Superradiance in extended media

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Superradiant properties of an extended uniformly excited medium, whose length exceeds the cooperative length, are examined. Numerical solution of the Maxwell-Bloch equations is used to show that there is a certain characteristic length that determines a new superradiant state. It takes the form of irregular oscillations and is characterized by a quadratic dependence of the intensity on the density of excited atoms. It is shown that a superradiant medium excited uniformly over some portion of it acts as a source of nondecaying 0π pulses.

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1. INTRODUCTION

Superradiance (SR), i.e., the coherent spontaneous decay of a system of excited radiators, is interesting not only as a possible means of producing a nonlaser source of strong coherent radiation, but also as a nontrivial physical phenomenon. This is the reason for the increased attention that SR has attracted in recent years.[1]

The appearance of a superradiant collective state is due to the phasing of the individual radiators during the initial noncoherent spontaneous decay. A quantum-mechanical description of field and medium is essential for a rigorous analysis of the initial stage of SR.[1] However, the basic features and characteristics of SR can be reproduced by considering the interaction between a classical electromagnetic field and quantum-mechanical oscillators (the quasiclassical approach) if a particular model of the initial stage of the process is chosen. This model cannot be chosen unambiguously and, depending on the parameters of the excited resonant medium and the method used to pump it,it can take the form of a source of polarized noise, an effective external field, or the initial angle of deflection of the Bloch vector.[2,3] The quasiclassical description also enables us to take into account the influence of the spatial inhomogeneity of the emission and excitation on the emission kinetics and the shape and parameters of the SR pulse.

Different SR regimes appear, depending on the ratio of the length of a one-dimensional excited medium to its characteristic lengths l_c, l_p [see (6) and (8) below]. They include the single-pulse state, the regularly oscillating,[4] the Arecchi-Courtens oscillations,[5,6] and the state of irregular oscillation. Karnyukhin and Kuz'min[7] have analyzed the properties of SR systems of length comparable with the emission wavelength λ. In the present paper we examine the kinetics of SR in a uniformly excited medium whose length exceeds the length characteristic for regular oscillations and also discuss the kinetics of an SR medium a part of which is uniformly excited.

2. IRREGULAR SUPERRADIATION OSCILLATIONS

Superradiation by a uniform extended inverted medium of length l can be described approximately within the framework of the Bonifacio-Lugiato[4] mean-field theory which ignores the spatial homogeneity of the field during the emission process, if the initial conditions are homogeneous. This means that one can go over from the truncated Maxwell equation for the complex amplitudes to the simpler pendulum-type equation, for which the escape of the field from the medium is looked upon as effective damping. The properties of SR states are determined by the characteristic time $\tau = (\tau_R \tau_p)^{1/2}$, for the effective energy exchange between the field and the medium, the collective decay time[4] τ_R, and the time $\tau_p = l/c$ taken by a photon to traverse the medium. When $\tau_{p<\tau}$, the system emits a single field pulse after a delay

$$\tau_p \ll \tau,$$

where the area under this pulse is $\theta(t = t_2) = \pi$ and the deflection of the Bloch vector is

$$\theta(t) = \frac{d}{\hbar}\int_{t_1}^{t} E(t')\,dt', \tag{1}$$

d is the matrix element of the atomic dipole moment, and t_1 and t_2 are the time limits of the pulse.

When $\tau_p \gg \tau + \tau_D$ and the functions $\theta(x,t=0)$ and $E(x,t=0)$ are initially homogeneous there is an appreciable volume within which the atoms will radiate simultaneously. In the single-mode case, the process can be satisfactorily described by the equation for the undamped pendulum in the mean-field theory up to $t = \tau_p$. The solution takes the form of a series of identical pulses whose shape is close to the hyperbolic secant, so that this SR state can be referred to as an undamped regular oscillation. The length of each pulse is then determined not by the time τ_R but by τ, and is independent of l. The period of the field oscillations is equal to the period of the pendulum solution. The oscillating state is related to the periodic energy exchange between the field and the medium (Burnham-Chiao oscillations[8]). The restricted size of the region in which coherent decay takes place has been examined by Arecchi and Courtens[5] and, in this case, leads to a random phase of the pulses.

