Langmuir collapse under pumping and wave energy dissipation

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We use numerical methods to study the dynamics of a Langmuir caviton which results from the modulational instability of long-wavelength pumping at the plasma frequency. In the final stage of the collapse, when the Landau damping of the plasma oscillations becomes important, the caviton "burns up" and the Langmuir energy is absorbed by resonant electrons.

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1. INTRODUCTION

The Langmuir collapse phenomenon predicted in Ref. 1 plays a fundamental role in contemporary plasma turbulence theory,[2] as it guarantees the short-wavelength transfer of Langmuir oscillations to the absorption region. As there is no rigorous analytical solution, doubt has been expressed in a number of papers (see, e.g., Ref. 3) about the existence of supersonic collapse, i.e., collapse at a sufficiently high level of energy of the plasma oscillations, $E^2/16\pi nT > m/M$. To refute this conclusion it is important to carry out numerical collapse simulation in which solutions are constructed which describe the nonlinear dynamics of a packet of Langmuir waves, the formation of a caviton with plasmons (Langmuir wave quanta) trapped in it, and the transition of the caviton into the self-similar regime of explosive collapse.[4–6] Nonetheless, in a recently published paper[7] there was again reached, in our opinion, the erroneous conclusion that the dynamics of a packet of strong Langmuir waves is reversible. This was in fact based only upon a specific choice of initial conditions for which the collapse is impossible (see Sec. 2).

The problem of obtaining a more exact numerical solution of the hydrodynamic equations describing the nonlinear dynamics of a packet of Langmuir waves remains therefore of vital interest. The present paper is devoted just to this. A caviton is here considered under turbulence conditions, i.e., in the presence of a constantly acting pumping wave at the plasma frequency and of damping of the Langmuir wave in the short-wavelength region.

2. LANGMUIR COLLAPSE OF AN ISOLATED CAVITON. PUMPING WAVE AND COLLAPSE

The starting set of equations are the equations obtained for the first time by one of the present authors[1] for Langmuir oscillations, modified to take into account the presence of pumping energy in the oscillations and their resonant damping by particles:

$$2i\Delta\frac{\partial\psi}{\partial t}+\Delta^2\psi=\mathrm{div}\,(\delta n\nabla\psi)-E_0\frac{\partial\delta n}{\partial z}+2i\hat{\Gamma}\Delta\psi, \tag{1}$$

$$\frac{\partial^2\delta n}{\partial t^2}-\Delta\delta n=\Delta\left\{\left|E_0-\frac{\partial\psi}{\partial z}\right|^2+\left|\frac{\partial\psi}{\partial r}\right|^2\right\}. \tag{2}$$

In these equations ψ is the complex amplitude of the high-frequency potential

$$\varphi_l={}^1/{}_2\psi(t,\mathbf{r})\exp(-i\omega_p t)+\text{c.c.}$$

$\omega_p=(4\pi n_0e^2/m)^{1/2}$ is the plasma frequency, δn the change in the density in the slow quasi-neutral plasma motions, and E_0 the amplitude of the pumping wave at the plasma frequency and is directed along the z-axis. The pump is defined as the average electric field in the plasma:

$$\mathbf{E}_0=\frac{2}{R^2L}\int_0^L dz\int_0^R rdr\mathbf{E} \tag{3}$$

and, hence,

$$\int_0^L dz\int_0^R rdr\nabla\psi=0.$$

We assume that there is an energy source (electromagnetic wave, electron beam) which sustains the pumping at a constant level, i.e., $dE_0/dt=0$. Moreover, $\hat{\Gamma}$ is an integral operator which takes into account the Landau damping of short-wavelength plasma oscillations. When writing it down we took it into account that the caviton collapse is substantially anisotropic, i.e., $\partial/\partial z \gg \partial/\partial r$ (a collapsing caviton is always a dipole and is flattened in the direction of the dipole axis[4]). In that case we can assume the plasma oscillations to be one-dimensional when calculating the damping:

$$\hat{\Gamma}f(t,r,z)=\frac{1}{L}\int_0^L\Gamma(z-z')f(t,r,z')dz', \tag{4}$$

where $\Gamma(z)$ is the Fourier transform of the Landau damping rate:

