Nonlinear resonances on skipping electron trajectories near the surface of a metal

V. F. Gantmakher, G. I. Leviev, and M. R. Trunin

Institute of Solid State Physics, USSR Academy of Sciences
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A nonlinear effect is described that is due to the action of a magnetic field of frequency ω and amplitude $H_{\sim}$ on electrons moving in the skin layer along skipping trajectories. The power $P_{2\omega}$ of the reflected signal at the frequency 2ω is measured as a function of the external magnetic field. Resonances that are periodic in the reciprocal field arise in the signal when a certain threshold is reached. The resonances are observed when the amplitude of the alternating field is greater than the external field in which the resonance arises. The positions of the nonlinear resonances and the resonances on the magnetic surface levels do not coincide, but have equal anisotropy. The measurements are carried out with bismuth, indium, and tin specimens. A classical model of the effect is proposed in which the focusing of the electrons by a microwave field that decays exponentially in the interior of the metal is taken into account. The model describes the main experimental facts satisfactorily.

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1. INTRODUCTION

One of the main sources of nonlinear electromagnetic effects in metals is the change of the electron trajectories in the magnetic field of the wave. However, most of the phenomena of this type—the formation of current states and auto-oscillations, self-capture in helicon resonance,[2] appearnce of a current sheet traveling deep inside the skin layer[3]—have been observed at low frequencies $\omega \ll \Omega$, v_F/l ($\Omega = eH/mc$ is the cyclotron frequency, l is the free path length, v_F is the Fermi velocity). The magnetic field of the wave can be regarded here as quasistatic relative to motion of the electron, while the frequency ω governs only the spatial distribution of this field. Similar phenomena are possible in principle at high frequencies. Of importance here is not only the amplitude of the alternating magnetic field along the electron trajectory, but also its phase. The nonlinear cyclotron resonance observed in bismuth[4] is an example of such a high-frequency nonlinear effect.

The scale of the alternating fields in which the nonlinear effects arise is determined by the sensitivity of the linear response of the electron system to the presence of a constant magnetic field: nonlinear effects appear earliest where there is a strong dependence of the surface impedance on the external magnetic field. For example, the self-capture of helicon resonance[2] arises when the amplitude of the alternating field in the standing wave becomes of the order of the period of the quantum oscillations. For the nonlinear cyclotron resonance it is necessary that the alternating magnetic field in the skin layer be able to change the cyclotron frequency Ω significantly, taking the electron out of resonance or, on the other hand, bringing an initially nonresonant electron into resonance.[4] In bismuth, where this effect has been observed experimentally, the conditions are most favorable: the cyclotron resonances, due to the small effective mass m, arise in comparatively low magnetic fields and at the same time, these resonances are sufficiently narrow. In normal metals, the observation of the nonlinear cyclotron resonance will probably be more difficult.

In normal metals, an effect is known which leads to a strong non-monotonicity of the impedance just in weak magnetic fields.[5] This is resonance at the magnetic surface levels.[6] In the classical limit, these levels correspond to the so-called skipping electron trajectories. The periodic motion along them is the result of the multiple reflection of electrons from the surface, to which the magnetic field returns them. In the present work, we describe a nonlinear effect which acts because of the effect of the high-frequency magnetic field on skin-layer electrons located on trajectories of such a type.

The nonlinear resonance on the skipping trajectories has been observed in three metals: tin, indium, and bismuth. The experimental method and the results are set forth in Sec. 2 (for a preliminary communication on these experiments, see Ref. 7). The basic feature of the nonlinear resonances is that they are observed at amplitudes of the alternating field $H_{\sim}$ that are greater than the value of the constant field H. This fact is the basis of a short discussion of the quantum mechanical aspect of the problem in Sec. 3. In Sec. 4, we then set forth the classical model in detail, according to which the alternating field forms in the skin layer a preferred periodic trajectory, the frequency of motion along which depends basically on the weak constant field. Comparison with experiment, given in Sec. 5, shows that this model accurately describes the experimental facts and can serve as starting point for the treatment of the theory of this phenomenon.