The case $\tau_p \gtrsim \tau$ is described in mean-field theory by the equation for the damped pendulum, and its solution takes the form of damped regular oscillations. This description of SR is in qualitative agreement with experimental results (see, for example, Ref. 9) but, even in this case, satisfactory quan-

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titative description must rely on allowance for the spatial inhomogeneity of the field amplitude and of the excitation of the medium and, consequently, on the solution of the complete set of Maxwell-Bloch equations.[3,10] It will be shown later that a rearrangement of the spatial structure of the field during the emission process in an extended medium leads to a new SR regime, namely, irregular oscillations (IQ).

Consider SR in a continuous medium consisting of excited two-level atoms. The medium is in the shape of a rod, and a plane electromagnetic wave propagates in the direction of its axis. The influence of the wave traveling in the opposite direction will be neglected.[6] The quasiclassical Maxwell-Bloch equations for this system have the form

$$\Omega_x+c^{-1}\Omega_t=\alpha c^{-1}P, \tag{2a}$$

$$P_t=n\Omega-T_2^{-1}P, \tag{2b}$$

$$n_t=-\mathrm{Re}\,(P\Omega^*)-T_1^{-1}(n+1), \tag{2c}$$

where $\Omega=(d/\hbar)E(x,t)$, $E(x,t)$, and $P(x,t)$ are the slowly-varying complex amplitudes of the electric field and atomic polarization density, $n(x,t)$ is the inverted population of the atoms, T_1 and T_2 are the inversion and polarization relaxation times of the individual atoms; $\alpha=4\pi^2d^2cn_0/\hbar\lambda=\tau^{-2}$, n_0 is the density of the resonant atoms, and the subscripts x and t indicate the corresponding derivatives.

Equations (2b) and (2c) describe the motion of the Bloch vector $\mathbf{R}$ (pseudospin) under the influence of the field E. The components of this vector are normalized so that, initially, $\mathbf{R}^2=|P|^2+n^2=1$. When the processes in which we are interested occupy a time interval $\Delta t\ll T_1,T_2$, the modulus of $\mathbf{R}$ is conserved, and the solution can be sought in the form

$$P(x,t)=\sin\theta(x,t)\,e^{i\gamma},\quad n(x,t)=\cos\theta(x,t), \tag{3}$$

where $\theta(x,t)$ and $\gamma(x,t)$ are slowly varying functions. The simple relationship (1) between the angle of deviation of the Bloch vector and the field is valid in the case of exact resonance in the absence of dephasing between the dipoles and the field, i.e., when $\gamma(x,t)=0$. Equation (2) then assumes the simpler form

$$\theta_{xt}+c^{-1}\theta_{tt}=\alpha c^{-1}\sin\theta. \tag{4}$$

SR by a uniformly excited medium is of considerable interest as a way of producing short, powerful pulses. It is precisely uniform pumping that can concentrate maximum energy in the system. The corresponding initial conditions are

$$\theta(x,0)=\theta_0,\quad \theta_t(x,0)=0, \tag{5}$$

where $\theta_0=2N^{-1/2}$ and N is the total number of collectively radiating atoms. We note that we cannot then use the traditional transformation to retarded time (as in, for example, Ref. 10) in order to simplify (4), because the initial conditions (5) will then no longer be homogeneous in the retarded time.

Numerical integration of the equations was performed for parameter values corresponding to $\lambda=1\ \mu$m, $n_0=10^{12}$ cm^{-3}, and $T_1=10$ ns, since most of the experimental observations of SR were performed in gaseous media, using atomic transitions in the infrared range. However, it is clear that

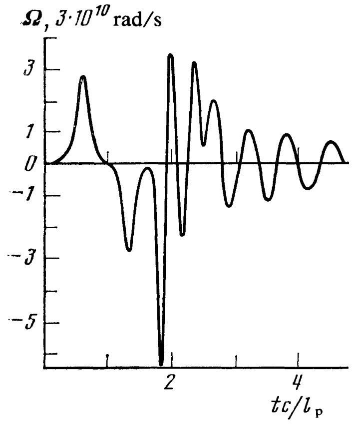

FIG. 1. Amplitude of emitted SR field for a medium of length $l=l_c+4\pi\tau c$ for the following parameters: $\theta_0=10^{-2}$, $\tau c=0.7$ cm, and $l_c=3.2$ cm.

the SR regimes are determined only by the relative values of the characteristic parameters of the process, and these will be indicated in what follows.