$$\Gamma(z)=\frac{1}{2\pi}\int dke^{ikz}\Gamma_k,$$

$$\Gamma_k=\frac{27\pi}{2n_0}\left(\frac{M}{m}\right)^2\frac{1}{k^2}\frac{\partial f}{\partial v}\bigg|_{kv=1}, \tag{4'}$$

and $f(v)$ is the distribution function of the electrons at resonance with the oscillations. We have used in Eqs. (1) to (4) the following scales for length, time, field, and density:

$$\bar{z}=3\left(\frac{M}{m}\right)^{1/2}r_D,\qquad \bar{t}=\frac{3M}{m}\omega_p^{-1},$$

$$\overline{\delta n}=\frac{m}{3M}n_0,\qquad \overline{\nabla\psi}=\left(\frac{16\pi}{3}n_0T\frac{m}{M}\right)^{1/2} \tag{5}$$

The number of plasmons

$$I_1=\int|\nabla\psi|^2d\mathbf{r}$$

is not an integral of motion of the original set of equations; this quantity is altered by the pumping and damping:

$$\frac{dI_1}{dt}=-\frac{i}{2}\mathbf{E}_0\int\nabla\psi\delta nd\mathbf{r}-\text{c.c.}+\int\hat{\Gamma}|\nabla\psi|^2d\mathbf{r}. \tag{6}$$

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The integral of motion of Eqs. (1) and (2), which is conserved when pumping and damping are present, has the form

$$I_2=\int\left\{|\Delta\psi|^2+\delta n|\nabla\psi-\mathbf{E}_0|^2+\frac{\delta n^2}{2}+\frac{\mathbf{v}^2}{2}\right\}d\mathbf{r}=\text{const},\tag{7}$$

where $\mathbf{v}$ is the ion velocity. When there is no pumping, the fact that the integral I_2 is negative is a sufficient condition for collapse. This problem is analyzed in detail in Ref. 5. If, however, the Langmuir caviton arises on the background of a constantly acting pumping (and this is just the situation realized in Langmuir turbulence), collapse turns out to be possible also when $I_2\geqslant 0$ (*vide infra*). The absence of collapse in the numerical simulation of the dynamics of a Langmuir packet, performed in Ref. 7, was connected precisely with the fact that they considered the case $E_0=0$ and initial conditions for which $\delta n=0$ and thus $I_2>0$.

The condition that I_2 be negative coincides as to order of magnitude with the condition for the occurrence of the modulational instability[8] of the Langmuir oscillations trapped in the caviton—a high-frequency-field formation from which the field-pressure force ejects the plasma. The collapse—the implosion of the caviton—is connected with the fact that the high-frequency pressure is negative and has the character of an eruption for which the formal singularity $|E|\to\infty$ is reached after a finite time. When $|E|\gg 1$ we can neglect on the left-hand side of Eq. (2) terms which do not contain time derivatives (the speed of the implosion is appreciably larger than the sound speed). The supersonic collapse is then, neglecting pumping and damping ($\Gamma\to 0$, $E_0\to 0$), described by the following self-similar solution of Eqs. (1), (2):

$$\psi\approx\frac{1}{(t_0-t)^{1-2/\alpha}}g(\xi)\exp\left(i\int\lambda(t)\,dt\right),$$

$$\delta n\approx\frac{V(\xi)}{(t_0-t)^{4/\alpha}}\qquad \xi=\frac{\mathbf{r}}{(t_0-t)^{2/\alpha}}\qquad \lambda(t)=\frac{\lambda_0}{(t_0-t)^{4/\alpha}}.\tag{8}$$

Here $\alpha=1, 2, 3$ is the dimensionality of the caviton. The solution (8) conserves the number of plasmons in the caviton: $I_1=\text{const}$. Using this solution in the right-hand side of Eq. (6) we easily verify that in the case of a three-dimensional caviton and $\Gamma\to 0$ the plasmon-number conservation condition used in obtaining (8) becomes asymptotic when there is pumping and is satisfied with greater accuracy the closer we approach the singularity $t\to t_0$:

$$I_1\approx\text{const}+O\left[(t_0-t)\exp\{3i\lambda_0(t_0-t)^{-1/3}\}\right].\tag{9}$$

A three-dimensional caviton thus loses the connection with the source when collapsing and in this sense the solution (8) is a universal one and is independent of the presence of pumping.

