Interference effects produced by a laser field in β decays and other processes not dependent on the presence of a field

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A compact expression for the total probability of a process perturbed by a laser field is investigated as a function of the field intensity and frequency, or of the parameters $\chi = F/F_0$ and $\xi = eF/\omega(2mI)^{1/2}$, where F is the field intensity, $F_0 = 2I(2mI)^{1/2}/e\hbar$ is the characteristic intensity determined by the decay energy I, m is the electron mass, and ω is the frequency of the field. In a weak field ($\chi \ll 1$), the probability can be represented by a perturbation theory series in e^2 or ξ^2 with coefficients that depend on (χ/ξ); when $\xi \gtrsim 1$ it is convenient to transform this series into a series in χ^2 with coefficients that depend on ξ^2. For $\xi > \xi_{-1} = 0.793$ (circularly polarized wave) the probability contains not only the terms of the perturbation series but also interference terms, which are nonanalytic at $\chi = 0$, oscillate rapidly with χ, and have amplitude $\sim\chi^4$ for β decays and $\sim\chi$ for $2\gamma \to e^+e^-$. For ξ near unity ($|\xi - 1| \lesssim \chi \ll 1$), the oscillations of an appreciable fraction of the interference terms cease. For $\xi \gg 1$, the probability contains both perturbation series terms as well as simplified interference terms, and it corresponds in the limit $\xi \to \infty$ to the process in a constant field. For the formation of the perturbation series terms de Broglie lengths are important, whereas for the interference terms barrier and larger lengths are important.

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1. INTRODUCTION

In Ref. 1 we considered the influence of a laser field on nuclear β decays on the basis of the differential distribution under the condition that the field intensity F satisfies $F \ll F_0 = 2I(2mI)^{1/2}/e\hbar$, where F_0 is the characteristic intensity determined by the decay energy I and the electron mass m, and that the field frequency satisfies $\omega \lesssim I/\hbar$. The decay probability, determined by the parameters

$$\chi = F/F_0, \quad \xi = eF/\omega(2mI)^{1/2}, \tag{1}$$

was represented by an asymptotic expansion in powers of χ^2 (which can be reproduced by perturbation theory) and small terms that are nonanalytic at $\chi = 0$ and have coefficients which depend on the frequency of ξ. The effective values of the quantum numbers of the final state were also determined, and a number of intermediate differential distributions was obtained.

Investigation by this method of the region in which $\xi \approx 1$ and effects of absorption from the wave and emission into the wave of a small number of photons are important proved to be difficult. However, if we are interested only in the total probability of the process, it can be obtained in accordance with the optical theorem as the imaginary part of the elastic scattering amplitude or directly from the square of the matrix element integrated over the quantum numbers of the final states before integration over the time. For the probability of the process one then obtains a comparatively simple expression in the form of an integral with respect to the time of the propagator of the light charged particle. This expression is used in the present paper to study interesting interference effects that are due to the field and take place at large distances from the region of formation of the process unperturbed by the field, at which it might appear that the field could no longer influence the total probability. The interference effects are described by terms nonanalytic at $\chi = 0$, and we shall see that for a number of processes they exceed the perturbation theory terms.

Restricting ourselves for simplicity to the case of a circularly polarized wave and a nonrelativistic situation, when the kinetic energy I of the final particles is much less than mc^2, we obtain the following expression for the total probability of the process in unit time (the system of units is such that $\hbar = c = 1$):

$$W_\lambda = c_\lambda \int_{-\infty}^{\infty} \frac{dv}{(v-i\varepsilon)^\lambda} \exp\left\{ i\beta\left[(1-\xi^2)v + \xi^2 \frac{\sin^2 v}{v} \right]\right\}, \tag{2}$$

$$v = \omega(t_2 - t_1)/2, \quad \beta = 2I/\omega = \xi/\chi, \quad \varepsilon \to +0.$$

Here, t_2 and t_1 are the times of emission and absorption of an electron in the amplitude of forward elastic scattering. The coefficient c_λ is proportional to the probability $W_{0,\lambda}$ of the process in the absence of the field, and its value can be readily recovered when the field is switched off, i.e., for $\xi = 0$:

$$c_\lambda = W_{0,\lambda} \frac{(-i\beta)^{1-\lambda}}{2\Gamma(1-\lambda)}. \tag{3}$$

The value of λ depends on the process. Thus, for the total probability of β decay (for example, $T \to He^3 + e^- + \bar{\nu}$) $\lambda = 9/2$; but if in the β decay the neutrino energy is fixed (I being understood as the previous value without the neutrino energy) or the process $\nu + T \to He^3 + e^-$ is considered, then $\lambda = 3/2$. The probability of production of a nonrelativistic e^+e^- pair by two γ photons in the presence of a circularly polarized wave is also given by the expression (2), in which it

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is necessary to replace the square of the charge e^2 by $2e^2$ and assume $\lambda = 3/2$. We do not take into account the Coulomb corrections.