2. EXPERIMENT

The experiment consists in the measurement of the nonlinear electromagnetic response of a metal as a function of the value of the weak external magnetic field H, applied parallel to its surface. The metal was placed in the field of an electromagnetic wave of high amplitude at a frequency $\omega/2\pi \approx 9.3$ GHz and the power $P_{2\omega}$ of the second harmonic 2ω was measured in the reflected signal.

The sample was located at the bottom of a bimodal cylindrical resonator, tuned to the frequencies ω and 2ω. At the

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frequency ω the mode E_{010} was excited in the resonator. The dependence of the amplitude of the magnetic field H_ω of this mode on the distance r to the axis of the resonator is described by a Bessel function of first order:

$$H_\omega(r)\approx 1.72 H_\sim J_1(2.405r/R), \quad H_\sim\approx 8.5\cdot 10^3 (P_\omega Q/\omega)^{1/2}, \qquad (1)$$

where $R = 12$ mm is the radius of the resonator, Q it is quality factor, and P_ω is the power fed to the resonator (in watts). According to (1), the field $H_\omega(r)$ is equal to zero on the axis of the resonator, reaches a maximum $H_\sim$ at a radius $r\approx 9$ mm and falls off to 0.9 H at the wall $r = R$. The samples had the shape of disks of diameter 17.8 mm so that the periphery of the sample was in a very strong alternating magnetic field. The maximum power which would be fed to the resonator from the magnetron, operating in the pulse regime, reached $P_\omega = 500$ W. At a quality factor $Q\approx 2000$, this corresponds to the amplitude $H_\sim = 35$ Oe. In what follows, we refer everywhere to the field $H_\sim$ on the periphery of the sample, calculated from Eq. (1).

The construction of the resonator and the measurement system have been described in detail in Ref. 4. The magnetic field was created by means of a set of Helmholtz coils. The earth's field was compensated for with an accuracy to within 1%. The experiments with bismuth were carried out at $T\approx 1.5$ K. Here the resonator was filled with superfluid helium. Because of the superconducting transition in tin and in indium, the experiments in them were carried out above the λ point. In order to avoid the noise produced by boiling helium, the resonator was filled with a heat exchange gas. Under these conditions, the sample was overheated relative to the helium bath by about 0.15 K, which could be measured from the superconducting transition of the sample. In what follows, the temperature is indicated with account of this correction.

The samples of all three metals were prepared by growth from the melt in dismountable quartz forms. Their surface was not subjected to additional treatment. All in all, the effect was observed on five samples of bismuth with the normal $\mathbf{n}\|C_3$, and on six samples of tin with $\mathbf{n}\|[001]$, $\mathbf{n}\|[100]$ and $\mathbf{n}\|[110]$. Of the three samples of indium investigated, the surface of which looked the same, narrow lines were observed only on the one with $\mathbf{n}\|[011]$. On the other two, they were weak and smeared out.

Figures 1 and 2 show examples of the experimental record of the signals $P_{2\omega}(H)$. The form of the resonance structure on these recordings depends on the value of the field $H_\sim$. At comparatively small amplitudes of $H_\sim$, there are generally no resonances. Here, however, we observed nonresonant generation of the second harmonic (the upper curve in Fig. 1) so that the absence of resonances was certainly not connected with the low sensitivity. The resonance appear at a threshold value of the field $H_\sim$, which depends on the resonant field $H_\sim$: the peaks at low fields appear at smaller amplitudes of $H_\sim$ (Fig. 3). Upon subsequent increase in $H_\sim$, the resonance in weak fields disappear, being masked by the background of the nonresonant signal.

