Kinetics of a quantum particle in a long metallic wire

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The problem of the conductivity of long metallic wires is considered. The calculations are carried with the aid of the one-dimensional supermatrix σ model. The transfer matrix technique is used. For arbitrary external frequencies the problem is reducible to the solution of two or three partial differential equations. Analytic solutions are obtained in the limits of low and high frequencies. The density-density correlator is computed, and it is shown that all the states of the system are localized. The permittivity is determined. The magnetic and spin-orbit interactions increase the localization length, but leave the form of the density-density correlator at low frequencies unchanged.

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1. INTRODUCTION

Recently Thouless[1] predicted the localization of the electrons in long metallic wires. A similar assertion was made by Abragams *et al.*[2] The arguments in these papers are based on the use of the scaling-law hypothesis. According to Thouless,[1] the only quantity determining the behavior of the system is the total resistance of the sample. In sufficiently long wires the resistance should increase exponentially with increasing sample length L. The transition from short wires with linear length dependence of the resistance to long wires with exponential dependence occurs at some characteristic value L_c of the length. The resistance of a sample with such length is approximately equal to 10 kΩ. To Thouless,[1] the wire thickness is not important, and all the predictions in his paper are made for both chains and thick wires. At the same time, these models are, from the formal point of view, entirely different. We have for one-dimensional chains well-developed methods[3–6] that allow us compute the frequency dependence of the conductivity.

Weller *et al.*[7] have analyzed with the aid of Berezinskiĭ's method[3] a system of N coupled chains, and have shown that, for low frequencies, the problem reduces to the solution of Berezinskiĭ's equations for a single chain. Therefore, the conductivity goes to zero as $\omega \to 0$. The model of coupled chains has also been considered by Dorokhov,[8] who computed the dependence of the localization length on the number of chains. Anderson *et al.*[9] have considered a model of N conducting channels with the aid of Landauer method. These authors also draw the conclusion that the electrons are localized. In principle, the models studied in these papers can be used to describe wires as well. But the number of chains or channels should not be large (or, conversely, the mean free path should be long); otherwise we cannot begin with a description in terms of the individual chains. This limitation corresponds to sufficiently thin or sufficiently pure wires.

Below we shall consider the opposite limit of sufficiently thick metallic wires. We assume that the following conditions are fulfilled:

$$\tau\varepsilon_0 \gg 1, \quad \tau\varepsilon_0 \ll p_0^2 S, \tag{1}$$

where S is the cross-section area and ε_0 and p_0 are the Fermi energy and momentum.

The first of the inequalities (1) implies that we are considering the case of weak localization. The second inequality in (1) corresponds to a situation in which the reciprocal electron-impurity scattering time is much greater than the distance between the levels of the transverse quantization. In this limit, classical diffusion occurs over small distances. The opposite case is apparently considered in Refs. 7–9. In the limit (1) of sufficiently thick wires being considered the system is three-dimensional in almost every respect. Only the sample geometry remains one-dimensional. The three-dimensional character of the Fermi surface leads to the usual three-dimensional integrals over the electronic lines in perturbation theory. The one-dimensional character of the sample geometry is unimportant for the integrals over the electronic lines. But the one-dimensional character of the geometry leads to a contribution from the diffusional modes that diverge at low frequencies.[10] Therefore, the contribution of such modes must be taken into account exactly in the investigation of the conductivity problem in wires. Here the method of supersymmetry developed by one of the present authors[11] is very convenient. In this method the problem of the kinetics of a quantum particle in a disordered potential reduces to the problem of the investigation of the supermatrix σ model. The inequalities (1) constitute conditions that are rigid enough to allow the use of such a σ model in the model for a wire being considered. The one-dimensional character of the sample geometry leads to a one-dimensional σ model. In the following sections the one-dimensional σ model corresponding to the conductivity problem is solved exactly with the aid of the transfer matrix method. It is shown that all the states in the model in question are localized. The density-density correlator is computed. The effect of the magnetic and spin-orbit interactions is considered.

2. WIRES AND THE ONE-DIMENSIONAL SUPERMATRIX σ MODEL

In Ref. 11 one of the present authors develops for the purpose of investigating the behavior of a quantum particle

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in a weak random potential a supersymmetry method based on the writing of the Green functions in the form of path integrals over the boson and fermion variables. Let us briefly give the main points in the derivation of the supermatrix σ model that arises in this method, and show how the one-dimensional character of the sample geometry makes such a σ model one-dimensional.

Let us consider a long metallic sample with a finite transverse dimension a. Assume that the dimension a is much greater than the atomic dimensions. The quantity a can be arbitrarily large if we are investigating the low-frequency behavior of a sample at absolute zero. At finite temperatures the dimension a should not exceed the critical diffusion length $L_D = (D\tau_\varepsilon)^{1/2}$, where τ_ε is the inelastic scattering time and D is the diffusion coefficient. The sample may contain randomly distributed impurities. The allowed energy values are the eigenvalues of the Schrödinger equation

$$H\varphi_k=\varepsilon_k\varphi_k, \quad H=\tilde{H}+H_1, \quad \langle H_1\rangle=0, \tag{2}$$

where the φ_k are the eigenfunctions corresponding to the energy ε_k. The angle brackets in (2) denote averaging over the positions of the impurities, $\tilde{H}$ is the regular part of the Hamiltonian (the kinetic energy and the interaction with the external field), and H_1 is the interaction with the impurities. We assume that the Hamiltonian H_1 in (2) can include both the scattering by the ordinary impurities and the scattering by the magnetic and spin-orbit impurities.

It is convenient to describe the behavior of an electron in a random potential with the aid of the retarded density-density correlator

$$X_\omega=\int_0^\infty dt e^{-i\omega t}\langle\langle[\rho(r,t),\rho(0,0)]\rangle\rangle=\int_{-\infty}^{\infty}(n_\varepsilon-n_{\varepsilon+\omega})K(\varepsilon,\omega)\frac{d\varepsilon}{2\pi}. \tag{3}$$

Using the single-particle retarded G^R and advanced G^A Green functions, we can reduce the function $K(\varepsilon,\omega)$ to the form

$$K(\varepsilon,\omega)=\langle G_\varepsilon^R(r,r')G_{\varepsilon+\omega}^A(r',r)\rangle,$$
$$G^{R,A}=\sum_k\frac{\varphi_k(r)\varphi_k^*(r')}{\varepsilon-\varepsilon_k\pm i\delta}. \tag{4}$$

In Eqs. (3) and (4) the symbol $\langle\ldots\rangle$ denotes averaging over the impurities and the double brackets correspond to the addition of thermodynamic averaging.