2. SLOWLY VARYING FIELD OF ARBITRARY INTENSITY

For arbitrary χ and $\xi \gg 1$ we can expand $\sin^2 v$ in (2) in powers of v:

$$W_\lambda = c_\lambda \int_{-\infty}^{\infty} \frac{dv}{v^\lambda} \exp\left\{ i\beta\left(v - \frac{1}{3}\xi^2 v^3 \right)\right\} \left[1 + i\frac{2}{45}\beta\xi^2 v^5 + \ldots \right]. \tag{4}$$

The integrals here can be expressed in terms of the Airy function and its derivative. Indeed, using the representations

$$\Phi(y) = \frac{1}{2}\int_{-\infty}^{\infty} dz \exp\left\{ i\left(zy + \frac{1}{3}z^3 \right)\right\}, \tag{5}$$

$$2^{2/3}\Phi^2(-2^{-2/3}u) = \int_{-u}^{\infty} \frac{dz}{(z+u)^{1/2}}\Phi(z) = 2\int_0^{\infty} dt\,\Phi(t^2 - u), \tag{6}$$

we substitute (5) with $y = t^2 - u$ in the right-hand side of (6) and integrate over t, obtaining

$$\int_{-\infty}^{\infty} \frac{dz}{z^{1/2}} \exp\left\{ i\left(uz - \frac{1}{3}z^3 \right)\right\} = 4e^{i\pi/4} 2^{-1/3} \pi^{-1/2} \Phi^2(-2^{-2/3}u). \tag{7}$$

By differentiating with respect to u and integrating by parts, we readily obtain other integrals of this type, connected by the recursion relation

$$\int_{-\infty}^{\infty} dz z^{\nu-1} \exp\left\{ i\left(uz - \frac{1}{3}z^3 \right)\right\} = -\frac{i}{\nu}\int_{-\infty}^{\infty} dz (z^{\nu+2} - uz^\nu) \exp\left\{ i\left(uz - \frac{1}{3}z^3 \right)\right\}. \tag{8}$$

Using these relations in (4), we obtain for $\lambda = 3/2$

$$W_{3/2} = W_{0,3/2}\frac{1}{\pi}\left\{ \frac{y\Phi^2 + \Phi'^2}{y^{1/2}} + \frac{\chi}{15\xi^2} \times\left[\Phi\Phi' - \frac{2}{3}y\Phi'^2 + \frac{2}{3}y^2\Phi^2 \right]\right\}. \tag{9}$$

$$W_{3/2} = W_{0,3/2}\left\{ 1 + \frac{1}{8}\chi^2 + \frac{7\chi^4}{48\xi^2} + \ldots - \frac{1}{2}\chi\left[\cos\frac{2}{3\chi} - \frac{2}{45\xi^2\chi}\sin\frac{2}{3\chi} \right] + \ldots \right\}.$$

Similarly

$$W_{9/2} = W_{0,9/2}\frac{1}{\pi}\left\{ \frac{y\Phi^2 + \Phi'^2}{y^{1/2}} + \frac{21}{4}\chi^2 y^{1/2}\Phi^2 + \frac{15}{4}\chi^2\frac{\Phi'^2}{y^{1/2}} - \left(1 + \frac{7\chi^2}{12\xi^2} \right)\chi\Phi\Phi' \right\}, \tag{10}$$

$$W_{9/2} = W_{0,9/2}\left\{ 1 + \frac{35}{8}\chi^2 - \frac{7\chi^4}{48\xi^2} + \ldots + \frac{7\chi^3}{24\xi^2}\left[\cos\frac{2}{3\chi} - \frac{\chi}{12}\sin\frac{2}{3\chi} \right] + \ldots \right\}, \quad \chi \ll 1. \tag{10'}$$

In Eqs. (9) and (10),

$$\Phi = \Phi(-y), \quad \Phi' = \Phi'(-y), \quad y = (2\chi)^{-2/3}. \tag{11}$$

Note that replacing I in (9) by $I(1-\eta)$, where η is the neutrino energy in units of I, and multiplying this expression by $(1-\eta)^{1/2}\eta^2 d\eta$, we obtain up to a constant factor the differential distribution with respect to the neutrino energy for β decay. Integration of this expression with respect to η from 0 to ∞ again gives (10).