In all three metals, the periodicity in the reciprocal field is tracked with excellent accuracy:

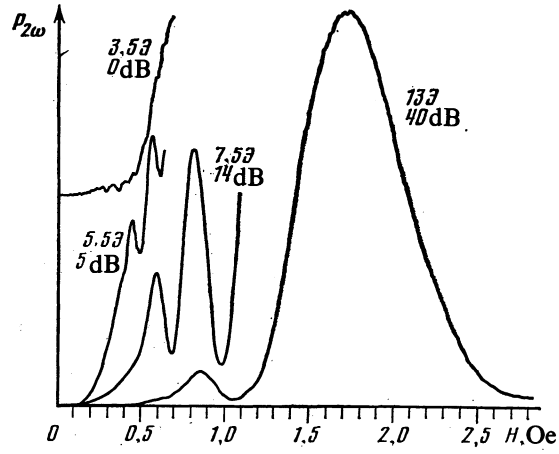

FIG. 1. Example of recordings of nonlinear resonances in bismuth ($\mathbf{n}\|C_3$, $\mathbf{H}\|C_1$, $T = 1.5$ K). Near each of the curves is shown the value of $H_\sim$ and the additional damping in the input attenuator of the detection system, in relative units. The curve with $H_\sim = 3.5$ Oe is displaced upward.

$$H_n = H_1/n, \quad n = 1, 2, 3, \ldots, \qquad (2)$$

while the value of H_1 depends weakly on $H_\sim$. In Fig. 4, this periodicity together with the $H_1(H_\sim)$ is shown for indium. A similar graph for tin ($\mathbf{n}\|[001]$, $\mathbf{H}\|[100]$) is shown in Ref. 7. Relation (2) determines the radical difference between the nonlinear resonances and the resonances in the impedance on the surface levels.

In all thre metals, the position of the lines H_n depended on the direction of the field H in the plane of the sample. As is seen from Fig. 5, the anisotropy of the nonlinear resonance in indium in the (011) plane corresponds exactly to the an-

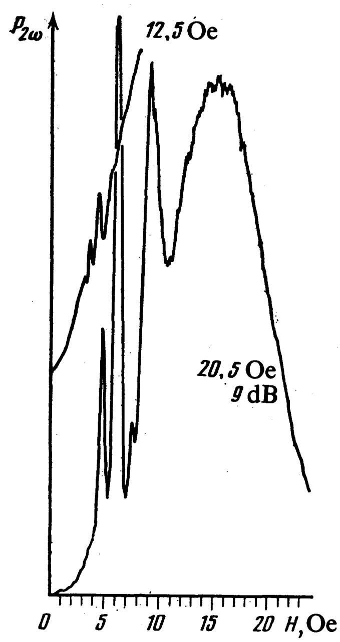

FIG. 2. Example of the recording of nonlinear resonances in indium ($\mathbf{n}\|[011]$. The field H in the plane of the sample makes an angle of 40° with the [100] axis, $T = 3.4$ K). The curve with $H_\sim = 12.5$ Oe is displaced upward.

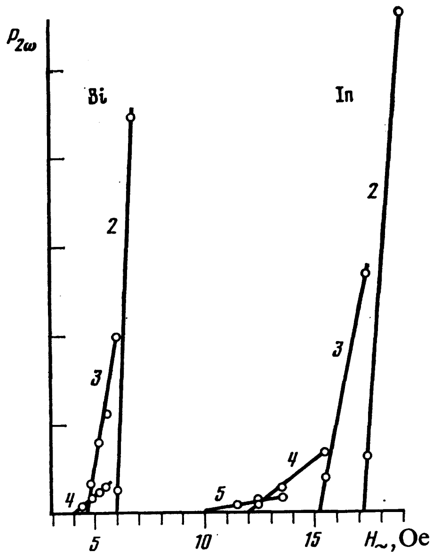

FIG. 3. Dependence of the amplitude of the resonance lines on $H_\sim$ for bismuth and indium. Near the straight lines is shown the number n of the corresponding resonance.

isotropies of the resonances at the surface levels,[8] although the location of the resonant peaks is different. The angular dependences in bismuth agree equally well: on the sample with $\mathbf{n}\|C_3$ the nonlinear resonances are shifted according to the law $H_n(\varphi) = H_n(0)/\cos\varphi$, where φ is the angle between $\mathbf{H}$ and the C_1 axis; near $\mathbf{H}\|C_2$ two systems of lines from two different ellipsoids are seen (cf. Ref. 5).

