h^3}\left|\frac{eH}{c}\right|\int_{\varepsilon_{Fn}} dp_z\oint dt\Lambda_{\alpha z}(t)\int_0^\infty d\xi v_\beta(t-\xi)$$

$$\times\exp\left[-(\nu-i\omega)\xi+ik\int_{t-\xi}^{t} v_z\, dt_z\right]. \quad (3)$$

The summations in (2) and (3) are over the entire multiply connective Fermi surface; ε_{Fn} is the Fermi surface of the electrons (or holes) of the surface n; h is Planck's constant $\mathbf{p}$, $\mathbf{v}$, and ν are the momentum, velocity, and collision frequency for the group n; $\alpha, \beta = x, y$; t is the time of motion of the quasiparticle over the orbit in the magnetic field $\mathbf{H}$ and is determined by the equation of motion

$$d\mathbf{p}/dt=e/c[\mathbf{vH}]; \quad \Lambda_{ik}(\mathbf{p})=\lambda_{ik}(\mathbf{p})-\langle\lambda_{ik}(\mathbf{p})\rangle;$$

$\Lambda_{ik}(\mathbf{p})$ is the deformation-potential tensor for the group n; $\langle\lambda_{ik}\rangle$ is the value of $\lambda_{ik}(\mathbf{p})$ averaged over the entire Fermi surface. Equation (1) contains the Fourier transform $\mathscr{E}_s(k)$ of the electric field $E_s(z)$, obtained by solving simultaneously the Maxwell and kinetic equations:

$$\mathscr{E}_s(k)=2\int_0^d dzE_s(z)\cos kz=[-2E_s'(0)+2E_s'(d)\cos kd$$

$$+2kE_s(d)\sin kd][k^2-i4\pi\omega c^{-2}\sigma_s(k)]^{-1}. \quad (4)$$

The prime denotes differentiation with respect to z; the quantities $E_s(0)$ and $E_s(d)$ are the values of the field $E_s(z)$ at $z=0$ and $z=d$. The problem of the distribution of $E_s(z)$ in a metal in a normal field $\mathbf{H}$ was solved in many papers.[17–20] It was shown that

$$E_s(z)=E_{s\,\mathrm{sk}}(z)+E_{s\,\Gamma\mathrm{K}}(z)+E_{s_{\mathrm{br}}}(z).$$

The spin component of the field is here

$$E_{s\,\mathrm{sk}}(z)\propto\exp(-z/\delta)E_s'(0);$$

$\delta \ll d$ is the depth of the skin layer. The anomalous-penetration field (the Gantmakher-Kaner component[17]) $E_{s\,\mathrm{GK}}(z)$ appears if the function $\sigma_s(k)$ has branch points. The field $E_{s\,\mathrm{br}}(z)$ differs substantially from $E_{s\,\mathrm{sk}}(z)$ if the dispersion equations

$$D_s(k)\equiv k^2-i4\pi\omega c^{-2}\sigma_s(k)=0$$

admit of a solution that defines a weakly damped wave. These two components attenuate exponentially over distances of the order of the electron mean free path l. (Under conditions of diffuse reflection of the electrons,[20] all three components interact with one another and this interaction determines the relation between the amplitudes of the different components.) We shall be interested in the amplitudes of the sound on the face $z=d$, i.e., $u_s(d,H)\equiv u_s(H)$. It is convenient to calculate the integrals by transforming to the complex domain and closing the contour in the upper half-plane. The functions $\sigma_s(k)$ and $\eta_s(k)$ can have branch points. Therefore the amplitude $u_s(H)$ is represented in the form of a sum of the integrals along the edges of a cut from these branch points to infinity and of the residues of the poles of the integrands. Two types of pole appear. One is the root of the equation $k_1^2 - q_s^2 = 0$; the other are the roots of the dispersion equation $D_s(k_i)=0$. The following can be shown: 1) the residue of the pole k_1 contains the factor $\exp(-d/L_{\mathrm{ac}})$ (L_{ac} is the damping length of the sound wave); 2) residues of the poles k_i contain factors of the type $\exp(-d/\delta)$ or

