Atomic dispersion in a light field

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A characteristic of the motion of a resonant atom in the field of a standing light wave is the existence of two trajectories of motion. The Landau-Zener transitions from one trajectory to the other lead to interference among the states of the translational motion. In the paper the dispersion law for an atom in a standing-wave field is found with allowance for the Landau-Zener transitions. In the above-barrier motion the interference effects lead to density-of-states oscillations with the Doppler frequency. In the subbarrier energy region strong mixing of the states of the continuous and discrete spectra occurs which manifests itself in anomalous behavior of the allowed (forbidden) band widths in the quasiclassical region. The physical manifestations of these effects are discussed.

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INTRODUCTION

A characteristic of the motion of atoms in a resonant light field is the strong coupling between the internal and translational degrees of motion. This coupling depends on the parameters of the external field, and determines the spectral (optical) and dispersion (mechanical) properties of the atom. The mixing of the optical and mechanical characteristics of the atom is the most noticeable in an inhomogeneous field.

In contrast to a traveling wave, which mixes only the two states of the translational motion with momenta $\mathbf{p}$ and $\mathbf{p}+\mathbf{k}$ (Ref. 1), a standing wave allows the occurrence of many-photon processes that intermix groups of states with different momenta.

The motion of an atom in an inhomogeneous field can be described with the aid of an effective potential determined by the interaction of the induced dipole moment with this atom. A two-level atom possesses two potentials in accordance with the two possible signs of the dipole moment. At exact resonance the dipole moment is a constant quantity equal to the transition matrix element, and the equations of motion split up strictly into two independent scalar Schrödinger equations with resonant potentials $\pm V(x)$. At large detunings of the external field frequency the dipole moment adiabatically follows the field, and the equations of motion in the quasiclassical approximation split up into two scalar equations with nonresonant potentials $\pm U(x)$ (Refs. 2 and 3).

In the region of convergence of the $\pm U(x)$ terms (near the nodes of the standing wave), the adiabatic character of the motion can be destroyed, and, as a result of a Landau-Zener (LZ) transition, the atom is found in a state that is a superposition of adiabatic states. In this case we can only speak of the probability of the atom's having a definite potential and a definite trajectory of motion. In the present paper we study atomic dispersion in the field of a standing light wave with allowance for the LZ transition.

The motion of an atom in two potentials (i.e., along two trajectories) at the same time leads to interference among the states of the translational motion. This interference manifests itself, in particular, in oscillations of the density of states during the above-barrier motion. In the subbarrier energy region the LZ transitions lead to the intermixing of the states to the continuous and discrete spectra corresponding to the nonresonant potentials. As a result, the allowed (forbidden) band widths are not exponentially small in the quasiclassical parameter, in contrast to the case of a scalar particle (i.e., one without internal degrees of freedom) moving in a definite potential.

The questions touched upon in the present paper are directly related to the theory of gas lasers operating on narrow transitions, to the spectroscopy of weak-signal absorption in the presence of a high-intensity standing wave, and to the problem of atomic scattering by a standing light wave.

Let us note that a similar physical picture connected with LZ transitions arises in the magnetic-breakdown problem,[4] as well as in the case of electron transitions in semiconductors in the field of a standing wave.[5]

EQUATIONS OF MOTION

In the field of a standing wave

$$E(x)e^{-i\Delta t}, \qquad E(x)=E_0 \sin kx,$$

having a small detuning Δ relative to an atomic transition frequency, the Hamiltonian of a two-level atom has the form ($\hbar=1$):

$$\hat{H}=-\frac{1}{2m}\frac{\partial^2}{\partial x^2}-V(x)\sigma_3+\frac{\Delta}{2}\sigma_1,$$

$$V(x)=V_0 \sin kx, \qquad V_0=dE_0, \tag{1}$$

where $\sigma_{1,3}$ are the Pauli matrices. In this representation the amplitudes of the probability for finding the atom in the ground and excited states are equal to $(\psi_+ \pm \psi_-)\sqrt{2}$. We shall, bearing in mind the case of narrow atomic resonances,

