Role of coherence effects in the inelastic interaction of fast particles with nuclei

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The possibility of separating the interactions that occur between elementary particles and nuclei into coherent and incoherent processes and the validity of the single-particle approximation in the analysis of the behavior of these particles in a nucleus allow a constructive solution of the problem of describing the incoherent processes of interaction of particles with nuclei with the use of dispersion relations and the diagrammatic technique. The thus arising equations for the single-particle Green functions describing the behavior of the single-particle component in the nucleus can be solved in the quasiclassical approximation. This circumstance allows us to formulate an approach to the description of nuclear reactions that employs the methods of kinetic theory. It is shown that this approach allows the analysis of the behavior of a fast particle in a nuclear reaction. A comparison of the results obtained with the experimental data leads to the conviction that the present approach allows the description of both the experimental data on the inelastic interaction of pions with a nucleus and the processes of pion and nucleon production on nuclei by intermediate energy protons.

1. INTRODUCTION

One of the most interesting problems of nuclear physics is the problem of describing the behavior of elementary particles in nuclear matter, a problem which reduces to a considerable degree to the problem of describing the propagation of these particles inside a nucleus and of describing their interaction with the nucleons of the nucleus. In principle, this problem can be solved within the framework of a theory that uses dispersion relations and the diagrammatic technique.[1] To constructively solve this problem, we can use the fact that all the particle-nucleus interaction processes can be separated into coherent processes, i.e., processes that do not cause a change in the state of the nucleus and are responsible largely for the propagation of the particles through the nucleus, and incoherent processes.[2] For the description of the coherent scattering of elementary particles of sufficiently high energies by nuclei, we have a multiple-scattering theory[3] that correctly reflects the main features of this process. The description of the incoherent processes requires both the correct consideration of the collisions of the particle with the nucleons of the nucleus and the consideration of the effect of the coherent propagation of the particles, and is therefore a more complicated problem. The present paper is devoted to the solution of this problem and the application of the results obtained to the description of the inelastic interaction of pions and nucleons with nuclei. To solve the problem, we shall use the method of single-particle Green functions under the assumption that the collisions between the particles inside the nucleus have a random character.

We shall formulate the picture of the scattering of an incoming particle by the nucleons of the nucleus in the language of multiple scattering and rescattering of waves. The wave ψ corresponding to the initial particle can be coherently scattered by the nucleons of the nucleus, which, as we assume, are arranged randomly. The averaged—over the scattering centers—coherent sum wave $\bar{\psi}$ can be described by a Schrödinger equation with a complex potential[4]:

$$\left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-V\right)\bar{\psi}=0. \tag{1a}$$

Besides the coherent scattering, which leaves the system of scatterers unchanged, there occurs between the incoming wave and the nucleons of the nucleus an incoherent interaction that leads to a change in the state of the randomly distributed nucleons in the nucleus and, hence, to a change in the state of the nucleus. Such a description, in which each particle-scattering event with a nonzero scattering angle leads to the inelastic excitation of the nucleus in an incoherent manner, implies that there exists a connection (known as the fluctuation-dissipation theorem) between the coherent and incoherent scattering processes, i.e., that the damping of the incident wave in the medium is entirely due to inelastic-collision processes.

In terms of the single-particle wave function ψ, this means that the part $\delta\psi=\psi-\bar{\psi}$ of the wave function which is responsible for the incoherent scattering should, on being averaged over the random scatterers, vanish, i.e., $\overline{\delta\psi}=0$. Then if we write Eq. (1a) for $\bar{\psi}$, then the equation for ψ should differ from (1a) by a term that takes account of the incoherent collisions:

$$\left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-V\right)\psi=j. \tag{1b}$$

Then we should, on averaging over the randomly distributed scatterers, set $\bar{j}=0$. Thus, we have separated the collision processes into two parts: the coherent, $V\psi$, and incoherent, j, parts.