$\exp(-d/L_{br})$ (L_{br} is the damping length of the natural electromagnetic mode of the metal and is connected with the quantity l; 3) the integrals along the edges of the cut are characterized by the presence of a factor $\exp(-d/l)$ (the analogy with the Gantmakher-Kaner effect should be pointed out). The sound amplitude takes the simplest form if the conditions

$$L_{ac}>d>l,\ L_{br},\delta. \tag{5}$$

are realized. Otherwise it is necessary to take into account the residues of all the poles and the integrals along the edges of the cut, and the $\mathbf{u}(H)$ curves will constitute a superposition of lines, each of which has a complicated dependence on H. The inequalities (5) were satisfied in the experiments of Refs. 11–13. If these inequalities hold, the main contribution to the integral (1) is determined by the pole $k_1 = q_s$, and the amplitude takes the form

$$\begin{aligned} u_s(H) &= \alpha_s q_s(\rho\omega^2)^{-1}\{Hc^{-1}\,\mathrm{sgn}\, s\sigma_s(q_s, H) \\ &\quad + q_s\eta_s(q_s, H)\}\mathscr{E}_s(q_s, H)\exp iq_s d \\ &= T(q_s, H)\mathscr{E}_s(q_s, H)\exp iq_s d. \end{aligned} \tag{6}$$

The factor $\alpha_s = (1/2)(1 + i\cot q_s d)$ describes the effect of multiple reflection of sound from the faces. If it is assumed that $2d > L_{ac}$, then $\alpha_s = 1$. The transformation coefficient is

$$T(q_s, H) = \sum_n T^{(n)}(q_s, H),$$

where $T^{(n)}$ is a coefficient governed by the group n of the conduction electrons. Under condition (5) we have $E_s(0) > E_s(d)$.

3. The character of the $u_s(H)$ dependence is determined by the functions $\mathscr{E}_s[\sigma_s(q_s, H)]$ and $T(q_s, H)$ and by their competition. Attention should be called to a resonance effect contained in (6) and (4). The conditions for the existence of a weakly damped wave in a metal are of the form

$$\mathrm{Im}\,\sigma_s<0, \quad |\mathrm{Im}\,\sigma_s|\gg|\mathrm{Re}\,\sigma_s|. \tag{7}$$

If the wave velocity (7) is comparable with that of the sound, intersection of the spectra of the acoustic oscillations and of the natural electromagnetic mode is possible:

$$\mathrm{Re}\, q_s^2=|4\pi\omega c^{-2}\,\mathrm{Im}\,\sigma_s|, \tag{8}$$

i.e., resonance is effected between the phonons and the weakly damped wave. It can be seen from (4) that the resonance line takes the form of a Lorentz curve whose width is determined by the damping $|\mathrm{Re}\,\sigma_s|$ of the electromagnetic mode. In a magnetic field $\mathbf{H}\|\mathbf{q}$, under certain conditions and relations between the frequencies ω, Ω, and ν, helicons and dopplerons can exist in the metal (Ω is the cyclotron frequency[21,18,19]). Consequently resonances of phonons with these waves can be observed in electromagnetic generation of sound. Equation (6) is valid for DPR because of the weak coupling of these waves. The helicon-phonon coupling is strong, therefore (6) cannot be regarded as correct for helicon-phonon resonance (HPR). This resonance was investigated earlier for the case of transformation of electromagnetic and acoustic waves.[22,23]

4. To investigate the function $u_s(H)$ it is necessary to calculate the conductivity components $\sigma_s(q_s, H)$ and $\eta_s(q_s, H)$. After integration with respect to t and ξ, the conductivity $\eta_s(q_s, H)$ takes the form

$$\eta_s(q_s, H) = -\frac{4\pi e}{h^3}\sum_n\int_{\varepsilon_{Fn}} m\,dp_z\sum_{l=-\infty}^{\infty}\Lambda_{sz}^{(l)}(p_z)\tilde{v}_x^{(-l)}(p_z) \times\frac{q_s\bar{v}_z(p_z)}{(l\Omega+\omega+i\nu)^2-q_s^2\bar{v}_z^2(p_z)}. \tag{9}$$