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neglect the damping of the states. We are interested in the stationary states

$$\hat{H}\psi=\varepsilon\psi, \quad \psi=\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \tag{1'}$$

satisfying the quasiperiodicity condition

$$\psi(x+\pi/k)=\pm\exp(i\pi p/k)\sigma_1\psi(x), \tag{2}$$

where p is the quasimomentum. Such a formulation of this condition is possible because of the invariance of the Hamiltonian under a shift by half a period π/k of the field with simultaneous interchange of the spinor components.[6,7]

Below we shall consider the case of a strong field, when the atom traverses over a period of the Rabi oscillations a distance v/V_0 that is small compared to the wavelength of the light:

$$V_0\gg kv. \tag{3}$$

Then we can use the quasiclassical solutions to the Schrödinger equation:

$$\psi_1, \psi_1^*=\begin{pmatrix}w\\ u\end{pmatrix}\frac{1}{p_1^{1/2}}\exp\left(\pm i\int p_1\,dx\right),$$

$$\psi_2, \psi_2^*=\begin{pmatrix}u\\ -w\end{pmatrix}\frac{1}{p_2^{1/2}}\exp\left(\pm i\int p_2 dx\right), \tag{4}$$

$$p_{1,2}(x)=[2m(\varepsilon\mp U)]^{1/2}, \quad U(x)=\frac{\Delta}{2}\left[1+\left(\frac{2V}{\Delta}\right)^2\right]^{1/2}$$

$$u=[{}^1/_2(1+V/U)]^{1/2}, \quad w=[{}^1/_2(1-V/U)]^{1/2}.$$

The quantities $\pm U(x)$ play the role of effective potentials for the atom in the case of finite detunings. At exact resonance (i.e., for $\Delta=0$) the atom has, according to (1), the potentials $\pm V(x)$. These potentials are shown in Fig. 1. Let us emphasize that the picture reduces to motion of the particle in a definite potential only at exact resonance. For finite detunings the atom possesses a definite nonresonant potential $+U(x)$ or $-U(x)$ only until the quasiclassical character is destroyed and the atom goes over, as a result of the LZ effect, from one trajectory to the other. The probability for such transitions attains its highest value near the field nodes, where the distance between the $\pm U(x)$ terms is smallest and equal to Δ. This occurs in the region of distances δx near the field nodes of the order of $\delta x\sim\Delta/V_0k\ll 1/k$. If the reciprocal transit time of the atom through this region is comparable to the term separation, i.e., if $v/\delta x\sim\Delta$, then the probability for a LZ transition is of the order of unity. Hence we define the characteristic frequency Δ_0 of the LZ transition as

$$\Delta_0=(V_0kv)^{1/2}, \quad kv\ll\Delta_0\ll V_0. \tag{5}$$

The frequency Δ_0 depends on the particle energy, and is the boundary between two characteristic frequency regions. For $\Delta\gg\Delta_0$ the LZ-transition probabilities are exponentially small, and the atom possesses a definite (nonresonant) potential $U(x)$ or $-U(x)$. For small detunings, i.e., for $\Delta\ll\Delta_0$, the transition probability is close to unity, and the atom possesses resonant potentials $\pm V(x)$. For $\Delta\sim\Delta_0$ the particle does not possess a definite potential, and the effects of the interference of the motions in the various potentials manifest themselves most strongly.

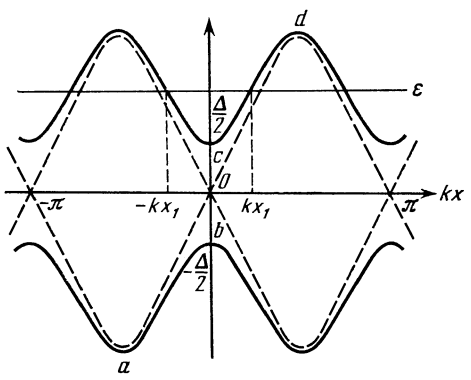

FIG. 1. Nonresonant potentials $\pm U(x)$ (continuous curves) and resonant potentials $\pm V(x)$ (dashed curves).