Figure 1 shows a graph of the escaping field as a function of time. It was obtained by numerical solution of (4), subject to the initial conditions given by (5). The specimen was assumed to be long enough to ensure that all the above SR states were realized. Apart from undamped and damped regular oscillations at the beginning and end of the emission process, the graph also shows the presence of sharp, irregular-oscillation peaks with irregular positions and large amplitudes. We emphasize that the condition $\gamma(x,t)=0$ imposed above exludes the Arecchi-Courtens random oscillations. On the one hand, this enables us to propose for the random oscillations a mechanism that is not related to the dephasing of radiation emitted by different portions of the medium and, on the other, it enables us to investigate this process in its "pure form."

Let us now examine systematically the evolution of the IO regime. The important parameter here is the cooperative length

$$l_c\approx\ln(2\pi/\theta_0)\,(4\pi cT_1/n_0\lambda^2)^{1/2}, \tag{6}$$

which determines the size of the region in which the coupled (coherent) decay of different atoms takes place.[6] The transit time l_c/c is equal to the SR delay time, so that a system of length $l\ll l_c$ will radiate coherently. The minimum value of l_c is equal to τc and is reached for systems in an initial Dicke state $\theta_0\approx\pi/2$, in which all the atoms decay simultaneously. When $\theta_0\ll1$, there is an increase in the cooperative length l_c, but the decay of the different portions of the specimen does not occur simultaneously. The resulting spatial inhomogeneity of the field is irregular in character and is also determined by the reabsorption condition $\tau_p>\tau$. This state corresponds to damped regular oscillations of the emitted radiation (Fig. 2).

When l_c is constant, a change in the length of the specimen leads to a change in the area η_1 under the first SR peak. For $l\ll\tau c$, the area is $\theta_1=\pi$, but it increases to 2π when the length reaches a certain characteristic value $l=l_p$ [see (8)]. The increase in θ_1 can be followed directly in Fig. 2 by

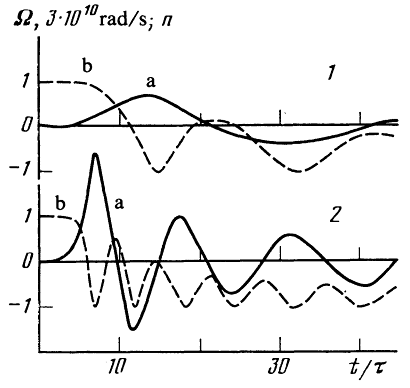

FIG. 2. Time dependence of field (a) and inversion (b) at $x = l$ for different $l < l_p$ (regular damped oscillations): 1) $l = \tau c$; 2) $l = l_c$. The values of the parameters θ_0, τ and l_c are the same as in Fig. 1.

inspecting the curves showing the inversion of the atoms at the edge of the specimen. At the time at which the peak emerges, the inversion is $n(x = l, t = t_2) = \cos\theta_1$. We note that when $l = l_c$, we have $\cos\theta_1 < 1$, so that this length is insufficient for the formation of a pulse of area 2π and maximum amplitude (compare this with Ref. 6).

Further increase in $l > l_c$ leads to the appearance of an extended region $l_0 \approx l - l_c$ of simultaneous homogeneous decay (Fig. 3) which, as follows from (4), can be described by the undamped-pendulum equation

$$\theta_{tt} = \tau^{-2} \sin\theta. \qquad (7)$$

It is clear from Figs. 1 and 3 that regular undamped oscillations are, in fct, observed only for a time $0 \leqslant t \leqslant l_0/c + \tau_D = l/c$. They are 2π pulses and have a repetition period that is a function of θ_0 and is equal to half the

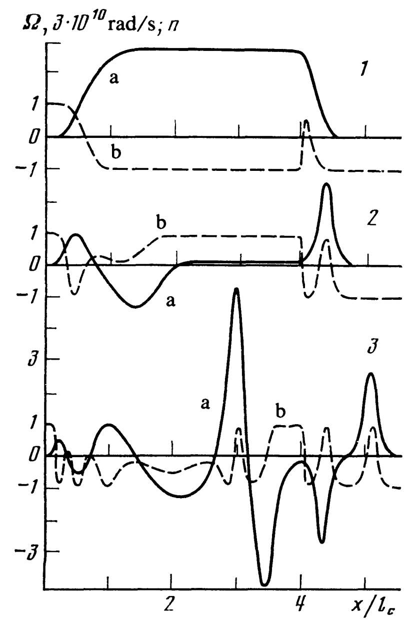

FIG. 3. Spatial distribution of the field (a) and inversion (b) in a medium with homogeneously pumped region $x \leqslant l = 4l_c$ and unexcited region $x > l$ at different times: 1) $t = \tau_D + \pi\tau$, 2) $t = l_p/s$, 3) $t = \tau_p$.

