Self-oscillating processes in thermal action of a concentrated energy flux on metals

N. N. Rykalin, A. A. Uglov, I. V. Zuev, V. I. Skobelkin, and S. V. Selishchev

Moscow Institute of Electronic Engineering
(Submitted 5 May 1983)
Zh. Eksp. Teor. Fiz. **85**, 1953–1961 (December 1983)

It is stated that thermal action of a concentrated energy flux (CEF) with specific power 10^5–10^7 W/cm^2 on a metal gives rise to thermal oscillations of the surface temperature and of the vapor-cloud density near the metal surface. Equations for the frequency and amplitude of the self-oscillations are obtained and the calculated and experimental data are compared. It is shown that this effect is a characteristic property of heating of matter by a CEF and is due to the screening of the CEF by the vapor of the material and to the instability of the free laminar outflow of the vapor.

PACS numbers: 44.50. + f, 79.20.Ds, 68.40. + e

INTRODUCTION

The action of a concentrated energy flux (CEF) on metals produces a host of interesting and meaningful physical effects. Some of these phenomena, which are directly connected with the thermal action of a beam on a metal, i.e., with production of a temperature field in the latter, have not yet been sufficiently well studied. This has been pointed out in the survey by Bunkin and Tribel'skiĭ.[1]

It is known[2–4] that screening of a beam by the matter evaporated from the surface of a substance plays a substantial role in the heating process. It was found that under certain condition this screening excites self-oscillations.[5–7]

Thus, when a metal is heated by time-invariant CEF larger than a certain critical value, the character of the variation of the metal surface temperature differs substantially from the usual one: it does not tend to be constant but fluctuates about a stationary value. This is due to the onset of self-oscillations of the metal temperature and of the density of its vapor produced in the course of the heating. The self-oscillation mechanism consists in screening of the CEF by vapor of the material and in instability of the laminar free outflow of the vapor from the heating zone. The excitation threshold, amplitude, and frequency of the self-oscillations depend on the energy-flux parameters, on the thermophysical properties of the metal, on the gas-dynamic properties of its vapor, and on the interaction of the energy flux with the vapor.

Since the temperature field determines the thermophysical processes in the target (melting, evaporation, hydrodynamic phenomena in the melt, and others), study of the behavior of the self-excitation of the temperature-field oscillations allow us to examine the physics of these processes from a qualitatively new viewpoint. This leads, in particular, to the conclusion that there exist resonant "stabbing" regimes, which were observed in experiment in the case of an electron beam.

1. FORMULATION OF PROBLEM AND BASIC PHYSICAL ASSUMPTIONS

A number of experimental investigations of the thermal action on metals by a CEF (electron beam,[4–8] laser beam[9–11]) with specific power 10^5–10^7 W/cm^2 have shown the following: (a) If the CEF is constant in time, oscillations are produced in the physical parameters that describe the beam + metal system, viz., the vapor flux, the intensity of the optical radiation, electron emission from the beam-action zone, and others. (b) A critical value of the CEF exists for the excitation of the oscillations. The actual critical value depends on the type of metal and on the beam parameters. This behavior is typical of self-oscillating systems.[12]

We make the following assumptions. The motion of the vapor-melt and melt-solid phase boundaries is due to the melting and evaporation of the material. We neglect them in comparison with the beam-energy dissipation by heat conduction (in other words, we introduce an effective heat-conduction coefficient that takes into account all the mechanisms whereby heat is transferred out of the beam-interaction region).

We locate the origin on the matter-vapor interface and direct the z axis counter to the motion of the beam particles. We consider the kinetics of the vapor only in the half-space $z > 0$ (see Fig. 1).