ABOVE-BARRIER ENERGIES

Let us consider the region of energies $\varepsilon>V_0$ corresponding to the above-barrier motion. If the particle energy is not too close to V_0, we can neglect the above-barrier reflection, which leads to exponentially small mixing of the states ψ_1 and ψ_1^*, ψ_2 and ψ_2^*. The LZ transitions mix the states ψ_1 and ψ_2 (ψ_1^* and ψ_2^*); therefore, the general solution for particles traveling in the positive direction has the form

$$\psi=c_1\psi_1+c_2\psi_2.$$

In the neighborhood of a node of the standing wave we can linearize the potential, i.e., set $V(x)\approx V_0kx$, lower the order of the equations, assuming the velocity to be constant and equal to $v=(2\varepsilon/m)^{1/2}$, and reduce them to the standard form

$$iv\,d\psi/dx=(-V_0kx\sigma_3+\Delta\sigma_1/2)\psi.$$

The solutions to this equation determine the unitary LZ-transition matrix $\hat{T}$, which gives the transformation of the coefficients of the general solution on going through a field node:

$$\begin{pmatrix}c_1\\ c_2\end{pmatrix}\to\hat{T}\begin{pmatrix}c_1\\ c_2\end{pmatrix}, \quad T_{11}=(1-R^2)^{1/2}e^{-i\chi}, \quad T_{12}=R=e^{-\pi\xi},$$

$$\chi=\pi/4+\arg\Gamma(1-i\xi)-\xi\ln(e/\xi), \quad \xi={}^1/_8(\Delta/\Delta_0)^2. \tag{6}$$

Using the quasiperiodicity condition (2), we obtain the following dispersion equation:

$$\sin(\pi p/k-S_0)=\pm(1-R^2)^{1/2}\sin S,$$

$$S_0={}^1/_2(S_1+S_2), \quad S={}^1/_2(S_1-S_2)-\chi, \quad S_{12}=\int_0^{\pi/k}p_{1,2}dx. \tag{7}$$

The two signs in (7) ensure the periodicity of the energy: $\varepsilon(p+k)=\varepsilon(p)$.

In the limiting case of large detunings, i.e., for $\Delta \gg \Delta_0$, the two dispersion branches $p = kS_{1,2}/\pi + nk$ correspond to independent motions of the atom in the potentials $\pm U(x)$. For small detunings, i.e., for $\Delta \ll \Delta_0$, we obtain the following dispersion equation:

$$\frac{\pi p}{k} = S_0 \pm \frac{\pi^{1/2}}{2}\frac{\Delta}{\Delta_0}\sin S.$$

The first term on the right-hand side of this relation determines the dispersion in the resonant potentials $\pm V(x)$,[1)] when the particle energy is doubly degenerate (for each sign of p). The second term, which is proportional to Δ, lifts this degeneracy.

In the general case, when $\Delta \sim \Delta_0$, the quantity $R \sim 1$, and the mixing of the trajectories is the most appreciable. The dispersion equation (7) has two branches, which no longer correspond to motions in definite potentials. The densities of states corresponding to these branches have the form

$$\frac{\pi}{k}\frac{dp}{d\varepsilon} = \frac{dS_0}{d\varepsilon} \pm \frac{(1-R^2)^{1/2}\cos S}{(\cos^2 S + R^2 \sin^2 S)^{1/2}}\frac{dS}{d\varepsilon}. \qquad (8)$$

Here we have taken into account the fact that the quasiclassical phases S_0 and S are large and that, as functions of ε, they vary more rapidly than R. It is easy to verify that the group velocity $d\varepsilon/dp$ is always positive.

The density of states of each branch oscillates about the quantity $dS_0/d\varepsilon$, which corresponds to the density of states in the resonant potential (see Fig. 2). These oscillations are due to the interference of the states during the motion along the different trajectories in the potentials $\pm U(x)$, and are determined by the difference between the phases S_1 and S_2. The number of oscillations coincides with the number of zeros of $\cos S$, and, since $S(\varepsilon)$ decreases monotonically with energy, it is equal to

$$S(\varepsilon = V_0)/\pi \sim V_0/kv \sim (mV_0/k^2)^{1/2} \gg 1.$$

The relative amplitude of the oscillations is of the order of unity if $R \sim 1$. In particular, for energies of the order of the barrier height, i.e., for $\varepsilon \sim V_0$, these characteristics manifest themselves at detuning values $\Delta \sim V_0(k^2/mV_0)^{1/4}$, which, in essence, does not differ very much from V_0. The characteristic period of the oscillations coincides with the Doppler frequency kv. The density-of-states oscillations do not occur at high particle energies, when $kv \gtrsim V_0$.