In the present section we report the results of a numerical simulation of Langmuir collapse—the transition of a caviton into the self-similar regime, the characteristics of this regime, and the influence of the pumping on the collapse process.

We solved the set of Eqs. (1) to (4) numerically in the rectangular region $0\leqslant z\leqslant L$, $0\leqslant r\leqslant R$ with first-kind boundary conditions for all functions with respect to z and with the condition that derivatives with respect to r vanish at the boundaries of the region:

$$\psi(0,r)=0,\qquad \frac{\partial\psi}{\partial z}(L,r)=0,\qquad \left.\frac{\partial\psi}{\partial r}\right|_{r=0,R}=0,$$

$$\frac{\partial\delta n}{\partial z}(0,r)=0,\qquad \delta n(L,r)=0,\qquad \left.\frac{\partial\delta n}{\partial r}\right|_{r=0,R}=0.\tag{10}$$

We first simulated the nonlinear dynamics of a dipole caviton when there is no pumping or damping. The initial high-frequency charge was chosen in the form

$$\rho=-\Delta\psi=\rho_0 z w^{1/2},\quad w>0,$$

$$\rho=-\Delta\psi=0,\quad w<0,\tag{11}$$

$$w=1-r^2/a^2-z^2/b^2.$$

The initial change in the density and its time-derivative were given as follows:

$$\delta n(0,r,z)=-\lambda|\nabla\psi|^2,\qquad \frac{\partial\delta n}{\partial t}(0,r,z)=0.\tag{11'}$$

We show in Fig. 1 the dynamics of the collapse for the parameter values $a=1.2$, $b=0.6$, $\rho_0=4.0$, $\lambda=0.24$. The present calculation differs from earlier ones[4,5] by its high accuracy, in particular by the large number of points along r and z: in the rectangle $L\times R=(\eta/2)\times\pi$ we took 128×32 points. Nonetheless the results for the collapse of an isolated caviton are close to those obtained in Refs. 4 and 5. The increase in the field amplitude is, within the framework of the applicability of the model considered, unlimited—E^2 increases when one goes from Fig. 1a to Fig. 1b at least by a factor 10^3. In the course of time the increase takes on the character of an explosive collapse according to the law:

$$|E|^2\propto(t_0-t)^{-2},\qquad |\delta n|\propto(t_0-t)^{-4/3},$$

which agrees with the solution (8) for $\alpha=3$ (Fig. 2). In the final self-similar stage of the collapse the caviton changes

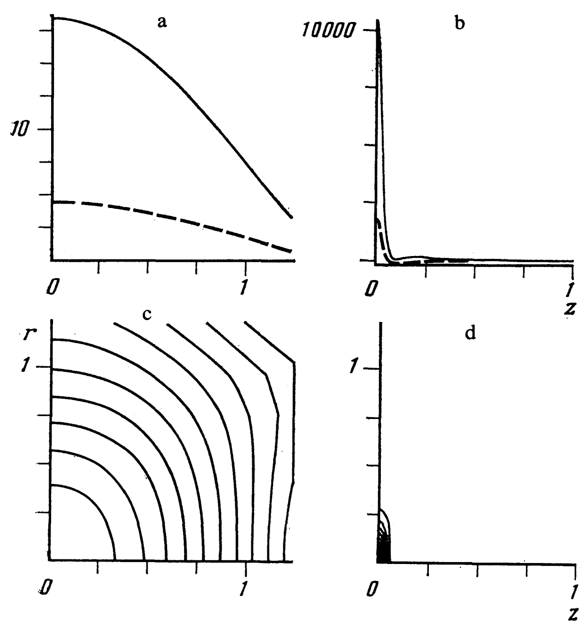

FIG. 1. Dynamics of free Langmuir collapse. a,b) $|E|^2$ (solid curves) and $-\delta n$ (dashed curves) as functions of z for $r=0$: a) $t=0$ and b) $t=1.3$; c,d) lines of constant $|E|^2$ at the same times.