As in Ref. 11, we write the Green-function product $G_\varepsilon^R G_{\varepsilon+\omega}^A$ in the form of an integral over commuting and anticommuting variables:

$$\langle G_{\varepsilon+\omega}^R(r,r')G_\varepsilon^A(r',r)\rangle$$
$$=\int\langle\psi_1^1(r)\bar{\psi}_1^1(r)\psi_3^2(r')\bar{\psi}_3^2(r')e^{-L}\rangle D\psi, \tag{5}$$

where the Lagrangian L has the form

$$L=i\int\bar{\psi}(X)\left(-H_0+\frac{1}{2}\omega\Lambda+H_1\right)\psi(X)\,dX. \tag{6}$$

In Eqs. (5) and (6), $X=\{r,\alpha\}$ describes the coordinate and spin of the particle and ψ is a supervector having as its components the classical boson and fermion variables. In the case of spinless particles ψ has eight components, which can be written in the form

$$\psi^a=\begin{pmatrix}u^a\\v^a\end{pmatrix}, \quad u^a=\frac{1}{\sqrt{2}}\begin{pmatrix}\chi^{a*}\\\chi^a\end{pmatrix}, \quad v^a=\frac{1}{\sqrt{2}}\begin{pmatrix}s^{a*}\\s^a\end{pmatrix}, \tag{7}$$
$$\bar{\psi}=(C\psi)^T, \quad a=1,2.$$

In the expressions (7) the superscript a indicates affiliation with the retarded or advanced Green functions, χ and χ^* are the anticommuting Grassman variables (the algebra of these variables can be found in Ref. 12), and s and s^* are complex numbers. The letter T denotes the operation of transposition. The elements of the matrix C have the form

$$C^{ab}=\Lambda^{ab}\begin{pmatrix}c_1&0\\0&c_2\end{pmatrix}, \tag{8}$$

where Λ is a diagonal matrix with elements $\Lambda^{11}=-\Lambda^{22}=1$. The matrices c_1 and c_2 are equal to

$$c_1=\begin{pmatrix}0&-1\\1&0\end{pmatrix}, \quad c_2=\begin{pmatrix}0&1\\1&0\end{pmatrix}.$$

In the absence of external fields, H_0 in (6) has the form

$$H_0=-\varepsilon+\frac{1}{2m}\frac{\partial^2}{\partial r^2}.$$

If only elastic scattering by ordinary impurities is possible in the metal, then $H_1=U(r)$, where $U(r)$ is the potential of the impurities. We assume that the potential $U(r)$ is a random quantity, distributed according to the δ-correlated Gaussian law

$$\langle U(r)\rangle=0, \quad \langle U(r)U(r')\rangle=\frac{1}{2\pi\nu\tau}\delta(r-r'), \tag{9}$$

where τ is the mean free path and $\nu=mp_0/2\pi^2$ is the density of states.

We carry out the averaging in (5) with the aid of (9). The expression (5) has the same form after, as before, the averaging if by the Lagrangian L we mean the following quantity:

$$L=\int\left[-i\bar{\psi}H_0\psi+\frac{1}{4\pi\nu\tau}(\bar{\psi}\psi)^2+\frac{1}{2}i(\omega-i\delta)\bar{\psi}\Lambda\psi\right]dr. \tag{10}$$

It is shown in Ref. 11 that the supersymmetry possessed by the Lagrangian (10) at zero frequencies is spontaneously broken, as a result of which there arise the averages $Q_{\alpha\beta}=\langle\psi_\alpha\bar{\psi}_\beta\rangle$. The quantity Q is a supermatrix containing both boson and fermion elements. This matrix satisfies the conditions for charge and Hermitian conjugation invariance:

$$Q=\bar{Q}=KQ^+K, \quad K=\begin{pmatrix}1&0\\0&k\end{pmatrix}, \tag{11}$$

where k is a superelement of the form

$$k=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

As in Ref. 11, we mean here by the operation of charge and Hermitian conjugation the following transformations:

$$\bar{Q}=CQ^TC^T, \quad Q^+=(Q^T)^*.$$

The symbols T and * denote supertransposition and complex conjugation. After separating the averages in the $(\bar{\psi}\psi)^2$ term in (10), we can write the Lagrangian L in the form

$$L=\int\left[\bar{\Psi}\left(-iH_0+\frac{1}{2}i(\omega-i\delta)+\frac{Q}{2\tau}\right)\Psi+\frac{\pi\nu}{4}\mathrm{SSp}Q^2\right]dr, \tag{12}$$

where $\mathrm{SSp}M\equiv\mathrm{Sp}\,kM$ for any matrix M. The supertrace SSp plays in the theory of supermatrices the same role played by the trace in the theory of ordinary matrices.

For low impurity concentrations in not very thin wires, such that the conditions (1) are fulfilled, the eigenvalues of the supermatrix Q fluctuate little, and are determined by the saddle-point value of the free-energy functional corresponding to the Lagrangian L (12). At the same time, for $\omega=0$, the ground state is highly degenerate. The general form of the supermatrix Z corresponding to the ground state can be written as follows:

$$Q=W+\Lambda(1-W^2)^{1/2}, \tag{13}$$

where

$$W=\begin{pmatrix}0 & Q^{12}\\ -Q^{12} & 0\end{pmatrix}, \quad Q^{12}=i\begin{pmatrix}a & i\sigma\\ \rho^+ & ib\end{pmatrix}. \tag{14}$$

The matrices a, b, σ, and ρ are 2×2 matrices, the elements of a and b being ordinary numbers and those of σ and ρ being Grassman variables. The a, b, σ, and ρ matrices that ensure the fulfillment of the condition (11) have the form

$$a=\begin{pmatrix}a_1 & a_2\\ -a_2^* & a_1^*\end{pmatrix}, \quad b=\begin{pmatrix}b_1 & b_2\\ b_2^* & b_1^*\end{pmatrix},$$
$$\sigma=\begin{pmatrix}\sigma_1 & \sigma_2\\ -\sigma_2^* & -\sigma_1^*\end{pmatrix}, \quad \rho=\begin{pmatrix}\rho_1 & \rho_2\\ -\rho_2^* & -\rho_1^*\end{pmatrix}. \tag{15}$$

The strong degeneracy of the ground state at zero frequencies lead to the existence of Goldstone modes connected with the Q fluctuations. These modes and their interaction are described by a nonlinear generalized σ model, in whose free energy the frequency plays the role of an external field[11]

$$F=\tfrac{1}{8}\pi\nu\int\mathrm{SSp}[D(\nabla Q)^2-2i(\omega-i\delta)\Lambda Q]dr, \tag{16}$$

where $D=\overline{v_x^2}\,\tau$ is the diffusion coefficient.