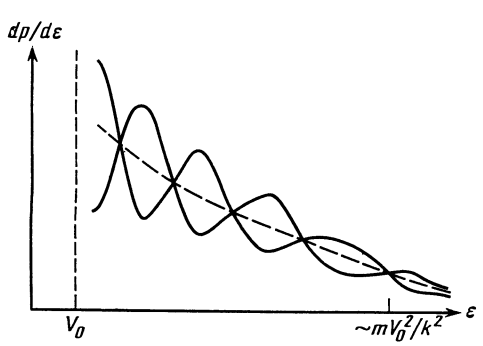

FIG. 2. The density of states for the two dispersion branches at above-barrier energies $\varepsilon > V_0$.

It is noteworthy that the effective particle mass

$$\frac{1}{m^*} = \frac{1}{2}\frac{d}{d\varepsilon}\left(\frac{dp}{d\varepsilon}\right)^{-2}$$

as a function of the energy is an alternating-sign quantity in the region of the density-of-states oscillations. And wherever the phase S is a multiple of π, the effective mass becomes infinite, which leads to a sharp reduction in the rate of spreading of the wave packet.

In the high-energy limit, $\varepsilon \gg V_0$, the problem can be considered in the prescribed-motion approximation at all coordinates (and not just in the neighborhoods of the standing-wave nodes). Then the correction $\delta\varepsilon = \varepsilon - p^2/2m$ to the free-particle energy is the quasienergy of a two-level system with level spacing Δ in the periodic field $V_0 \sin kvt$, where $v = (2\varepsilon/m)^{1/2}$. In this case $\delta\varepsilon$ depends on the Doppler frequency kv, and coincides with the results of Refs. 6, 7, and 9.

The effect of the density-of-states oscillation can be measured directly from the shape of the absorption line for a weak test signal in a gas of resonant atoms located in the field of a high-intensity standing wave, since the dependence of the absorption coefficient on the frequency is largely determined by the densities of the states between which the transition occurs, while the matrix elements depend weakly on the frequency. A resonance structure with characteristic energy scale kv ("dopplerons") has been experimentally observed by Oka *et al.*[10,11] in a standing wave. Theoretically, these questions for the case of weak fields and large detunings are discussed in Ref. 12. In our paper we study the other limiting case, namely, the case of small detunings and strong fields [the condition (5)].

SUBBARRIER ENERGIES

In the subbarrier region $\varepsilon < V_0$ we can distinguish several characteristic energy intervals. For positive and not too low energies the passage through the field nodes can be considered in the prescribed-motion approximation, which significantly simplifies the problem. In this case the particle moves above the potential barrier $-U(x)$, and can be found in the potential wells corresponding to the potential $U(x)$. In the absence of transitions from one potential to the other, we should have either bounded motion with a discrete spectrum in the field $+U(x)$, or unbounded motion with a continuous spectrum in the field $-U(x)$ (up to the exponentially small quasiclassical effects of the subbarrier crossing and above-barrier reflection). Allowance for the LZ transitions and the reflections at the turning points $\pm x_1$ during the motion in the potential $U(x)$ leads to a situation in which the solution is a superposition of all the four functions (4). In this case ψ_1 and ψ_1^* intermix at the points $\pm x_1$, while, as before, ψ_1 and ψ_2, ψ_1^* and ψ_2^* intermix in the neighborhood of a field node. These processes can be considered separately if the turning points are located sufficiently far away from the field nodes, so that $x_1 \sim \varepsilon/kV_0 \gg \delta x \sim \Delta/kV_0$, i.e., $\varepsilon \gg \Delta$. This inequality guarantees also the constancy of the particle velocity in the LZ transition region. Furthermore, the above-barrier reflec-

tion in the potential $-U(x)$ can be neglected if the quasiclassicality condition for the motion near a field node is fulfilled, i.e., if $1/mv \sim 1/(m\varepsilon)^{1/2} \ll \delta x$. Thus, the quasiclassicality condition for the motion and the condition for the constancy of the particle velocity near a field node are fulfilled at energy and detuning values satisfying the inequalities

$$\Delta_1 \ll \varepsilon < V_0, \quad \Delta_1(\Delta_1/\varepsilon)^{1/2} \ll \Delta \ll \varepsilon, \quad \Delta_1 = (V_0^2 k^2/m)^{1/3}. \tag{9}$$

In this case the resonance detuning Δ can be either higher or lower than the characteristic LZ transition frequency $\Delta_0(\varepsilon)$, (5).

