Anomalous electron tunneling in a magnetic field near a surface

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Sub-barrier electron tunneling is known to be significantly weakened by a transverse magnetic field. In particular, in a strong magnetic field and at large distances x from an impurity, the wave function across the field in a semiconductor takes instead of the usual form $\psi\sim\exp(-x/a)$ the form $\psi\sim\exp(-x^2/4\lambda^2)$, where λ is the magnetic length. It is shown that if the tunneling is along the surface of a crystal and across a magnetic field parallel to it the interaction between the electron and the surface results in an anomalous transparency, i.e., the effect of the magnetic field is much weaker. In particular, the wave function of a donor near the surface takes the form $\psi\sim\exp(-x/b)$ where x is the coordinate along the surface and across the field, and the length b differs from λ only by a logarithmic factor. The asymptotic wave functions of donors in a film in a magnetic field parallel to the surface is investigated as a function of the film thickness. It is shown that the hopping conduction along such a film and across the field is determined by a thin subsurface layer in which the electrons tunnel by "using" the interaction with the surface. The conductivity is then exponentially larger, and the field dependence weaker, than in a bulky sample.

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1. INTRODUCTION

A magnetic field is known[1] to alter greatly the asymptotic behavior of the wave functions of the impurity state in semiconductors. Whereas in the absence of a magnetic field the wave function of a state with energy $E<0$ is of the form $\psi\sim\exp(-r/a)$, where $a=\hbar/(2m|E|)^{1/2}$ is the effective Bohr radius and m is the effective mass, in a weak magnetic field, i.e., at $\lambda\gg a$ and $\rho\ll\lambda^2/a$, we have

$$\psi(\mathbf{r})\propto\exp\left(-\frac{r}{a}-\frac{r\rho^2a}{24\lambda^4}\right), \tag{1}$$

and in a strong field, i.e., at $\lambda\ll a$ or at $\lambda\gg a$ put $\rho\gg\lambda^2/a$, we have

$$\psi(\mathbf{r})\propto\exp\left(-\frac{|z|}{a_H}-\frac{\rho^2}{4\lambda^2}\right). \tag{2}$$

Here $\lambda=(c\hbar/|e|H)^{1/2}$ is the magnetic length, $\mathbf{H}$ is the magnetic field whose direction in this paper is always the z axis, $a_H=\hbar/(2mE_H)^{1/2}$ and E_H are the effective Bohr radius and the ionization energy in the field H, and ρ is the distance from the point $\mathbf{r}$ to the z axis. Equations (1) and (2) are used to describe the giant positive magnetoresistance in the region of hopping conduction over impurities.

The alteration of the asymptotic wave functions by a magnetic field can be interpreted as the consequence of an additional potential barrier produced by the magnetic field. Indeed, if the vector potential is chosen in the form $\mathbf{A}=(1.2)\mathbf{H}\times\mathbf{r}$ the influence of the magnetic field reduces in the Schrödinger equation for an electron in the ground state to addition of a parabolic potential barrier in the form $\hbar^2\rho^2/8m\lambda^4$, which we shall call a magnetic barrier. As a result, the summary potential relief takes the form shown by the dashed line in Fig. 1, and the electron has to tunnel not only under a barrier of height E_H, due to the negative energy, but also under the magnetic barrier. If the latter alters the exponent of the function ψ the field is called weak. Otherwise it is said to be strong.

It has been shown in Refs. 2 and 3 that if the electron encounters other impurity centers as it tunnels through the field, the sub-barrier scattering by them weakens considerably the influence of the magnetic field on the tunneling. At sufficiently large distances the wave functions fall off in a manner qualitatively different from (1) and (2), namely $\psi\propto\exp(-\rho/b)$, where b is a length that depends on the impurity density and on the field. It can be stated that in each scattering act the electron imparts to the impurities a momentum in the direction perpendicular to $\mathbf{r}$ and $\mathbf{H}$ and consequently knocks down the center of the Landau oscillator and decreases the magnetic potential to zero. The magnetic barrier acquires as a result a sawtooth shape (Fig. 1) and no longer increases monotonically with increasing r. The influence of the field reduces then effectively only to some lifting of the bottom of the band. It is clear from this interpretation that randomness in the disposition of the scattering centers plays no role and that the described phenomenon should be more general in character.

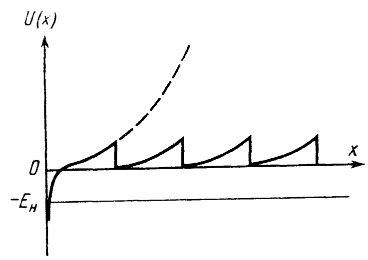

FIG. 1. Effective tunnel barrier in the presence of a magnetic field without allowance for the electron scattering (dashed line) and with allowance for scattering (solid line). The energy level of the tunneling electron is shown by a horizontal line.

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Imagine a crystal bounded from the left by a surface perpendicular to the y axis, with a magnetic field parallel to the surface and directed along z. We examine how the wave function of an impurity located near the crystal surface falls off in the x direction (which is parallel to the surface and perpendicular to the field). We shall show that the influence of the surface leads to an anomalously small decrease of the wave function, like $\psi \propto \exp(-x/b)$. This phenomenon can be interpreted as a shift of the oscillator center in the x direction by the electron scattered from the surface and imparting to it a momentum component p_y. (It is in essence the quantum analog of the classical motion of a magnetized electron along a wall by striking it periodically.) It is interesting that at $\lambda \ll a$ the value of b for this problem differs only logarithmically from the magnetic length λ. We shall obtain below an analogous asymptotic expression for a film bounded by surfaces $y = \text{const}$, as a function of the distances from the impurity to the surfaces and of the film thickness d. It will be shown that if the film is thick enough, to be able to tunnel over a large distance x along the film the electron first tunnels to the nearest surface, next along it (giving up momentum to it) to the required coordinate x, and tunnels next across the film to the end point (Fig. 2).