A detailed comparison of the location of the nonlinear and linear resonances in bismuth at $\mathbf{H}\|C_1$ is shown in Fig. 6. At the top is shown the spectrum of the linear resonances, while the height of the spikes reflects approximately the relative amplitude of the resonance lines at the same frequency, observed in Ref. 5, and also in Ref. 9, where this same spectrum is given for a frequency about 4 times higher. The numbers indicate the values of p and q entering in Eq. (3) below. In the middle of the graph, the positions of the lines H_n are shown for various amplitudes of $H_\sim$ and the periodicity of (2) in the reciprocal field is quite evident. Finally, the locations of the fundamental lines at the frequency 2ω, calculated according to Eq. (3), is shown at the bottom.

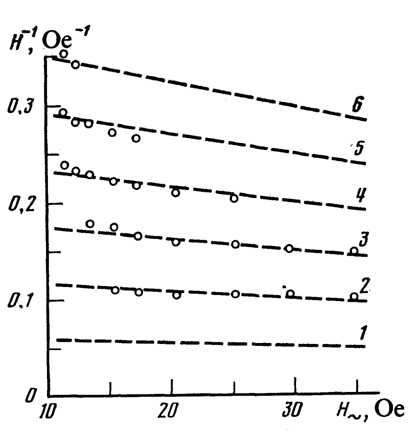

FIG. 4. Position of the resonance lines in indium ($\mathbf{n}\|[011]$, the field $\mathbf{H}$ makes an angle of 40° with the [100] direction) at various amplitudes of $H_\sim$. The dots indicate the approximations of the experimental points by Eq. (2) with H_1^{-1} linearly dependent on $H_\sim$.

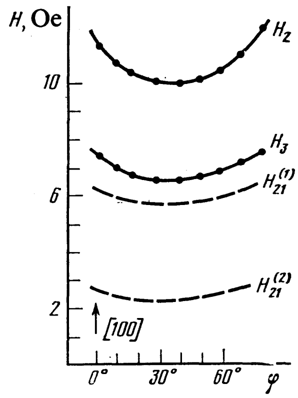

FIG. 5. Shift in the position of two nonresonances in indium upon rotation of the field $\mathbf{H}$ in the (011) plane of the sample. For comparison, the anisotropy of the linear resonances at the same frequency are shown for the two series observed in this plane (from Ref. 8).

All the experimental facts can be summed up in the following fashion:

a) the nonlinear resonances are periodic in the reciprocal field: $H_n = H_1/n$;

b) the fields H_n are not identical with the fields of the linear rsonances H_{pq}, but have the same anisotropy;

c) the resonances appear at a threshold value of $H_\sim$, while the value of the threshold $H_{th}^{(n)}$ is greater the smaller the value of n, i.e., the stronger the field H_n;

d) the inequality $H_{th}^{(n)} > H_n$;

e) in the region above threshold H_n increases slightly with increase in $H_\sim$.

3. QUANTUM-MECHANICAL APPROACH

The resonances at magnetic surface levels, observed in the impedance, are described on the basis of the solution of

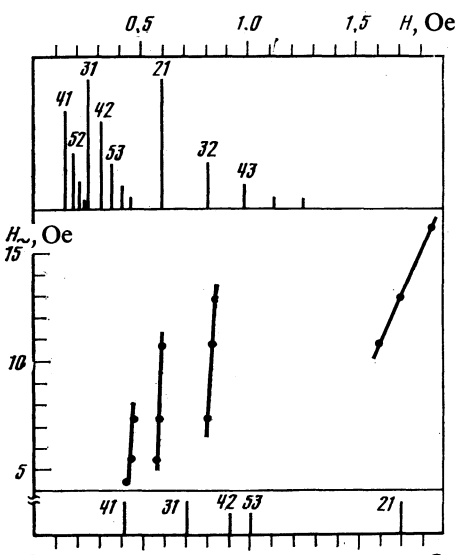

FIG. 6. Comparison of the positions of the linear and nonlinear resonances in bismuth ($\mathbf{n}\|C_3$, $\mathbf{H}\|C_1$).

