Stochastic self-oscillations of carrier density in a semiconductor with impurities

V. L. Bonch-Bruevich and Le Vu Ki

Moscow State University

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The kinetics of the capture of hot carriers in a semiconductor with shallow impurities and with recombination traps of two types is considered. The electron heating is due to flow of direct current. In the equation for the field intensity in the sample, however, the displacement current is also taken into account. The conditions under which stochastic self-oscillations of the carrier density and of the field intensity set in (with the static differential conductivity remaining positive) are indicated.

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§ 1. INTRODUCTION, BASIC EQUATIONS

The problem of carrier behavior in a semiconductor containing shallow donors and two types of impurity with deeper levels, in the presence of a heating electric field, was considered long ago.[1] It was shown that self-oscillation of the density of the free carriers (and hence of the current density) sets in under certain conditions, owing to the periodically changing rates of the impurity-to-band transition. No account was taken, however, of the displacement current, i.e., an algebraic connection between the field intensity and the current density was assumed. We consider here the same problem with the displacement current taken into account. The phase space of the system becomes thus three-dimensional, so that the ensuing possibilities are greatly increased.[2–4]

We introduce the following notation: N_i is the density of the impurity of the i-th type ($i = 1,2$), $N = N_1 + N_2$, $N_i = Ny_i$, $n = Nx$ is the conduction-electron density, $N_i^- = Nx_i$ is the density of the type-i impurity levels occupied by electrons, $N_d = Ny$ is the density of the shallow donors (with allowance for the possible compensation), $c_i = s_i N^{-1}$ is the coefficient of electron capture on a level of type i in the absence of electron heating, and $n_i = N\nu_i$ are quantities known in the statistics of electrons in semiconductors, with dimensionality of density, and depend on the position of the i-th level (the product $c_i n_i$ describes the rate of the thermal and optical electronic transitions from impurity atoms of the i-th type into the band). The capture coefficients under conditions of electron heating will be designated $c_i f_i$; in a homogeneous sample the functions f_i can be regarded as dependent on the electric field strength $E = \mathscr{E} L^{-1} x_3$ ($\mathscr{E}$ is the emf of the dc battery, and L is the sample length along the current). The recombination-kinetics equations for the dimensionless variables x_1 and x_2 take then the same form as in Ref. 1:

$$x_i = s_i\{x f_i(x_3)(y_i - x_i) - \nu_i x_i\} \equiv \mathscr{P}_i s_i. \tag{1}$$

The equation for the field intensity in the one-dimensional problem considered by us is easily obtained from Maxwell's equations. It is necessary only to take it into consideration that the sample itself can have a capacitance C. It follows from an examination of the equivalent circuit in Fig. 1 that

$$EL + \left(\sigma E + \frac{\varepsilon^*}{4\pi}\dot{E}\right) SR = \mathscr{E}. \tag{2}$$

Where $\sigma = en\mu$ is the electric conductivity of the sample (n, and possibly μ, depends on E), $\varepsilon^* = \varepsilon + 4\pi CL/S$, S is the sample cross section, and R is the load resistance. Putting

$$4\pi L/\varepsilon^* SR = \beta, \quad e\mu NSRL^{-1} = \alpha, \tag{3}$$

we obtain for the dimensionless variable x_3

$$x_3 = \beta[1 - (1 + \alpha x) x_3] \equiv \beta \mathscr{P}_3. \tag{2'}$$

Neglect of the displacement current corresponds to taking the limit as $s\tau_M(x) \to \infty$, where s is the larger of the quantities s_1 and s_2, and τ_M is the effective Maxwellian relaxation time, given by

$$\tau_M^{-1}(x) = (4\pi/\varepsilon^*)[\sigma(x) + L/SR]. \tag{4}$$

The value of x that must be substituted here depends on the formulation of the problem (see § 5 below).

Equations (1) and (2) must be supplemented by the local neutrality condition. The latter take in dimensionless variables the form

$$x_1 + x_2 + x = y. \tag{5}$$

Next, by definition,

$$y_1 + y_2 = 1, \; 0 \leqslant x_i \leqslant y_i, \; x_3 \geqslant 0, \; x \geqslant 0. \tag{6}$$

§ 2. SINGULAR POINTS

As can be seen from Eqs. (1), at any value of x_3 the integral curves on the boundaries of the triangle OAB (see

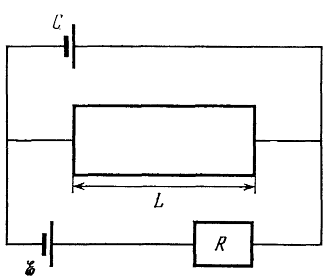

FIG. 1. Equivalent circuit corresponding to Eq. (2).