The expression for $\sigma_s(q_s,H)$ is obtained from (9) by replacing $[-\Lambda_{sz}^{(l)}q_s\bar{v}_z]$ with $[ev_s^{(l)}(p_z)(l\Omega+\omega+i\nu)]$. We have introduced the symbols m for the cyclotron mass, $\bar{v}_z(p_z)$ for the electron drift velocity along the z axis averaged over the cyclotron period,

$$\bar{v}_z(p_z, t)=\bar{v}_z(p_z)+\Delta v_z(p_z, t);$$

$v_s^{(l)}$ and $\Lambda_{sz}^{(l)}$ for the expansion coefficients of the functions

$$v_s(t)\exp\left[iq_s\int_0^t\Delta v_z dt_1\right],$$

$$\Lambda_{sz}(t)\exp\left[iq_s\int_0^t\Delta v_z dt_1\right]$$

in Fourier series, and $\tilde{v}_x^{(l)}$ for the Fourier-expansion coefficients of the function

$$v_x(t)\exp\left[-iq_s\int_0^t\Delta v_z dt_1\right].$$

The presence of an α-fold symmetry of the Fermi surface about the $z\|\mathbf{H}$ axis leads to a choice of l (Ref. 24) in the sum (9)

$$l=l_s=k\alpha+\mathrm{sgn}\, s; \quad k=0;\ \pm1;\ \pm2. \tag{10}$$

An exact calculation of the integrals with respect to p_z for an arbitrary Fermi surface at any value of H is impossible. It is convenient to break down the entire H region into interval. In the first interval (strong fields), in which the inequality

$$q\bar{v}_{z\,\mathrm{ext}}/\Omega<1, \tag{11}$$

holds for all groups, the local approximation holds for $\sigma_s(q_s, H)$ and $\eta_s(q_s, H)$. In the second interval (weak fields), where

$$q\bar{v}_{z\,\mathrm{ext}}/\Omega>1. \tag{12}$$

The nonlocal approximation holds for all carrier groups ($\bar{v}_{z\,\mathrm{ext}}$ is the extremal value of $\bar{v}_z(p_z)$). If the values of $\bar{v}_{z\,\mathrm{ext}}$ for the different groups differ significantly, a certain broad intermediate interval is produced, in which the condition (12) holds for only one group, while the conditions (11) are valid for all the remaining ones. It is known that in this region the metal can contain dopplerons.[19,20,15] Therefore the DPR (8) of the generated-sound amplitude should be expected here. DPR for electromagnetic generation of sound was observed in tungsten.[12,15]

5. In the weak-field region (12) the contribution made to sound generation by the deformation mechanism turns out to be larger than of the induction mechanism. The denominators of $\sigma_s(H)$ and $\eta_s(H)$ have a resonant character if the following condition is satisfied

$$\nu \ll \Omega. \quad (13)$$

When the Fermi surface has no sections for which the electron density of state would be extremal, integration with respect to p_z averages out these singularities. The functions $u_s(H)$ vary slowly in this case with changing H. Exact calculations of $u_s(H)$ were made for alkali metals[3–5] and aluminum.[6] It was shown that $|u_x(H)|$ is equal to a constant as $H \to 0$ and decreases to a small value at $\Omega / q\bar{v}_{z0} \lesssim 1$ ($\bar{v}_{z0}$ is the value of $v_z(p_z)$ at the limiting point on the Fermi surface. The function $|u_y(H)|$, which is equal to zero at $H = 0$, has a complicated dependence on H and on the parameter ql. This behavior of the $|u_\alpha(H)|$ curves is common to all metals. For an exact determination of $|u_\alpha(H)|$ in this metal it is necessary to know the shape of its Fermi surface, the function $\Lambda_{\alpha z}(\mathbf{p})$, and the relations between the parameters ω, ν, Ω, and $q\bar{v}_{z0}$.

If the Fermi surface has a section that singles out electrons with extremal displacements along the vector **H** during the cyclotron period, $\bar{v}_{z\,\text{ext}}$, a large group of carriers moves in phase and under certain conditions it interacts with the wave. This leads to the appearance of DSCR in the functions $\sigma_s(H)$ and $\eta_s(H)$. Its conditions are of the form

$$\omega - q_s \bar{v}_{z\ ext} = l_s \Omega, \quad (14)$$

where l_s is the integer (10). The contribution of the remaining electrons is smaller, owing to dephasing, and determines the nonresonant behavior, described above, of the $|u_\alpha(H)|$ curves. At low frequencies

$$\omega < \nu \quad (15)$$

the resonance is spatial in character.