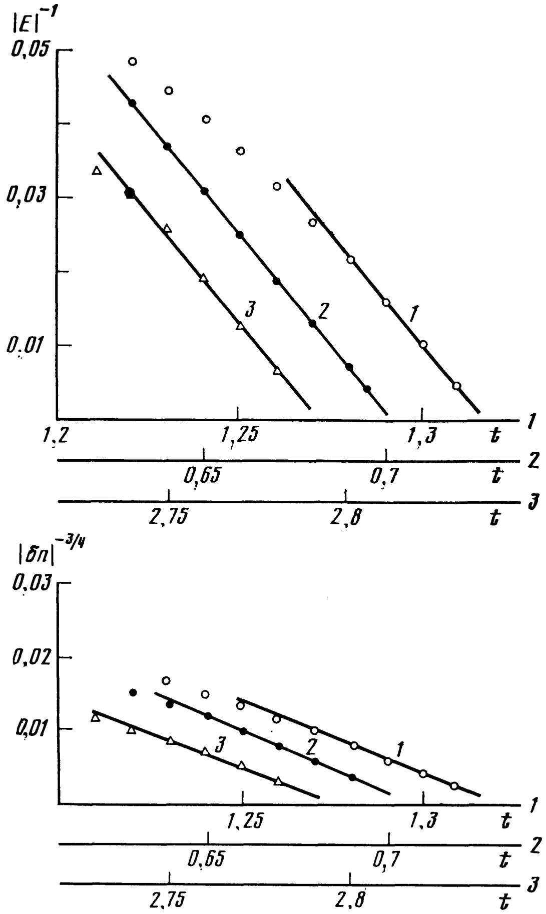

FIG. 2. Self-similar t-dependence of $|E|^2$ and $|\delta n|^{-3/4}$ [the straight lines correspond to Eqs. (8)]: 1) for the variant shown in Fig. 1; 2) for the variant corresponding to the initial conditions (12) in a pumping field $E_0 = 2.0$; 3) for the variant shown in Fig. 3.

from an initially isotropic to an appreciably anisotropic one—flattened along the z axis with $k_\perp/k_z \approx 0.2$.

The dynamics of the Langmuir collapse when there is pumping at the plasma frequency was studied for greatly differing initial conditions. For instance, we considered the collapse arising from the initial conditions for the field and the density corresponding to a quasi-planar soliton:

$$\rho = -\Delta\psi = -\left(\frac{\alpha_0}{\sqrt{2}} r + \beta_0\right) \frac{\mathscr{E}_0^2 \,\mathrm{sh}(\mathscr{E}_0 z/\sqrt{2})}{\sqrt{2}\,\mathrm{ch}^2(\mathscr{E}_0 z/\sqrt{2})},$$

$$\delta n = -(\alpha_0 r + \beta_0) \frac{\mathscr{E}_0^2}{\mathrm{ch}^2(\mathscr{E}_0 z/\sqrt{2})} \qquad (12)$$

with $\mathscr{E}_0 = 4.0$, $\alpha_0 = -1/30$, $\beta_0 = 1$ and the pumping amplitude $E_0 = 2$. As time goes on the stage of explosive collapse is reached when the picture of the spatial distribution of the field and density repeats to a large extent the one shown in Fig. 1 with the only difference that in the case considered the caviton turns out to be appreciably more anisotropic: $k_\perp/k_z < 0.1$. In agreement with the analysis given above the growth of the field and of the density when there is pumping is, as before, described by the same self-similar relations (8). This is illustrated in Fig. 2, from which it follows that the presence of pumping, without changing the law of the self-similar collapse, affects only the time taken by the caviton to reach the self-similar regime.

In the presence of pumping a caviton may result from the modulational instability of the pumping wave, starting from an initially sinusoidal perturbation of the field and of the density, as illustrated in Fig. 3. We show in that figure the dynamics of a packet of Langmuir waves arising from a weak (initial amplitude $\varepsilon = 10^{-2}$) initial modulation of the pumping field $E_0 = 2$, while the wavelength of the modulation lies in the modulational instability region $\lambda = 3$. In that case we changed the boundary conditions with respect to z and used the periodicity condition instead of (6), and also chose the region $L \times R = \pi \times \pi$.

The modulational instability leads to a deepening of the density well, to a localization of the electric field in the density minimum, and, finally, to the formation of a collapsing caviton.

It is important that the case shown in Fig. 3 illustrates the appearance of collapse in the presence of pumping starting from initial conditions with $I_2 > 0$ (for the initial conditions used $I_2 = 2\times 10^{-2}$). In the self-similar collapse stage the law governing the growth of the field and density is as before given by Eq. (8) and only the stage where the self-similar regime is reached depends on the initial conditions.