For this region of the parameters the dispersion equation has the form

$$(1-R^2)^{1/2}\cos\varphi\cos(\pi p/k) = \pm{}^1/_2[\cos(S+\varphi)+(1-R^2)\cos(S-\varphi)], \tag{10}$$

$$S = S_2 + \chi, \quad \varphi = \varphi_1 - \chi, \quad \varphi_1 = \int_{-x_1} p_1 dx.$$

The phase φ_1 corresponds to quasiclassical motion in the potential $U(x)$ between the turning points. The phase S_2 corresponds to motion in the potential $-U(x)$ between the points $(0,\pi/k)$. The phase $S+\varphi$ corresponds to motion in the resonant potential.

For $\Delta \ll \Delta_0$ we have

$$1-R^2 = \frac{\pi}{4}\left(\frac{\Delta}{\Delta_0}\right)^2 \ll 1,$$

and the motion of the atom reduces to oscillations in the resonant potential. In the zeroth approximation in Δ/Δ_0 the quasiclassical energy levels are defined in the usual manner:

$$S(\varepsilon_n) + \varphi(\varepsilon_n) = \pi(n+{}^1/_2).$$

In the next order in Δ these levels smear into bands with dispersion

$$\delta\varepsilon_n(p) = \pm\frac{\Omega}{\pi^{1/2}}\frac{\Delta}{\Delta_0}\cos\varphi(\varepsilon_n)\cos\frac{\pi p}{k}. \tag{11}$$

Here $\Omega = \pi\partial\varepsilon/\partial(S+\varphi)$ is the classical oscillation frequency of a particle with energy ε_n in the potential well $V(x)$, and, in order of magnitude, $\Omega \sim kv \sim k(V_0/m)^{1/2}$.

Notice that the widths of the bands are determined not by the exponentially small tunneling probability, but by the fact that the wave functions of a particle with energy $\varepsilon > 0$ moving in the potentials $V(x)$ and $-V(x)$ strongly overlap. The frequency of the transition between the degenerate states in the two neighboring wells [the coefficient of $\cos(\pi p/k)$ in (11)] is the matrix element of the interaction energy Δ. The band widths $\delta\varepsilon_n$ are much smaller than the level spacing Ω. The formula (11) is in accord with perturbation theory in terms of Δ (Ref. 13).

In the quasiclassical limit Ω and Δ_0 do not change when n is changed by unity, but the phase $\varphi(\varepsilon_n)$ changes significantly. This leads to an irregular variation of the band widths. In particular, for φ close to $\pi(n+1/2)$ we have $\delta\varepsilon_n \to 0$, and then we must take the next corrections in Δ/Δ_0 into consideration. Such a situation obtains when the levels in the potentials $V(x)$ and $|V(x)|$ coincide.

For $\Delta \gg \Delta_0$ we have $R \ll 1$, and Eq. (10) assumes the form

$$\pm\cos(\pi p/k) = (1+R^4/8)\cos S - {}^1/_2 R^2 \operatorname{tg}\varphi \sin S. \tag{12}$$

If $|\cos S|$ is not too close to unity and $\tan\varphi$ is finite, then $p \approx kS/\pi + nk$, which corresponds to motion of the particle in the nonresonant potential $-U(x)$. At energies for which $S(\varepsilon_m) = m\pi$, there arise forbidden bands whose width is (if $R^2\tan\varphi \ll 1$) equal to