period of the pendulum (7). The halfwidth of the peaks is then a minimum, given by $\tau = (8\pi T_1/cn_0\lambda^2)^{1/2}$. To provide a clearer illustration of the soliton properties of the pendulum solution, Fig. 3 shows the behavior of the emerging SR pulses in an identical resonance medium. As can be seen, the pulses propagate without change of shape and without absorption. This process may also be looked upon as decay of the initial excitation into solitons in the extended medium.

The restriction on the interval of time within which undamped oscillations exist occurs because the trailing edge of the uniform-field region moves with velocity c. On the other hand, the rate of energy escape from the system is determined by the pulse propagation velocity $v < c$ in the absorbing resonant medium (see Ref. 11, p. 99 of transition). For the values of the medium parameters that we have used, $v = 0.5c$, so that a substantial fraction of the energy remains in the specimen after the end of the pendulum regime. It is clear from Fig. 1 that it is precisely at this time that the irregular oscillations appear, so that the reason for them is the residual excitation of the medium. Figure 3 shows the evolution of the strong spatial inhomogeneity of the residual field and excitation. The displacement of the homogeneous inversion region occurs more slowly than the motion of the field envelope. Amplification of the residual inhomogeneous field in the medium is possible because of secondary inversion of atoms, and this amplification is a maximum under complete inversion, i.e., when at least the first emitted peak is a 2π pulse. Thus, in the problem with homogeneous initial excitation, the irregular oscillations appear only when the length of the emitted pulses T_{pul} is short enough, or when the length l_0 is long enough to ensure that the 2π pulse succeeds in developing as a result of the complete decay of a portion of the homogeneous field. Numerical analysis of (4) shows that, with good accuracy, $T_{\text{pul}} \approx 2\pi\tau$. The condition for the presence of irregular oscillations is then written in the form

$$l \geqslant l_p = l_c + 2\pi\tau c. \qquad (8)$$

When $l < l_p$, only damped regular oscillations are observed (see Fig. 2).

Figure 1 illustrates the case $l = l_p + 2\pi\tau c$, so that the irregular oscillations follow the two 2π pulses. The damped oscillations at the end of the emission are due to the decay of a small excited region at the left-hand edge of the specimen (see lower curves in Fig. 3).

Numerical analysis of the IO pulses has shown that their intensity is a quadratic function of n_0, whereas the fact that the area and width of the 2π pulses are constant shows that their amplitude is proportional to τ^{-1}, i.e., the intensity is a linear function of n_0. The number of IO peaks and their amplitudes depend on the order of the inequality given by (8). The search for an accurate analytic expression for the IO pulses is greatly complicated by the fact that they evolve in a highly perturbing amplifying medium and do not succeed in reaching a stationary state (see last section, below).

Thus, the SR regime examined above differs from the Arecchi-Courtens oscillations and is not directly related to noncoherent (unphased) decay of different portions of the specimen, so that both mechanisms of formation of random oscillations must be taken into account in the analysis of superradiance. Spatial inhomogeneities of the field and excitation of the medium are important for IO, so that the equations of the mean-field theory cannot, in principle, describe the evolution of this SR regime.

3. SUPERRADIATION BY AN INHOMOGENEOUSLY EXCITED MEDIUM

The traditional approach to the theoretical analysis of SR is to consider the properties of the radiation produced by the fully excited medium (excitation by homogeneous or progressive pump). On the other hand, in practice, it is also possible to achieve sufficient excitation in a portion of the medium. The resulting SR pulse will then traverse a region of resonant absorption before it escapes from the specimen. Let us denote the length of the excited region by l, and that of the unexcited region by l_1. It then follows from the last section that pulses of undamped regular oscillations, which are 2π pulses, do not vary even for $l_1 > \tau c$ (Fig. 3). However, when $l < l_p$, the area under the pulses is less than 2π in the damped-oscillation regime, so that they undergo appreciable transformation over the length l_1.