DSCR in metals can be due to various groups on the Fermi surface. For example:

a) carrier groups of limiting points (the Kjeldaas edge 25 for an elliptic limiting point);

b) carrier groups of inflection sections, 26 where

$$\partial^2 S/\partial p_z^2 = 0, \quad \text{i.e.,}\ \partial\bar{v}_z/\partial p_z = 0,$$
$$\bar{v}_z(p_z) = v_{z\,\text{ext}}, \quad v_z = (2\pi m)^{-1}\partial S/\partial p_z, \quad (16)$$

and $S(p_z)$ is the area of the intersection of the Fermi surface by the plane $p_z = \text{const}$;

c) carrier groups of central sections, where $\bar{v}_z(p_z) = 0$.[27]

The power and shape of the DSCR depends on the type of singularity of the functions $\sigma_s(H)$ and $\eta_s(H)$. The most peaked resonances occur in case b).

The contributions made to the coefficients $\sigma_s(H)$ and $\eta_s(H)$ by group (16) is the region of the l resonance are described by the formulas

$$\sigma_{s\ \text{res}} = \frac{4\pi^2 c}{h^3}\left|\frac{e}{H}\right| \sum_{l>0} \left\{ \frac{m^2 p_z}{(l\beta)^{1/2}} [v_s^{(l)}\, \tilde{v}_x^{(-l)} g(\Delta_s + i\gamma) + v_s^{(-l)}\, \tilde{v}_x^{(l)} g(\Delta_s - i\gamma)] \right\}_{p_z = p_{z\,\text{ext}}}, \quad (17a)$$

$$\eta_{s\ \text{res}} = \frac{4\pi^2 c}{h^3 |H|} \sum_{l>0} \left\{ \frac{m^2 p_z}{(l\beta)^{1/2}} [\Lambda_{sz}^{(l)}\, \tilde{v}_x\ \ g(\Delta_s + i\gamma) - \Lambda_{sz}^{(-l)}\, \tilde{v}_x^{(l)} g(\Delta_s - i\gamma)] \right\}_{p_z = p_{z\,\text{ext}}}. \quad (17b)$$

Here

$$\gamma = \frac{\nu}{\Omega}, \quad \beta = \frac{1}{2}\left[\frac{p_z^2}{\bar{v}_z}\frac{\partial^2 \bar{v}_z}{\partial p_z^2}\right]_{p_z = p_{z\,\text{ext}}},$$

$$\Delta_s = l_s - \frac{q_s \bar{v}_{z\,\text{ext}}}{\Omega}.$$

The function $g(x) = x^{-1/2}$ describes resonance lines. The value of g near resonance, when $|\Delta| \ll \gamma \ll 1$, is larger by $\gamma^{-1/2}$ times than the value of g far from resonance, when $|\Delta| \gg \gamma$. The linewidth is determined by the parameter γ. The integer $l = l_s$ is different for two polarizations. Therefore even the first peaks in the "+" and "−" polarizations differ in amplitude, which decreases with increasing l. The number of observed resonance peaks depends on the crystal symmetry. If the Fermi surface has several resonance groups (16), the functions $\sigma_s(H)$ and $\eta_s(H)$ constitute superpositions of the resonance curves (27).

Notice the features of DSCR in electromagnetic generation of sound. At low wave frequencies (15) the following inequality holds at (12):

$$4\pi\omega c^{-2}|\sigma_s| > q_s^2. \quad (18)$$

The amplitude $u_s(H)$ takes then the form

$$u_s(H) = -E'(0)\, i q_s^2 c^2 (2\pi\rho\omega^3)^{-1} \exp[iQ_s(H)d - \Gamma_s(H)d] F_s(H), \quad (19)$$

$$F_s(H) \equiv \frac{\eta_{s\ \text{res}}(H) + \eta_{s\,\text{mon}}(H)}{\sigma_{s\ \text{res}}(H) + \sigma_{s\,\text{mon}}(H)}, \quad Q_s \equiv \operatorname{Re} q_s,$$
$$\Gamma_s \equiv \operatorname{Im} q_s, \quad E'(0) \equiv E'_x(0).$$