The investigated asymptotic behavior of the wave function makes it possible to describe a number of observable phenomena. It is known[4] that MOS structures are doped with sodium ions located on the interface between the oxide and the semiconductor. The potential experienced by the electron near such an ion consists of an attracting Coulomb potential in the right-hand half-space and of an infinite potential wall on the left. This potential contains an electron level, so that the sodium atoms play the role of surface donors. At low temperatures one investigates two-dimensional hopping conductions over these donors. In a magnetic field parallel to the structure surface, our results have made it possible to find the exponential dependence of the hopping resistance on the donor density and on the field strength.

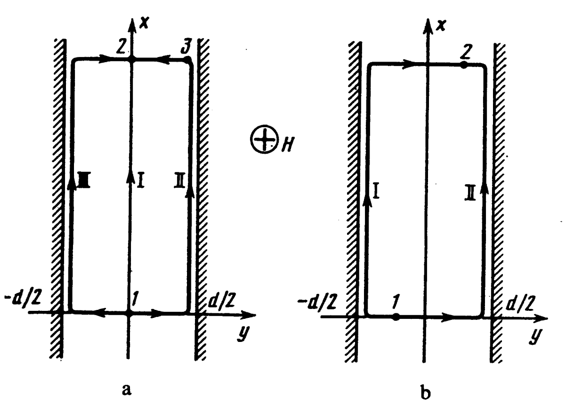

FIG. 2. Paths of tunneling from point 1 to point 2 in a thick film ($d \gg \lambda L$), where L is the logarithm (11a). The magnetic field is perpendicular to the plane of the figure. a) Impurity center located at the origin (point 1). If x (the coordinate of point 2) satisfies the inequality $x \gg d \gg \lambda L$, the tunneling is along the paths II and III. If $aL \ll x \ll d$, the tunneling is along the path I. b) The impurity center (point 1) is not in the symmetry plane of the film. At $x \gg d \gg aL$ it is necessary to choose the larger of the probability amplitudes of tunneling along the paths I and II.

We turn now to films that are doped uniformly in the volume. We investigate below the hopping conductivity of such films in a strong magnetic field parallel to the surface. It turns out that their conductivity is determined by impurities located in a thin subsurface layer of thickness less than the average distance between the impurities in the volume. In other words, the subsurface layer shunts the film volume. The point is that since the impurities are close to the surface their wave functions overlap much more strongly than of those far from the surface. We emphasize that unlike in Ref. 2 we are dealing here with ordinary hopping conduction with hops to the nearest neighbors.

Let us dwell on another possible effect of the crystal surface on the asymptotic wave functions in a magnetic field, an effect that has no bearing on the impurities and hopping conduction and will not discuss further below. Imagine a metal-insulator-metal tunnel function in the form of two plates in contact with two opposite faces (bases) of a short dielectric parallelepiped. Let the x axis be perpendicular to the bases and the z and y axes perpendicular to the lateral faces. Let the junction be located in a magnetic field directed along the z axis, and let its strength be such that it has a noticeable effect on the tunneling transparency. According to our results the electrons can then tunnel more effectively near the faces $y = \text{const}$ than through the middle of the parallelepiped base. In other words, the faces $y = \text{const}$ can shunt the tunnel junction, acting as it were as "banisters" for the electrons and altering the thickness dependence of the transparency. Similar banisters can be dislocation lines joining the bases of the parallelepiped.

Before we proceed to finding concrete asymptotic forms, we shall describe our general approach to these problems. We are interested only in the argument of the exponential of the ground-state wave function of an impurity located at a point $\mathbf{r}'$. If the impurity-potential radius is small enough compared with the tunneling distance, it can be assumed that the argument of the exponential of the wave function hardly differs from that of the green function $G_E(\mathbf{r};\mathbf{r}')$ of the Schrödinger equation that takes into account exactly the potential of the surface or of the film and the influence of the magnetic field, but does not contain the impurity potential. We shall therefore calculate below only the argument of the exponential of the Green function $G_E(\mathbf{r};\mathbf{r}')$, i.e., writing the Green function in the form $G_E(\mathbf{r};0) = G_0 e^{-g}$, we shall obtain the value of g.

2. FILM WITH PARABOLIC POTENTIAL

We consider first a simple model of the film and specify its potential in the form

$$U = m\Omega^2 y^2/2 = \hbar^2 y^2/2m\Lambda^4, \tag{3}$$

where $\Lambda = (\hbar/m\Omega)^{1/2}$ is the effective thickness of the film. The potential can be of this form when practically all the film electrons are captured by surface levels[5] or depart to the metallic electrodes adjacent to the film surfaces,[6] leaving in the film a positive space charge of free donors. We begin with this case because in a magnetic field directed along the z axis the potential (3) admits of an exact solution for free electrons.