the quantum-mechanical problem of the motion of an electron in a magnetic field near a specularly reflecting wall. All the hopping trajectories of the electrons with Fermi energy ε_F which do not have a component of the velocity along the magnetic field, i.e., which are located on an extremal cross section of the Fermi surface, form a single-parameter family. They can be classified, for example, by the reflection angle α or by the area S of the segment cut off by the surface from the Larmor circle. In correspondence with the solution of the Schrödinger equation, the only allowed trajectories are those for which the magnetic flux through the segment is equal to an integer number of magnetic flux quanta. This determines the discrete spectrum of states of the electron near the surface of the metal. The spectrum depends on the magnetic field $\varepsilon_p = \varepsilon_p(H)$. The weak electromagnetic field leads, in accord with perturbation theory, to resonance transitions among these states. These transitions are usually observed in measurements of the dependence of the surface impedance Z on the magnetic field H. A formula connecting the resonance frequency with the field,

$$\omega = \left(\frac{\Omega^2\varepsilon_F}{\hbar}\right)^{1/3}(a_p - a_q), \quad a_p = \left[\frac{3\pi}{2}\left(p - \frac{1}{4}\right)\right]^{2/3};$$

$$p, q = 1, 2, 3, \dots, \tag{3}$$

has been well confirmed for most metals.[5,8–11]

An increase in the amplitude of the electromagnetic field should lead to the appearnce, in the next order of perturbation theory, of a nonlinear response near the resonance lines (3). We did not succeed in observing this effect, probably because of inadequate sensitivity of the detection system to the frequency of the second harmonic. The experiments described in the present work were carried out at very high amplitude of the alternating field $H_\sim$, much higher than the characteristic values of the constant field. Under these conditions, the perturbation theory is known beforehand to be inapplicable.

Since we find ourselves in the resonance region of frequencies and fields, where resonant transitions are observed at small amplitudes, the natural condition that the width of the stationary levels be much less than the separation $\hbar\omega$ between them means that the lifetime on the level is much greater than the period of change of the field ω^{-1}. Therefore, the spectrum of the surface states cannot follow the instantaneous values of the alternating field. Moreover, the high-frequency field H_ω inside the segment of the classical trajectory is extremely nonuniform. All this means that under the influence of a strong high-frequency field, the spectrum of the near-surface states should be highly deformed. The experimental results also indicate such a radical deformation (see Figs. 5 and 6).

At the present time, we cannot propose a consistent quantum model for the description of the experimental results. Therefore, taking into account for the complexity of the corresponding quantum-mechanical problem, we have undertaken to propos an explanation of the observed nonlinear resonances in terms of classical trajectories.

4. CLASSICAL MODEL

Initial resonant trajectory. Soon after the discovery of oscillations of the impedance in a weak field, an experiment[5] was undertaken to explain its occurrence on the basis of consideration of classical parameters of the hopping trajectory. The electron, specularly reflecting from the surface at a small angle α carries out periodic motion in the magnetic field with frequency $(\pi/\alpha)\Omega$. Equating this frequency to the external frequency, we obtain the relation

$$n(\pi/\alpha)\Omega = \omega, \quad n = 1, 2, 3, \dots. \tag{4}$$

This equation alone, however, is insufficient for the determination of the resonance fields. It is necessary to apply an additional condition on the angle α. Therefore, we have assumed that, upon leaving the surface, the electron moves in phase with the wave:

$$v_F\alpha = \omega/k. \tag{5}$$

Because of the strong damping of the wave in the skin layer, the quantity k does not have a definite value. The relation $k = \delta^{-1}$ usually used for estimates (δ is the skin depth) determines α only in order of magnitude: $\alpha \approx \omega\delta/v_F \approx 10^{-2}$ and cannot be the cause of the resonance condition.

We shall show how we can get around this shortcoming of the model under conditions of a strong field $H_\sim$. The fact that generatiaon of the second harmonic is sensitive to the value of the constant magnetic field, in spite of the presence of the larger alternating field, indicates that responsible for the effect should be electrons whose interaction with the alternating magnetic field is minimal. There are such electrons. Those electrons which start out from the surface $x = 0$ at the time $t = 0$, when the node of the field H_ω is located on the surface, and then move into the interior along with the nodal plane $H_\omega = 0$, leave the skin layer without experiencing the influence of the alternating magnetic field.