The density-density correlator can accordingly be written in the form

$$K=-2(\pi\nu)^2\int Q_{13}^{12}(r)Q_{31}^{21}(r')e^{-F[Q]}DQ. \tag{17}$$

At sufficiently low temperatures, when the diffusion length determined by the inelastic processes is greater than the transverse dimensions of the system, only the states with coordinate-independent Q are important. The higher harmonics lead to a sharp increase in the energy, and can therefore be discarded. Therefore, to study the low-frequency behavior of the system, we need only consider the one-dimensional σ model described by the formula (16). A simple analysis shows that the frequency should satisfy the following inequality: $\omega \ll D/S$. At high frequencies the corrections to the classical diffusion are three-dimensional, and their contribution is small.

In deriving the formulas (16) and (17) we made essential use of the conditions (1), which allow us to limit ourselves to the consideration of the saddle-point value of the matrix Q, and to replace the sums of the type

$$\sum_m(\varepsilon_m-\varepsilon_0+iQ/2\tau)^{-1}$$

that arise in the calculations by integrals over $\xi=\varepsilon_m-\varepsilon_0$.

The symmetry of the matrix Q defined by the formulas (15) corresponds to a model in which the disorder is created by potential impurities (model I). The application of an external magnetic field leads to the following substitution in the Lagrangian (16):

$$\nabla Q\to\nabla Q+ic^{-1}eA[Q,\tau_3],$$

where τ_3 is the matrix

$$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

in the space of the matrices a, b, σ, and ρ. As a result, the symmetry of the ground state is lowered, and some of the modes acquire a gap even at zero frequency. Such modes can be neglected in the limit $\omega \ll DeH/c$. The expressions (13), (14), and (16) have the same form in this case if by a, b, σ, and ρ we mean the following matrices (model IIa):

$$a=\begin{pmatrix}a_1 & 0\\ 0 & a_1^*\end{pmatrix}, \quad b=\begin{pmatrix}b_1 & 0\\ 0 & b_1^*\end{pmatrix},$$
$$\sigma=\begin{pmatrix}\sigma_1 & 0\\ 0 & -\sigma_1^*\end{pmatrix}, \quad \rho=\begin{pmatrix}\rho_1 & 0\\ 0 & -\rho_1^*\end{pmatrix}. \tag{18}$$

To allow for the magnetic impurities and the spin-orbit interactions we must double the dimensions of ψ and Q because of the allowance for the spin structure. These interactions also lower the symmetry of the Lagrangian (16), and freeze out some of the diffusion modes. In the presence of magnetic impurities the supermatrix Q is defined, as in the case with a magnetic field, by the formulas (18). But now the quantities a_1, b_1, σ_1, and ρ_1 are 2×2 matrices proportional to the 2×2 unit matrix (model IIb). The model IIb coincides with the model IIa if we double in the latter the free energy and the density-density correlator given by the formulas (16) and (17). If there are no magnetic interactions, but there are spin-orbit interactions present, the matrices a, b, σ, and ρ in (14) have the form (model III)

$$a=\begin{pmatrix}a_1 & a_2\\ a_2^* & a_1^*\end{pmatrix}, \quad b=\begin{pmatrix}b_1 & -b_2\\ b_2^* & b_1^*\end{pmatrix},$$
$$\sigma=\begin{pmatrix}\sigma_1 & \sigma_2\\ -\sigma_2^* & \sigma_1^*\end{pmatrix}, \quad \rho=\begin{pmatrix}\rho_1 & \rho_2\\ -\rho_2^* & \rho_1^*\end{pmatrix}. \tag{19}$$

The expressions written above show that the problem of computing the density-density correlator reduces to the problem of investigating a one-dimensional supermatrix σ model with symmetry that depends on the magnetic and spin-orbit interactions. In the following sections we shall carry out such an investigation with the aid of the transfer matrix method.

3. REDUCTION OF THE ONE-DIMENSIONAL σ MODEL TO PARTIAL DIFFERENTIAL EQUATIONS

The transfer matrix method is very effective in the investigation of the one-dimensional classical models.[13,14] The generalization to the case of systems whose free energies contain both boson and fermion variables does not present any difficulties.[15] The use of this method allows us to solve the problem exactly. Let us, using the usual procedure, reduce the functional integral (17) to a definite integral over the supermatrices:

$$K(x_1,x_2)=-2\pi^2\nu\tilde{\nu}\int\Gamma(0,x_1;Q_0,Q_1)\,(Q_1)_{13}^{12}\Gamma(x_1,x_2;Q_1,Q_2)$$

$$\times(Q_2)_{31}^{21}\Gamma(x_2,L;Q_2,Q_0)\,dQ_0\,dQ_1\,dQ_2, \tag{20}$$

$$\text{where}=\tilde{\nu}=\begin{cases}\nu S, & \text{models I, IIa}\\ 2\nu S, & \text{models IIb, III}\end{cases} \tag{20a}$$

The kernel Γ in (20) satisfies the Schrödinger equation

$$\left(-\frac{\partial}{\partial x}+\mathscr{H}(Q)\right)\Gamma(x,x';Q,Q')=\delta(x-x')\delta(Q-Q'), \tag{21}$$

where the δ function in the right member of Eq. (21) appears after the passage to the limit $\alpha\to 0$ in the following expression:

$$\exp\left(-\frac{\mathrm{SSp}(Q-Q')^2}{\alpha}\right).$$

In principle, the Hamiltonian $\mathscr{H}$ can be written in a differential form, but this requires a particular parametrization of the supermatrix Q, and the corresponding formulas turn out to be very unwieldy. To simplify the calculations, let us retain for some time the integral form corresponding to the Schrödinger equation. In this representation the Hamiltonian is given by the following expression:

$$\left[\mathscr{H}+\frac{1}{4}i\pi\tilde{\nu}(\omega-i\delta)\,\mathrm{SSp}\Lambda Q\right]\Phi(Q)$$

$$=\lim_{\alpha\to 0}\frac{1}{\alpha}\int\exp\left[-\frac{\pi\tilde{\nu}}{8\alpha}\mathrm{SSp}\,D(Q-Q')^2\right][\Phi(Q')-\Phi(Q)]\,dQ, \tag{22}$$

where $\Phi(Q)$ is an arbitrary matrix function. The evaluation of the integral on the right-hand side of Eq. (22) in the limit $\alpha\to 0$ would lead to the differential form of the Hamiltonian.