We shall be interested in the wave function of an impurity located at the origin. The corresponding Green function is

$$G_E(\mathbf{r};0)=\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dk_z}{2\pi}\int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\left[\frac{\hbar^2k_z^2}{2m}+E_n(k_x)-E\right]^{-1}$$

$$\times e^{ik_zz}e^{ik_xx}\varphi_n^*(y-y_0)\varphi_n(-y_0), \tag{4}$$

where

$$\varphi_n(y-y_0)$$

$$=\frac{1}{(2^nn!)^{1/2}}H_n\left(\frac{y-y_0}{\mathscr{L}}\right)\exp\left[-\frac{1}{2\mathscr{L}^2}(y-y_0)^2\right]\frac{1}{\pi^{1/4}\mathscr{L}^{1/2}} \tag{5}$$

is the eigenfunction of a one-dimensional oscillator of frequency ω,

$$E_n=(n+{}^1/_2)\hbar\omega+\hbar^2k_x^2/2m^*, \tag{6}$$

k is the wave vector of the electron,

$$m^*=m\left(1+\frac{\omega_c^2}{\Omega^2}\right),\quad y_0=\frac{\omega_c^2}{\omega^2}\lambda^2k_x,\quad \mathscr{L}=\left(\frac{\hbar}{m\omega}\right)^{1/2},$$

$$\omega=(\omega_c^2+\Omega^2)^{1/2},\quad \omega_c=\frac{|e|H}{mc},\quad E=\frac{\hbar\omega}{2}-E_H. \tag{7}$$

We calculated the integral of (4) at $\hbar\omega_c \gg E_H$. In this case, at an arbitrary value of Λ, the relation $\hbar\omega \gg E_H$ is valid. Using this fact for $\mathbf{r} = (x,0,0)$, i.e., in a direction perpendicular to the magnetic field and in the symmetry plane of the film, we obtain from (4)

$$g(x)=x^2/4\lambda^2,\quad x\ll\Lambda^2/a_H, \tag{8a}$$

$$g(x)=\left(\frac{x}{a_H}\right)\left(\frac{m^*}{m}\right)^{1/2},\quad x\gg\Lambda^2/a_H. \tag{8b}$$

Comparing (8) with (2) we see that at $x \ll \Lambda^2/a_H$ the asymptotic relation takes the usual form. In the region $x \gg \Lambda^2/a_H$ the magnetic potential is periodically "dropped" over a length Λ^2/a_H and g is proportional to x. Equation (8b) can also be interpreted in the following manner. In Eq. (6) the term $\hbar^2k_x^2/2m^*$ can be interpreted as the kinetic energy of motion along the x axis. Then m^* is the effective mass for motion in the x direction, and takes into account the influence of the potential (3) and of the magnetic field. This means that the wave function of a state with ionization energy E_H should decrease like $\exp(-x/b)$, where

$$b=\hbar/(2m^*E_H)^{1/2}=a_H(m/m^*)^{1/2}.$$

3. ASYMPTOTIC WAVE FUNCTION OF AN IMPURITY LOCATED NEAR A CRYSTAL BOUNDARY

We consider now the calculation of the asymptotic wave function of the ground state of an impurity center located near a crystal surface specified in the form of an infinite potential wall. The coordinate axis perpendicular to the surface is designated y, the field is directed along the z axis, and we seek the asymptotic form along x. We assume that $\lambda \ll a$.

We consider first the case when the impurity center is located exactly on the surface. We shall show that in this case the wave function of an electron located at the center is concentrated mainly over a distance of the order of $\lambda \ln^{1/2}(\hbar\omega_c/E_H)$ from the surface. In fact, in the absence of an impurity center, y (the oscillator coordinate) is a conserved quantity, and the wave function is a superposition of two confluent hypergeometric functions. The conditions under which the wave function vanishes far from the surface and on the crystal surface can be shown to lead to the following expression for the correction to the zero-level energy on account of the interaction with the surface:

$$\Delta E(y)=\frac{1}{\sqrt{\pi}}\hbar\omega_c\left(\frac{y}{\lambda}\right)\exp\left(-\frac{y^2}{\lambda^2}\right), \tag{9}$$

where y is the distance from the surface to the oscillator center. Equation (9) is valid if $y \gg \lambda$. The ground-state energy has a minimum if the maximum of the wave function is centered at a distance $\tilde{y}$ from the surface, defined by the equation

$$\Delta E(\tilde{y})=\beta E_H. \tag{10}$$

In (10), β is an unknown constant of the order of unity ($\beta < 1$). Using Eqs. (9) and (10) we obtain for $\tilde{y}$ the expression

$$\tilde{y}=\lambda L, \tag{11}$$

where we have introduced

$$L=\ln^{1/2}(\hbar\omega_c/\sqrt{\pi}\beta E_H). \tag{11a}$$

We note that Eq. (11) is valid in the logarithmic approximation, i.e., it is assumed that $L \gg 1$. To calculate the asymptotic wave function it is convenient to locate the origin at the point $\tilde{y}$ (we direct the y axis along the outward normal to the surface).