The intensity of the interaction of the beam particles with the vapor at an arbitrary point of the half-space $z > 0$ is proportional to the vapor density at this point. Since the vapor density is a maximum at the target surface (near the

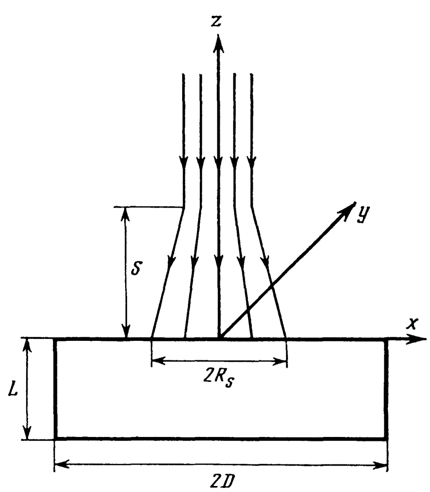

FIG. 1.

beam spot) and tends to zero on the periphery, we can single out a near-surface region that makes a substantially larger contribution to the interaction of the beam with the vapor than the remainder of the half-space. (It is shown in the Appendix that the "size" of the vapor cloud is $S \approx 10R$, where R is the beam radius.)

The size of the vapor cloud is ~ 1–10^{-1} cm. The thermal velocity of the vapor atoms at temperatures on the order of 5000 K is 10^5–10^6 cm/sec. The relaxation time of the vapor density in the vapor cloud is then longer than the 10^{-4}–10^{-6} sec. Consequently the beam-particle distribution function has time to attune itself to the vapor-density distribution.

We denote by $-E_1(z)$ the beam energy flux through a unit area in a unit time along the z axis at $z>0$. We define a flux energy absorption coefficient α_1 (hereafter simply "absorption coefficient") and a coefficient α_2 of the beam-particle-flux scattering (hereafter simply "scattering coefficient") by the equations

$$dE_1/dz = -\alpha_1 \rho(t, \mathbf{r}) E_1, \tag{1}$$

$$dI_1/dz = -\alpha_2 \rho(t, \mathbf{r}) I_1, \tag{2}$$

where I_1 is the beam-particle flux and $\rho(t,\mathbf{r})$ is the vapor density. Then

$$E_1(0) = E_0 \exp\left(-\int_0^S \alpha_1 \rho \, dz\right), \tag{3}$$

$$I_1(0) = I_0 \exp\left(-\int_0^S \alpha_2 \rho \, dz\right). \tag{4}$$

The coefficients α_1 and α_2 depend on the parameters that describe the incident beam (energy for electrons, wavelength for photons) and the parameters that describe the state of the vapor cloud (density, temperature, degree of ionization, etc.).

We define on the basis of Eq. (4) the defocusing R_S of the beam on account of scattering (see Fig. 1):

$$\iint_{\Omega_S} \left[\exp\left(-\int_0^S \alpha_2 \rho(t, \mathbf{r})\, dz\right)\right] d\sigma = \pi R^2, \quad \Omega_S = \{x^2+y^2 \leqslant R_S^2;\ z=0\}. \tag{5}$$

With allowance for all the assumptions, the equation system that describes the target heating takes the form

$$c_s \rho_s \frac{\partial T}{\partial t} = \lambda_s \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{6}$$

$$\mathbf{r} \in \{x^2+y^2 \leqslant D^2;\ -L \leqslant z \leqslant 0\},\quad T_s = T(t, 0, 0, 0),\quad T(0, \mathbf{r}) = T_0,$$

$$T(t, x^2+y^2=D^2; z) = T(t, x, y, -L) = T_0, \tag{7}$$

$$\lambda_s \frac{\partial T}{\partial z}\bigg|_{z=0} = f_s(T) + E_2 \Omega_s, \tag{8}$$

$$E_2 = k(T_s, E_1(0)) E_0 \exp\left(-\int_0^S \alpha_1 \rho(t, \mathbf{r})\, dz\right), \tag{9}$$

$$\frac{\partial \rho}{\partial t} + \nabla \mathbf{j} = 0,\quad \mathbf{j} = \rho \mathbf{u}, \quad \mathbf{r} \in \{(-\infty, \infty); (-\infty, \infty); [0, \infty)\},\quad \rho(0, \mathbf{r}) = \rho_0, \tag{10}$$

where T is the temperature of the metallic target, c_S the specific heat, ρ_s the density, λ_s the thermal conductivity, D the target radius, L the target thickness, $f_s(T)$ a function that determines the heat extraction from the metal surface, $k(T_s, E_1(0)) < 1$ the coefficient of energy absorption by the surface, which takes into account the reflection of the incident beam from the surface, and E_0 the energy flux density produced by the energy source (see Fig. 1).