Here Γ_s is the coefficient of sound damping, due to all electron groups, in the metal. The coefficient $\Gamma_s(H)$ was considered under DSCR conditions in Refs. 26 and 28. $\eta_{s\,\text{mon}}$ and $\sigma_{s\,\text{mon}}$ denote the monotonic parts of the quantities

$$\sum_h \eta_s^{(n)} \text{ and } \sum_h \sigma_s^{(n)}.$$

Near each resonance, the shape of the line $u_s(H)$ is determined by competition between three rapidly varying functions: $\exp[-\Gamma_s(H)d]$, $\eta_{s\,\text{res}}(H)$ and $\sigma_{s\,\text{res}}(H)$. If the contribution from the resonance group (16) to conductivity exceeds the contribution from all the remaining groups, $\sigma_{s\,\text{res}} > \sigma_{s\,\text{mon}}$, the function $F_s(H)$ can be represented as

$$F_s(H) = \frac{\eta_{s\ \text{res}}}{\sigma_{s\ \text{res}}} + \frac{\sigma_{s\ \text{res}}\eta_{s\,\text{mon}} - \sigma_{s\,\text{mon}}\eta_{s\ \text{res}}}{(\sigma_{s\ \text{res}})^2}.$$

The first term leads to the monotonic part of $u_s(H)$. The second term is proportional to g^{-1} and has a minimum at resonance. It is either added to the monotonic part if $(\sigma_{s\,\text{res}}\eta_{s\,\text{mon}} - \sigma_{s\,\text{mon}}\eta_{s\,\text{res}}) > 0$, or subtracted from it if

$(\sigma_{s\,\mathrm{res}}\eta_{s\,\mathrm{mon}} - \sigma_{s\,\mathrm{mon}}\eta_{s\,\mathrm{res}}) < 0$. If the contribution to the conductivity from the nonresonant groups is larger than $\sigma_{s\,\mathrm{res}}$, then

$$F_s(H) \approx \frac{\eta_{s\,\mathrm{mon}}}{\sigma_{s\,\mathrm{mon}}} - \frac{\sigma_{s\,\mathrm{res}}\eta_{s\,\mathrm{mon}} - \sigma_{s\,\mathrm{mon}}\eta_{s\,\mathrm{res}}}{(\sigma_{s\,\mathrm{mon}})^2}.$$

The first term describes the monotonic course of $u_s(H)$. The second term, proportional g, increases abruptly at resonance. Depending on the parameters of the metal, i.e., on the ratios of the quantities $\sigma_{s\,\mathrm{res}}\eta_{s\,\mathrm{mon}}$ and $\sigma_{s\,\mathrm{mon}}\eta_{s\,\mathrm{res}}$, the second term is subtracted or added to the monotonic term $u_s(H)$. A resonant change of the damping coefficient $\Gamma_s(H)$ can lead to a difference between the amplitudes of the resonances $|u_s(H)|$ from the amplitudes of the peaks $|F_s(H)|$.

It is clear from all the foregoing that DSCR can manifest itself in electromagnetic generation of sound either in the form of maxima $(\sigma_{s\,\mathrm{res}}\eta_{s\,\mathrm{mon}} - \sigma_{s\,\mathrm{mon}}\eta_{s\,\mathrm{res}} < 0)$, or in the form of minima $(\sigma_{s\,\mathrm{res}}\eta_{s\,\mathrm{mon}} - \sigma_{s\,\mathrm{mon}}\eta_{s\,\mathrm{res}} > 0)$ of the amplitude $|u_s(H)|$. The singularities of the DSCR take place for both amplitude components $|u_x(H)|$ and $|u_y(H)|$. In experiments on tungsten,[11,13] the $|u_\alpha(H)|$ lines were superpositions of systems of maxima due to different groups (16).

If the inequality

$$q_s^2 > 4\pi\omega c^{-2}|\sigma_s|, \tag{20}$$

is satisfied in a metal, as is possible at frequencies $\omega \gg \nu$, we have

$$\mathscr{E}_s(q_s) \approx -2E'(0)\,q_s^{-2}$$

and the sound amplitude DSCR line shape is determined by the shape of the deformation-conductivity line (17b)

$$u_s(H) = (\rho\omega^2)^{-1}\exp[iQ_s(H)d - \Gamma_s(H)d][\eta_{s\,\mathrm{mon}}(H) + \eta_{s\,\mathrm{res}}(H)]. \tag{21}$$

The amplitude of the peaks $\eta_{s\,\mathrm{res}}(H)$ is modulated by the factor $\exp[-\Gamma_s(H)d]$, i.e., by the DSCR of the damping coefficient.