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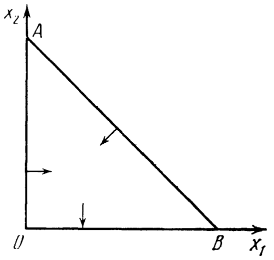

FIG. 2. Concerning the instability of infinity. The line AB is determined by the equation $x^1 + x^2 = y$. The arrows indicate the directions of the integral curves (the lower arrow should be directed inward).

Fig. 2) are directed into the triangle. Next, as $x_3 \to \infty$ we have $\dot{x}_3 < 0$. Thus, the infinity is absolutely unstable in the problem considered.

The singular points in the finite region are defined by the equations

$$x_i = xy_i f_i (f_i x + \nu_i)^{-1}, \quad i=1,\ 2, \tag{7}$$

$$x_3 = (1+\alpha x)^{-1}, \tag{8}$$

$$F(x) = x\left\{1+\sum_{i=1,2} y_i f_i (f_i x+\nu_i)^{-1}\right\} = y. \tag{9}$$

Obviously, $F(0) = 0$, $F \to \infty$ as $x \to \infty$.

We shall be interested in the case when there is only one singular point. To this end it suffices to have the function $F(x)$ monotonic: $F'(x) > 0$. It can be directly verified that this is the case if (the meaning of the a_3 will be made clear in § 3 below)

$$\begin{aligned} a_3 = \beta s_1 s_2 \{ & y_1 \nu_1 f_1 (xf_1+\nu_1)^{-1}(xf_2+\nu_2)(1+\alpha x-\alpha x_3 \varphi_1) \\ & + y_2 \nu_2 f_2 (xf_2+\nu_2)^{-1}(xf_1+\nu_1)(1+\alpha x - \alpha x_3 \varphi_2) \\ & + (x_1 f_1+\nu_1)(x_2 f_2+\nu_2)(1+\alpha x)\} > 0. \end{aligned} \tag{10}$$

Here

$$\varphi_i = x\frac{d \ln f_i(x_3)}{dx_3}. \tag{11}$$

Obviously the condition that a_3 be positive does not contain the parameter β.

We note two limiting cases in which the solution of (9) is easily obtained.

a). High generation rate: let

$$x \ll \nu_i f_1^{-1}. \tag{12a}$$

Then

$$x \approx y\left(1+\sum_{i=1,2} y_i f_i \nu_i^{-1}\right)^{-1}. \tag{13a}$$

The conditions for the applicability of this expression are clear from its comparison with (12a).

b). Low generation rate: let

$$x \gg \nu_i f_i^{-1}. \tag{12b}$$

Then

$$x \approx y-1. \tag{13b}$$

This expression is valid at a sufficiently high donor density: $y - 1 \gg \nu_i f_i^{-1}$.

§ 3. STABILITY OF SINGULAR POINTS

We denote the solutions of the system (7)–(9) by x_s, $x_{i,s}$ ($i = 1, 2, 3$). We put

$$x = x_s + \delta x(t), \quad x_i = x_{i,s} + \delta x_{i,s}(t)$$

and note that according to (5) $\delta x = -(\delta x_1 + \delta x_2)$. We assume $\delta x_i = C_i \exp(\lambda t)$ and carry out in (1) and (2) the standard linearization with respect to the δx_i. After elementary but somewhat laborious algebra we obtain a secular equation for the determination of the numbers λ:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0. \tag{14}$$

Here a_3 is given by Eq. (10), in which it is necessary to replace x and x_3 by x_s and $x_{3,s}$,

$$a_1 = \sum_{i=1,2} s_i[\nu_i y_i f_i (x_s f_i+\nu_i)^{-1} + x_s f_i + \nu_i] + \beta(1+\alpha x_s), \tag{15}$$

$$\begin{aligned} a_2 = {} & s_1 s_2 \{ y_1 \nu_1 f_1 (x_s f_1+\nu_1)^{-1}(x_s f_2+\nu_2) + y_2 \nu_2 f_2 (x_s f_2+\nu_2)^{-1} \\ & \times (x_s f_1+\nu_1) + (x_s f_1+\nu_1)(x_s f_2+\nu_2)\} + \beta\{s_1 y_1 \nu_1 f_1 (x_s f_1+\nu_1)^{-1} \\ & \times (1+\alpha x_s - \alpha x_{3,s}\varphi_1) + s_2 y_2 \nu_2 f_2 (x_s f_2+\nu_2)^{-1}(1+\alpha x_s - \alpha x_{3,s}\varphi_2) \\ & + (1+\alpha x)[s_2(x_s f_2+\nu_2) + s_1(x_1 f_1+\nu_1)]\}. \end{aligned} \tag{16}$$

Obviously, $a_1 > 0$ and the coefficients a_2 and a_3 can become negative if at least one of the quantities φ_1 or φ_2 is positive and is large enough. As can be seen from the definition (11), this corresponds to capture of carriers by a repelling impurity. Such a situation was investigated in its time in connection with the domain-instability phenomenon[5]; here, however, we are interested in a case in which this instability does not occur.