A CEF having an intensity 10^5–10^7 W/cm^2 produces a temperature 5×10^3 K in the beam-action zone. At these temperatures the saturated vapor density is such that the atom mean free path in the vapor becomes smaller than the size of the beam spot (the latter is usually 10^{-1} to 10^{-2} cm). The interaction of the atoms with one another must therefore be taken into account in the gasdynamics of the vapor,[2] i.e., a gas jet escapes from the spot where the beam acts and produces a vapor cloud (flare) near the metal surface.

Free laminar escape of gas becomes unstable at small Reynolds numbers.[13] Stochastic density and velocity pulsations are present in the gas jet, and the most energetic of them are pulsations with a scale determined by the size of the entire flux as a whole.[14] In this case this is the size of the vapor cloud.

The use of gasdynamic equations that describe the instability of the gas cloud and its evolution is rather complicated, for an important role is played in this case the fact that the motion is three-dimensional.[15]

We use the following approach to the description of the vapor kinetics. For instability to occur in the freely escaping vapor the Reynolds number must be at least larger than unity:

$$v \rho S/\eta > 1, \tag{11}$$

or

$$\rho S^2/\eta - S/v > 0, \tag{12}$$

where v is the velocity and η is the viscosity of the vapor. We assume that the pulsation is produced within a time τ_b and vanishes after a time τ_d, where

$$\tau_b \approx \rho S^2/\eta,\quad \tau_d \approx S/v. \tag{13}$$

The dependence of the vapor stream near the target surface on its characteristics and on the surface temperature can be written in various forms that take into account both the free outflow of the vapor as well as its diffusion over the periphery. We have chosen a relation [Eq. (15)] shown by analysis to lead to agreement with the experimental data without a rigorous allowance for other important factors such as development of turbulence of the vapor stream, condensation, etc., which would complicate without justification the mathematical description of the phenomenon.

Assume that

$$\mathbf{j} = (0, 0, j), \tag{14}$$

$$j = n(T_s(t-\tau))\, v(T_s) - \frac{\eta(T_s)}{\rho} \frac{\partial \rho}{\partial z}, \tag{15}$$

$$\tau(z) = z/v(T_s) - \rho z^2/\eta(T_s), \tag{16}$$

$$n(T_s) = \frac{A_n}{T_s} \exp\left(-\frac{B_n}{T_s}\right),\quad \frac{\partial \rho}{\partial z}\bigg|_{z=S} = 0. \tag{17}$$

Here T_s is the surface temperature at the beam spot, $n(T)$ is the saturated vapor pressure, A_n and B_n are constants determined from the temperature dependence of the saturated-vapor pressure, and $v(T_s)$ the thermal velocity of the vapor atom in the surface region.

The first term in (15) corresponds to the limiting case of outflow of a rarefied gas, and the second to the limiting case of self-diffusion of a dense gas. We assume that (15) yields a good approximation also in the intermediate region. The satisfactory agreement between the calculation and the experimental data justifies this assumption which, generally speaking, should be corroborated by the gasdynamic equations.