$$\delta\varepsilon_m = \Omega^{(-)}R^2/\pi|\cos\varphi|, \tag{13}$$

where $\Omega^{(-)} = \pi\partial\varepsilon/\partial S$ is the classical frequency for the above-barrier motion in the potential $-U(x)$. Thus, for large detunings the forbidden-band widths are exponentially small in the parameter $(\Delta/\Delta_0)^2$, and do not depend on the phase φ of the motion in the potential $U(x)$. Forbidden bands arise under the condition $\varphi(\varepsilon_n) = \pi(n+1/2)$ as well. These energy values correspond to the discrete levels in the potential $U(x)$ in the absence of LZ transitions. As to the widths of these forbidden bands, they are given by the formula (13) with $\Omega^{(-)}$ replaced by $\Omega^{(+)} = \pi\partial\varepsilon/\partial\varphi$ and $\cos\varphi$ by $1/\sin S$. In the particular cases in which $\cos\varphi$ and $\sin S$ vanish simultaneously, the forbidden band has a width that is proportional to the first power of R:

$$\delta\varepsilon = \frac{2R}{\pi}(\Omega^{(-)}\Omega^{(+)})^{1/2}.$$

In the general case $\Delta \sim \Delta_0$ we have $R \sim 1$, and the spectrum consists of allowed and forbidden bands having the same order of magnitude Ω. By varying the field frequency we can reconstruct the spectrum from the discrete spectrum for small detunings to the superposition of the continuous and the discrete spectra in the limit of large detunings.

The atomic dispersion corresponding to Eq. (12) is depicted in Fig. 3. The branches *1* and *4* correspond to the case when $\tan\varphi$ is finite. The density of states here varies basically like $dS/d\varepsilon$, and has the usual square-root singularities at the band edges. The bands *2* and *3* correspond to the case when $\cos\varphi$ in this energy range passes through zero. In this case there occurs, as it were, a splitting of one band with characteristic width kv into two bands with the same overall width. The dispersion branches *2* and *3* have in the small R

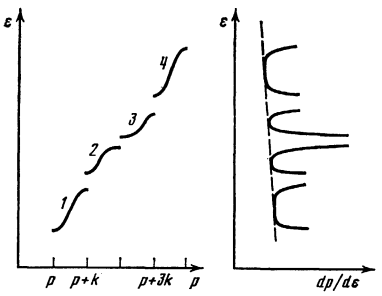

FIG. 3. Characteristics of the dispersion and the density of states in the subbarrier energy region $\Delta/2 < \varepsilon < V_0$.

region sections with very small slopes ($\sim vR^2$), which results in a high density of states. Evidently, this is a manifestation of the discrete spectrum of the particle in the potential $U(x)$ in a background of the continuous spectrum in the potential $-U(x)$.

Let us consider the energies ε lying in the interval $(-\Delta/2, \Delta/2)$, when the particle executes above-barrier motion in the potential $-U(x)$. For sufficiently large detunings, i.e., for $\Delta \gg \Delta_1$, the spectrum is continuous, if we neglect the exponentially narrow forbidden bands due to the above-barrier reflection. Thus, the width of the forbidden bands for ε close to zero is of the order of

$$\Omega^{(-)} \exp\left[-\text{const}\,(\Delta/\Delta_1)^{3/2}\right].$$

The detuning function in the argument of the exponential function is the result of the square-root behavior of the potential barrier in the vicinity of a field node: $-U(x) \approx [\Delta^2/4 + (V_0 kx)^2]^{1/2}$. We see that, as the detuning decreases, the forbidden-band widths increase and attain a value of the order of Ω.

Finally, at energies $\varepsilon < -\Delta/2$ the particle possesses a discrete spectrum that is broadened as a result of the tunneling. The strong dependence of the tunneling probability on the frequency is noteworthy. For $\Delta = 0$ the particle passes under the barrier $V(x)$, while in the case of small, but finite detunings a barrier $-|V(x)|$ of significantly smaller width and height has to be surmounted.

Certain characteristics of the spectrum of an atom in the field of a standing wave have been numerically analyzed by Letokhov and Minogin.[14]

THE CASE OF SMALL DETUNINGS

When $|\varepsilon| \ll V_0$ and $|\Delta| \ll \Delta_1$, it may turn out that the velocity of the atom is not constant in the neighborhood of an LZ transition; in particular, the turning points may be close to a field node (see Fig. 1). Such a situation can be considered with the aid of perturbation theory in terms of Δ/Δ_1.