A singular point is stable if all three roots of (14) satisfy the condition Re $\lambda < 0$. According to the known Hurwitz criterion, the necessary and sufficient conditions for this are the inequalities

$$a_1 > 0, \quad a_1 a_2 - a_3 > 0, \quad a_3 > 0. \tag{17}$$

The first of them is always satisfied, the last can also be violated, but we are interested here in the case $a_3 > 0$. The second inequality of (17) can be reduced to the form (we omit for brevity the subscript s of x):

$$\begin{aligned} & \sum_{i=1,2} s_i[\nu_i y_i f_i (xf_i+\nu_i)^{-1} + xf_i + \nu_i] \\ & \quad \times \Big\{ s_1 s_2 [y_1 \nu_1 f_1 (xf_1+\nu_1)^{-1}(xf_2+\nu_2) \\ & + y_2 \nu_2 f_2 (xf_2+\nu_2)^{-1}(xf_1+\nu_1) + (xf_1+\nu_1)(xf_2+\nu_2)] + \beta(1+\alpha x) \\ & \quad \times \Big[\sum_{i=1,2} s_i (y_i \nu_i f_i (xf_i+\nu_i)^{-1} + xf_i + \nu_i) + \beta(1+\alpha x)\Big]\Big\} \\ & \quad > \alpha\beta(1+\alpha x)^{-1} \\ & \times \Big\{ \sum_{i=1,2} \varphi_i s_i \nu_i y_i f_i (x_i f_i+\nu_i)^{-1} \Big[\sum_{i=1,2} s_i (\nu_i y_i f_i (xf_i+\nu_i)^{-1} + xf_i + \nu_i) \end{aligned}$$

$$+\beta(1+\alpha x)\Big]+s_1s_2\nu_1\nu_2y_1y_2f_1f_2(xf_1+\nu_1)^{-1}(xf_2+\nu_2)^{-1}(\varphi_1+\varphi_2)\Big\}. \quad (18)$$

In contrast to (10), the inequality sign in (18) depends on the value of β (i.e., on the load resistance). Consequently the condition (10) can in principle be made compatible with an inequality inverse to (18). Using again the Hurwitz criterion, we easily verify that the singular point considered is in this case a saddle-node or a saddle-focus; the corresponding surface of the unstable motions is two-dimensional.

§ 4. EXPLICIT FORM OF THE INSTABILITY CONDITION

The instability condition obtained by reversing the sign in (18) is too cumbersome in general. We simplify it by noting that contributing to the onset of the instability are the values $\beta \gg s_1, s_2$. We assume also for simplicity that the coefficient of capture by level 2 depends relatively little on the field intensity (i.e., in final analysis, on the electron energy), leaving only the terms with φ_1. The condition for the instability of the singular point takes then the form

$$\sum_{i=1,2} s_i[\nu_iy_if_i(xf_i+\nu_i)^{-1}+xf_i+\nu_i] < \alpha x(1+\alpha x)^{-2}s_1y_1\nu_1f_i(xf_1+\nu_1)^{-1}\frac{d\ln f_1}{dx_3}\bigg|_{x_3=(1+\alpha x)^{-1}} \quad (19)$$

(here $f_2 = 1$)

When particularizing the inequality (19) it must be remembered that we assume the field and charge-density distributions to be spatially homogeneous. This can be ensured by considering the conditions under which the sample differential conductivity σ_d is positive. This is certainly the situation in the case (13b) If the field dependence of the carrier mobility is neglected (as is justified in fields that are not too strong). The inequality (19) takes then the form

$$(y-1)(f_1+s_2s_1^{-1})+(y_1\nu_1+s_2s_1^{-1}y_2\nu_2)(y-1)^{-1} < \alpha y_1\nu_1[1+\alpha(y-1)]^{-2}\frac{d\ln f_1}{dx_3}\bigg|_{x_3=[1+\alpha(y-1)]^{-1}}. \quad (19')$$

On the other hand, under the same assumptions the conditions $a_3 > 0$ yields

$$\alpha\nu_1y_1[1+\alpha(y-1)]^{-2}\frac{d\ln f_1}{dx_3}<(y-1)f_1 +[\nu_1y_1+\nu_2y_2f_1](y-1)^{-1}. \quad (20)$$

The inequalities (19') and (20) are compatible if

$$s_2s_1^{-1}f_1^{-1}<\nu_2y_2[(y-1)^2+\nu_2y_2]^{-1}, \quad (21)$$

i.e., the cross section for capture by "passive" traps (type 2) should not be too large. We note that inequalities (20) and (21) become stronger with increasing f_1.