Let us expand the kernel in terms of the orthonormal eigenfunctions Ψ_n of the Hamiltonian $\mathscr{H}$ in the form

$$\Gamma(x,x';Q,Q')$$

$$=\theta(x'-x)\sum_n\Psi_n(Q)\,\Psi_n^*(Q')\exp\{-E_n(x'-x)\}. \tag{23}$$

The normalization conditions can be written in the form

$$\int\Psi_n^*(Q)\,\Psi_n(Q)\,dQ=1. \tag{24}$$

The functions Ψ_n and Ψ_n^* can be either ordinary functions or Grassman variables. Therefore, the order in which they have been written down in the expression (24) is important.

Substituting the expressions (23) and (24) into (20), and performing a Fourier transformation with respect to the coordinate, we obtain

$$K(\omega,k)$$

$$=-2\pi^2\nu\tilde{\nu}\sum_n\int\Psi_n(Q)\,Q_{13}^{12}\,[P_{k31}^{(n)}(Q)+P_{-k31}^{(n)}(Q)]\,e^{-E_nL}\,dQ, \tag{25}$$

where L is the sample length.

The function $P_k^{(n)}(Q)$ is connected with the kernel Γ by the following relation:

$$P_k^{(n)}(Q)$$

$$=\int\Gamma(x_1-x_2;Q,Q')\,Q'^{21}\Psi_n(Q')\,e^{(-ik+E_n)(x_1-x_2)}\,dQ'\,d(x_1-x_2). \tag{26}$$

Multiplying Eq. (21) by $Q'^{21}\Psi_n(Q')$, and integrating both sides of this equation over Q', we obtain after performing a Fourier transformation the equation

$$(ik-E_n+\mathscr{H})P_k^{(n)}(Q)=Q^{21}\Psi_n(Q). \tag{27}$$

As we have already mentioned, the $\Psi_n(Q)$ and E_n are the eigenfunctions and eigenvalues of the Schrödinger equation

$$\mathscr{H}\Psi_n=E_n\Psi_n. \tag{28}$$

For the subsequent calculations, it is convenient to go over to the "polar" coordinates of the supermatrix Q (Ref. 16). These coordinates are given by the following expressions:

$$Q=U\begin{pmatrix}\cos\hat{\theta} & i\sin\hat{\theta}\\ -i\sin\hat{\theta} & -\cos\hat{\theta}\end{pmatrix}\bar{U}, \tag{29}$$

where

$$\hat{\theta}=\begin{pmatrix}\theta_{11} & 0\\ 0 & \theta_{22}\end{pmatrix},\quad U=\begin{pmatrix}u & 0\\ 0 & v\end{pmatrix}.$$

The elements θ_{11} and θ_{22} are equal to

$$\theta_{11}=\begin{pmatrix}\theta & 0\\ 0 & \theta\end{pmatrix},\quad \theta_{22}=i\begin{pmatrix}\theta_1 & \theta_2\\ \theta_2 & \theta_1\end{pmatrix},$$

$0<\theta<\pi,\ \theta_1>0,\ \theta_2>0$ model I,

$$\theta_{11}=\begin{pmatrix}\theta & 0\\ 0 & \theta\end{pmatrix},\quad \theta_{22}=i\begin{pmatrix}\theta_1 & 0\\ 0 & \theta_1\end{pmatrix},$$

$0<\theta<\pi,\quad \theta_1>0$ models II,

$$\theta_{11}=\begin{pmatrix}\theta_1 & \theta_2\\ \theta_2 & \theta_1\end{pmatrix},\quad \theta_{22}=i\begin{pmatrix}\theta & 0\\ 0 & \theta\end{pmatrix},$$

$\theta>0,\ 0<\theta_1<\pi,\ 0<\theta_2<\pi$ model III.

The supermatrices u and v in (29) satisfy the following conditions

$$\bar{u}u=\bar{v}v=1,\quad \bar{u}=u^+,\quad \bar{v}=kv^+k. \tag{30}$$

In the integral (22) the important values of Q' are those close to Q. In this limit the supertrace of $(Q-Q')^2$ in the index of the exponential function can be written in the form

$$\mathrm{SSp}(Q-Q')^2=\Delta F_0+\Delta F_1, \tag{31}$$

where

$$\Delta F_0 = 2\,\mathrm{SSp}\,(\Delta\hat{\theta})^2,$$

$$\Delta F_1 = 2\,\mathrm{SSp}\,[(\Delta u\cos\tilde{\theta})^2 + (\Delta v\cos\tilde{\theta})^2 + 2\Delta u\sin\tilde{\theta}\Delta v\sin\tilde{\theta} - (\Delta u)^2 - (\Delta v)^2],\quad \Delta\tilde{\theta} = \hat{\theta}' - \hat{\theta},$$

$$\Delta u = -\Delta\bar{u} = -\ln(\bar{u}u'),\ \Delta v = -\Delta\bar{v} = -\ln(\bar{v}v'),\quad \tilde{\theta} = {}^1/_2(\hat{\theta}+\hat{\theta}').$$

The quadratic form (31) has been written with the accuracy that we must have in order to obtain the first and second derivatives in the Schrödinger equation.

The absence of weighting denominators in (20) is a consequence of the use of integrals taken over commuting and anticommuting variables. In the limit $L \to \infty$ only the ground state makes a contribution to the sum over n in the expression (20). The right-hand side of this expression is finite, and does not vanish only when the energy E_0 of the ground state is equal to zero. Let us assume that the wave function Ψ_0 of the ground state possesses a high symmetry with respect to transformations in the space of the supermatrices Q, and that it depends only on the "radial" variables $\hat{\theta}$. For such functions the normalization conditions can be written in the form

$$\int \Psi^2(\hat{\theta})\,dQ = \Psi^2(0) = 1. \tag{32}$$

The relation (32) can be proved in the following manner. Let us consider the integral

$$I(\beta) = \int f(\theta)\,e^{\beta\,\mathrm{SSp}(\Delta Q)}\,dQ, \tag{33}$$

where $f(\hat{\theta}) \to f(0) \neq 0$ for $\theta \to 0$.

On the face of it, it seems to be convenient to evaluate the integral (33), using the polar coordinates (29). But the Jacobian that arises when we go over to these variables (see Ref. 16) is singular at $\hat{\theta} \to 0$. Therefore, the $\hat{\theta}$ integral becomes divergent. On the other hand, the integrand does not depend on the matrices u and v. This leads to the vanishing of the contribution when we perform the integration over the anticommuting variables. Consequently, we arrive at an indeterminacy of the type $0\cdot\infty$. At the same time, the integration over $\hat{\theta}$ in the $\partial I(\beta)/\partial\beta$ integral does not lead to divergences. The subsequent integration over the anticommuting variables in this integral yields zero, i.e.,

$$\partial I(\beta)/\partial\beta = 0; \tag{34}$$

consequently, we find that $I(\beta) = c$, where c does not depend on β.