2. INSTABILITY OF THE HEATING PROCESS

We assume for simplicity that the coefficients α_1 and α_2 do not depend on the temperature and on the density of the vapor, and that the coefficient $k(T_s, E_1) = 1$. Averaging the system (5)–(17) over the spatial coordinates, we reduce it to a system of two ordinary first-order differential equations:

$$\frac{dT}{dt} = -\frac{g}{L_e^2}(T-T_0)+Q\exp(-\alpha_1\rho S), \qquad (18)$$

$$\frac{d\rho}{dt}=\frac{v}{S}[n(T)-n(T(t-\tau(S)))]-\frac{\eta}{\rho S^2}[\rho-n(T)], \qquad (19)$$

where g is the effective thermal-diffusivity coefficient, Q is the effective heating intensity, and L_e is the effective "thermal" length. For sufficiently large t we can write

$$n(T)-n(T(t-\tau))\approx\tau dn/dt. \qquad (20)$$

Let us test the solution (18)–(20) for stability. From the linear theory of stability we obtain the criterion for the oscillation excitation:

$$\alpha_1\rho_e S\left(\frac{v_e\rho_e S}{\eta_e}-1\right)\frac{g}{L_e^2}\left(\frac{B_n}{T_e}-1\right)\left(1-\frac{T_0}{T_e}\right) - \frac{g}{L_e^2}-\frac{\eta_e}{\rho_e S^2}>0. \qquad (21)$$

Here $T_e, \rho_e = (nT_e)$ constitute the stationary solution of the system (18), (19).

No oscillations are excited if the vapor absorbs very little of the incident-beam energy, $\alpha_1 + 0$, or if the viscosity is very high, $\eta\to\infty$. Since the saturated vapor density $n(T_e)$ decreases with decreasing temperature T_e much more rapidly than the viscosity $\eta(T_e)$ and the velocity $v(T_e)$, there exists a value T_b such that at $T_e < T_b$

$$S/v_e-\rho_e S^2/\eta_e>0. \qquad (22)$$

It follows from (21) and (22) that no oscillations are excited at $T_e < T_b$. It is easy to verify that the equality $T_e = T_b$ corresponds to an atom-vapor mean free path of the order of S, so that the behavior of the viscosity for $T_s < T_b$ becomes incorrect. Thus the transition to the limit as $\eta\to 0$ corresponds to the transition $T_e\to T_b$ in (21).

The criterion (21) does not take into account the dependence of the relaxation time of the temperature in the target, $(g/L_e^2)^{-1}$, on the frequency of the excited oscillations. To take this effect into account we must consider the case when the wavelength of the perturbation of the temperature field is substantially less than the target thickness.

Let us consider the stability of the stationary solution of the following system of equations:

$$\frac{\partial T}{\partial t}=g\frac{\partial^2 T}{\partial z^2},\quad T(0,z)=T_0,\quad T_s=T(t,0),$$
$$z\in(-\infty,\ 0],\ T(t,\ -\infty)=T_0,$$
$$\lambda_s\frac{\partial T}{\partial z}\Big|_{z=0}=E_0\exp(-\alpha_1\rho S),$$
$$\frac{d\rho}{dt}=\left(1-\frac{v\rho S}{\eta}\right)\frac{dn}{dt}-\frac{\eta}{\rho S^2}[\rho-n(T_s)].$$

We introduce the notation

$$\gamma=v_e\rho_e S/\eta_e,\quad p=\eta_e/\rho_e S^2,$$
$$H_2=\alpha_1\rho_e S\frac{E_0}{\lambda_s T_{se}}\exp(-\alpha_1\rho_e S)=\alpha_1\rho_e S\frac{1}{T_{se}}\frac{\partial T_e}{\partial z}\Big|_{z=0}$$

in the approximation where $\gamma\gg 1$ the condition for the oscillation excitation is

$$g[H_2(\gamma-1)(B_n/T_{se}-1)]^2>4p,$$

and the largest growth rate will be possessed by the mode

$$\delta T=A_1\exp\left\{g\left[H_2(\gamma-1)\left(\frac{B_n}{T_{se}}-1\right)\right]^2 t +H_2(\gamma-1)\left(\frac{B_n}{T_{se}}-1\right)z\right\}.$$

Thus, the effective thermal length decreases far beyond the self-oscillation excitation threshold (at E_0 much larger than the threshold value, and accordingly at large $T_e\gg 1$ and $\gamma\gg 1$):

$$L_e=[H_2(\gamma-1)(B_n/T_{se}-1)]^{-1}, \qquad (23)$$

and the oscillations of the target temperature field become localized near the region where the material becomes heated by the beam.