The DSCR resonance lines (19) and (21) can be easily distinguished from the curves of the phonon resonances with a weakly damped wave (8): their linewidth is considerably smaller and they are located in the region of weaker magnetic fields, where the wave damping is large.

Equations (19) and (21) describe DSCR in the case of resonances due to groups a) and c) (it is necessary to substitute in them the corresponding values of $\eta_{s\,\mathrm{res}}$ and $\sigma_{s\,\mathrm{res}}$). The DSCR manifest themselves in the form of jumplike increases of $\eta_s(H)$ and $\sigma_s(H)$, which lead to small increments of the amplitude $u_s(H)$ of the excited sound.

6. In the local limit of strong magnetic fields (11) the shape of the Fermi surface exerts no influence on the character of the $u_s(H)$ dependence. What matters is whether the metal is compensated or not. The asymptotic forms of $\sigma_s(H)$ and $\eta_s(H)$ are

$$\sigma_s(H) = i\,\mathrm{sgn}\,s\sum_n \frac{N\,\mathrm{sgn}\,mec}{(1+i\tilde{\gamma}\,\mathrm{sgn}\,s)H}[1+a_1\varkappa_s^2], \qquad N_e \neq N_h. \tag{22}$$

In a compensated metal, $\sigma_s(H)$ is considerably smaller because of cancellation of the principal term—of the Hall conductivity:

$$\sigma_s(H) = i\,\mathrm{sgn}\,s\sum_n \frac{N\,\mathrm{sgn}\,mec}{(1+i\tilde{\gamma}\,\mathrm{sgn}\,s)H}[a_1\varkappa_s^2 - i\tilde{\gamma}\,\mathrm{sgn}\,s], \quad N_e = N_h; \tag{23}$$

$$\eta_s(H) = \frac{i}{q_s}\sum_n \frac{eN\mu}{|m|}a_2\varkappa_s^2. \tag{24}$$

$$\tilde{\gamma} = (\nu - i\omega)/\Omega, \quad \varkappa_s = q_s v\,\mathrm{sgn}\,s/(1+i\tilde{\gamma}\,\mathrm{sgn}\,s),$$

N_e and N_h are the numbers of the electrons and holes, $N = V_p/h^3$, and V_p is the volume in the momentum space occupied by the group D; $|\varkappa_s| \ll 1$ in Eqs. (22)–(24); $v = \bar{v}_{z\,\max}$; a_1 and a_2 are numerical coefficients that depend on the concrete shape of the Fermi surface ($a_1 = a_2 = 1/5$ for a sphere). To calculate $\eta_s(H)$, the true value of $\Lambda_{sz}(p_z, t)$ was replaced by the model-dependent value

$$\Lambda_{sz}(p_z, t) = -\mu v_s(t, p_z)\bar{v}_z(p_z).$$

Substituting (22)–(24) in (6) and (4) we obtain the function $u_s(H)$. The simplest form of $u_s(H)$ occurs in the limiting cases (18) and (20). The condition (20) corresponds to stronger fields than (18). We assume that the inequalities (11), (13), (15) and (18) or (20) are satisfied. Inasmuch as in the experiments[13,14] the sound was assumed to have linear polarization, we present the answers for the functions $u_x(H)$ and $u_y(H)$.

A. Uncompensated metal ($N_e \neq N_h$). In strong magnetic fields, for which the quasiclassical approach developed here is valid, the condition realized is (18). The sound generation is determined by the induction mechanism, and the deformation mechanism yields in comparison a contribution smaller by a factor $(qv/\Omega)^{-2} \gg 1$. The amplitude of the sound is of the form

$$u_y(H) \approx \frac{qc}{2\pi\rho\omega^3}E'(0)e^{iqd}H, \tag{25}$$

where $q = q_+ = q_-$. The component $u_y(H)$ varies in accord with a linear law, and $u_x(H)$ is smaller than $u_y(H)$ by a factor $(qv/\Omega)^{-2}$. This result coincides with the result for an alkali metal.[1–4] In magnetic fields (20), quantum theory is necessary. The character of the $u_y(H)$ line, however is obvious. The monotonic part $u_y(H)$ tends to a constant value. Superimposed on it are quantum oscillations of the conductivity $\hat{\sigma}(H)$, with small amplitude. At a value $H = H_{\mathrm{hp}}$, where $q^2 = |i4\pi\omega c^{-2}\sigma_-(H_{\mathrm{hp}})|$, helicon-phonon resonance takes place.