We now consider an exponentially damped wave with a wave vector $k = k' + ik''$:

$$H_\omega = H_\sim \exp(-k''x)\sin(\omega t - k'x), \quad \mathbf{H}_\omega \| z. \tag{6}$$

In the normal skin effect, $k'' = k'$. In the anomalous skin effect, the field is damped not exponentially but the correct relation between the real and imaginary parts of the impedance $Z = R + iX$ is obtained if we set $k'' = 3^{1/2}k'$ in (6). The velocity of motion of the nodal plane $H_\omega = 0$ along the x axis in the wave (6) has a precise constant value $v = \omega/k'$, which does not depend on the value of k' nor on the relation between k' and k''. Thanks to the fact that we are interested not in "motion in phase with the wave," but "motion along with the nodal plane," the value of k in (5) turns out to be accurately determined, and not connected with the Fourier expansion of the function (6).

Thus, there exists a trajectory, moving along which the electrons are not subject to the action of the alternating magnetic field, at least so long as they do not leave the skin layer (we shall consider their return to the skin layer separately below). Requiring that the electrons on this trajectory, moving inside the metal in the constant field $\mathbf{H}\|z$ and returning to the surface after an integer number of periods n, we obtain the resonance condition from (4) and (5):

$$H_n=\frac{1}{\pi}\left(\frac{c\omega^2 p_F}{ev_F{}^2k'}\right)\frac{1}{n}, \quad n=1,2,3,\ldots \tag{7}$$

(p_F) is the radius of the orbit in momentum space).

Focusing by the exponentially decaying electromagnetic field. In order that the electrons found on the resonance trajectory (4), (5) make a significant contribution to the skin current, they must be of sufficient number. Therefore, it is an important fact that the alternating magnetic field in the skin layer can bring about focusing of the beam of the trajectories close to resonance, i.e., can transfer to the resonance trajectory electrons that boil off the surface at the time $t = 0$ at an angle close to but not equal to α. Actually, these electrons are acted upon by a force

$$F=\frac{e}{c}v_F\frac{\partial H}{\partial x}(x-x_0), \quad x_0=v_F\alpha t, \tag{8}$$

directed almost exactly along the x axis (since $|v_x|\ll|v_y|\approx v_F$ for the considered electrons). The sign of the force F depends on the signs of the derivative $\partial H/\partial x$ and the velocity v_y. the sign of v_y on the hopping trajectory has a definite value, which is closely connected with the direction of the field, while the sign of the derivative $\partial H/\partial x$ close to the nodal plane $H_\omega = 0$ is different in the two half periods. At one time in the period, the force F is directed against the deviation $u = x - x_0$ from the resonance trajectory. Then the equation of motion in the direction perpendicular to this trajectory has the form

$$\partial^2u/\partial\tau^2+b^2e^{-2\tau}u=0, \tag{9}$$

where

$$2\tau=k''x=\omega\left(\frac{k''}{k'}\right)t, \quad b^2=\frac{H_\sim}{H_b}, \quad H_b=\frac{\pi}{4}\left(\frac{k''}{k'}\right)^2 H_1. \tag{10}$$

Its general solution is written down in terms of the Bessel function of zero order:

$$u=C_1J_0(be^{-\tau})+C_2Y_0(be^{-\tau}). \tag{11}$$

We are interested in the transformation of the diverging beam into a parallel one. From the condition that $\partial u/\partial\tau = 0$ at $\tau = \infty$, we obtain the result that such a focusing takes place at values of the amplitude of the field

$$H_\sim=b_s{}^2H_b, \tag{12}$$

where H_b is defined in accord with (10) and (7), while b_s is the zero of the Bessel function $J_0(b)$, $S = 1,2,\ldots$. Of practical interest is the first term $b = 2.405$.