3. WEAK FIELD OF MODERATE OR HIGH FREQUENCY

If the field is weak, $\chi \ll 1$, and the frequency is not low, i.e., $\beta \lesssim 1$, then $\xi \equiv \chi\beta \ll 1$. This is the simplest case of perturbation theory. Expanding the exponential in (2) in a series in ξ^2 and writing $\sin^2 v$ in exponential form, we arrive at the integrals

$$\int_{-\infty}^{\infty} dt (t - i\varepsilon)^\nu e^{i\alpha t} = 2\Gamma(1+\nu)/(-i\alpha)^{1+\nu}, \quad \varepsilon \to +0. \tag{12}$$

By means of this expression, we obtain from (2) a representation of W_λ as a series in powers of ξ^2 with coefficients that depend on β:

$$W_\lambda = W_{0,\lambda}\{1 + \xi^2 k_{1,\lambda}(\beta) + \xi^4 k_{2,\lambda}(\beta) + \ldots\}, \tag{13}$$

$$k_{1,3/2}(\beta) = -{}^1/_2 - {}^1/_3\beta^2 + {}^1/_6\beta^{1/2}(\beta+2)^{3/2} + {}^1/_6\beta^{1/2}(\beta-2)^{3/2}, \tag{14}$$

$$k_{1,9/2}(\beta) = -{}^7/_2 - {}^1/_9\beta^2 + {}^1/_{18}\beta^{-5/2}(\beta+2)^{9/2} + {}^1/_{18}\beta^{-5/2}(\beta-2)^{9/2}. \tag{15}$$

If $\beta < 2$, the terms with $(\beta - 2)^\lambda$ in (14) and (15) must be omitted, since the contour with respect to t in (12) can in this case be readily closed below, where there are no singularities within the contour.

4. WEAK FIELD OF LOW FREQUENCY

More interesting is the case of a weak field of low frequency, $\chi \ll 1$, $\beta \gg 1$. In this case $\xi \gtrsim 1$, i.e., this parameter is not small. Nevertheless, the contribution to W in (13) made by perturbation theory, i.e., by the terms $\sim\xi^2, \ldots$, is retained, remains small, and takes the form of an expansion in powers of χ^2, since for $\beta \gg 1$ the terms $\xi^{2n} k_n(\beta)$ can be conveniently represented in the form $\chi^{2n}\beta^{2n}k_n(\beta)$ and the functions $\beta^{2n}k_n(\beta)$, which are finite at $\beta^{-1} = 0$, can be expanded in powers of the small parameter $\beta^{-2} = \chi^2/\xi^2$. In this case, the coefficients $c_n(\xi)$ of the resulting series in χ^2,

$$1 + \xi^2 k_1(\beta) + \xi^4 k_2(\beta) + \ldots = 1 + \chi^2 c_1(\xi) + \chi^4 c_2(\xi) + \ldots, \tag{16}$$

will be polynomials of degree $n - 1$ in ξ^{-2}.

The perturbation theory terms are formed at $v \sim \beta^{-1}$, i.e., times and lengths

$$t \sim I^{-1}, \quad l \sim (2mI)^{-1/2}, \tag{17}$$

which are characteristic for the formation of the process unperturbed by the field, are important for them.

When ξ is not small, not only the perturbation terms contribute to (2) but also the neighborhoods of the saddle points. These contributions are associated with the interference of electron waves traveling along different paths, and here it is the barrier distances, much greater than the characteristic de Broglie wavelength, that are important. Interference terms in decay probabilities were obtained for the first time in Ref. 2.

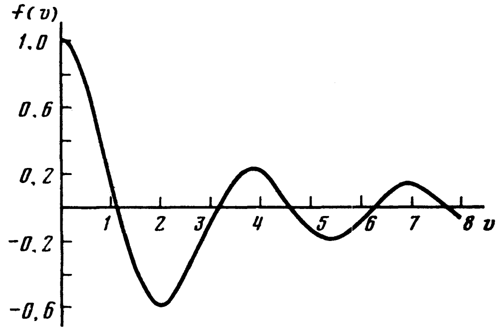

FIG. 1.