At a large distance from the center the asymptotic wave function is given by Eq. (4), in which $\varphi_n(y - y_0)$ is the wave eigenfunction of a one-dimensional oscillator in a state n, satisfying the zero boundary condition on the crystal surface and corresponding to the eigenvalue $E_n(k_x)$. The quantum number k_x specifies the position of the center of the oscillator $y_0 = \lambda^2k_x$. The ground-state energy E of an electron at the center is in this case

$$E=\hbar\omega_c/2+\Delta E(\tilde{y})-E_H. \tag{12}$$

We shall calculate $G_E(\mathbf{r};0)$ at points on the x axis, i.e., we put in (4) $y = z = 0$. It can be easily seen that at $\hbar\omega_c \gg E_H$ it suffices to retain in the sum over in (4) only the term with $n = 0$. In addition, as will be shown by the subsequent calculations, the typical y_0 in (4) satisfy the inequalities $0 < y_0 \ll \lambda \ll \tilde{y}$ (y_0 is reckoned here from the origin). It suffices therefore to take the function $E_0(k_x)$ in the form

$$E_0(k_x)=\frac{\hbar\omega_c}{2}+\frac{\hbar\omega_c}{\sqrt{\pi}}\left(\frac{\tilde{y}-y_0}{\lambda}\right)\exp\left[-\frac{(\tilde{y}-y_0)^2}{\lambda^2}\right]$$

$$\approx\frac{\hbar\omega_c}{2}+\Delta E(\tilde{y})\exp\left[\frac{2\tilde{y}y_0}{\lambda^2}\right], \tag{13}$$

where $\Delta E(\tilde{y})$ is given by Eq. (9) with $y = \tilde{y}$. In addition, since $\tilde{y} \gg \lambda$, we can choose the function $\varphi_0(-y_0)$ in the same form as for a free electron in a magnetic field in the absence of a surface:

$$\varphi_0(-y_0)\propto\exp(-y_0^2/2\lambda^2). \tag{14}$$

Substituting (12)–(14) in (4) and introducing a new integration variable $\varkappa = y_0/\lambda = \lambda k_x$ we obtain

$$G_E(x;0)\propto\int_{-\infty}^{\infty}dk_z\int_{-\infty}^{\infty}d\varkappa\left\{\frac{\hbar^2k_z^2}{2m}+E_H+\Delta E(\tilde{y})\right.$$
$$\left.\times\left[\exp\left(\frac{2\tilde{y}\varkappa}{\lambda}\right)-1\right]\right\}^{-1}\exp\left(\frac{ix\varkappa}{\lambda}\right)\exp(-\varkappa^2). \tag{15}$$

Since $\Delta E(\tilde{y})\sim E_H$, the typical $\varkappa$ in (15) will be of order λ/y (in which case the typical y_0 are of order $\lambda^2/\tilde{y}\ll\lambda$). Using the smallness of the variable $\varkappa$ compared with unity in the essential integration region in (15), we replace the function $\exp(-x^2)$ in the numerator of (15) by unity. Closing the path of integration with respect to $\varkappa$ by an infinitely remote semicircle in the upper or lower (depending on the sign of x) $\varkappa$ half-plane, we find that the integral is determined by the residues of the integrand at the first-order poles closest to the real axis:

$$\varkappa_{1,2}=\frac{\lambda}{2\tilde{y}}\left[\ln\left(\frac{\hbar^2k_z^2/2m+E_H}{\Delta E(\tilde{y})}-1\right)\pm i\pi\right]. \tag{16}$$

The more remote poles make an exponentially small contribution. It is easy to verify that integration with respect to k_z does not alter the form of the exponential factor of the wave function. We thus arrive at the conclusion that the argument g of the exponential at a large distance from the center is of the form

$$g=\pi|x|/2\lambda L. \tag{17}$$

We note that if the center is not on the surface but at a distance smaller than or of the order of λL from the surface, we get the same asymptotic form (17). It will be more convenient to consider the case when the distance from the center to the surface is much larger than λL somewhat later.

4. IMPURITY CENTER IN A FILM

In this section we consider the problem of calculating the asymptotic wave function of an impurity center located in a film bounded by two infinite potential walls and in a magnetic field. As before, the y axis is perpendicular to the film plane and the magnetic field is along the z axis. The film boundaries are located at $y=\pm d/2$. The magnetic field is regarded as strong in the sense that $\hbar\omega_c\gg E_H$. The argument g of the exponential will be investigated as a function of the parameter $\eta=d/2\lambda$. We defer the calculation to the end of this section and present rightaway the results and their qualitative interpretation. We locate the impurity center at the origin. For a remote point $\mathbf{r}=(x,y,0)$ the exponent $g(x,y)$ takes then the form

$$g(x,y)=\frac{|x|}{a_H},\quad \eta\ll1, \tag{18a}$$

$$g(x,y)=\frac{|x|}{d}\left[\frac{2E_H}{\Delta E(0)}\right]^{1/2}+\frac{y^2}{2\lambda^2},\quad 1\ll\eta\ll L, \tag{18b}$$

$$g(x,y)=\frac{\pi}{2}\frac{|x|}{\lambda L}+(\eta-L)^2+\frac{y^2}{2\lambda^2}-\frac{|y|}{\lambda}(\eta-L),$$
$$L\ll\eta\ll\frac{|x|}{\lambda} \tag{18c}$$

$$g(x,y)=\frac{x^2+y^2}{4\lambda^2},\quad L\ll\frac{|x|}{\lambda}\ll\eta. \tag{18d}$$

In Eq. (18b),

$$\Delta E(0)=\frac{2}{\sqrt{\pi}}\hbar\omega_c\eta e^{-\eta^2}.$$

It can be seen from (18) that in the case of a film with abrupt boundaries the expression for $g(x,y)$ is substantially different, depending on the ratio of the lengths d and λL. If $d\ll\lambda L$, the function $g(x,y)$ is similar to that for a film with a parabolic potential. If, however, the film is thick enough, i.e., $d\gg\lambda L$, the function $g(x,y)$ acquires, besides the terms that depend on x and y, a term that depends only on the film thickness [see (18c)]. This fact can be quite easily understood.