It is now clear that, to describe the initial-particle-energy-averaged inelastic processes due to the incoherent scattering, we should investigate the behavior of the $\overline{\delta\psi\delta\psi}$. Then, knowing from Eq. (1a) the function $\bar{\psi}$, we can determine the correlation function $\overline{\psi\psi}=\bar{\psi}\bar{\psi}+\overline{\delta\psi\delta\psi}$ of the system.

Such a description of the interaction of an incoming

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particle with a nucleus, in which we limit ourselves to the consideration of the single-particle modes of excitation of the system, is valid in the case when the particle's mean free path l is significantly longer than the correlation length r_c.

Let us note that, in the theory of finite Fermi systems[5] and in many-body theory,[6] the optical model is justified by separating from the eigenenergy operator in the equation for the Green function the part that is a smooth function of the energy. We shall use a similar recipe to describe the coherent processes, assuming that averaging over the energy is equivalent to averaging over the random scatterers.

To compute the correlation function $\overline{\psi\psi} = \overline{\psi^-\psi^-} + \overline{\delta\psi\delta\psi}$, we shall use the methods of quantum statistical mechanics developed in Refs. 5,7–11.

2. DETERMINATION OF THE SINGLE-PARTICLE GREEN FUNCTIONS AND THEIR CONNECTION WITH THE CROSS SECTION. THE EQUATIONS OF MOTION

To determine the single-particle Green function, which contains all the single-particle information about the interaction of the incoming particle with the nucleons of the nucleus, let us use the Heisenberg representation of the particle creation and annihilation operators, $\Psi^+(\mathbf{r},t)$ and $\Psi(\mathbf{r},t)$, and let us introduce it as the average over the state Φ_E of the total Hamiltonian which describes the scattering of the incoming particle by the nucleons of the nucleus:

$$G(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)=\begin{cases} G^<(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2), & t_1<t_2, \\ G^>(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2), & t_1>t_2, \end{cases} \tag{2}$$

where (see, for example, Ref. 9)

$$\begin{aligned} G^<(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)&=\mp i\langle\Psi^+(\mathbf{r}_2,t_2)\Psi(\mathbf{r}_1,t_1)\rangle, \\ G^>(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)&=-i\langle\Psi(\mathbf{r}_1,t_1)\Psi^+(\mathbf{r}_2,t_2)\rangle. \end{aligned} \tag{3}$$

Here and below $c=\hbar=1$; the lower sign pertains to fermions; the upper sign, to bosons. The physical interpretation of the Green functions (3) consists in the following: $G^>(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)$ gives the amplitude of the probability of finding at the point $\mathbf{r}_1$ at the moment t_1 a particle added at the point $\mathbf{r}_2$ at the moment t_2, while $G^<(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)$ is the amplitude of the probability that the removal of a particle at $\mathbf{r}_2$, t_2 will lead to its being absent from the point $\mathbf{r}_1$ at t_1. It should be noted that, for $t_1=t_2$, the quantity $-iG^<(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)$ is the density matrix of the system, while the quantity

$$\langle n(\mathbf{r},t)\rangle=-iG^<(\mathbf{r},t,\mathbf{r},t) \tag{4}$$

is the mean particle density at the point $\mathbf{r},t$.

Let us go over to the Wigner representation for the Green functions, for which purpose we make the change of variables

$$\begin{aligned} &\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2, \quad t=t_1-t_2, \\ &\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2, \quad T=(t_1+t_2)/2 \end{aligned} \tag{5}$$

and, performing a Fourier transformation with respect to $\mathbf{r}$ and t, we obtain

$$\begin{aligned} G^<(\varepsilon,\mathbf{p},\mathbf{R},T)&=\int d\mathbf{r}\,dt\exp(-i\mathbf{pr}+i\varepsilon t)\,(\pm iG^<(\mathbf{r},t,\mathbf{R},T)), \\ G^>(\varepsilon,\mathbf{p},\mathbf{R},T)&=\int d\mathbf{r}\,dt\exp(-i\mathbf{pr}+i\varepsilon t)\,(iG^>(\mathbf{r},t,\mathbf{R},T)). \end{aligned} \tag{6}$$