3. SELF-OSCILLATIONS IN A BEAM + EVAPORATING MATTER (VAPOR) + SUBSTANCE SYSTEM

We shall calculate the amplitude and frequency of the self-oscillations by starting from the system (18, 19). After a number of approximations that make the calculations less cumbersome, we obtain

$$dx_2/dt=-hx_2-Hx_1, \qquad (24)$$

$$dx_1/dt=[(\gamma-1)h+p]x_2+[(\gamma-1)H-p]x_1 -px_1x_2+px_1^2+px_2x_1^2-px_1^3, \qquad (25)$$

where

$$h=\left(1-\frac{T_0}{T_e}\right)\left(\frac{B_n}{T_e}-1\right)\frac{g}{L_e^2},\quad H=\alpha_1\rho_e Sh,$$
$$\rho(t)=\rho_e+\rho_e x_1(t),\quad n(t)=\rho_e+\rho_e x_2(t),$$
$$\omega_0^2=p(H+h),\quad \varepsilon=[(\gamma-1)H-h-p]/\omega_0.$$

From the theorem for bifurcation in the vicinity of a complex single focus it follows that the system (24), (25) has a stable limit cycle at $\varepsilon>0$.[16] The parameter ε is equal to the ratio of the growth rate of the oscillation amplitude to the cyclic frequency of the oscillations near the equilibrium position. If $\varepsilon\ll 1$, the self-oscillations are close to harmonic. At $\varepsilon\gg 1$ relaxation oscillation set in.

For the case $\varepsilon \ll 1$ we have from (24) and (25) in the first-order approximation of the averaging method[17]

$$\rho(t)=\rho_e-\frac{h}{H}\rho_e A_e \cos\omega_e t+\frac{\omega_0}{H}\rho_e A_e \sin\omega_e t,$$

$$n(t)=\rho_e+\rho_e A_e \cos\omega_e t,$$

$$A_e=2\left[\frac{[(\gamma-1)H-h-p]}{p[2h/H+3h^2/H^2+3\omega_0^2/H^2]}\right\}^{1/2},$$

$$\omega_{1e}=\omega_0+\frac{Hp}{\omega_0}\left[\frac{3}{8}\frac{h^2}{H^2}\left(1+\frac{h}{H}\right)+\frac{1}{8}\frac{\omega_0^2}{H^2}\left(1+3\frac{\omega_0}{H}\right)\right]A_e^2.$$

We shall analyze the relaxation oscillations by the method described in Ref. 18. At $\varepsilon \gg 1$ the variable x_1 is fast and x_2 is slow. The variation of x_1 is determined by the linear terms near the origin and by the cubic ones at appreciable distances. Since the existence of a limit cycle at $\varepsilon > 0$ has been proved, we simplify the calculations by neglecting the quadratic terms in the right-hand side of (25). In the discontinuous-oscillations approximation the period of the oscillations at $h \gg H$ is

$$T_{r1}\approx 1.6h^{-1},$$

and the amplitudes are

$$x_{2max}=-x_{2min}\approx 0.67(H/h)(\gamma H/p)^{1/2},$$

$$x_{1max}=-x_{1min}\approx 1.2(\gamma H/p)^{1/2}.$$

Similarly, at $H \gg h$,

$$T_{r2}\approx H^{-1}(H/h)^{1/2}\ln(H/h)^{1/2},$$

$$x_{2max}=-x_{2min}\approx 0.5(H/h)(\gamma h/p)^{1/2},$$

$$x_{1max}=-x_{1min}\approx 0.25(\gamma H^2/ph)^{1/2}.$$

In the results we must take into account the dependence of the "thermal" effective length on the frequency of the exciting oscillations via Eq. (23).

4. DISCUSSION OF RESULTS

Calculation of the frequency of the self-oscillations by means of the derived equations leads to satisfactory agreement with the experimental results.[4,8,10,11] The frequencies of the self-oscillations range from 10^2 to 10^4 Hz and the amplitudes are in the range $(1–5)\times 10^2$ K.

