Time evolution of the particle distribution function in a high-temperature plasma

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The time evolution of the single-particle distribution function of the particles in a nonrelativistic plasma in the absence of an external field is studied. A linear differential equation that describes the high-energy part of the distribution function is derived. Approximate analytic solution of this equation yields the thermalization (Maxwellization) time of particles in the region of energies $\varepsilon \gg kT: t_\varepsilon \approx 0.64(\varepsilon/kT)^{3/2} t_0$, where t_0 is the relaxation time in the region of the characteristic mean energies ($\varepsilon \sim kT$). The significance of the results is discussed for the example of the γ luminosity of the accretion plasma around a black hole.

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1. INTRODUCTION

In problems associated with the study of the various physical processes in a nondegenerate hot plasma, great importance attaches to the rate at which thermal equilibrium is established for the different components of the plasma. The characteristic thermalization times of the electron, t_{ee}, and ion, t_{zz}, components, and also the time of equalization of the temperatures between these components, t_{ez}, have been investigated in detail (see, for example, the reviews[1–3]). However, much still remains unclear about the nature of and the time required for the establishment of thermal equilibrium of the high-energy (relative to the mean energies) tail of the Maxwell distribution, though it is obvious that the establishment of this equilibrium is a slower process than the thermalization in the region of the characteristic mean energies.[4] Moreover, in many dynamical problems it is precisely the time evolution of the high-energy tail of the distribution function that plays the most important part.

In the present paper, we investigate this problem for nonrelativistic systems.

2. KINETIC EQUATION FOR HOMOGENEOUS SYSTEMS IN THE APPROXIMATION OF WEAK COUPLING

It is well known that in a nondegenerate fully ionized plasma the kinetic stage in the evolution of the system can be divided into three time intervals with different scales, in which there is successively establishment of thermal equilibrium of the electron component, the ion component, and then between these two components of the plasma. This circumstance makes it possible to reduce the investigation of the behavior of the complete ensemble to independent study of the evolution of each of the components.

We consider a nonrelativistic system of N identical particles described by the Hamiltonian

$$\hat{H}=\sum_{i=1}^{N}\left[\frac{\hat{p}_i^2}{2m}+\frac{1}{2}\sum_{i\neq j}^{N}\hat{V}(\mathbf{r}_i-\mathbf{r}_j)+\hat{U}(\mathbf{r}_i)\right], \tag{1}$$

where m is the mass, $\mathbf{r}$ is the radius vector, $\hat{\mathbf{P}}$ is the momentum operator, $\hat{V}$ is the interaction operator, and $\hat{U}$ is the operator of an external field. In the kinetic stage of the evolution, when it is possible to describe the system by means of the single-particle distribution function $f(\mathbf{p},\mathbf{r},t)$, an integral Fokker-Planck equation holds (see, for example, Ref. 5):

$$\frac{\partial f}{\partial t}+\frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{r}}-\frac{\partial U}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{p}}=-\frac{\partial J_i}{\partial p_i}, \tag{2}$$

where $\mathbf{J}$ is the vector functional with components

$$J_i\equiv J_i(\mathbf{p},\mathbf{r},t)=C\int d\mathbf{p}_1|\mathbf{p}-\mathbf{p}_1|^{-3}[(\mathbf{p}-\mathbf{p}_1)^2\delta_{ik}-(\mathbf{p}-\mathbf{p}_1)_i(\mathbf{p}-\mathbf{p}_1)_k]\times\left(f\frac{\partial f_1}{\partial p_{1k}}-f_1\frac{\partial f}{\partial p_k}\right)_{\mathbf{r}_1\to\mathbf{r}} \tag{3}$$

$$C=\frac{m}{8\pi}\int_0^\infty q^3V_q^2\,dq,\qquad V_q=\int V(\mathbf{r})e^{-i\mathbf{q}\mathbf{r}}\,d\mathbf{r}. \tag{3a}$$