To compute c we can consider the limit $\beta \to \infty$. In this limit the function $f(\hat{\theta})$ in the expression (33) can be replaced by its value $f(0)$ at the origin. Using the formula (13), retaining in the limit $\beta \to \infty$ only the term linear in W in the matrix Q, and evaluating the Gaussian integrals, we obtain

$$\int f(\hat{\theta})\,e^{\beta\,\mathrm{SSp}(\Delta Q)}\,dQ = f(0). \tag{35}$$

The relation (35) is valid for any β and any f functions that decrease at infinity. The relation (32) can easily be obtained from the identity (35).

Assuming that the wave function Ψ_0 of the ground state depends only on $\hat{\theta}$, we can write the Schrödinger equation for this function in its explicit form. To do this, we set $\Phi(Q) = \Psi_0(\hat{\theta})$ in Eq. (22), and integrate over u and v. In the limit $\alpha \to 0$ all the integrals that arise in (22) are Gaussian with respect to Δu and Δv. Evaluating these integrals, we reduce the right-hand side of Eq. (22) to the form

$$\lim_{\alpha\to 0}\frac{1}{\alpha}\left(\frac{\bar{\nu}D}{2\alpha}\right)^q \int \exp\left[-\frac{1}{4}\pi\bar{\nu}D\,\mathrm{SSp}\,(\Delta\hat{\theta})^2\right] \times J(\hat{\theta}')J^{-1}(\tilde{\theta})\,[\Psi_0(\hat{\theta}') - \Psi_0(\hat{\theta})]\,d\hat{\theta}', \tag{36}$$

where $q = \frac{3}{2}$ in the models I, III, and $q = 1$ in the models IIa, IIb.

The Jacobian $J(\theta')$ in the expression (36) is a result of our going over to the variables $\hat{\theta}$, u, and v (29), while $J^{-1}(\tilde{\theta})$ appears as a result of the integration of the exponential function (22), (31) over Δu and Δv. The explicit form of the Jacobians $J(\theta)$ can be found in Ref. (16). Going over to the variables λ, λ_1, and λ_2 with the aid of the formulas $\lambda = \cos\theta$, $\lambda_1 = \cosh\theta_1$, and $\lambda_2 = \cosh\theta_2$ (model I), or $\lambda = \cos\theta$ and $\lambda_1 = \cosh\theta_1$ (model II), or $\lambda = \cosh\theta$, $\lambda_1 = \cos\theta_1$, and $\lambda_2 = \cos\theta_2$ (model III), we rewrite the corresponding Jacobians in the form

$$J_\lambda = \frac{|1-\lambda^2|}{(\lambda_1^2+\lambda_2^2+\lambda^2-2\lambda\lambda_1\lambda_2-1)^2}\ \text{models I and III},\qquad J_\lambda = \frac{1}{(\lambda_1-\lambda)^2}\ \text{model II}. \tag{37}$$

Expanding $J(\theta')$, $J^{-1}(\tilde{\theta})$, and $\Psi_0(\hat{\theta}')$ in powers of $\hat{\theta}' - \hat{\theta}$, and evaluating the Gaussian integrals, we find in the limit $\alpha \to 0$ that

$$\mathscr{H}_0\Psi_0 = 0,$$

$$\mathscr{H}_0 = -\frac{1}{2\pi\bar{\nu}D}\left[\frac{1}{J_\lambda}\frac{\partial}{\partial\lambda}\left(J_\lambda|1-\lambda^2|\frac{\partial}{\partial\lambda}\right) + \sum_m \frac{1}{J_\lambda}\frac{\partial}{\partial\lambda_m}\left(J_\lambda|1-\lambda_m{}^2|\frac{\partial}{\partial\lambda_m}\right)\right]$$

$$+ i(\omega - i\delta)\pi\bar{\nu}\left|\prod_m \lambda_m - \lambda\right| = 0. \tag{38}$$

The subscript m in (38) can assume the values 1 and 2 in the models I and III, and is equal to 1 in the model II.

As has already been noted, only the ground state makes a contribution to the sum (25). Therefore, to compute the density-density correlator $K(\omega,k)$, we should solve Eq. (27) with the ground-state wave function in the right member and the ground-state energy $E_0 = 0$. Let us seek the matrix $P_k^{(0)}$ satisfying such an equation in the form

$$P_k = -iuR_k\bar{v}\cdot 2\pi\bar{\nu}D,\quad R_k = \begin{pmatrix} R_{11k} & 0 \\ 0 & R_{22k}\end{pmatrix}, \tag{39}$$

where in the model I

$$R_{11k} = \begin{pmatrix} (1-\lambda^2)^{1/2}f_k & 0 \\ 0 & (1-\lambda^2)^{1/2}f_k\end{pmatrix},$$

$$R_{22k} = i\begin{pmatrix} (\lambda_1{}^2-1)^{1/2}\lambda_2 f_{1k} & (\lambda_2{}^2-1)^{1/2}\lambda_1 f_{2k} \\ (\lambda_2{}^2-1)^{1/2}\lambda_1 f_{2k} & (\lambda_1{}^2-1)^{1/2}\lambda_2 f_{1k}\end{pmatrix},$$

in the model II

$$R_{11k}=\begin{pmatrix}(1-\lambda^2)^{1/2}f_k & 0\\ 0 & (1-\lambda^2)^{1/2}f_k\end{pmatrix},$$

$$R_{22k}=i\begin{pmatrix}(\lambda_1^2-1)^{1/2}f_{1k} & 0\\ 0 & (\lambda_1^2-1)^{1/2}f_{1k}\end{pmatrix}$$

and in the model III

$$R_{11k}=\begin{pmatrix}(1-\lambda_1^2)^{1/2}\lambda_2 f_{1k}(1-\lambda_2^2)^{1/2}\lambda_1 f_{2k}\\ (1-\lambda_2^2)^{1/2}\lambda_1 f_{2k}(1-\lambda_1^2)^{1/2}\lambda_2 f_{1k}\end{pmatrix},$$

$$R_{22k}=i\begin{pmatrix}(\lambda^2-1)^{1/2}f_k & 0\\ 0 & (\lambda^2-1)^{1/2}f_k\end{pmatrix}.$$