We assume for the sake of argument

$$f_1=1+{}^1/{}_2\gamma x_3^2. \quad (22)$$

Such an expression is naturally obtained, e.g., for sufficiently weak fields, when $1/2\gamma x_3^2 \ll 1$. In particular, under conditions when the generalized Davydov distribution[6] is valid and the energy and momentum are respectively scattered by longitudinal acoustic phonons and by a charged impurity, we have[7]

$$\gamma=2L^2\left(0{,}72+\frac{1}{3}\ln W_BT^{-1}\right)\mathscr{E}^{-2}E_0^2. \quad (23)$$

Here W_B is the Bohr energy in the crystal, T is the lattice temperature, $E_0^2 = 16s^2/3\pi\mu_L{}^2$, s is the speed of sound, and μ_L is the mobility due only to scattering by longitudinal acoustic phonons.

In the case (22), the inequalities (19') and (20) take the form

$$(y-1)(1+s_2s_1^{-1})+(y-1)^{-1}[y_1\nu_1+y_2\nu_2s_2s_1^{-1}] <\gamma\alpha y_1\nu_1[1+\alpha(y-1)]^{-3}<y-1+(y-1)^{-1}(y_1\nu_1+y_2\nu_2), \quad (24)$$

and the condition for their compatibility is obtained from (21) by setting f_1 equal to unity. The left-hand inequality in (24) may turn out to be quite stringent. Actually the function $\alpha[1 + \alpha(y - 1)]^{-3}$ does not exceed $4/27\,(y - 1)$ and we have assumed that $\nu_1 \ll (y - 1)f_1$. We note, however, that expression (22) is valid only under conditions close enough to equilibrium, i.e., this expression is the least advantageous in the sense of realizing the instability condition. In stronger fields the dependence of the capture coefficient on the field intensity becomes appreciably stronger (exponential) and the inequality (21) becomes much less stringent. An analysis of the ensuing conditions calls, however, for specifying many factors and is apparently meaningful only as applied to specific material. What matters to us is only that the situation considered is not as exotic as might appear at first glance.

§ 5. PHYSICAL CONSEQUENCES

Under conditions when the only singular point is a saddle-node or a saddle-focus, and infinity is absolutely unstable, only one possibility can be realized, namely an integral curve that emerges at some initial instant of time from a singular point (or its vicinity) should return to the same place. In a two-dimensional case such a situation is impossible; in systems with phase space of more dimensions, however, it is admissible. We see that allowance for the displacement current turns out to be fundamental in the considered problem, for otherwise we would obtain a dynamic system of second order rather than third. According to the statement made in § 1, the parameter that determines the role of the displacement current is $[s\tau_M(x)]^{-1}$, where x is the characteristic value of the dimensionless carrier density. In this case, obviously, it is given by Eq. (13b). Thus, the stochastic instability considered above should vanish if

$$\frac{4\pi}{\varepsilon^*s}\left\{e\mu(N_d-N)+\frac{L}{SR}\right\}\to 0. \quad (25)$$

It was assumed above (§ 4) that $\beta \gg s$, i.e., $4\pi L(\varepsilon^*SRs)^{-1} \gg 1$. Assuming as an estimate $L = 1$ cm, $S = 1$ cm^2, $\varepsilon^* = 20$, and $s = 10^7$ sec^{-1} (the last value may be an overestimate), we see that the indicated strong equality is satisfied at $R < 10^2\ \Omega$. On the other hand, under the conditions indicated the left-hand side of (25) can become small if $R \gg 10^2\ \Omega$, and $N_d - N \gg \varepsilon^*/4\pi e\mu s \sim 10^{10}$ cm^{-3} (at $\mu = 10^4$ cm^2/V·sec).

To avoid misunderstandings we note that the role of the displacement current turns out to be so fundamental only in a system with two types of impurity. If there are more than two different types of capture the allowance for the displacement current is in principle not obligatory (although it may remain important for quantitative reasons).

So long as the system has a solitary singular point of the saddle-node or saddle-focus type, either periodic or stochastic periodic oscillations must occur in the system. Since in fact there is always an "extraneous" noise not accounted for in the dynamic system (1), (2), the later possibility seems much more probable. It corresponds to open trajectories that return to the region of the singular point. This should manifest itself in observations as additional current noise.

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