In the momentum representation the equations (1′) with the potential linearized in the neighborhood of a field node reduce to a system of two first-order differential equations. For $\Delta = 0$ these equations are independent, and their solution is elementary. For small Δ the solutions can be constructed with the aid of perturbation theory in terms of Δ/Δ_1. Matching them with the quasiclassical solutions (4) outside the field nodes, and imposing the quasiperiodicity condition (2), we obtain the following dispersion relation:

$$\cos\varphi = \pm\pi^{1/2}\frac{\Delta}{\Delta_1}\Phi\left(-\frac{2\varepsilon}{\Delta_1}\right)\cos\left(\frac{\pi p}{k}\right), \qquad \varphi = \int [2m(\varepsilon - V)]^{1/2}\,dx. \tag{14}$$

Here φ is the quasiclassical phase in the resonant potential between the turning points and Φ is the Airy function, which in the present case is the overlap integral of the wave functions of the degenerate states in the resonance potentials $\pm V(x)$. When $\varepsilon \gg \Delta_1$, the formulas (10) and (14) coincide.

From (14) we obtain the following expression for the atomic dispersion $\varepsilon_n + \delta\varepsilon_n(p)$ in the form

$$\delta\varepsilon_n(p) = \pm\frac{\Omega}{\pi^{1/2}}\frac{\Delta}{\Delta_1}\Phi\left(-\frac{2\varepsilon_n}{\Delta_1}\right)\cos\left(\frac{\pi p}{k}\right), \qquad \cos\varphi(\varepsilon_n) = 0. \tag{15}$$

For positive energies the overlap integral oscillates, while for $\varepsilon_n < 0$ and $|\varepsilon_n| \gg \Delta_1$ it decreases exponentially:

$$\Phi(2|\varepsilon_n|/\Delta_1) \approx (2|\varepsilon_n|/\Delta_1)^{-1/4}\exp\left[-\tfrac{2}{3}(2|\varepsilon_n|/\Delta_1)^{3/2}\right].$$

This is the subbarrier transmission coefficient in the potential $-|V(x)|$.

Let us emphasize that the widths of the levels with negative energy very critically depend on Δ. Indeed, for $\Delta = 0$ the level width is determined by the tunneling of the particle through the barrier $V(x)$. In this case the barrier height is $\gtrsim V_0$ and the width is $\gtrsim \pi/k$, so that the transmission coefficient is $\sim\exp[-\text{const}(mV_0/k^2)^{1/2}]$. We see that the index of the exponential function contains, in comparison with the case of finite detunings, the additional large factor $(V_0/|\varepsilon|)^{3/2}$.

DISCUSSION

A characteristic of the motion of a two-level atom in a standing wave is the presence of two trajectories of motion. The transitions from one trajectory to the other lead to interference between the states of the translational motion.

In the region of above-barrier energies this manifests itself in density-of-states oscillations with the Doppler frequency (the formula (8)). In the subbarrier energy region $\Delta/2 < \varepsilon < V_0$ there occurs a strong intermixing of the states of the continuous and discrete spectra. In the case of large Δ this intermixing manifests itself in the fact that there occur in the background of a relatively smooth behavior of the density of states spikes corresponding to quasidiscrete energy levels. At negative energies the possibility of the atom's moving in the different potentials causes the level width to depend very critically on the detuning.

In a strong field such a picture is described by the quasiclassical approximation. Far from the standing-wave nodes, where the field is strong and varies slowly, the atoms are in a superposition of states corresponding to definite trajectories. Near the field nodes the LZ transition intermixes these states. In this way we can describe the particle dispersion in a broad range of values of the parameters of the problem. Two regions of energy and detuning values are an exception. For $\varepsilon \sim \Delta \sim \Delta_1$ the particle motion in the vicinity of a node is not quasiclassical. It can be shown that in this case the transition matrix is determined by a scalar Schrödinger equation with an effective potential in the form of a polynomial of the fourth degree. In the other range $\Delta \sim V_0 \sim$ kv of the parameters of the problem the LZ transition region essentially coincides with the wavelength of the field. Since the motion of the atom at such high energies can be considered to be prescribed, the problem reduces to the solution of the standard equations for a two-level system in a monochromatic field with frequency kv.