In the expressions (2) and (3), summation over repeated Latin indices is understood. In addition, for brevity we have introduced the notation $f\equiv f(\mathbf{p},\mathbf{r},t)$ and $f_1\equiv f(\mathbf{p}_1,\mathbf{r}_1,t)$. In the special case when the mean changes in the physical quantities in each elementary interaction are small compared with their initial values, Eq. (2) can be reduced to a linear differential Fokker-Planck equation. It should also be noted that the expression (3) for the collision integral $L^{(2)}=-\partial\mathbf{J}/\partial\mathbf{p}$ is valid when:

a) the characteristic inhomogeneity scales for the system are appreciably greater than the range of the forces between the particles;

b) the mean interaction energy $\langle V\rangle$ is much leaa than the mean kinetic energy $\sim kT$; thus, for the Coulomb interaction this means that $e^2n^{1/3}\ll kT$, i.e., $n\ll 5\cdot 10^{37}\cdot(kT/m_ec^2)^3\,\mathrm{cm}^{-3}$.

We now consider the special but nevetheless very interesting case of a homogeneous and isotropic system. Under these conditions, the distribution function f depends only on the modulus p of the momentum and on the time $t: f=f(p,t)$. In addition, it is obvious from general considerations that the vector $\mathbf{J}=\mathbf{J}(\mathbf{p},t)$ must be parallel to the vector $\mathbf{p}$, whose direction is the only one that is distinguished on space. This makes it possible to simplify significantly the collision integral $L^{(2)}$ by expressing the divergence of $\mathbf{J}$ in spherical coordinates and bearing in mind that $|\mathbf{J}|=J_p$:

$$L^{(2)}=-\mathrm{div}_{\mathbf{p}}J=-\frac{\partial}{p^2\partial p}J_pp^2.$$

Integrating (3) over the directions of $\mathbf{p}_1$, we obtain

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$$L^{(2)}=\frac{8\pi C}{3p^2}\frac{\partial}{\partial p}\left(\int\limits_0^p\frac{p_1^3}{p}+\int\limits_p^\infty p^2\right)\left(f_1\frac{\partial f}{\partial p}-f\frac{\partial f_1}{\partial p_1}\frac{p}{p_1}\right)p_1dp_1. \tag{4}$$

Integrating (4) over p_1, we arrive at the collision integral in the form

$$L^{(2)}(p,f)=\frac{8\pi Cn}{3p^2}\Big[\frac{m\varepsilon(<p)}{2\pi}\frac{\partial}{\partial p}\left(p^{-1}\frac{\partial f}{\partial p}\right)+\frac{3w(<p)}{4\pi}\frac{\partial f}{\partial p}$$

$$+\frac{3p^2 f}{n}+\left(\frac{\partial}{\partial p}p^2\frac{\partial f}{\partial p}\right)\int\limits_p^\infty\frac{p_1f_1}{n}dp_1\Big], \tag{5}$$

$$n=4\pi\int\limits_0^\infty p_1^2f_1dp_1,\quad w(<p,t)=\frac{4\pi}{n}\int\limits_0^p p_1^2f_1dp_1, \tag{6}$$

where n is the concentration of the particles, and $w(<p,t)$ is the probability of finding a particle with momentum less than p at time t; further

$$\varepsilon(<p,t)=\frac{4\pi}{n}\int\limits_0^p\frac{p_1^4}{2m}f_1dp_1 \tag{7}$$

is the mean energy (per particle) of the particles that at the time t have momentum less than p.

Thus, for an isotropic and spatially homogeneous ($U=0$) system of N identical particles we obtain an equation for the single-particles distribution function:

$$\partial f/\partial t=L^{(2)}(p,f), \tag{8}$$

where $L^{(2)}$ is determined by Eqs. (4)–(7).

The operator $L^{(2)}$ (4) can be reduced to a linear differential form for the region of the variable p greater than the characteristic mean value $\tilde p$. Strictly speaking, in fact, we require fulfillment of only the condition

$$\exp\left(\frac{\tilde p^2-p^2}{\tilde p^2}\right)\ll 1,$$

where $p^2\gtrsim 3\tilde p^2$.