We consider the contribution of the saddle points to the integral (2), written in the form

$$\int_{-\infty}^{\infty} \frac{dv}{v^{\lambda}} \exp[iAg(v)], \qquad (18)$$

$$g(v)=(\xi^{-2}-1)v+v^{-1}\sin^2 v, \quad A=\beta\xi^2=\xi^3/\chi.$$

For the derivatives of $g(v)$, we have

$$g'(v)=\xi^{-2}-1+f(v), \quad f(v)=\frac{\sin v}{v}\left(2\cos v-\frac{\sin v}{v}\right), \qquad (19)$$

$$g''(v)=\frac{2}{v}\left[\left(\cos v-\frac{\sin v}{v}\right)^2-\sin^2 v\right]. \qquad (20)$$

The saddle points $v=v_c$ are roots of the equation $g'(v)=0$ or

$$f(v)=1-\xi^{-2}. \qquad (21)$$

A graph of the even function $f(v)$ for $v>0$ is shown in Fig. 1. At the points $v=0$, 2.04, 3.82,...this function attains an absolute maximum, an absolute minimum, a second maximum, etc., these being equal to 1, -0.586, $+0.229$, etc. The right-hand side of Eq. (21) attains the same values at $\xi_{+1}=\infty$, $\xi_{-1}=0.794$, $\xi_{+2}=1.139$, etc. Thus, for $1.139<\xi<\infty$ there is only one pair of real saddle points $\pm v_1(\xi)$, while for $\xi<\xi_{-1}=0.794$ there are none at all. It is readily seen that the complex saddle points make an exponentially small contribution to the probability, and we shall ignore them.

As ξ approaches unity, the number of saddle points increases and becomes infinite at $\xi=1$. It is now convenient to denote the increasing (but finite for $\xi\neq 1$) sequence of saddle points by $v_s(\xi)$, $s=1,2,3.\ldots$ In the limit $\xi=1$, this sequence becomes infinite, and

$$v_s(1)=s\frac{\pi}{2}\left(1-\frac{2}{\pi^2 s^2}-\frac{10}{3\pi^4 s^4}-\ldots\right), \quad s=1,3,5,\ldots, \qquad (22.1)$$

$$v_s(1)=s\pi/2, \quad s=2,4,6,\ldots. \qquad (22.2)$$

These are roots of the equations

$$\tan v=2v, \quad v\neq 0; \qquad (23.1)$$

$$\sin v=0, \quad v\neq 0, \qquad (23.2)$$

into which Eq. (21) decouples for $\xi=1$. It can be seen from (20) that $g''(v_s)<0$ at saddle points with odd s and $g''(v_s)>0$ at points with even s. In the simplest case, when the domains of influence of the saddle points do not overlap, for the total contribution to the integral (18) from the pair of saddle points $\pm v_s, v_s>0$, we obtain

$$\left(\frac{2\pi}{A|g_s''|}\right)^{1/2} 2e^{i\pi\lambda/2}v_s^{-\lambda}\cos\left(Ag_s\pm\frac{\pi}{4}-\frac{\pi\lambda}{2}\right),$$
$$g_s\equiv g(v_s), \quad g_s''\equiv g''(v_s)\gtrless 0. \qquad (24)$$

The upper and lower signs refer to $g_s''\lessgtr 0$, or, respectively, even and odd s. If the expression (24) vanishes, we can obtain deeper expansion terms.

By summation over v_s with allowance for the perturbation theory terms we obtain

$$W_{3/2}=W_{0,3/2}\left\{1+\frac{1}{8}\chi^2-\chi^4\left(\frac{105}{128}-\frac{7}{48\xi^2}\right)+\ldots\right.$$
$$+\frac{1}{2\xi^2}\chi\left[-\sum_{s=1,3,5,\ldots} v_s^{-3/2}\left(\frac{2}{|g_s''|}\right)^{1/2}\cos(Ag_s)\right.$$
$$\left.\left.+\sum_{s=2,4,\ldots} v_s^{-3/2}\left(\frac{2}{g_s''}\right)^{1/2}\sin(Ag_s)\right]+\ldots\right\}, \qquad (25)$$

$$W_{9/2}=W_{0,9/2}\left\{1+\frac{35}{8}\chi^2+\chi^4\left(\frac{35}{128}-\frac{7}{48\xi^2}\right)+\ldots\right.$$
$$+\frac{105}{16\xi^5}\chi^4\left[\sum_{s=1,3,5,\ldots} v_s^{-9/2}\left(\frac{2}{|g_s''|}\right)^{1/2}\sin(Ag_s)\right.$$
$$\left.\left.+\sum_{s=2,4,\ldots} v_s^{-9/2}\left(\frac{2}{g_s''}\right)^{1/2}\cos(Ag_s)\right]+\ldots\right\}. \qquad (26)$$