3. LANGMUIR COLLAPSE AND LANDAU DAMPING. ABSENCE OF STATIONARY CAVITONS

In the final stage of the collapse, when the oscillations trapped in the caviton have a sufficiently short wavelength, Landau damping of the plasma oscillations becomes important and this diminishes the pressure of the plasmons trapped in the caviton. We explained above that this pressure is the cause of the collapse, so that in principle Landau damping may lead to a stabilization of the collapse. As there is no analytical solution of this problem we performed a numerical simulation of the corresponding problem using Eqs. (1) to (4).

In the case when the Landau damping is due to plasma electrons with a Maxwellian distribution function

$$f(v) = 3\left(\frac{M}{2\pi m}\right)^{1/2} \exp\left(-\frac{9M}{2m} v^2\right), \qquad (13)$$

the damping cannot halt the explosive collapse of the dipole caviton. In this case the structures of the electric field and of the density are close to that shown in Fig. 1; the collapse law remains self-similar up to amplitudes $|E|^2 \approx 4\times 10^4$, which corresponds to $|E|^2/16\pi n_0 T \approx 6$ when the initial hydrodynamic equations, in fact, cease to be valid and nonetheless the collapse continues. This result is the consequence of the three-dimensional geometry of the caviton, when the short-wavelength transfer of plasma oscillations connected with the deepening of the density well proceeds more slowly than the growth of the amplitude of the electric field in its center, as follows from the self-similar solution (8). Using this solution one obtains easily for the characteristic wave number in the Langmuir spectrum the following relation which is satisfied with good accuracy in the numerical experiment (see Ref. 2):

$$3k^2 r_D^2 \approx \frac{|\delta n|}{n_0} \sim \left[\frac{E^2(0)}{16\pi n_0 T}\right]^{1/3} \left[\frac{E^2}{16\pi n_0 T}\right]^{2/3}. \qquad (14)$$

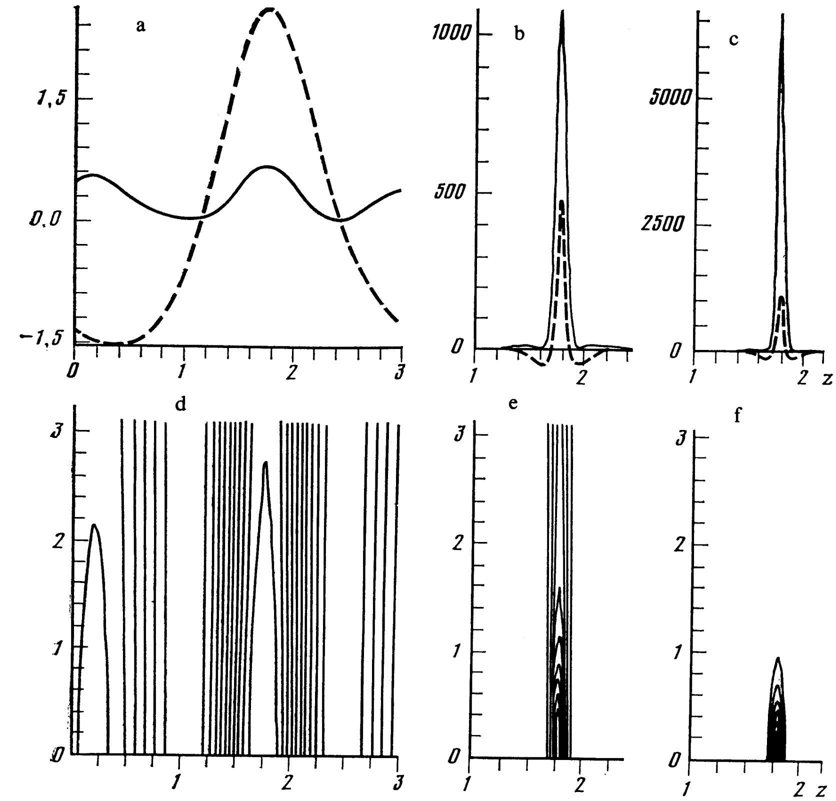

FIG. 3. Dynamics of Langmuir collapse occurring due to the action of a pump $E_0 = 2.0$ from the initially sinusoidal modulation of field and density in the plasma. a,b,c) $|E|^2$ (solid curves) and $-\delta n$ (dashed curves) as functions of z for $r=0$: a) $t=1.45$, b) $t=2.74$, c) $t=2.77$; d,e,f) lines of constant $|E|^2$ at the same times.