Indeed, for $p^2\gg\tilde p^2$ the main contribution to the collision integral $L^{(2)}$ (4) is determined by the region of integration $p_1\lesssim\tilde p$, since the disribution function f decreases rapidly at large p; thus, in the limiting case $t\to\infty$ it is obvious that $f(p)\to\exp(-p^2/2mkT)$. Then on the right-hand side of the expression (4) we can replace the limit of integration p by ∞, obtaining

$$L^{(2)}(p,f)\approx\frac{2Cn}{3p^2}\left[2m\bar\varepsilon\frac{\partial}{\partial p}\left(p^{-1}\frac{\partial f}{\partial p}\right)+3\frac{\partial f}{\partial p}\right],$$

$$\bar\varepsilon=\frac{4\pi}{n}\int\limits_0^\infty\frac{p^4}{2m}fdp, \tag{9}$$

where $\tilde\varepsilon=\varepsilon(<\infty)$ is the mean kinetic energy per particle. Since this mean energy is conserved in a conservative system, we can go over to the new independent dimensionless variables

$$z=p^2/2m\theta,\quad \tau=t/t_0, \tag{10}$$

$$\theta={}^2/_3\bar\varepsilon,\quad t_0=(m\theta)^{3/2}/\sqrt{2}Cn. \tag{11}$$

Thus, Eq. (8) is reduced to the form

$$\frac{\partial f}{\partial\tau}=\frac{1}{\sqrt z}\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}+f\right). \tag{12}$$

It is obvious that the Maxwell distribution $f=e^{-z}$ satisfies the stationary equation (12). The quantity θ, which occurs implicity in Eq. (12), is the temperature that is established in the system as $t\to\infty$. It is in this sense that we shall in what follows speak of the temperature of the plasma even when it (or rather its high-temperature tail) is still far from the equilibrium state.

To elucidate the evolution in time of the energy distribution of the system, it is expedient to make the substitution $f=ge^{-z}$, after which we obtain

$$\frac{\partial g}{\partial\tau}=\frac{1}{\sqrt z}\left(\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial z}\right). \tag{13}$$

The funtion

$$g=e^z f(z,\tau) \tag{14}$$

determines the degree of thermalization at the point z at the time t and in the limit $\tau\to\infty$ tends to unity.

Equation (12) for the high-energy "tail" of the distribution function $f(p,t)$ was obtained earlier in a different way by Gould.[3] However, the derivation of Eq. (12) in Ref. 3 used the assumption, not stricly obvious, that it is permissible to expand the distribution function f in the collision integral with respect to the parameter $\Delta p/p$, whereas in the present paper Eq. (12) is derived more consistently. In particular, we are now able to determine correctly the region of applicability of this equation: $z\geqslant 3$.

3. SOLUTION OF THE KINETIC EQUATION FOR THE HIGH-ENERGY PART OF THE DISTRIBUTION FUNCTION

Suppose that at the initial time the ensemble consists of particles with energy ε. We find the time evolutin of the high-energy part of the distribution function $f=ge^{-z}$. For this, we must solve Eq. (13) in the region $z_0\leqslant z<\infty$ with the initial condition

$$g(z,0)=0 \tag{15a}$$

and boundary condition

$$g(z_0,\tau)=\mu(\tau). \tag{15b}$$

Going over to the new variable u, we obtain

$$\frac{\partial g}{\partial\tau}=\frac{\partial^2 g}{\partial u^2}-b\frac{\partial g}{\partial u}\qquad b\approx(u^{-1/5}-{}^1/_5u^{-1}),\quad u={}^4/_5z^{5/4}. \tag{16}$$