Recall that the anomalous transparency effect considered in the present paper is due to the fact that the electron, imparting momentum to the film walls, shifts the center of the Landau oscillator. It seems sufficiently evident that to give up momentum effectively, in the case of a thick film with abrupt boundaries, the electron must come close to the film boundaries. In fact, examine Eq. (18c) at $y=0$. The function $\exp[-g(x,0)]$ describes the probability amplitude of electron tunneling from an impurity center to a remote point $r=(x,0,0)$. It follows from (18c) that out of all the possible paths of tunneling from point 1 to point 2 in a thick film ($d\gg\lambda L$) the electron will choose the paths marked II and III in Fig. 2(a). Indeed, Eq. (18a) can be treated in the following manner: the electron tunnels from point 1 first to a point located at a distance of the order of λL from the wall (this process is described by the factor $\exp[-\frac{1}{2}(\eta-L)^2]$), and then dropping the magnetic barrier, tunnels along the wall at a distance (the factor $\exp(-\pi x/2\pi L)$), and finally tunnels again from the wall to the point 2 (again the factor $\exp[-\frac{1}{2}(\eta-L)^2]$). We note that the factor describing the tunneling along the wall is exactly equal to the exponential factor obtained by us earlier in Sec. 3 for the case when the center is near the surface of a semi-infinite crystal [see (17)].

Clearly the tunneling paths II or III will be preferable to the path I (along the x axis) only if the magnetic barrier, which must be overcome on path I is thicker than the barrier that must be overcome when tunneling to the surface of the film (i.e., at $x\gg d/2$). If the inverse condition holds ($x\ll d/2$), however, the tunneling is along path I and the asymptotic form in this case [see (18d)] coincides with the asymptotic form of a strong field in the absence of the film potential (2). We note that it follows from the arguments just presented that at $x\gg d/2\gg\lambda L$ the wave functions at points located near the film surfaces should be exponentially large compared with its values at points on the axis, for in order to land at a point located near the surface (e.g., point 3 on Fig. 2a) it is necessary to overcome only once a magnetic barrier of width

$\approx d/2$. Indeed, from (18c) at $y\neq 0$ we find that the wave function in the asymptotic region is centered mainly near the film surfaces, and its value near the surface is larger than the value on the x axis by a factor $\exp(\eta^2/2)$, which precisely describes the probability amplitude of electron tunneling from point 3 to point 2.

Let us now dwell briefly on the case when the impurity center is not in the symmetry plane of the film but is shifted towards one of the film boundaries. Then, if $d\lesssim \lambda L$, the electron is effectively located midway in the film (since its energy is then a minimum) and the asymptotic form will be the same (18a,b). If, however, the film thickness is much larger than λL, to find the asymptotic form at an arbitrary point 2 (the impurity as at point 2) it is necessary to compare the amplitudes of the tunneling probabilities along paths I and II (see Fig. 2b). The tunneling will follow the path for which the probability is larger and the asymptotic form is obtained by multiplying $\exp(-g)$, where g is given by (17), by the corresponding amplitudes for tunneling to the surface and from the surface of the film. In particular, if points 1 and 2 of Fig. 2b have the same y coordinate, the argument $g(x,y)$ of the exponential is given by

$$g(x,y)=\frac{\pi}{2}\frac{|x|}{\lambda L}+\left(\eta-L-\frac{|y|}{\lambda}\right)^2. \tag{19}$$

The foregoing is correct, of course, only if the x coordinate of the point at which the asymptotic form is sought is much larger than the film thickness.

Finally, if the center is near the surface of a semi-infinite crystal at a distance from it large compared with λL, then in Fig. 2b there remains only path I (the surface is on the left) and to find the asymptote at point 2 for this path it is necessary to go through the procedure described above.

We devote the end of this section to the mathematical derivation of Eqs. (18). The asymptotic wave function is determined by Eq. (4), where now $\varphi_n(y-y_0)$ is the wave function of one-dimensional oscillator in a state n, satisfies zero boundary conditions on the film surfaces, and corresponds to the eigenvalue $E_n(k_x)$. Next, just as in Sec. 3, we include in the sum over n only the term with $n=0$ (since $\hbar\omega_c \gg E_H$). The form of the functions $\varphi_0(y-y_0)$ and $E_0(k_x)$ in (4) differs substantially, depending on the value of the parameter η.

We consider first the case $\eta\gg 1$. In the absence of an impurity the y coordinate of the center of the oscillator y_0 is preserved and, as can be shown, the zeroth-order energy correction due to the interaction with the film surfaces takes the form

$$\Delta E(y_0)=\frac{1}{\sqrt{\pi}}\hbar\omega_c\left[\left(\eta-\frac{y_0}{\lambda}\right)\exp\left\{-\left(\eta-\frac{y_0}{\lambda}\right)^2\right\}+\left(\eta+\frac{y_0}{\lambda}\right)\times\exp\left\{-\left(\eta+\frac{y_0}{\lambda}\right)^2\right\}\right]. \tag{20}$$