This representation is convenient in that it allows us to determine the quantum-mechanical distribution function

$$\begin{aligned} f(\mathbf{p},\mathbf{R},T)&=\int\frac{d\varepsilon}{2\pi}G^<(\varepsilon,\mathbf{p},\mathbf{R},T) \\ &=\int d\mathbf{r}e^{-i\mathbf{pr}}\left\langle\Psi^+\left(\mathbf{R}-\frac{\mathbf{r}}{2},T\right)\Psi\left(\mathbf{R}+\frac{\mathbf{r}}{2},T\right)\right\rangle \end{aligned} \tag{7}$$

Strictly speaking, the quantum-mechanical distribution function is not a positive definite quantity on account of the uncertainty relations. The physical meaning of the distribution function is contained in the integral of this function: if we integrate (7) over the momenta, we obtain the particle-density distribution as a function of the coordinates, while if we integrate over the coordinates we obtain the momentum distribution of the particles in the system. In the classical limit the function (7) goes over into the normal classical distribution function. Thus, it can be seen that the function $G^<(\varepsilon,\mathbf{p},\mathbf{R},T)$ characterizes the density of the particles with momentum $\mathbf{p}$ and energy ε at the space-time point $\mathbf{R},T$. The function $G^>(\varepsilon,\mathbf{p},\mathbf{R},T)$, in its turn, characterizes the density of the possible $(\varepsilon,\mathbf{p})$ states at the point $\mathbf{R},T$, provided they are not occupied.

It is clear that, in a nuclear reaction, all the states of the continuous spectrum will get out of the nucleus by the moment of time $T\to\infty$. Therefore, the cross section for inclusive reactions, e.g., $(N,N'x)$, where N is a nucleon, can be written in the form

$$\frac{d\sigma}{dp}=\lim_{T\to\infty}\frac{1}{I}\int f(\mathbf{p},\mathbf{R},T)\,d\mathbf{R}, \tag{8}$$

where I is the incident flux. It should be noted that, if the form of the interaction between the particles in the final state is known, then possession of complete information about the single-particle Green functions allows us to describe the production of complex particles (e.g., in pickup reactions) as well.

In a real experiment we deal with an ensemble consisting of a particle beam and the nuclei of the target. Such a system is not in a definite eigenstate Φ_E of the Hamiltonian, and is rather described by the density matrix corresponding to the distribution of the particles of the incident beam. Therefore, it will be more correct to average (3) over the distribution, which, for definiteness, can be approximated by the Lorentz function and represented in the form

$$G_E^<(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)=\frac{\mathrm{Sp}(\rho\Psi^+(\mathbf{r}_2,t_2)\Psi(\mathbf{r}_1,t_1))}{\mathrm{S\ pp}}, \tag{9}$$

where

$$\rho=\frac{1}{\pi}\frac{\Delta E/2}{(H-E_0)^2+(\Delta E/2)^2}, \tag{10}$$

while H is the total Hamiltonian of the system nucleus plus incoming particle. Strictly speaking, the expression (10) contains the Hamiltonian H_0 in which the interaction between the incoming particle and the nucleons of the nucleus is neglected. The replacement of H_0 by H is equivalent to the replacement of the averaging over the energy of the initial particle by the averaging over the ensemble of states of the system nucleus + particle. This basic assumption that the computation of the energy-averaged Green function G_E can

be replaced by the computation of the Green function $\overline{G}$ averaged over the ensemble of random scatterers allows us to use the methods of mutiple scattering theory.

Let us introduce the Green function

$$\overline{G}^{<}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = \mp i\langle\overline{\Psi^{+}(\mathbf{r}_2, t_2)\Psi(\mathbf{r}_1, t_1)}\rangle, \tag{11}$$

where the bar denotes averaging over the ensemble of random scatterers. Let us separate out from Ψ and Ψ^{+} the parts $\overline{\Psi}$ and $\overline{\Psi}^{+}$ averaged over this ensemble of particles. Then the Green function (11) can be represented in the form

$$\begin{aligned}\overline{G}^{<}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) &= \mp i[\langle\overline{\Psi}^{+}(\mathbf{r}_2, t_2)\overline{\Psi}(\mathbf{r}_1, t_1)\rangle + \overline{\delta\Psi^{+}(\mathbf{r}_2, t_2)\delta\Psi(\mathbf{r}_1, t_1)}\rangle] \\ &= \bar{g}^{<}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) + g^{<}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2).\end{aligned} \tag{12a}$$