Since $u\geqslant u_0=4/5z_0^{5/4}\gg 1$ and $u^{-1/5}$ is a slow function, we can, considering the solutins of Eq. (16) with the initial and boundary conditions (15) in the region $u_0\leqslant u\leqslant\gamma u_0$, where $\gamma\sim 10$, regard b in a first approximation as a constant, and replace it by some mean value in the given region. Then Eq. (16) becomes a heat conduction equation with constant coefficients, the solution of which (for a semi-infinite line) is well known (see, for example, Ref. 6). Indeed, after the substitution

$$g=\varphi(x,\tau)\exp\left[\frac{bx}{2}-\frac{b_2}{4}\tau\right],$$

where $x=u-u_0$, Eq. (16) reduces to

$$\partial\varphi/\partial\tau=\partial^2\varphi/\partial x^2 \tag{17}$$

with the initial and boundary conditions

$$\varphi(x,0)=0,\quad \varphi(0,\tau)=\mu(\tau)e^{b^2/4\tau}. \tag{18}$$

The solution of this equation can be written in the form

$$\varphi(x,\tau)=\frac{1}{2\sqrt{\pi}}\int_0^\tau \frac{x}{(\tau-y)^{3/2}}\exp\left(\frac{b^2}{4y}-\frac{x^2}{4(\tau-y)}\right)\mu(y)\,dy. \tag{19}$$

To find the concrete solution $\varphi(x,\tau)$, it is necessary to know the boundary function $\mu(\tau)$, which, strictly speaking, must be determined from the solution of the kinetic energy in the general case for $z\sim 1$. However, if we assume that the thermalization at the point z_0 takes place directly at the initial time, this does not lead to a significant distortion of the distribution function for $z\sim\gamma z_0\gg z_0$ $(u\gg u_0)$. Therefore, setting $\mu(\tau)=1$ and making some manipulations in the expression (19), we find

$$g(x,\tau)=\frac{1}{\sqrt{\pi}}\left(\int_{v_-}^{\infty}e^{-v^2}dv+e^{bx}\int_{v_+}^{\infty}e^{-v^2}dv\right), \tag{20}$$

where

$$x=\frac{4}{5}\left(z^{5/4}-z_0^{5/4}\right),\quad b\approx\left\langle\frac{1}{u^{1/5}}-\frac{1}{5u}\right\rangle,\quad v_\pm=\frac{x\pm b\tau}{2\sqrt{\tau}}.$$

Noting that for $x\gg 1$ the lower limit in the second integral in (20) is greater than unity, and that the error function $\Phi(s)$ decreases very rapidly for large s, we can ignore the second term in (20) compared with the first for any τ. Substituting for the mean value in b the corresponding value for the main term $u^{-1/5}$ at the point $z^*=z/2$, we obtain $b\to 1.24/z^{1/4}$. Noting also that

$$x={}^4/_5\left(z^{5/4}-z_0^{5/4}\right)\approx{}^4/_5 z^{5/4},\quad z\gg 1,$$

we find

$$g(z,\tau)\approx\frac{1}{\sqrt{\pi}}\int_{a(z,\tau)}^{\infty}e^{-v^2}dv, \tag{21}$$

$$a(z,\tau)\approx\frac{2}{5}\,\frac{z^{3/2}-1.55\tau}{\sqrt{\tau}\,z^{1/4}}. \tag{22}$$

Since in the limit $\tau\to\infty$ the function $a(z,\tau)$ tends to $-\infty$, we conclude directly from (21) that g, which determines the degree of thermalization of the system of particles in the region of z, tends to unity, as was to be expected.

The characteristic thermalization time at the given point can be determined from the condition $g(z,\tau)=0.5$, as a result of which we arrive at the condition $a(z,\tau)=0$, i.e.,

$$\tau_z\approx 0.64z^{3/2}. \tag{23}$$

It should be noted that the obtained approximate solution (21), (22) does not in fact depend on z_0. Moreover, the thermalization time for the region $z\sim\gamma z_0$, where $\gamma\sim 10$, is appreciably greater than τ_{z_0} (by $z^{3/2}$ times). Therefore, the assumption $g(z_0,\tau)\equiv\mu(\tau)=1$ made above is justified for any z_0 used in the finding of the approximate solution $g(z,\tau)$ in the region $z\sim 10z_0$. Thus, the restriction $\gamma\sim 10$ required above is also not critical.