We represent the physical mechanism of the self-oscillations in the following manner. For each fixed value of the CEF, the screening of the beam by the evaporating matter makes the stationary temperature T_s lower than its maximum value T_m in the absence of screening. Assume that the temperature in the beam spot has increased on account of the CEF fluctuations. The dissipative heat-extraction processes then cause the surface temperature to tend to a stationary value T_e. Since the surface temperature has risen, the evaporation of matter from the surface increases. The change of the density of the vapor clouds is determined by the competition of two processes: the rate of accumulation of vapor in the vapor and the rate of spreading of the cloud through outflow of the vapor over the periphery. Let the stochastic pulsation of the vapor cloud be such that during the positive surface temperature rise the vapor-cloud density becomes so much smaller than the stationary value ρ_e that the increase of the intensity of heating the target matter as a result of the decrease of the screening of the beam by the vapor will predominate over the rate of heat dissipation. The spot-temperature rise will increase, i.e., the stationary state will become unstable. The amplitude of the excited oscillations is limited: the temperature cannot exceed T_m; with increasing vapor density the rate of spreading increases and the screening increases.

Since the vapor flow, electron emission, optical-radiation emission from the action zone, etc., depend on the surface temperature of the material, oscillations of the spot temperature cause these quantities likewise to oscillate.

Equations (6) and (8) correspond to heating of the target material by a surface heat source (the heating of the metal by a laser or by an electron beam). The approach described, however, is valid also in the case of heating by a three-dimensional source. One can likewise not exclude in principle the dependences of the coefficients α_1, α_2, and k on the vapor temperature and density, or on the form of the CEF.

CONCLUSION

The existence of self-oscillations of the temperature of a substance and of its vapor is a characteristic property of heating of a substance by a concentrated energy flux. From our point of view it is of interest to develop further the results of the present paper along the following directions.

Analysis of the influence of the self-oscillations on the shape of the penetration zone, i.e., clarification of the role of the self-oscillations in the existence of the "stabbing" zone.[1,4] This calls for consideration of spatially three-dimensional (two-dimensional) heat conduction equations for solid and liquid phases in the target material (the Stefan problem), and for the use of the mathematical approach proposed in the present paper to describe the interaction between the vapor and the beam and of the gasdynamics of the vapor.

Analysis of the influence of self-oscillations on the motion of the melt-vapor phase boundary, including a study of the influence of the plasma in the vapor, of condensation in the vapor, and others on the parameters of the self-oscillations, on the basis of the gasdynamic equations and in final analysis of the kinetic microscopic equations. It is appropriate to note here that in the vapor cloud the length of the perturbation wave that has the largest growth rate is approximately equal to the size of the cloud, since the beam particle interacts on its way to the surface of the material with the entire thickness of the vapor, and consequently short-wave perturbations (shorter than the size of the cloud) will not be effective.

Generally speaking, it is of interest to investigate the effect of self-oscillations on physical phenomena that are connected with the effectiveness of heat transfer from the beam-action region. In this case one can expect formation in the target material of a spatial region that has some physical property predominantly in the beam direction, as is the case in the "stabbing" effect.

APPENDIX

Let the evaporation region be a circle of radius R. We obtain the distribution of the vapor density on the z axis, using the cosine law. Let the temperature of the surface in the spot be constant and equal to T. The vapor mass flux along the z axis is then

$$j(z)=n(T)v(T)\int_0^R\int_0^{2\pi}\frac{\cos^2\theta}{\pi(r^2+z^2)}r\,dr\,d\varphi,\quad \cos^2\theta=\frac{z^2}{r^2+z^2}.$$

Consequently

$$j(z)=n(T)v(T)R^2/(R^2+z^2).$$

Assuming that $u_z\approx v(T)$, we have

$$\rho(z)=n(T)R^2/(R^2+z^2).$$

Then

$$m_1=\int_0^{10R}\rho(z)\,dz=Rn(T)\,\mathrm{arctg}\,10,$$

$$m_2=\int_{10R}^{\infty}\rho(z)\,dz=Rn(T)\left[\frac{\pi}{2}-\mathrm{arctg}\,10\right],$$

$$m_2/m_1\approx 7\cdot 10^{-2}.$$

The greatest effect on the beam is thus exerted by a vapor "column" approximately $10\,R$ high.