Note that for $\xi\sim 1$ (but not too near unity) we have $v_s\sim 1$, $g_s''\sim 1$. Therefore, the interference terms (24) in the integral (18) have the order $A^{-1/2}\sim X^{1/2}$ independently of λ. But in the probabilities (25) and (26) they are of order χ and χ^4, repectively, because of the strong dependence on β of the coefficient c_λ.

For $\xi\gg 1$, there is only the one pair of saddle points $\pm v_1$, and

$$v_1=\frac{1}{\xi}\left(1+\frac{1}{9\xi^2}+\ldots\right), \quad Ag(v_1)=\frac{2}{3\chi}\left(1+\frac{1}{15\xi^2}+\ldots\right)$$

$$g''(v_1)=-\frac{2}{\xi}\left(1-\frac{1}{3\xi^2}+\ldots\right). \qquad (27)$$

In this case, the probabilities (25) and (26) go over into the probabilities for a weak, slowly varying field; see (9′) and (10′). If the frequency corrections are ignored altogether, we obtain the probabilities in a weak constant field, which agree with the results of Refs. 3 and 2 for the decays $\pi^0\to e^+e^-$ and $\pi\to\pi^0 e\nu$ if the energy release in these decays is assumed to be small, i.e., we go to the nonrelativistic limit, and in the expression for $W_{3/2}$ the square of the charge e^2 is replaced by $2e^2$.

For $\xi\gtrsim 1$ and $\chi\ll 1$ the $v\sim\xi^{-1}$ are important for the formation of the interference terms, i.e., times and lengths

$$t\sim(2mI)^{1/2}/eF\sim I^{-1}\chi^{-1}, \quad l\sim I/eF\sim(2mI)^{-1/2}\chi^{-1}, \qquad (28)$$

which are of the order of the time of motion and distance to the barrier and exceed by χ^{-1} times the time and length of formation of the unperturbed process.

5. CEASING OF THE OSCILLATIONS OF THE INTERFERENCE TERMS FOR SELECTED VALUES OF THE PARAMETER ξ

In accordance with (21), the positions of the saddle points are determined solely by the parameter ξ. As a rule, the contributions from the saddle points give functions that oscillate rapidly with varying χ and ξ, since $2I/\omega = \xi/\chi \gg 1$. However, one can find values of ξ at which not only $g'(v_s) = 0$ but also $g(v_s) = 0$. For these selected values of ξ, the contributions from some saddle points do not oscillate with variation of χ. Let us consider how this happens.

We note first that at a saddle point

$$g(v_s)=2\sin v_s\left(\frac{\sin v_s}{v_s}-\cos v_s\right). \qquad (29)$$

Thus, $g(v_s) = 0$ if $v_s(\xi)$ is equal to a nonvanishing root w of one of the equations

$$\tan w=w, \qquad (30.1)$$

$$\sin w=0. \qquad (30.2)$$

At saddle points $v_s(\xi)$ satisfying (30.1) or (30.2),

$$g''(v_s)=-\frac{2v_s}{1+v_s^2}<0, \qquad (31.1)$$

$$g''(v_s)=\frac{2}{v_s}>0. \qquad (31.2)$$

To each saddle point $v_s(\xi)$ satisfying (30.1) there corresponds in accordance with (21)

$$\xi^2=\xi_s^2=1/\sin^2 v_s=1+\cot^2 v_s=1+1/v_s^2, \qquad (32)$$

i.e., it has a corresponding value ξ_s always greater than unity. The roots of Eq. (30.1) are determined by[4]

$$w=w_s=s\frac{\pi}{2}\left(1-\frac{4}{\pi^2 s^2}-\frac{32}{3\pi^4 s^4}-\dots\right),\quad s=3,5,7,\dots \qquad (33)$$

the smallest root $w_3 = 4.49$ corresponding in accordance with (32) to the largest $\xi_3^2 = 1.05$. The proximity of this value to unity means that with roots w_s of Eq. (33) there can coincide only the saddle points $v_s(\xi)$ having the same odd $s \geq 3$, since it is only these that for ξ near unity have values near w_s [cf. (22.1) with (33)].