Here $E(0)$ is the initial value of the electric field in the collapsing caviton; we have assumed that initially the pressure balance condition $|\delta n(0)|T \approx E^2/16\pi$ is satisfied in such a caviton. It follows from (14) that at the limit of the applicability of the hydrodynamic equations (the limit $E^2 \approx 16\pi n_0 T$ is determined by the appearance of plasmon-plasmon interactions) the main part of the energy of the plasma oscillations is concentrated already in the long-wavelength region of the spectrum, and Landau damping is not able to halt the collapse.

In real plasma turbulence the role of Landau damping can increase because the absorption of plasma oscillations leads to an acceleration of the plasma electrons and to the formation of "tails" in their distribution function, so that the absorption region is shifted to the long-wavelength part of the spectrum.

To simulate that process we considered collapse under such circumstances that the resonant electrons had a velocity distribution in the shape of a two-temperature Maxwell distribution:

$$f(v)=3\left(\frac{M}{2\pi m}\right)^{1/2}\left[\exp\left(-\frac{9M}{2m}v^2\right)+\alpha_T\exp\left(-\frac{9M}{2m\beta_T}v^2\right)\right]; \tag{15}$$

here $\alpha_T = n_T/n_0$ is the fraction of particles in the "tail" and $\beta_T = T_T/T_0$ is the tail temperature. We note that in that case the initial set of Eqs. (1), (2) is modified by taking into account the contribution from the tail particles to the dispersion of the Langmuir oscillations. The dimensionless set of Eqs. (1), (2) remains unchanged but the scales of the various quantities are changed as follows:

$$\bar{z}=3(1+\alpha_T\beta_T)\left(\frac{M}{m}\right)^{1/2} r_D, \quad \bar{t}=\frac{3M}{m}(1+\alpha_T\beta_T)\,\omega_p^{-1},$$

$$\overline{|\nabla\psi|^2}=\frac{16\pi n_0 T m}{3M(1+\alpha_T\beta_T)}, \quad \overline{\delta n}=\frac{n_0 m}{3M(1+\alpha_T\beta_T)}.$$

We show in Fig. 4 the dynamics of the collapse of a dipole caviton with the same initial conditions as in Eq. (12) but with $E_0 = 0$ and the presence of damping by the "tail" electrons with $\alpha_T = 0.02$, $\beta_T = 25$. During the collapse the caviton reaches the self-similar regime but at $E^2/16\pi n_0 T \approx 1/3$ Landau damping stops the collapse. After that the energy of the plasma oscillations in the caviton decreases rapidly down to a value $E^2/16\pi n_0 T \sim 10^{-2}$ and the caviton continues for a while to intensify in energy, after which it becomes a source of sound waves diverging from it.

Finally, we show in Fig. 5 the dynamics of Langmuir collapse under those conditions where it exists in plasma turbulence. The collapsing caviton is formed from the initially sinusoidal field modulation as the result of the modulational instability of the pumping wave with amplitude $E_0 = 2$. The initial amplitude of the modulation is $\varepsilon = 10^{-2}$ and the wavelength of the modulation $\lambda = 3$ lies in the modulational-instability region for the chosen value of E_0. As a result of the collapse the amplitude of the electric field increases according to the self-similar law up to a value $E^2/16\pi n_0 T \approx 1/4$, after which the energy of the plasma oscillations decreases to a value $E^2/16\pi n_0 T \sim 1/150$ due to the Landau damping by tail electrons with $\alpha_T = 0.02$, $\beta_T = 25$. Sound waves are emitted from the caviton in which the balance between the gaskinetic and plasma pressures is violat-