In deriving the Hamiltonian $\mathscr{H}_0$, (38), we used the fact that the ground-state wave function Ψ_0 does not depend on the matrices u and v. The expression (39) for the function P_k, which contains only the first powers of u and v, allows us to derive the corresponding Hamiltonian for the function P_k, which Hamiltonian depends only on $\hat{\theta}$. Let us substitute $\Phi(Q)=P_k$ into Eq. (22), and integrate first over Δu and Δv. Using the expansions

$$u'=u\left[1-\Delta u+\frac{1}{2}(\Delta u)^2\right],\quad v'=v\left[1-\Delta v+\frac{1}{2}(\Delta v)^2\right],$$

which follow from the expressions (31), we reduce the right-hand side of Eq. (22) to the form

$$v\left[\mathscr{H}_0R_k-\frac{1}{2}\langle(\Delta v)^2\rangle_1 R_k-\frac{1}{2}R_k\langle(\Delta u)^2\rangle_1+\langle\Delta vR_k\Delta u\rangle_1\right]\bar{u}. \tag{40}$$

The symbol $\langle\cdots\rangle_1$ denotes the following averaging:

$$\langle\ldots\rangle_1=\frac{\int(\ldots)e^{-F_1}d(\Delta u)\,d(\Delta v)}{\int e^{-F_1}d(\Delta u)\,d(\Delta v)}.$$

The matrices u and v are separated in (40) from the $\hat{\theta}$ variables. Therefore, the computation of the averages in (40) and the substitution of these expressions into Eq. (22) give rise to differential equations containing only the variables θ, θ_1, and θ_2. Such a procedure is similar to the one used in quantum-mechanical problems with centrosymmetric potentials. In such problems the angular variables separate, but give rise to additional terms in the radial part. The model II contains two independent variables, λ and λ_1, and two unknown functions f_k and f_{1k}. Carrying out the corresponding calculations, we obtain

$$\begin{aligned}&2\pi\tilde{\nu}D(ik+\mathscr{H}_0)f_k+2\lambda\frac{\partial f_k}{\partial\lambda}+2(\lambda_1^2-1)J_\lambda(f_k-f_{1k})=\Psi_0,\\ &2\pi\tilde{\nu}D(ik+\mathscr{H}_0)f_{1k}-2\lambda_1\frac{\partial f_{1k}}{\partial\lambda_1}+2(1-\lambda^2)J_\lambda(f_{1k}-f_k)=\Psi_0.\end{aligned} \tag{41}$$

The analogous equations for the models I and III are more complicated. After rather tedious transformations we find

$$2\pi\tilde{\nu}D(ik+\mathscr{H}_0)f_k+2(-1)^s\lambda\frac{\partial f_k}{\partial\lambda}+4[\lambda_2|\lambda_1^2-1|$$
$$\times\beta(\lambda_1,\lambda,\lambda_2)(f_k-f_{1k})+\lambda_1|\lambda_2^2-1|\beta(\lambda,\lambda_2,\lambda_1)(f_k-f_{2k})]=\Psi_0, \tag{42}$$

$$2\pi\tilde{\nu}D(ik+\mathscr{H}_0)f_{ak}-2(-1)^s\left[\lambda_a\frac{\partial f_{ak}}{\partial\lambda_a}+\frac{|\lambda_b^2-1|}{\lambda_b}\frac{\partial f_{ak}}{\partial\lambda_b}\right]$$
$$+4\left[\frac{\lambda_a|\lambda_b^2-1|}{\lambda_b}\beta(\lambda_a,\lambda_b,\lambda)(f_{bk}-f_{ak})\right.$$
$$\left.+\frac{|1-\lambda^2|}{\lambda_b}\beta(\lambda_a,\lambda,\lambda_b)(f_{ak}-f_k)\right]=\Psi_0.$$

In the equations (42), $s=0$ for the model I and $s=1$ for the model III. The subscripts a and b can assume the values 1 and 2, with $a\neq b$. The three-variable function $\beta(x,y,z)$ is equal to

$$\beta(x,y,z)=\frac{-2xy+z-z^3+zx^2+zy^2}{(x^2+y^2+z^2-2xyz-1)^2}. \tag{43}$$

Substituting (39) into Eq. (25), and integrating over u and v, we reduce the density-density correlator to the form

$$K(\omega,k)=\bar{K}(\omega,k)+\bar{K}(\omega,-k), \tag{44}$$

where in the model II

$$\bar{K}=4\pi^3\nu\tilde{\nu}^2D\int_1^\infty\int_{-1}^1\Psi_0J_\lambda((\lambda_1^2-1)f_{1k}+(1-\lambda^2)f_k)\,d\lambda_1\,d\lambda,$$

and in the models I and III

$$\bar{K}=4\pi^3\nu\tilde{\nu}^2D\int\int\int\Psi_0J_\lambda(|\lambda_1^2-1|\lambda_2^2f_{1k}+|\lambda_2^2-1|\lambda_1^2f_{2k}$$
$$+|1-\lambda^2|f_k)\,d\lambda_1\,d\lambda_2\,d\lambda. \tag{45}$$

In the expression (45) the integration is performed over the region $1<\lambda_1<\infty$, $1<\lambda_2<\infty$, $-1<\lambda<1$ for the model I and over the region $-1<\lambda_1<1$, $0<\lambda_2<1$, $1<\lambda<\infty$ for the model III. The formulas (32), (38), (41), (42), (44), and (45) completely solve the problem of the kinetics of a quantum particle in a long metallic wire for all the types of symmetry. Unfortunately, Eqs. (32), (38), (41), and (42) are very complicated, and can be solved analytically only in the limiting cases of high ($\omega\gg(\tilde{\nu}^2D)^{-1}$) and low ($\omega\ll(\tilde{\nu}^2D)^{-1}$) frequencies. In the intermediate region the results can be obtained only numerically. In the next section we shall consider the limiting cases of high and low frequencies.