Equation (20) is valid if the distance from the center of the oscillator to each of the surfaces is large compared with the magnetic length: $\eta-y_0/\lambda\gg 1$, $\eta+y_0/\lambda\gg 1$. Since (as we shall see later) the characteristic values of y_0 in the integral (4) are such that these conditions are satisfied, the functions $E_0(k_x)$ and $\varphi_0(y-y_0)$ will be taken in the form

$$E_0(k_x)=\frac{\hbar\omega_c}{2}+\Delta E(y_0),\quad \varphi_0(y-y_0)=\exp\left[-\frac{1}{2\lambda^2}(y-y_0)^2\right], \tag{21}$$

where $\Delta E(y_0)$ is given by (20). The function $\varphi_0(y-y_0)$ is of the same form as for a free electron in a magnetic field in the absence of the film potential. The ground-state energy E of an electron on an impurity, which enters in Eq. (4), is equal to

$$E=\hbar\omega_c/2+\Delta E(0)-E_H, \tag{22}$$

where $\Delta E(0)$ is the increase of the energy of the zeroth level in the absence of an impurity, due to the interaction with the film walls, in the case when their distances to the oscillator center are equal (this quantity is given by Eq. (20) with $y_0=0$).

Substituting (20)–(22) in (4) and introducing a new integration variable $\varkappa=y_0/\lambda=\lambda/k_x$, we obtain the following expression for the Green function $G_E(\mathbf{r};0)$ in the case $\eta\gg 1$:

$$G_E(\mathbf{r};0)\propto\int_{-\infty}^{\infty}dk_z e^{ik_z z}\int_{-\infty}^{\infty}d\varkappa\left\{\frac{\hbar^2k_z^2}{2m}+E_H+\Delta E(0)\left[e^{-\varkappa^2}\left(\operatorname{ch}2\varkappa\eta-\frac{\varkappa}{\eta}\times\operatorname{sh}2\varkappa\eta\right)-1\right]\right\}^{-1}e^{ix\varkappa/\lambda}e^{-\varkappa^2}e^{y\varkappa/\lambda}e^{-y^2/2\lambda^2}. \tag{23}$$

We calculate next the function $G_E(\mathbf{r},0)$ at $z=0$. Since $\eta\gg 1$, we are restricted in (23) to the inequality $\Delta E(0)\ll\hbar\omega_c$. Within the framework of this inequality there are several parametrically different cases that are realized at different film thicknesses d.

1) Let the film thickness be such that the inequalities $\hbar\omega_c\gg\Delta E(0)\gg E_H$, are satisfied or, equivalently, $1\ll\eta\ll L$. According to (23), the characteristic values of $\varkappa$ are then such that the inequalities $\varkappa\ll 1/\eta\ll 1\ll\eta$ are satisfied. Then, using the fact that $|y|<d/2$, we rewrite (23) in the form

$$G_E(x,y;0)\propto\int_{-\infty}^{\infty}dk_z\int_{-\infty}^{\infty}d\varkappa\left\{\frac{\hbar^2k_z^2}{2m}+E_H+\Delta E(0)\,2\eta^2\varkappa^2\right\}^{-1}\times e^{ix\varkappa/\lambda}e^{-y^2/2\lambda^2}. \tag{24}$$

The integral with respect to $\varkappa$ in (24) is determined by the residue of the integrand at its poles in the complex $\varkappa$ plane. We find ultimately that the argument of the exponential of the wave function takes in this case the form (18b).

2) Let now the width of the film be such that η lies in the interval $L\ll\eta\ll x/\lambda$ (the first of these inequalities corresponds to $\Delta E(0)\ll E_H$). To calculate the integral with respect to $\varkappa$ in (23) we choose the contour shown in Fig. 3 in the complex $\varkappa$ plane (we consider for the sake of argument the case $x>0$). The integrals along the segments I and II vanish when these segments are moved away to infinity. The integral along segment II can be easily seen to be proportional to $\exp(-tx^2/\lambda^2)$, where t is a constant of the order of unity. At the considered values of η this term makes an exponentially small contribution. Therefore the integral (23) is determined by the residues of the integrand at the first-order poles:

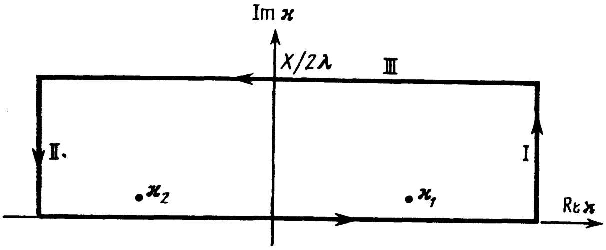

FIG. 3. Contour of integration in Eq. (23).

$$\varkappa_{1,2}=\frac{i\pi}{2}\ln^{-1/2}\left[\frac{\hbar\omega_c}{\sqrt{\pi}(E_H+\hbar^2k_z^2/2m)}\right]$$
$$\pm\left[\eta-\ln^{1/2}\frac{\hbar\omega_c}{\sqrt{\pi}(E_H+\hbar^2k_z^2/2m)}\right].$$

After finding these residues, we rewrite (23) in the form

$$G_E(x,y;0)\propto\int_{-\infty}^{\infty}dk_z\left(\frac{\hbar^2k_z^2}{2m}+E_H\right)^{-1}$$
$$\times e^{-y^2/2\lambda^2}\left[\exp\left(i\frac{|x|}{\lambda}\varkappa_1-\varkappa_1^2+\frac{y\varkappa_1}{\lambda}\right)\right.$$
$$\left.-\exp\left(i\frac{|x|}{\lambda}\varkappa_2-\varkappa_2^2+y\frac{\varkappa_2}{\lambda}\right)\right].\qquad(25)$$

From (25) we ultimately obtain for the argument of the exponential the expression (18c).