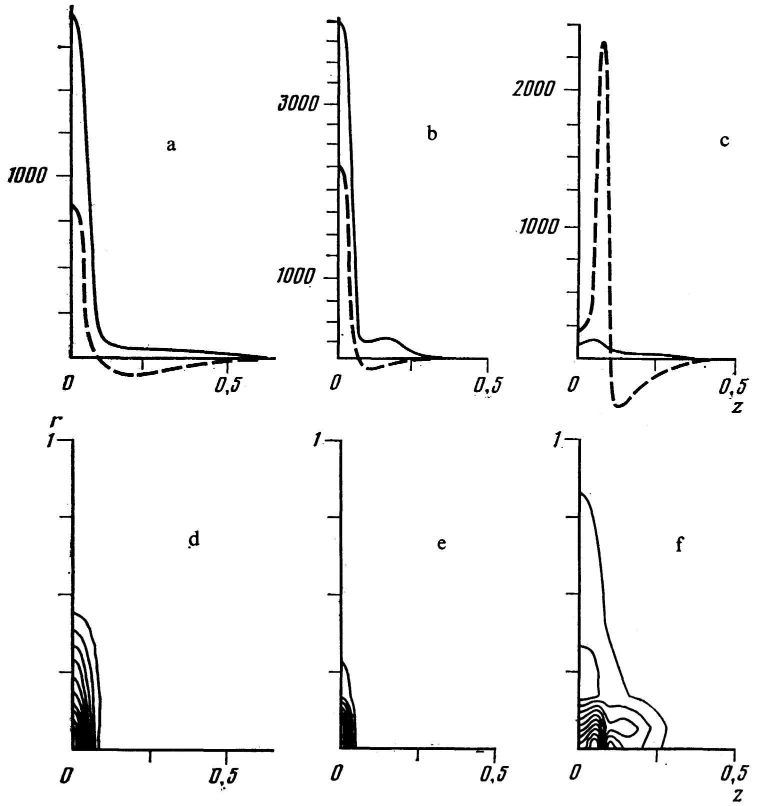

FIG. 4. Dynamics of Langmuir collapse from an initially soliton-shaped field and density distribution in the presence of Landau damping for a two-temperature Maxwellian electron distributin function. a,b,c) $|E|^2$ (solid curves) and $-\delta n$ (dashed curves) as functions of z for $r=0$; a) $t=2.482$, b) $t=2.509$, c) $t=2.596$; d,e,f) lines of constant $|E|^2$ at the same times.

ed. The caviton thus completely "burns up" and transfers the Langmuir energy trapped by it to the short-wavelength absorption region. The quasi-stationarity of the turbulence is guaranteed by the continuous creation of cavitons that transfer the energy dissipated from the pumping wave to the region of effective Landau damping. On the other hand, it was observed in Ref. 9 that stationary cavitons of Langmuir waves are formed as the result of the evolution of the collapse. From our point of view the discrepancy between these results can easily be explained on the basis of physical considerations. In the present paper we simulated the collapse of a real three-dimensional caviton. According to the solution (8) the collapse of such a caviton proceeds with increasing speed. As the collapse proceeds the role of the ion inertia increases and the dispersed ions continue by inertia to deepen the caviton even under conditions when the dissipation of Langmuir energy is effective. At the same time, in two-dimensional cavitons considered in Ref. 9, the collapse proceeds with a constant speed and there is no inertial dispersal of ions. Moreover, the use in that paper of a model mass ratio $M/m=25$ appreciably narrowed the inertial range of the turbulence (the region between source and absorption). It is also important that the smallness of the growth rate of the beam instability which starts off the Langmuir turbulence in the numerical simulation[9] led to a slow energy dissipation in the turbulence, and the modulational instability in that paper remained therefore close to threshold. The formation of a quasi-stationary lattice of solitons under similar conditions was also observed in a one-dimensional numerical simulation.[10]

One should emphasize that the results of Refs. 9 and 10 do not at all contradict the conclusion about the existence of Langmuir collapse. The collapse, i.e., the implosion of a Langmuir caviton, was observed also in those papers. However, due to the fact that they were close to threshold for the modulational instability the effective collision frequency which characterizes the speed of the dissipation of the energy of the Langmuir oscillations was small and the collapse remained a rather rare event and under those conditions a quasi-stationary lattice of cavitons was formed. One can thus conclude that the quasi-stationary caviton picture observed in Ref. 9 is a consequence of the peculiarities of the statement of the numerical experiment and does not reflect the real features of three-dimensional Langmuir turbulence. This problem is discussed in more detail in Ref. 10.