Under our experimental conditions, the focusing action of the field ceases somewhat earlier, at the depth $\bar{x}$, where the amplitude of the field H is comparable with the external field H:

$$H_\sim e^{-k''\bar{x}}=H. \tag{13}$$

The condition that a parallel beam be formed at this depth has the form

$$J_0(b)\,Y_1(b\lambda)-Y_0(b)J_1(b\lambda)=0, \quad \lambda=e^{-k''\bar{x}/2}=(H/H_\sim)^{1/2}<1. \tag{14}$$

The solution of this equation determines the function $b(\lambda)$, i.e., it specifies the connection between $H_\sim$ and H (see Fig. 7, which shows the solution (14) with minimum $H_\sim$ for the given H; the solution tends to b_1 as $H\to 0$).

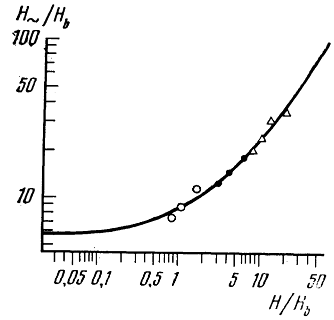

FIG. 7. Dependence of the field of focusing on the value of the constant field H, obtained from Eq. (14). For the procedure of comparison with the experimental data and the choice of the parameter H_b for all the metals, see Sec. 5 in the text (+ —indium, H_b = 0.5 Oe; ○—bismuth, H_b = 0.5 Oe; ●—tin, H_b = 1 Oe).

Return to the skin layer. In its return to the skin layer, the electron cannot escape the action of the strong field. However, inasmuch as the electron moves toward the wave, this field changes rapidly. The angle $\Delta\alpha$ through which this field turns the electron is proportional to $\int H_\omega dt$, taken along the trajectory. The coefficient of proportionality is determined from the fact that the field H_1 turns the electron by the angle α in the time π/ω. Taking the field distribution in the form (6), we obtain the result that the electron, coming to the surface at the instant of time $t = 0$, when the node of the wave H_ω is located there, is turned through an additional angle:

$$\frac{\Delta\alpha}{\alpha}=\varkappa\frac{H_\sim}{H_1}, \tag{15}$$

where $\varkappa = 2/5\pi$ at $k'' = k'$ and $\varkappa = 2/7\pi$ at $k'' = 3^{1/2}k'$.

The smallness of the angle $\Delta\alpha$ means that after reflection at an angle $\alpha + \Delta\alpha$ the electron will again be caught in a resonant trajectory because of the focusing effect.

Thus, we have constructed a periodic trajectory in the fields H and H_ω. Outside the skin layer, the electron moves along the arc of a circle with radius $R = v_F/\Omega$, which intersects the surface at the angle α—the portion a in Fig. 8. Then in the portion b, under the action of the actual field H_ω it returns to the surface with a steep slope, while in the portion c, as a result of focusing, it again goes over to a hopping trajectory that is somewhat displaced relative to the initial trajectory.

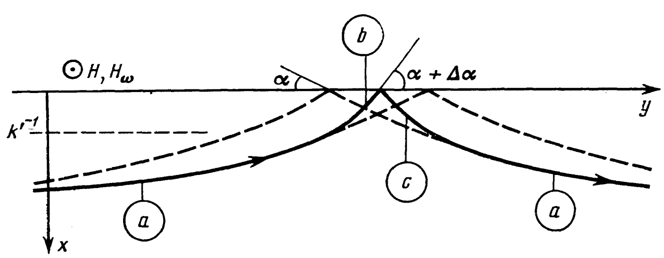

FIG. 8. Hopping trajectories without the alternating field H_ω (dotted) and in the presence of a field H_ω of sufficiently high amplitude (solid curve).

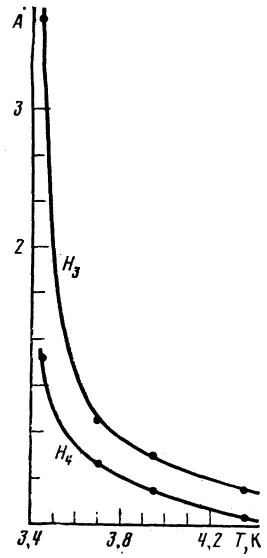

FIG. 9. Temperature dependence of the amplitude of the nonlinear resonances H_3 and H_1 in indium ($\mathbf{n}\|[011]$, the field $\mathbf{H}$ makes an angle of 40° with the [100] direction).