4. THE DENSITY-DENSITY CORRELATOR AND THE PERMITTIVITY

Let us first consider the high-frequency limit $\omega\gg(\tilde{\nu}^2D)^{-1}$. Naturally, the use of the transfer matrix method is not necessary for the computations in this region. It is much easier to use perturbation theory in terms of W, which is applicable in a space of any dimension. But the calculation in the high-frequency region allows us to verify the equations obtained, and to demonstrate how the transfer matrix method works. At high frequencies the important values of λ, λ_1, and λ_2 are the values close to unity. Therefore, it is convenient to use the following variables:

$$\begin{aligned}&\lambda_1=1+x_1,\ \lambda_2=1+x_2,\ \lambda=1-x\ \text{(model I)},\\ &\lambda_1=1+x,\ \lambda=1-x\ \text{(model II)},\\ &\lambda_1=1-x_1,\ \lambda_2=1-x_2,\ \lambda=1+x\ \text{(model III)}.\end{aligned} \tag{46}$$

The dominant contribution to the integrals with respect to x_1, x_2, and x in the (44) and (45) is made by the region

$x_1 \sim x_2 \sim x \sim (D\tilde{\nu}^2\omega)^{-1/2}$. In the leading approximation, we can integrate over all positive x_1, x_2, and x in (44) and (45). Expanding Eqs. (37), (38), (41), and (42) in powers of the small variables x_1, x_2, and x, and solving these equations in this limit, we obtain in the model II

$$\Psi_0 = e^{-(x_1+x)\gamma}, \quad f_k \approx f_{1k} = \frac{1}{2\pi\tilde{\nu}D} \frac{1}{ik+(i\omega/D)^{1/2}} e^{-(x_1+x)\gamma},$$

and in the models I and III

$$\Psi_0 = e^{-\gamma(x_1+x_2+x)}, \quad f_k \approx f_{1k} \approx f_{2k} = \frac{1}{2\pi\tilde{\nu}D} \frac{e^{-(x_1+x_2+x)\gamma}}{ik+(i\omega/D)^{1/2}}, \tag{47}$$

where $\gamma^2 = (\pi\tilde{\nu})^2 i(\omega - i\delta)$.

The ground-state wave function satisfies the normalization condition (32). Substituting the expressions (47) into the integrals (44), (45), and integrating over x, x_1, and x_2, we find in the leading approximation for all the models the correlator

$$K(\omega, k) = \frac{4\pi\nu}{i\omega + Dk^2}. \tag{48}$$

The formula (48) describes classical diffusion.

Let us now consider the more interesting case $\omega \ll (\tilde{\nu}^2 D)^{-1}$. In this limit the dominant contribution is made by the region $\lambda_1 \gg 1$, $\lambda_2 \gg 1$, $\lambda \sim 1$ in the model I, the region $\lambda_1 \gg 1$, $\lambda \sim 1$ in the model II, and the region $\lambda_1 \sim 1$, $\lambda_2 \sim 1$, $\lambda \gg 1$ in the model III. The Eqs. (38), (41), and (42) and the integrals (44), (45) are also much simpler in these regions of the variables. We can neglect the function f_k and retain only f_{1k} in the formulas (38), (41), and (44) for the model II. In the formulas (38), (42), and (45), it is sufficient to retain only f_{1k} and f_{2k} in the model I and f_k in the model III. Let us seek the solutions to these equations in the leading approximation in the form:

in the model I

$$\Psi_0 = \Psi_0(\lambda_1\lambda_2), \quad f_{1k} = f_{2k} = \Phi_k(\lambda_1\lambda_2),$$

in the model II

$$\Psi_0 = \Psi_0(\lambda_1), \quad f_{1k} = \Phi_k(\lambda_1), \tag{49}$$

in the model III

$$\Psi_0 = \Psi_0(\lambda), \quad f_k = \Phi_k(\lambda).$$

Carrying out a change of variables in accordance with the formulas

$$z = i\omega\pi^2\tilde{\nu}^2 D\lambda_1\lambda_2 \quad \text{(model I)},$$
$$z = i\omega \cdot 2\pi^2\tilde{\nu}^2 D\lambda_1 \quad \text{(model II)} \tag{50}$$
$$z = i\omega \cdot 2\pi^2\tilde{\nu}^2 D\lambda \quad \text{(model III)}$$

and evaluating the integrals over the remaining variables, we obtain the following equations in place of (38), (41), (42), (44), and (45):

$$-z\frac{d^2\Psi_0}{dz^2} + \Psi_0 = 0,$$

$$-\frac{d}{dz}\left(z^2\frac{d\Phi_k}{dz}\right) + ikL_c\Phi_k + z\Phi_k = \Psi_0(z), \tag{51}$$

$$K(\omega, k) = \frac{4\pi\nu A(k)}{i\omega}, \quad A(k) = \int_0^\infty (\Phi_k(z) + \Phi_{-k}(z))\Psi_0(z)\,dz.$$

In the expressions (51) the length L_c is equal to

$$L_c = \begin{cases} \pi\tilde{\nu}D & \text{(model I)} \\ 2\pi\tilde{\nu}D & \text{(model II and III)} \end{cases} \tag{52}$$

Remembering the formulas (20a) for $\tilde{\nu}$, we rewrite the expression for L_c in the explicit form, applicable for all the three models:

$$L_c = \begin{cases} \pi\nu SD & \text{(model I)}, \\ 2\pi\nu SD & \text{(model II)}, \\ 4\pi\nu SD & \text{(model III)}. \end{cases} \tag{53}$$

Thus, the density-density correlator is given by identical expressions in all the three models. Only the characteristic lengths L_c are different. The solutions $\Psi_0(z)$ and $\Phi_k(z)$ should decrease at infinity. Furthermore, the function $\Psi_0(z)$ should satisfy the boundary condition

$$\Psi_0(0) = 1. \tag{54}$$

The condition (54) is a consequence of the fact that Eq. (38) possesses at $\omega = 0$ an exact solution $\Psi_0(\lambda_1, \lambda_2, \lambda) = 1$, which satisfies the normalization condition (32). Then, in terms of the z variables, the limit $\omega \to 0$ is just the limit corresponding to the condition (54). Furthermore, the integral for $A(k)$ in the expression (51) should converge if the dominant contribution arises from the region $z \sim 1$. Consequently, the function should by integrable. The other possible functions, for which the region $\lambda, \lambda_1, \lambda_2 \sim 1$ is the important region in the integrals (44), (45), give contributions of higher order in ω.

The equations (51) with the above-indicated boundary conditions coincide exactly with the low-frequency limit of Berezinskiĭ's equations,[3] which were derived for the problem of the kinetics of a particle in a one-dimensional disordered chain, if we replace the length L_c in the equations (51) by the mean free path l. Solving the equations (51), we find

$$\Psi_0(z) = 2z^{1/2}K_1(2z^{1/2}),$$

$$\Phi_k(z) = \frac{2}{z^{\alpha/2}}\left[I_\alpha(2z^{1/2})\int_{2z^{1/2}}^\infty \xi K_\alpha(\xi)K_1(\xi)\,d\xi + K_\alpha(2z^{1/2})\int_0^{2z^{1/2}} \xi I_\alpha(\xi)K_1(\xi)\,d\xi\right], \tag{55}$$

where K_1, K_α, and I_α are Bessel functions and $\alpha = (1 + 4ikL_c)^{1/2}$.