Thus, for ξ^2 in the interval $1 < \xi^2 \leq 1.05$ there exists a sequence of ξ_k^2 values for which the first term of the sum with $s = k \geq 3$ in the probability $W_{3/2}$ ceases to oscillate as χ varies, becoming instead a constant and leading to a linear negative function of χ exceeding the perturbation theory corrections. In the probability $W_{9/2}$, the corresponding terms vanish, since they contain, not $\cos(Ag_s)$, but $\sin(Ag_s)$, and therefore in the order χ^4 there is no such effect.

The saddle points $v_s(\xi)$ can satisfy the second equation (30.2) only at $\xi = 1$, being equal to its roots with the same even s:

$$v_s(1)=w_s=s\pi/2,\quad s=2,4,\dots\ . \qquad (34)$$

Since $g''(v_s) = 4/\pi s > 0$ at these points, at $\xi = 1$ the entire second sum over the even s in the probability $W_{9/2}$ reduces to the number $\pi^{-4}\zeta(4) = 1/90$, leading to the additional term $(7/96)\chi^4$. The terms of the first sum decrease with increasing s as s^{-4}, remaining rapidly oscillating functions of χ. Both sums are formed, as before, at barrier distances by virtue of the good convergence of the series in s.

In the probability $W_{3/2}$ at $\xi = 1$ the second sum vanishes, and the terms of the first, decreasing as s^{-1}, oscillate with varying χ. However, $\xi = 1$ is a limit point of the sequence of values ξ_k, $k = 3, 5,\dots$, for which, as we have shown, the term with $s = k$ of the first sum becomes a constant. One can show that for large $k \sim \beta \gg 1$ the other terms with $s \sim k$ cease to oscillate, but allowance for them by the method of deepest descent becomes unsuitable because of the strong overlapping of the regions of influence of the distant saddle points. It is also difficult to sum these weakly decreasing terms. All this makes it necessary to use a different method to calculate the interference terms in $W_{3/2}$ at $\xi = 1$.

We represent the integral (18) in the form of the sum $J_0 + J_+ + J_-$ of three integrals over the intervals $-V<v<V$, $V<v<\infty$, and $-\infty<v<-V$, respectively, taking V large: $\chi^{-1/2} \ll V \ll \chi^{-1}$. Then the contribution J_0 to $W_{3/2}$ is given by the expression (25) with $v_s < V$, and for $R = J_+ + J_-$ we have at $\xi = 1$

$$R=\int_V^\infty \frac{dv}{v^{3/2}}\left[\exp\left(i\beta\frac{\sin^2 v}{v}\right)-i\exp\left(-i\beta\frac{\sin^2 v}{v}\right)\right]. \qquad (35)$$

We represent R as a sum of integrals over intervals of length $\pi/2$ and introduce in each of them the variable $x = 2v - n\pi$ instead of v:

$$R=(2i)^{1/2}\sum_{n=N}^{\infty}\int_{-\pi/2}^{\pi/2}\frac{dx}{(n\pi+x)^{3/2}}\sin\left(\beta\frac{1-(-1)^n\cos x}{n\pi+x}-\frac{\pi}{4}\right),$$
$$N=2V/\pi+1/2. \qquad (36)$$

Ignoring the corrections of order N^{-1}, we replace $n\pi + x$ by $n\pi$. Then

$$R\approx(2i)^{1/2}\pi\sum_{n=N}^{\infty}(n\pi)^{-3/2}$$
$$\times\left[J_0(z)\sin(z-\pi/4)+(-1)^n E_0(z)\cos(z-\pi/4)\right], \qquad (37)$$
$$z=\beta/n\pi,$$

where $J_0(z)$ and $E_0(z)$ are Bessel and Weber functions. Important in the sum are $n \sim \beta \gg 1$, i..e, $z \sim 1$. Therefore, the sum can be replaced by an integral, and the term with $E_0(z)$ can be ignored because of the rapid oscillation of $(-1)^n$. Then

$$R\approx(2i)^{1/2}\beta^{-1/2}\left[\int_0^\infty\frac{dz}{z^{1/2}}J_0(z)\sin\left(z-\frac{\pi}{4}\right)\right.$$
$$\left.-\int_{\beta/N\pi}^\infty\frac{dz}{z^{1/2}}J_0(z)\sin\left(z-\frac{\pi}{4}\right)\right]. \qquad (38)$$