Figure 1 shows the curves $g(z,\tau)$ for some values of τ. It can be seen from the figure that the high-energy part $(z\gg 1)$ of the distribution function for $\tau<\tau_z$ is strongly suppressed

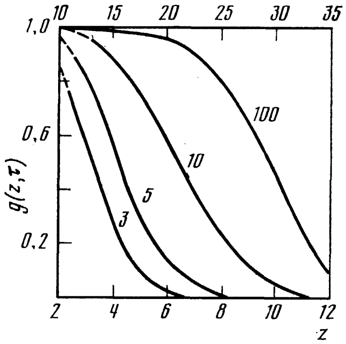

FIG. 1. Degree of thermalization $g(z,\tau)$ as a function of $\tau=t/t_0$ and $z=E/kT$. The values of τ are given next to the curves.

compared with the Maxwell distribution. It is therefore obvious that the fulfillment of the condition $\Delta t\sim t_0$, where Δt is the characteristic lifetime of the plasma, is not yet sufficient for one to be able to use the Maxwell distribution to calculate the rates at which physical processes take place in the plasma.

Hitherto, we have discussed the evolution of the distribution function without particularizing the physical processes that lead to the thermalization of the plasma. The only parameter that depends on the interaction process and occurs implicitly in Eq. (12) is the scale factor t_0, which is a relaxation time. Its value is determined by the expressions (11) and (3a). In the special case of the Coulomb interaction, when $V(r)=e^2/r$, we obtain the well-known result for the relaxation time (see, for example, Ref. 3):

$$t_0=(kT)^{3/2}m^{1/2}/2\sqrt{2}\pi e^4\Lambda n, \tag{24}$$

where Λ is the Coulomb logarithm, and m is the mass of the colliding particles.

For proton-proton collisions there are not only the Coulomb forces but also the nuclear forces, which become important once $kT\gg 1$ MeV. The total cross section of elastic proton-proton scattering in the energy interval 10–300 MeV can be well approximated in the form[7]

$$\sigma_{pp}{}^N=\sigma_{nn}{}^N=10{,}63/\beta^2-29{,}92/\beta+42{,}4\quad[\text{mb}], \tag{25}$$

where $\beta=v/c$, and v is the relative velocity of the colliding particles.

To estimate the relaxartion time t_0^N due to the nuclear forces, we use the well-known relation

$$t_0\approx 1/\langle\sigma_t v\rangle n, \tag{26}$$

where

$$\sigma_t=\int(1-\cos\vartheta)\,d\sigma \tag{27}$$

is the so-called transport cross section, which determines the rate of energy transfer.

In the center-of-mass system, the angular distribution of the scattered nucleons is almost isotropic,[7] and, therefore, $\sigma_t\approx(1/3)\sigma_{pp}^N$. Accordingly,

$$t_0{}^N\equiv\frac{q(T)}{n}\approx\frac{10^{16}}{n}\left[0{,}61\left(\frac{m_pc^2}{kT}\right)^{1/2}+7{,}34\left(\frac{kT}{m_pc^2}\right)^{1/2}-3\right]^{-1}\ [\text{sec}]. \tag{28}$$

It follows from comparison of the expressions (28) and (24) that for $kT \sim 10$ MeV the nuclear relaxation time is about one and a half orders of magnitude shorter than the relaxation time due to Coulomb collisions. With increasing temperature, the ratio t_0^N/t_0^C becomes even smaller. Thus, despite Gould's assertions, elastic nuclear collisions lead to thermalization of a plasma with temperature $kT \geqslant 1$ MeV much more rapidly than the Coulomb interaction, dispite the long-range nature of the latter. Therefore, for a high-temperature ion plasma the nuclear forces are dominat. This tends to equalize the relaxation times τ_{pp} and τ_{ee}, whereas τ_{pe} remains appreciably larger than τ_{pp} and τ_{ee}. Therefore, independent consideration of the evolution in time of the distribution functions of the electrons and protons is even more justified.

The time delay of the thermalization of the high-energy tail of the distribution function will influence the formation of the radiation spectrum of the plasma, which is associated with both the electron and the ion component. We shall consider this question for the special example of the γ luminosity of a plasma due to the decay of secondary π^0 mesons, which have a high production threshold.