B. Compensated metal ($N_e = N_h$). For a compensated metal, the induction and deformation mechanisms of sound generation make contributions of the same order. The conditions (18) and (20) can be realized in the quasiclassical approximation. We then have the inequality

$$|\varkappa^2| \gg |\tilde{\gamma}|. \tag{26}$$

The sound amplitude in the interval (18) takes the form

$$u_x(H) = \frac{|e|qe^{iqd}}{2\pi\rho\omega^3}E'(0)\frac{\sum_n N|m|v}{\sum_n Nm|m|v^2a_1}AH^2 \quad (\omega < \nu), \tag{27a}$$

$$u_y(H)=-\frac{qce^{iqd}}{2\pi\rho\omega^3}E'(0)(1+A)H;$$
$$A=\sum_n N\mu|m|v^2a_2 \Big/ \sum_n Nm|m|v^2a_1. \qquad (27b)$$

It follows from (27) that $|u_x|/|u_y|\sim|\gamma|/|\varkappa^2|\ll 1$. In the interval of very strong fields (20) the functions $u_x(H)$ and $u_y(H)$ are the following:

$$u_x(H)=-\frac{i}{H^2}\frac{2qc^2e^{iqd}}{\rho\omega^2|e|}E'(0)\sum_n(ma_1+\mu a_2)Nv^2|m|, \qquad (28a)$$

$$u_y(H)=\frac{i}{H}\frac{2ce^{iqd}}{\rho\omega^2 q}E'(0)\sum_n N|m|\nu. \qquad (28b)$$

In quantizing fields H, small quantum oscillations are superimposed on the monotonic course of the lines (28). The conditions under which Eqs. (27) and (28) were obtained correspond to the situation when the Alfven wave is strongly damped (its damping is small if $\omega\gg\nu$). Therefore the lines $|u_\alpha(H)|$ have no sharp resonant singularities. We present here $|u_\alpha(H)|$, since this quantity is observed in experiment. The character of the asymptotic relations (27) and (28) indicates that at the corresponding values of H the functions $|u_\alpha(H)|$ should have maxima. These maxima are explained by the fact that the skin-layer depth $\delta(H)$ becomes of the order of the sound wavelength. The described $|u_y(H)|$ dependence was observed in tungsten[12] and in tin.[13] It must be noted that the effect considered is similar to the size effect of Fischer and Kao,[29] wherein a maximum of rf-wave absorption exists in a compensated metal when $\delta(H)$ is close in magnitude to the plate thickness.

The dependence of the field H_{ext} corresponding to the maximum of $|u_y(H)|$ on the parameters of the metal is described by the formula

$$H_{ext}\approx\left\{(1+A)^{-1}4\pi s_{ac}^2\omega^{-1}\sum_n N|m|\nu\right\}^{1/2}, \qquad (29)$$

where s_{ac} is the velocity of the transverse sound in a metal. The quantity H_{ext} depends on the temperature like $\left[\sum_n \nu(T)\right]^{1/2}$. With increasing T the frequency ν increases and the position of the maximum of $|u_y(T)|$ moves into the region of stronger fields H. A similar effect was observed in Refs. 12 and 13. It appears that an investigation of the function $H_{ext}(T)$ will make it possible to determine the dependence of the averaged collision frequency ν on the temperature.

In conclusion I am deeply grateful to É. A. Kaner, A. P. Korolyuk, A. V. Golik, and V. I. Khizhnyĭ for helpful discussions of the work.