[1]F. V. Bunkin and M. I. Tribel'skiĭ, Usp. Fiz. Nauk **130**, 193 (1980) [Sov. Phys. Usp. **23**, 105 (1980)].

[2]S. I. Anisimov, Ya. A. Imas, G. S. Romanov, and Yu. V. Khodyko, Deĭstvie izlucheniya bol'shoĭ moshchnosti na metally (Action of High-Power Radiation on Metals), Nauka, 1970.

[3]I. V. Nemchinov, Izv. AN SSSR, seriya fiz. **46**, 1021 (1982).

[4]N. N. Rykalin, I. V. Zuev, and A. A. Uglov, Osnovy elektronno-luchevoĭ obrabotki (Principles of Electron-Beam Machining, Mashinostroenie, 1978).

[5]I. V. Zuev, S. V. Selishchev and V. I. Skobelkin, Dokl. Akad. Nauk SSSR **255**, 2371 (1980).

[6]I. V. Zuev, S. V. Selishchev and V. I. Skobelkin, Fiz. i khim. obrabotki mater. **6**, 3 (1980).

[7]N. N. Rrykalin, V. I. Skobelkin, I. V. Zuev, S. V. Selishchev and A. A. Uglov, Dokl. Akad. Nauk SSSR **257**, 99 (1981) [Sov. Phys. Dokl. **26**, 329 (1981)].

[8]V. A. Artemov and M. A. Vlasov, Zh. Tekh. Fiz. **48**, 193, (1978) [Sov. Phys. Tech. Phys. **23**, 113 (1978)].

[9]A. E. Kuznetsov, A. A. Orlov and P. I. Ulyakov, Kvantovaya Elektron. (Moscow) No. 1, 57 (1972) [Sov. J. Quantum Electron. **2**, 44 (1972)].

[10]N. N. Rykalin, A. A. Uglov, and M. B. Ignat'ev, Dokl. Akad. Nauk SSSR **265**, 1117 (1982) [Sov. Phys. Dokl. **27**, 628 (1982)].

[11]A. A. Uglov and M. B. Ignat'ev, Fizika plazmy **8**, 1285 (1982) [Sov. J. Plasma Phys. **8**, 732 (1982)].

[12]A. A. Andronov, A. A. Vitt and S. E. Khaikin, Theory of Oscillations, Addison-Wesley, 1966.

[13]G. Schlichting, Boundary-Layer Theory, McGraw, 1968.

[14]D. I. Blokhintsev, Akustika neodnorodnoĭ dvizhushcheisya sredy (Acoustics of Inhomogeneous Moving Medium), Nauka, 1981.

[15]P. Bradshaw, Complex Turbulent Flows, Russ. transl. in: Teoreticheskaya i prikladnaya mekhanika (Theoretical and Applied Mechanics), Mir, 1979.

[16]A. A. Andronov, E. A. Leontovich, I. I. Gordon, and A. G. Maĭer, Teoriya bifurkatsii dinamicheskikh sistem na ploskosti (Theory of Bifurcation of Dynamic Systems on a Plane), Nauka, 1967.

[17]P. S. Landa, Avtokolebaniya v sistemakh s konechnym chislom stepenei svobody (Self-Oscillations in Systems with Finite Number of Degrees of Freedom), Nauka, 1980.

[18]E. F. Mishchenko and N. Kh. Rozov, Differentsial'nye uravneniya s malym parametrom i relaksatsionnye kolebaniya (Differential Equations with Small Parameter, and Relaxation Oscillations), Nauka, (1975).

Translated by J. G. Adashko