On the whole the numerical simulation of Langmuir collapse performed in the present paper has confirmed the assumption stated in Refs. 2 and 5 that the collapsing caviton can serve as the elementary cell of plasma turbulence. The caviton is formed as a result of the modulational instability of the pumping wave in the long-wavelength region of the source. Collapsing self-similarly it transfers the Langmuir energy trapped by it to the short-wavelength region where this energy is absorbed by resonance electrons.

Moreover, in a number of cases, e.g., when the fast transfer of resonant electrons from the turbulence region makes the deformation of their distribution function inappreciable, Landau damping is unable to halt the collapse, and plasmon-plasmon interactions (the intrinsic non-linearity of the Langmuir oscillations) play an important role in

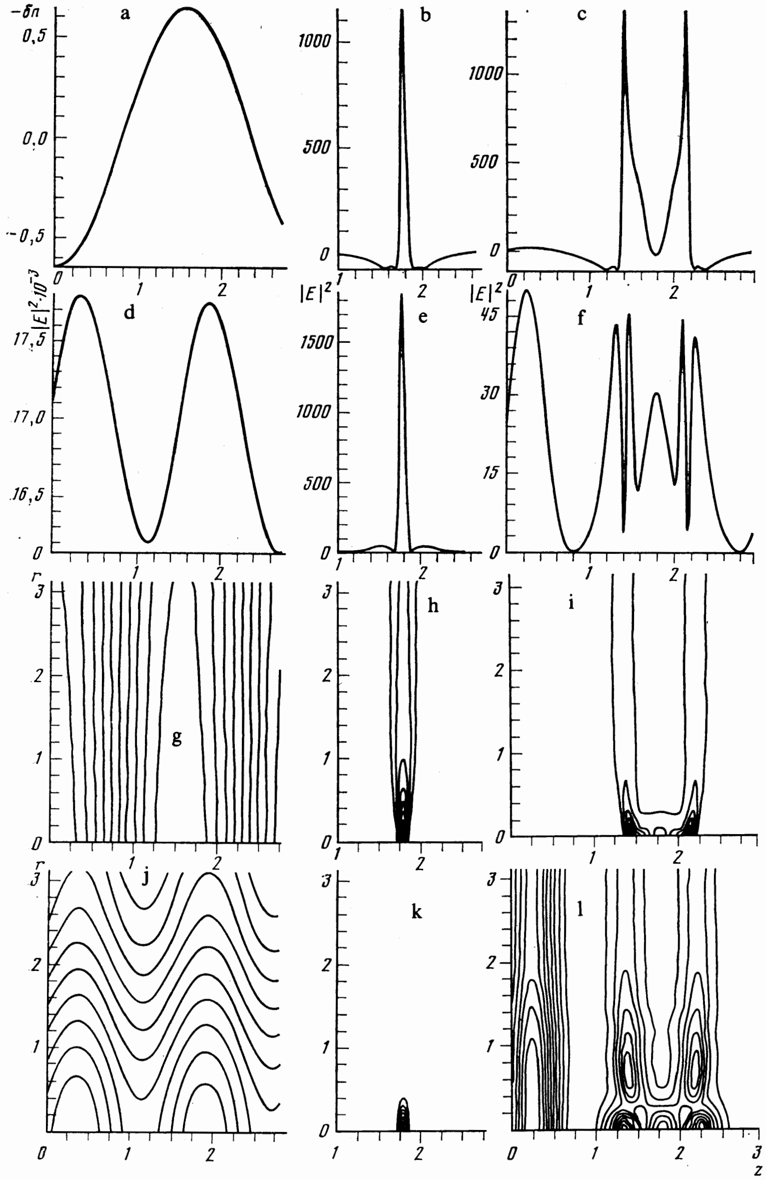

FIG. 5. Dynamics of Langmuir collapse occurring due to the action of a pump $E_0 = 2.0$ from an initially sinusoidal field and density modulation in the presence of damping for a two-temperature Maxwellian electron distribution function. a,b,c) $-\delta n$ as function of z for $r = 0$: a) $t = 0.01$, b) $t = 2.88$, c) $t = 3.28$; d,e,f) $|E|^2$ as function of z for $r = 0$ at the same times; g,h,i) lines of constant $|\delta n|$ at the same times; j,k,l) lines of constant $|E|^2$ at the same times.

the absorption of the oscillations in the short-wavelength region.

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