3) If the film thickness is such that the conditions $L\ll x/\lambda\ll\eta$, are satisfied we find, after integrating in (23) again along the contour shown in Fig. 3, that now the main contribution to the integral with respect to $\varkappa$ is made by segment III. Making in (23) the change of variable $\varkappa'=(ix+y)/2\lambda+\varkappa$ we obtain, as a result of integration along this segment, expression (18d) for the argument $g(x,y)$ of the exponential.

Finally, in the inverse limiting case of films that are thin compared with the magnetic length ($\eta\ll 1$) we can represent (4) in the form ($y=z=0$)

$$G_E(x;0)\propto\int_{-\infty}^{\infty}dk_z\int_{-\infty}^{\infty}d\varkappa\left\{\frac{\hbar^2k_z^2}{2m}+E_H+\frac{\hbar\omega_c}{2}\varkappa^2\right\}^{-1}e^{i\varkappa x/\lambda},\qquad(26)$$

where $\varkappa=k_x\lambda$. It follows from (26) that the argument of the exponential takes in this case the form (18a).

5. INFLUENCE OF SURFACE ON HOPPING CONDUCTION IN A MAGNETIC FIELD

In this section we solve the two hopping-conduction problems mentioned in the Introduction. The first, that of surface hopping conduction, corresponds, e.g., to the physical situation near the surface of an MOS structure doped with surface donors and is purely two-dimensional. Assume that donors are randomly located on the surface (x,z plane) of a pure crystal, and their number n per unit surface is small enough to satisfy the inequality $na^2\ll 1$. We consider hopping conduction over the nearest neighbors. In this case the resistivity is $\rho=\rho_3\exp(\varepsilon_3/kT)$. We calculate the value of ρ_3, known to be determined by the overlap of the impurity wave functions and to depend exponentially on the impurity density and on the magnetic field[1]:

$$\rho_3\propto\exp\xi_c(n,H).\qquad(27)$$

To calculate $\xi_c(n,H)$ by the percolation method[1] we must plot a curve on which $2g(x,z)=\xi$, where $g(x,z)$ is the modulus of the argument of the exponential of the wave function of the donor located at $x=z=0$. We must next calculate the ares $S(\xi)$ of the figure bounded by this curve and find ξ_c from the condition

$$nS(\xi_c)=B_c,\qquad(28)$$

where B_c is the critical number of bonds over which the percolation takes place. Using (17) for $g(x,0)$ and the fact that $g(0,z)=|z|/a_H$ we find that

$$S(\xi)=c\xi^2\lambda La_H,$$

where c is a numerical coefficient. From (28) and (27) we obtain then for the two-dimensional resistivity

$$\rho_{3\square}\propto\exp(\alpha na_H\lambda L)^{-1/2},\qquad(29)$$

where α is a numerical coefficient. If we repeat this calculation with the aid of Eq. (2), which does not take scattering from the wall into account, we obtain in (29) an exponent $\frac{2}{3}$ rather than $\frac{1}{2}$. Recognizing that under the conditions ($na^2\ll 1$, $\lambda\ll a$) considered we have the inequality $na_H\lambda L\ll 1$, we see that the influence of the surface on the asymptotic wave function decreases strongly the value of the magnetoresistance and changes its dependences on n and H. It must be borne in mind that the limiting case $\lambda\ll a$ considered above is realized in experiment only when the Bohr radius is small enough (InSb, InAs, etc.) For silicon MOS structures $\lambda\gg a$ at reasonable field values. In this case an examination of the asymptotic wave functions near the wall shows that they fall off like $\exp(-|x|/b)$, where $b=a(1-\gamma a^4/\lambda^4)$ and γ is a numerical coefficient, i.e., much more slowly than in accord with Eq. (1). As a result, the magnetoresistance $\rho_{3\square}(H)/\rho_{3\square}(0)$ in the case of surface hopping conduction should be proportional to $\exp(n^{-1/2}a^3\lambda^{-4})$, i.e., be considerably more stable than indicated by Eq. (1).

We consider now the second problem, hopping conduction of a film uniformly doped over its volume. Let N be the donor density and $Na_3\ll 1$, so that hopping conduction is produced. We assume that the film thickness is large compared with all the microscopic lengths ($d\gg N^{-1/3},a,\lambda L$) and that $\lambda\ll a$. In a magnetic field, the value of ρ_3 of a bulky sample is of the form[1]

$$\rho_3=\rho_0\exp\{q(Na_H\lambda^2)^{-1/2}\},\qquad(30)$$

where $q\approx 0.9$. Naturally, the resistivity in the interior of the film has the same value. We shall show now, however, that the subsurface layer has an exponentially smaller resistance and shunts the resistance of the film volume. To this end we consider near any of the film surfaces a layer of thickness Δy that satisfies the inequalities $\lambda L\ll\Delta y\ll N^{-1/3}$. Such a layer can be regarded as a two-dimensional system with $n(\Delta y)=\Delta yN$ impurities per unit area. According to (19) the value of the wave function of one impurity of the layer at another impurity, at large distances between them, differs from the function $\exp(-\pi|x|/2\lambda L)$ used in the derivation of (29) only by a factor of the order of $\exp\{-(\Delta y/\lambda)^2\}$ that describes the probability of tunneling first to the surface and then from the surface to a distance on the order of Δy. The square of this factor enters in the expression for the layer conductivity. We therefore obtain for the two-dimensional resistivity $\rho_{3\square}$ of the layer

$$\rho_{3\square}(\Delta y) \propto \exp\{(\alpha \Delta y N a_H \lambda L)^{-1/2} + 2(\Delta y/\lambda)^2\}. \tag{31}$$