The frequency $\Delta_0(\varepsilon)$ separates the small and large detunings, so that for $\Delta < \Delta_0$ the atom moves primarily in the

resonant potentials $\pm V(x)$, while for $\Delta > \Delta_0$ it moves primarily in the nonresonant potentials $\pm U(x)$.

As can be seen from (11), for $\Delta < \Delta_0$ the perturbation of the spectrum of the resonant potential is small compared to the level spacing Ω. The spectrum of an atom in a standing wave in the case of small detunings was recently considered by Compagno *et al.*[8] in the strong-coupling approximation. To determine the boundary of the resonance region, the perturbation of the spectrum was compared not with the level spacing, but with the level width, which in a strong field ($V_0 > k^2/2m$) is exponentially small. Such a determination of the boundary of the resonance region is artificial.

In conclusion, let us note that the interference effects considered by us can manifest themselves under different physical conditions. Besides the above-noted density-of-states oscillation effect, which is important in the problems of slow-atom spectroscopy, they manifest themselves in the bipotential scattering of an atomic beam by the field of a standing light wave.[15]

[1)]Strictly speaking, the phase S_0 corresponds to the motion of a particle in a potential composed of segments of nonresonant potentials with jumps of magnitude Δ at the field nodes[8] [the segments (ab) and (cd) in Fig. 1]. For $\Delta \ll \Delta_0$ allowance for the dependence of S_0 on Δ yields spectral corrections of the order of $(\Delta/\Delta_0)^2$, which are insignificant in comparison with the contribution of the resonant potential $V(x)$.

[1]A. I. Burshteĭn, A. B. Doktorov, and A. Yu. Pusen, Zh. Eksp. Teor. Fiz. **72**, 98 (1977) [Sov. Phys. JETP **45**, 52 (1977)].
[2]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **67**, 1660 (1974) [Sov. Phys. JETP **40**, 825 (1975)].
[3]A. P. Kazantsev, Usp. Fiz. Nauk **124**, 113 (1978) [Sov. Phys. Usp. **21**, 58 (1978)].
[4]A. A. Slutskin, Zh. Eksp. Teor. Fiz. **53**, 767 (1967) [Sov. Phys. JETP **26**, 474 (1968)].
[5]A. S. Aleksandrov, V. F. Elesin, A. N. Kremlev, and V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **72**, 1913 (1977) [Sov. Phys. JETP **45**, 1005 (1977)].
[6]V. P. Kraĭnov and V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **78**, 2204 (1980) [Sov. Phys. JETP **51**, 1104 (1980)].
[7]P. A. Braun and G. P. Miroshnichenko, Opt. Spektrosk. **49**, 1024 (1980).
[8]G. Compagno, J. S. Peng, and F. Persico, Phys. Rev. A **26**, 2065 (1965).
[9]V. D. Popov and V. P. Yakovlev, Tezisy dokladov na X Vsesoyuznom soveshchanii po teorii poluprovodnikov (Abstracts of Papers Presented at the Tenth All-Union Conf. on the Theory of Semiconductors), Vol. 2, Novosibirsk, 1980, p. 127; V. M. Dubovik, V. D. Popov, and V. P. Yakovlev, in: Vzaimodeĭstvie lazernogo izlucheniya s rezonansnymi sredami (Interaction of Laser Radiation with Resonant Media), Énergoizdat, 1982, p. 108.
[10]S. M. Freund, M. Römheld, and T. Oka. Phys. Rev. Lett. **35**, 1497 (1975).
[11]J. Read and T. Oka, Phys. Rev. Lett. **38**, 67 (1977).
[12]E. Kyrola and S. Stenholm, Opt. Commun, **22**, 123 (1977).
[13]A. P. Botin, A. P. Kazantsev, and V. S. Smirnov, Zh. Eksp. Teor. Fiz. **71**, 122 (1976) [Sov. Phys. JETP **44**, 63 (1976)].
[14]V. S. Letokhov and V. G. Minogin, Zh. Eksp. Teor. Fiz. **74**, 1318 (1978) [Sov. Phys. JETP **47**, 690 (1978)].
[15]A. P. Kazantsev, G. I. Surdutovich, and V. P. Vakovlev, Opt. Commun. **43**, 180 (1982).

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