Numerical solution of (4) for SR in a medium of length $l + l_1$ was performed for quasihomogeneous initial conditions [condition (5) with $\theta_0 = \pi$ within l_1]. Figure 4 shows the field and inversion (broken curves) as functions of time at different points in the unexcited medium for $l = 1.5 l_c < l_p$. As expected, the shape of the oscillations changes very substantially over the length $l_1 \sim \tau c$. The subsequent evolution of the field leads to the appearance of a stable localized pulse whose shape gradually changes at constant total energy and constant velocity of the center of mass. It is clear that the pulse of medium perturbation has the same features. The total area under the field envelope is zero. These properties

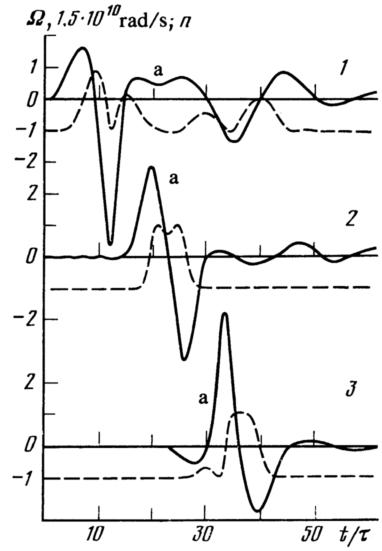

FIG. 4. Transformation of regular SR oscillations into the 0π pulse (solid curves) during propagation in a medium with resonance absorption: 1) $x = l + 1.5\tau c$, 2) $x = l + 4\tau c$, 3) $x = l + 6\tau c$. Characteristic parameters: $l_c = 1.5$ cm, $\tau c = 0.7$ cm.

are characteristic for a decaying Lamb 0π pulse.[12,13] It is readily verified that our solution is identical to the 0π pulse, for which an analytic expression is available. In fact, if we substitute $\sqrt{\alpha}(t - 2x/c) \to \xi, \sqrt{\alpha}t \to \rho$, we find that (4) becomes identical with the canonical sine-Gordon equation

$$\theta_{\xi\xi} - \theta_{\rho\rho} = \sin\theta.$$

This has two types of stable asymptotic solution, namely, kink ("loop") and breather ("breathing solution").[14]

The kink solution is

$$\theta = 4 \operatorname{arctg} \exp\left(\pm \frac{\xi - u\rho}{(1-u^2)^{1/2}}\right)$$

(where $u = 1 - 2v/c$, and v is the velocity of the center of mass) and corresponds to the stationary 2π pulse examined in the last section. The breather has the form

$$\theta = 4 \operatorname{arctg} \{\operatorname{tg}\nu \sin[\rho\cos\nu] \operatorname{sch}[(\xi - u\rho)\sin\nu]\}, \qquad (9)$$

where the parameter $0 < \nu < \pi/2$ determines the characteristic width and amplitude of the breather. Direct comparison of the function $\theta_t(\nu, u; x, t)$, obtained with the aid of (9), with the field amplitude shown in Fig. 4 enables us to conclude that, under the above conditions, the pulse of regular SR oscillations evolves toward a stable 0π pulse with $\nu \approx 1$.

Numerical analysis of systems with different l and, consequently, different areas θ_i under the ith oscillation bursts has shown that the evolution toward the 0π pulse is possible for $l \gtrsim \tau c$ (the corresponding areas are $\theta_1 \gtrsim 3\pi/2$, $\theta_2 \lesssim -\pi/2$). An increase in l leads to an increase in the amplitude and energy of the peaks, and the resulting breather is described by (9) with large ν and high velocity v. Moreover, the characteristic length for the transformation of the regular oscillations into the 0π pulse increases from τc to $3\tau c$. The latter value corresponds to the condition $l \simeq l_p$, for which two 0π pulses with different velocities are formed. Finally, the evolution of the IO pulses for $l_1 \gg \tau c$ leads to the formation of a chain of coupled nondecaying 0π pulses.

Thus, the SR pulse from an inhomogeneously excited medium in which $l, l_1 \gtrsim \tau c$ is a stable 0π pulse (or a set of 0π pulses) that propagates through the identical absorbing resonance medium without attenuation. The 0π pulse of SR can be observed, for example, in an experiment such as that reported by Gibbs *et al.*[15] if the cell containing the cesium vapor is oriented in the exciting laser beam in such a way that only a portion of the volume is pumped.

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