In accordance with formulas 6.731, 1, 2 in Ref. 5

$$\int_0^\infty\frac{dz}{z^{1/2}}J_0(z)\sin\left(z-\frac{\pi}{4}\right)=-\frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}. \qquad (39)$$

Using the fact that

$$\beta^{1/2}\ll N=2V/\pi+1/2\ll\beta, \qquad (40)$$

we obtain for the correction integral in the square brackets of

$$\int_B^\infty\frac{dz}{z^{1/2}}J_0(z)\sin\left(z-\frac{\pi}{4}\right)\approx\frac{\sin 2B}{2B(2\pi)^{1/2}},\quad B=\frac{\beta}{N\pi}\gg 1. \qquad (41)$$

Thus, for $\xi = 1$ and $\chi \ll 1$ the probability $W_{3/2}$ is determined by the expression (25), in which the sum over the even s is equal to zero, the sum over the odd s contains contributions of the saddle points $v_s < V$, which oscillate with χ, while the contribution from $|v| > V$ leads to the appearance in the square brackets in (25) of a negative constant term and a small oscillating term:

$$-\frac{1}{2}-\frac{\sin 2B}{2\pi B}, \quad B=\frac{\beta}{N\pi}\gg 1. \tag{42}$$

It is easy to show that for N satisfying the condition (40) the probability does not depend on N, i.e., the small oscillating term in (42) is compensated by the contributions of the saddle points v_s near V.

For $\xi \approx 1$, the most important result is the appearance in $W_{3/2}$ in the curly brackets of (25) of a nonoscillating correction term, which is equal to $-\chi/4$, exceeds the perturbation theory corrections, and is formed for $v \sim \beta \gg 1$, i.e., at times

$$t \sim \beta\omega^{-1} \sim I^{-1}\chi^{-2}, \tag{43}$$

which are χ^{-1} times greater than the barrier values and exceed by χ^{-2} times the formation time of the unperturbed process.

6. CONCLUSIONS

Thus, interference effects appear in the total probability of the process with a neutral particle in the final state ($\lambda = 9/2$) only in the terms $\sim \chi^4$, whereas they already appear in the terms $\sim \chi$ in the probability of the process in which the electron energy in the absence of the field is fixed ($\lambda = 3/2$). Naturally, when there is no integration over the energy of the neutral particle the interference and threshold effects are manifested more clearly. The latter are particularly strong for $|\xi - 1| \lesssim \omega/I \ll 1$, when the threshold number $I\omega^{-1}(\xi^2 - 1)$ of photons absorbed from the field becomes of order ± 1 and in the calculation of the corrections it is not only the absorption of a large number of photons that is important but also the absorption and emission into the wave of a small number of them. In the latter case, almost the entire energy of the process is used to form the effective electron mass. Leaving the region of formation of the unperturbed process, the electron oscillates in the field of the wave, returning repeatedly to the point of emission and interfering with an electron wave that leaves this region later.

A high degree of proximity of ξ to unity requires for fixed field F rather precise specification of the frequency:

$$|\Delta\xi| = \xi|\Delta\omega|/\omega \lesssim \omega/I.$$

For $\xi \approx 1$, this means that $|\Delta\omega| \lesssim \omega^2/I$, i.e., the effect requires for its observation times $t \gtrsim (\Delta\omega)^{-1} \gtrsim I\omega^{-2} \sim I^{-1}\chi^{-2}$ [cf. (43)].

Summarizing, we can say that the appearance in the total probability of interference corrections that oscillate with χ and are formed at barrier distances much greater than the de Broglie wavelength was expected; see Ref. 2. It is interesting that these terms can be more important than the corrections made by perturbation theory. For a constant field, an oscillating term of such type was obtained in Ref. 3. It is very interesting that some of the oscillating terms cease to oscillate with χ for special values of ξ near unity. It is also instructive that the corrections to the probability in a weak field calculated perturbatively are formed at distances of the order of the de Broglie wavelength of the electron, whereas much larger scales, determined by the magnitude of the field, are manifested in the formation of the interference corrections, which cannot be reproduced by perturbation theory. All these corrections are small to the extent that the field is small, i.e., they are determined by some power of the parameter χ.

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