4. LUMINOSITY OF A HIGH-TEMPERATURE NUCLEON PLASMA AS A RESULT OF THE PRODUCTION AND DECAY OF π^0 MESONS

At proton energies $\geqslant 150$ MeV in the center-of-mass system, the inelastic channel for pion production is opened. Since the pion production cross section up to ~ 500 MeV is appreciably smaller than the cross section for elastic proton-proton scattering, inelastic collisions will not significantly influence the evolution of the particle distribution function in a plasma with $kT \leqslant 100$ MeV. Nevertheless, the production and decay of π^0 mesons during a time $\sim 10^{-16}$ sec is of interest from the point of view of the γ luminosity of the plasma. Thus, for a Maxwellian plasma this mechanism of radiative cooling of nucleons becomes predominant at temperatures $kT \geqslant 20$ MeV.[10]

The rate of π^0 production in a plasma with Maxwellian distribution of the particles has been calculated by many authors.[11–14] The radiation spectrum has a characteristic maximum at $E_\gamma \approx 70$ MeV. Its shape depends weakly on the distribution function of the protons and is largely determined by the kinematics of the decay $\pi^0 \to 2\gamma$. Therefore, the plasma luminosity L_γ at the time t is determined directly by the rate of production of the π^0 mesons, and this rate does depend strongly on the proton distribution function. Therefore, for times

$$\Delta t < t_\pi \approx (\varepsilon_\pi/kT)^{3/2} t_0, \tag{29}$$

where $\varepsilon_\pi \approx 150$ MeV, the luminosity L_γ will be suppressed compared with the luminosity of a plasma in which the Maxwell distribution has been established. This can be seen in Fig. 2, in which we have plotted the plasma luminosity ratio

$$\varkappa = \frac{R_\pi(T, \tau)}{R_\pi(T, \infty)} \approx \frac{L_\gamma(T, \tau)}{L_\gamma(T, \infty)}$$

as a function of the time $\tau = t/t_0$. The calculations of $R_\pi(T, \tau)$ were made for the distribution $f = e^{-z}g$ on the basis of the π^0 production cross section compiled in Ref. 15. The calculations of $R_\pi(T, \infty)$ corresponding to the Maxwell distribution agree with the previously obtained results of Ref. 13.

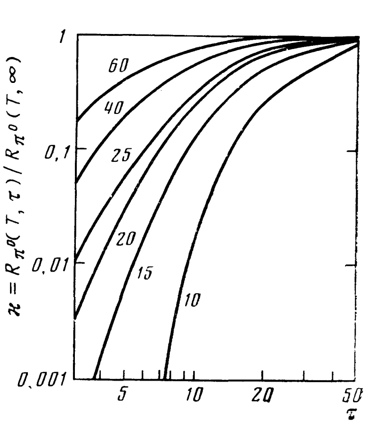

FIG. 2. Dependence of $\varkappa = R_\pi(T, \tau)/R_\pi(T, \infty)$ on the time $\tau = t/t_0$ and the plasma temperature kT. The values of the temperature kT (in MeV) are indicated next to the curves.

A high-temperature plasma ($T \geqslant 10^{11}$°K) can be formed near compact relativistic objects such as neutron stars and black holes (Refs. 11, 12, and 16–20). An important argument in favor of the existence of such an astrophysical plasma would be the detection of γ radiation due to the decay of secondary π^0 mesons.

To estimate the γ luminosity of an accretion plasma, one usually finds the mean energy of the hot nucleons in the region of generation of the radiation; further, under the assumption that the nucleons have an established Maxwellian distribution with $kT = (2/3) < \varepsilon >$, one calculates $R_\pi(T) \equiv R_\pi(T, \infty)$. However, the results obtained in this way will be correct only if

$$\Delta t \geqslant t_\pi, \tag{30}$$

where t_π is determined from Eq. (29), and Δt is the "lifetime" of the hot nucleon plasma. We discuss the fulfillment of this condition for two models of a high-temperature accretion plasma around a black hole.