The resistivity (31) has a sharp minimum at a layer thickness

$$\Delta y_{\rm opt} \approx \lambda (N L a_H \lambda^2)^{-1/5} \ll N^{-1/3}. \tag{32}$$

Substituting (32) in (31) we find the two-dimensional resistivity of this layer

$$\rho_{3\square}(\Delta y_{\rm opt}) \approx \rho_{0\square} \exp\{p(N a_H \lambda^2 L)^{-2/5}\}, \tag{33}$$

where p is a numerical coefficient. On the other hand, the resistivity of a square centimeter of the inner part of the film is given by (30), referred to d. Comparing this resistivity with (33) and recognizing that $N a_H \lambda^2 L \ll 1$ we conclude that the resistivity of films that are not too thick is determined by the surface layer and takes the form (33). The anomalous tunneling transparency near the surface therefore decreases substantially the resistivity of the film and weakens its dependence on the magnetic field.

6. CONCLUSION

We conclude by presenting one more interpretation, without the use of the magnetic-barrier concept, of the effect considered in the present paper as well as in Refs. 2 and 3, namely the anomalous transparency to tunneling across a magnetic field. This interpretation is based on the concept of the Feynman path integral. It is known[7] that to find the total amplitude of the probability of transition from point 1 to point 2 it is necessary to sum the transition amplitudes over all possible paths joining these points. The contributions of the individual paths are proportional here to $\exp(iS/\hbar)$, where S is shortened action for a given trajectory. Using the expression for the action S in an electromagnetic field[8] in the form

$$S = \int_1^2 \{2m[E - U(r)]\}^{1/2} dr + \frac{e}{\hbar c}\int_1^2 \mathbf{A}\, d\mathbf{r},$$

we find that in the case of electron tunneling with negative energy $-E_H$ in a potential $U = 0$ the amplitude of a transition along a given path is equal to

$$\exp\left(-\frac{\mathscr{L}_{12}}{a_H}\right)\exp\left(\frac{ie}{c\hbar}\Phi\right),$$

where $\mathscr{L}_{12}$ is the length of the path and Φ is the magnetic-field flux through the contour consisting of this path and the straight line joining points 1 and 2. We note that the influence of the magnetic field is contained entirely in the second factor. Owing to the presence of the oscillating factor $\exp(ie\Phi/\hbar c)$, when the transition amplitudes are summed over the different paths the waves propagating along the different paths interfere with one another, and this leads to an exponential dependence of the transparency on the magnetic field [see (1) and (2)]. The result of the presence of some scattering potential, e.g., an opaque crystal surface parallel to the straight line drawn from point 1 to point 2, is therefore clear. Some of the paths will be forbidden, and an unbalance in the path integral is produced thereby. The neighboring trajectories will suppress one another more weakly, so that the transition amplitude will increase exponentially. In the film, paths are forbidden by both walls (Fig. 4), so that in the limit of a very thin film $\Phi \approx 0$ for practically all the remaining trajectories, so that the magnetic field should play no role. This conclusion is confirmed by Eq. (18a), according to which at $d \ll \lambda$ the asymptotic wave function takes a "nonmagnetic" form.

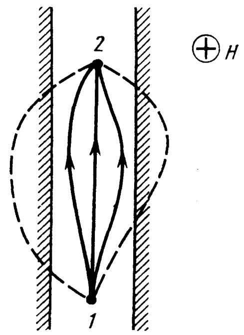

FIG. 4. Paths joining points 1 and 2. Forbidden paths are shown by dashed lines. The magnetic field is perpendicular to the plane of the figure.

Similarly, if an opaque screen with a narrow aperture is placed halfway between points 1 and 2 the total amplitude at point 2 will receive contributions from only those paths which pass through the aperture. It can be verified that in this case the increment to the argument of the exponential (1) in a weak field, and the entire argument of the exponential (2) in a strong field, will decrease roughly to one-half.

Multiple subbarrier scattering by impurities located in the region adjacent to the straight line joining points 1 and 2, which was considered in Refs. 2 and 3, is similar to the action, as it were, of an entire system of screens. It enhances the contribution of some paths and weakens that of others. The resultant unbalance increases the sum over the trajectories and enhances the transparency exponentially.[2,3]

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[1]B. I. Shklovskiĭ and A. L. Éfros, Élektronnye svoĭstva legirovannykh poluprovodnikov (Electronic Properties of Doped Semiconductors), Nauka, 1979.
[2]B. I. Shklovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 43 (1982) [JETP Lett. **36**, 51 (1982)].
[3]B. I. Shklovskiĭ and A. L. Éfros, Zh. Eksp. Teor. Fiz. **84**, 811 (1983) [Sov. Phys. JETP **57**, 470 (1983)].
[4]A. Harstein and A. B. Fowler, Surf. Scien. **73**, 19 (1978).
[5]M. Pepper, Phil. Mag. **37**, 83, 187 (1978).
[6]O. V. Konstantinov, O. N. Filatov, and A. Ya. Shik, Fiz. Tekh. Poluprov. **7**, 786 (1973) [Sov. Phys. Semicond. **7**, 539 (1973).
[7]R. P. Feynman and A. B. Hibbs, Quantum Mechanics and Path Integrals, McGraw, 1965.
[9]L. D. Landau and E. M. Lifshitz, Classical Theory of Fields, Pergamon, 1971.

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