5. COMPARISON WITH EXPERIMENT

In the analysis of the experimental results, we must first give attention to the strong and poorly controllable dependence of the amplitude of the resonances on the quality of the surface of the samples. In indium, we could only select experimentally those samples on which the resonances were narrow and strong. In tin, there was a clear correlation: the more the surface appears to be optically reflectig and perfect, the stronger were the resonances. In samples with an etched surface, the effect was not observed under otherwise similar conditions. The effect also depended strongly on the free path length of the electrons in the interior of the sample. The strong dependence of the amplitude of the resonances on the temperature, an example of which is shown in Fig. 9, indicates this. The resonance peaks, observed in a sample of indium (Fig. 2), almost completely disappeared after this surface was very slightly bent. With the exception of the temperature dependence, all the observations mentioned are qualitative, since we had no independent method of control of the surface at our disposition. However, they correspond exactly to what is known about the conditions of observation of resonances on surface levels in measurements of the surface impedance.[8,11] Therefore, there is no doubt that the observed resonances are connected with electrons that are specularly reflected from the surface at small angles. An additional confirmation of this—the coincidence of the dependence on the angle of rotation of the field—can be seen, for example, in Fig. 5.

The basic experimental facts enumerated at the end of Sec. 2 well describe the classical model.

The periodicity of the resonances in the reciprocal field has frequently been mentioned above. According to the records shown in Fig. 2, the value of H_1 for indium was equal approximately to 20 Oe. The value $H_1 = 12$ Oe was given in Ref. 7 for tin. Both these values agree in order of magnitude with the calculated estimates of Eq. (7). For bismuth, for which $H_1 \approx 1.7$ Oe at $\mathbf{n}\|C_3$ and $\mathbf{H}\|C$, very accurate calculated estimates can be made. If we use the value of the parameter v_F/v_F^3 measured in Ref. 5, the value $v_F = 1.0\times10^8$ cm/sec,[12] and the value

$$k'=(4\pi\omega/c^2)\,(R/|Z|^2)=0.6\cdot10^4\,\mathrm{cm}^{-1},$$

obtained from measurements of the impedance,[13] we obtain the value $H_1 = 1.1$ Oe.

The threshold for the appearance of the lines, and also their subsequently disappearance upon increase in the amplitude of the wave $H_\sim$, are in excellent agreement with the model of focusing by an exponentially damped field. It must be kept in mind here that because of the nonuniformity of the field H_ω on the surface of the sample, it is impossible to expect an abrupt disappearance of the lines upon increase in the field amplitude in the resonator. $H_{th}^{(n)}/H_n$ for one of the lines of the resonance series, we can, with the help of the graph in Fig. 7, determine the value of the parameter H_b. We can then verify how the points corresponding to other lines of the series lie on the theoretical curve. In other words, the slope of the $H_\sim(H)$ curve drawn through the point $(H_{th}^{(n)}, H_n)$ is compared with the theoretical value, after which the parameter H is determined on one of these points. As is seen from Fig. 7, for tin and for indium, we have obtained ideal agreement of the slopes, but not for bismuth.

The values of the parameters H_b shown in Fig. 7, are much smaller than those which could be expected on the basis of Eq. (10). Of course, the approximation of the field distribution in the anomalous skin effect by means of Eq. (6) is very rough. However, this is probably not the only difficulty. Evidently, we are dealing here with the self-action of the electromagnetic wave: focusing leads to a concentration of the skin current in the vicinity of the moving plane $H_\omega = 0$, and this in turn means an increase in the derivative $\partial H/\partial x$ near it and increase in the focusing. A similar effect of concentration of the skin current near the nodal plane in the normal skin effect has recently been observed experimentally.[3]

It should be noted that there is one clearly observed dependence in the experiment which does not find its explanation within the frame work of the model discussed here: the shift in the resonance lines in a stronger field in the case of increase in $H_\sim$. However, we shall show that this shortcoming of the model is not fundamental and that as a whole, the suggested model describes the experimental facts satisfactorily.

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