a) Spherically symmetric accretion

For standard spherical accretion, the proton temperature as a function of the distance r from the gravitational center is

$$\frac{kT}{m_p c^2} \approx 0{,}1 \frac{r_g}{r}\left(1 + \frac{\mu^2}{3}\right)^{-1},$$

$$r_g = 2GM/c^2 \approx 3 \cdot 10^6 \left(\frac{M}{10 M_\odot}\right) \text{ [cm]}, \tag{31}$$

where r_g is the gravitational radius, and $\mu \gtrsim 1$ is the Mach number. It follows from this that for $r \lesssim 10 r_g$ the nucleon temperature $T_p \approx (2/3) < \varepsilon >$ reaches values $\gtrsim 10$ MeV. Further, the time of falling of the plasma $\Delta t \sim t_{ff}$ and the proton concentration n_p at the distance r are (see, for example, Ref. 21)

$$\Delta t \sim t_{ff} = \frac{(\mu^2+3)^{1/2}}{\mu} \frac{r_g}{c} \left(\frac{r}{r_g}\right)^{3/2}, \tag{32a}$$

$$n_p \approx \frac{8}{3\sqrt{3}} \frac{\dot{m}_0}{\sigma_T \eta r_g} \left(\frac{r_g}{r}\right)^{3/2} \left(1+\frac{\mu^2}{3}\right)^{-3/2}, \tag{32b}$$

where σ_T is the Thomson cross section, $\dot{m}_0 = \dot{M}/\dot{M}_{cr}$, $\eta = L/L_{Ed}$, and

$$\dot{M}_{cr} = 2\pi r_g m_p c/\eta\sigma_T$$

is the critical accretion rate. We estimate the ratio of the relaxation time t_0^N (28) to the time of falling of the plasma:

$$\frac{t_0^N}{t_{ff}} \approx \frac{10^{16} q(T)}{n t_{ff}} \approx 10 q(T) \frac{\eta}{\dot{m}_0}. \tag{33}$$

Substituting the characteristic values $q \approx 5 (10 \leqslant kT \leqslant 100$ MeV), $\eta \approx 0.1$, $\dot{m}_0 \approx 0.1$, we find that $t_{ff} \ll t_0^N$, and, therefore, pion production in the plasma is strongly suppressed in the case of spherical accretion.

b) Disk accretion

Shapiro, Lightman, and Eardley[17] showed that it is possible to have a regime of disk accretion onto a black hole, in which the rapid cooling of the electron component due to Comptonization of low-frequency radiation results in the formation of a two-temperature plasma with $T_e \sim 10^9$ °K and $T_i \geqslant 10^{11}$ °K.

For disk accretion, the time of radial falling of the plasma is

$$\Delta t \sim \frac{R}{v_R} \approx 2 \left(\frac{r}{r_g}\right)^2 \frac{r_g \tau_T \eta}{c \dot{m}_0}, \tag{34}$$

where $\tau_T = nh\sigma_T$ is the optical thickness of the disk with respect to Thomson scattering, equal in order of magnitude to unity for the discussed model, and $h \approx 0.1r$ is the half-thickness of the disk. Using the characteristic value for $\eta \approx 0.06$ in the region of the main energy release of the disk, $r \leqslant 10 r_g$, where the high-temperature ion plasma is formed, we obtain

$$\frac{t_0^N}{\Delta t} \approx 2\dot{m}_0 \left(\frac{2}{\tau_T}\right)^2 \left(\frac{10 r_g}{r}\right)^2,$$

where, as before, $\dot{m}_0 = \dot{M}/\dot{M}_{cr}$. For appreciable γ luminosity of a Maxwellian plasma we require fulfillment of the condition (29), i.e.,

$$\left(\frac{10\text{ MeV}}{kT}\right)^{-3/2} \left(\frac{2}{\tau_T}\right)^2 \left(\frac{10 r_g}{r}\right)^2 \dot{m}_0 \geqslant 1. \tag{35}$$

Unfortunately, because of the large uncertainties in the values of the parameters that describe disk accretion,[22] it is at present impossible to say anything definite about the fulfillment of the condition (35) and, therefore, about the effective generation of pions in accretion disks around black holes.

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