The functions Ψ_0 and Φ_k, (55), decrease at infinity. The function Ψ_0 satisfies the condition (54), and the function Φ_k is integrable in the region of small z values.

Substituting the expressions (55) into the integral in (51), and carrying out the same calculations performed in Ref. 17, we find an expression for the density-density correlator at large distances $x \gg L_c$ and times $t \to \infty$:

$$K(\varepsilon, t\to\infty, x)\equiv p_\infty(x)$$
$$=\frac{1}{4\sqrt{\pi}L_c}\left(\frac{\pi^2}{8}\right)^2\left(\frac{4L_c}{|x|}\right)^{3/2}\exp\left(-\frac{|x|}{4L_c}\right). \quad (56)$$

The formula (56) shows that the correlator $p_\infty(x)$ decreases exponentially with a localization length given by the formulas (53). This fact proves the localization of all the states in the system.

In the region of low momenta $k\ll L_c^{-1}$ the function $A(k)$ has the form

$$A(k)=1-4\zeta(3)k^2L_c^2, \quad (57)$$

where $\zeta(3)$ is the Riemann zeta function.

The function $A(k)$ for low momenta determines the permittivity ε:

$$\varepsilon=-4\pi e^2\nu\frac{d^2A(k)}{dk^2}\Big|_{k=0}=32\zeta(3)e^2\nu L_c^2. \quad (58)$$

The formulas (56)–(58) have the same form as the corresponding formulas in the one-dimensional disordered chain model. But now the localization length $L_c\sim l(p_0^2S)$, i.e., significantly greater than the mean free path l.

In the high-frequency limit $\omega\gg D/L_c^2$, where the classical formula (48) is applicable, the conductivity σ is, according to the Einstein relation, equal to

$$\sigma=2\nu e^2D. \quad (59)$$

As the frequency ω decreases, the formula (59) goes over smoothly into the formula (58).

The frequency dependences given by the three models are all different. In particular, in the model III this dependence is nonmonotonic, since in this model the first correction to the conductivity in the high-frequency region is positive.[18]

The dependence of the localization length L_c (53) on the magnetic and spin-orbit interactions leads, according to the formula (58), to a corresponding dependence of the permittivity. It is not difficult to estimate the orders of magnitude of those values of the interaction strengths at which the transition occurs between the models I, IIa, IIb, and III. To do this, it is sufficient to consider the effect of these interactions on the first-order correction to the classical diffusion. An external magnetic field leads to a transition from the model I, which is invariant under time reversal and spin rotations, to the model IIa. The permittivity given by the model IIa is four times greater than the permittivity given by the model I. The transition between these models occurs in fields of strength $H\sim\Phi_0(S^{1/2}L_c)^{-1}$, where $\Phi_0=\pi\hbar c/e$ is the flux quantum. Upon the addition of a sufficiently large amount of magnetic or spin-orbit impurities (the models IIb and III), an amount which ensures the satisfaction of the inequality

$$\tau_s^{-1},\ \tau_{s0}^{-1}\gg D/L_c^2,$$

the permittivity increases by a factor of 16 in comparison with the case of a wire without magnetic or spin-orbit interactions.

The entire investigation carried out above is applicable at absolute zero, at which temperature inelastic processes do not occur. The interactions that cause inelastic scattering delocalize the electronic states, and reestablish classical conductivity. In such a situation lowering of the temperature should lead to an increase of the resistance. Experimentally, growth of the resistance with decreasing temperature in wires has been observed many times.[19–21] Unfortunately, all the measurements were performed at not very low temperatures, i.e., they were not performed in the temperature region where the corrections to the classical conductivity are small. It would be interesting to measure the permittivity (58) and compare this quantity with the diffusion coefficient D obtained from the formula (59), which is applicable in a higher-temperature region, where the localization effects are suppressed. It would also be interesting to investigate the external-magnetic-field dependence of the permittivity.

[1]D. J. Thouless, J. Phys. C **8**, 1803 (1975).
[2]E. Abragams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[3]V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **65**, 1251 (1973) [Sov. Phys. JETP **38**, 620 (1974)].
[4]A. A. Abrikosov and I. A. Ryzhkin, Zh. Eksp. Teor. Fiz. **71**, 1916 (1976) [Sov. Phys. JETP **44**, 1005 (1976)].
[5]N. F. Mott and W. D. Twose, Adv. Phys. **10**, 107 (1961).
[6]V. L. Berezinskiĭ and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **77**, 2499 (1979) [Sov. Phys. JETP **50**, 1209 (1979)].
[7]W. Weller, V. N. Prigodin, and Yu. A. Firsov, Phys. Status Solidi **110**, 143 (1982).
[8]O. L. Dorokhov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 259 (1982) [JETP Lett. **36**, 318 (1982)].
[9]P. W. Anderson, D. J. Thouless, E. Abragams, and D. S. Fisher, Phys. Rev. B **22**, 3519 (1980).
[10]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[11]K. B. Efetov, Zh. Eksp. Teor. Fiz. **82**, 872 (1982) [Sov. Phys. JETP **55** 514 (1982)].
[12]N. N. Bogolyubov and D. V. Shirkov, Vvedenie v teoriyu kvantovannykh poleĭ (Introduction to the Theory of Quantized Fields), Nauka, Moscow, 1976 (Eng. Transl., Interscience, New York, 1959).
[13]R. P. Feynman, Rev. Mod. Phys. **20**, 367 (1948).
[14]V. G. Vaks, A. I. Larkin, and S. A. Pikin, Zh. Eksp. Teor. Fiz. **51**, 361 (1966) [Sov. Phys. JETP **24**, 240 (1967)].
[15]T. Bohr and K. B. Efetov, J. Phys. C **15**, L249 (1982).
[16]K. B. Efetov, Zh. Eksp. Teor. Fiz. **83**, 833 (1982) [Sov. Phys. JETP **56**, 467 (1982)].
[17]A. A. Gologin, V. I. Mel'nikov, and É. I. Rashba, Zh. Eksp. Teor. Fiz. **69**, 327 (1975) [Sov. Phys. JETP **42**, 168 (1975)].
[18]S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[19]N. Giordano, in: Physics in One Dimension (Ed. by J. Bernasconi and T. Schneider), Springer-Verlag, Berlin.
[20]P. Chaudhari and H. U. Habermeier, Solid State Commun. **34**, 687 (1980).
[21]A. E. White, M. Tinkham, W. J. Skocpol, and D. C. Flanders, Phys. Rev. Lett. **48**, 1752 (1982).

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