[1]J. J. Quinn, Phys. Lett. **25A**, 522 (1967).
[2]A. R. Casanova, Phys. Rev. **178**, 1050 (1969).
[3]E. A. Kaner and V. L. Fal'ko, Zh. Eksp. Teor. Fiz. **64**, 1016 (1973) [Sov. Phys. JETP **37**, 516 (1973)].
[4]L. I. Babkin and V. Ya. Kravchenko, *ibid.* **67**, 1006 (1974) [**40**, 498 (1975)].
[5]N. C. Banic and A. W. Overhauser, Phys. Rev. B **16**, 3379 (1977); B **18**, 3878 (1978).
[6]N. C. Banic and A. W. Overhauser, Phys. Rev. B **17**, 4505 (1978).
[7]E. A. Kaner and V. L. Falko, Sol. St. Comm. **35**, 353 (1980).
[8]G. Turner, R. L. Thomas, and D. Hsu, Phys. Rev. B **3**, 3097 (1971).
[9]W. D. Wallace, H. R. Gaerttner, and B. W. Maxfield, Phys. Rev. Lett. **27**, 995 (1971).
[10]D. E. Chimenti, C. A. Kukkonen, and B. W. Maxfield, Phys. Rev. B **1**, 3228 (1974).
[11]A. V. Golik, A. P. Korolyuk, and V. I. Khishnyi, Fiz. Nizk. Temp. **8**, 994 (1982) [Sov. J. Low-Temp. Phys. **8**, 502 (1982).
[12]A. V. Golik, A. P. Korolyuk, and V. I. Khidjnii, Sol. St. Comm. **44**, 173 (1982).
[13]A. V. Golik, A. P. Korolyuk, V. L. Fal'ko, and V. I. Khizhnyĭ, Abstracts, 22nd All-Union Conf. on Low Temp. Phys., Kishinev, 1982, part II, p. 13.
[14]A. N. Vasil'ev, Yu. P. Gaidukov, and S. I. Zolotov, *ibid.*, p. 9.
[15]L. T. Tsymbal, Élektromagnitnye vozbuzhdeniya i dopleron-fononnyi rezonans w metallakh (Electromagnetic Excitations and Doppleron-Phonon Excitations in Metals), Doctoral Dissertation, Donetsk, 1982.
[16]V. V. Gudkov, Osobennosti rasprostraneniya poperechnykh ul'trazvukovykh voln v oblasti dopleron-fononnogo rezonansa vol'frame i indii (Features of Propagation of Transverse Ultrasonic Waves in the Region of Doppleron-Phonon Resonance in Tungsten and Indium), Candidate's dissertation, Sverdlovsk, 1981.
[17]V. F. Gantmakher and É. A. Kaner, Zh. Eksp. Teor. Fiz. **48**, 1572 (1965) [Sov. Phys. JETP **21**, 1053 (1965)].
[18]O. V. Konstantinov and V. G. Skobov, Fiz. Tverd. Tela (Leningrad) **12**, 2768 (1970) [Sov. Phys. Solid State **12**, 2237 (1971)].
[19]L. M. Fisher, V. V. Lavrova, V. A. Yudin, O. V. Konstantinov, and V. G. Skobov, Zh. Eksp. Teor. Fiz. **6**, 759 (1971) [Sov. Phys. JETP **33**, 410 (1971)].
[20]I. F. Voloshin, V. G. Skobov, L. M. Fisher, and A. S. Chernov, *ibid.* **80**, 183 (1981) [**53**, 92 (1981)].
[21]É. A. Kaner and V. G. Skobov, Usp. Fiz. Nauk **89**, 367 (1966) [Sov. Phys. Usp. **9**, 480 (1967)].
[22]V. G. Skobov and É. A. Kaner, Zh. Eksp. Teor. Fiz. **46**, 273 (1964) [Sov. Phys. JETP **19**, 189 (1964)].
[23]J. J. Quinn, Phys. Rev. Lett. **24**, 817 (1970).
[24]G. A. Kotkin, Zh. Eksp. Teor. Fiz. **41**, 281 (1962) [Sov. Phys. **14**, 201 (1961)].
[25]T. Kjeldaas, Phys. Rev. **113**, 1473 (1959).
[26]E. A. Kaner, V. G. Peschanskiĭ, and I. A. Privorotskiĭ, Zh. Eksp. Teor. Fiz. **40**, 214 (1961) [Sov. Phys. 147 (1961)].
[27]V. L. Gurevich, *ibid.* **37**, 71 (1959) [**10**, 51 (1960)].
[28]A. M. Grishin, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 370 (1982) [JETP Lett. **35**, 455 (1982)].
[29]H. Fischer and Yi-han Kao, Sol. St. Comm. **7**, 275 (1969).

Translated by J. G. Adashko