In this representation the function $\bar{g}^{<}$ describes the change, caused by the coherent processes, in the single-particle states, while $g^{<}$ describes the change caused by the incoherent processes. Similarly, we can introduce the Green function

$$\overline{G}^{>}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = \bar{g}^{>}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) + g^{>}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2). \tag{12b}$$

Thus, we have reduced the problem of describing nuclear reactions to the problem of computing the Green functions $\bar{g}^{<}, g^{<}, \bar{g}^{>}$, and $g^{>}$. Since we are primarily interested in the description of inelastic reactions caused by incoherent processes, our problem has been reduced to the problem of computing the functions $g^{<}$ and $g^{>}$.

Let us write the equations for Ψ and Ψ^{+} in the case of interest to us. We shall assume that at the initial moment of time we have a bound system of A particles (the target nucleus) and an imcoming particle, and that these particles move in a mean nuclear field $U(\mathbf{r})$. Furthermore, the particles of this system interact through a residual interaction. In this case the coherent interaction processes can be described by the equations

$$\begin{aligned}\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(\mathbf{r}_1)\right]\overline{\Psi}(\mathbf{r}_1, t_1) &= \int d\mathbf{r}_2\, dt_2 \Sigma^{-}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2)\overline{\Psi}(\mathbf{r}_2, t_2), \\ \left[-i\frac{\partial}{\partial t_2} + \frac{\nabla_2^2}{2m} - U(\mathbf{r}_2)\right]\overline{\Psi}^{+}(\mathbf{r}_2, t_2) &= \int d\mathbf{r}_1\, dt_1 \overline{\Psi}^{+}(\mathbf{r}_1, t_1)\Sigma^{+}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2).\end{aligned} \tag{13}$$

Here Σ^{-} and Σ^{+} are the mass operators that cause a change in the states of the system through the coherent processes. Notice that if $\Sigma^{\pm}$ are local operators, then they can be related to the optical potential. To take account of the coherent processes, we proceed in the same way as we did in the derivation of Eq. (1), i.e., we add randomly distributed currents $j^{-}(\mathbf{r}_1,t_1)$ and $j^{+}(\mathbf{r}_2,t_2)$ to the right-hand sides of (13), assuming that $\overline{j}^{-} = \overline{j}^{+} = 0$. Then we can derive for the functions $g^{<}$ and $g^{>}$ equations for the formulation of which we use the matrix form:

$$g^{<} = g^{+}\Sigma^{<}g^{-}, \quad g^{>} = g^{+}\Sigma^{>}g^{-}, \tag{14}$$

where

$$\begin{aligned}\Sigma^{<}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) &= \mp i\overline{\langle j^{+}(\mathbf{r}_2, t_2)j(\mathbf{r}_1, t_1)\rangle}, \\ \Sigma^{>}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) &= -i\overline{\langle j(\mathbf{r}_1, t_1)j^{+}(\mathbf{r}_2, t_2)\rangle},\end{aligned} \tag{15}$$

while g^{+} and g^{-} are distribution functions satisfying the equations

$$g^{\pm} = g_0^{\pm} + g_0^{\pm}\Sigma^{\pm}g^{\pm}. \tag{16}$$

Here g_0 is the Green function describing the collisionless motion of the particle in the field U:

$$\begin{aligned}\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(\mathbf{r}_1)\right]g_0^{-}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) &= \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(t_1 - t_2), \\ \left[-i\frac{\partial}{\partial t_2} + \frac{\nabla_2^2}{2m} - U(\mathbf{r}_2)\right]g_0^{+}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) &= \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(t_1 - t_2).\end{aligned} \tag{17}$$

Thus, we have obtained for the description of the incoherent processes four equations (14) and (16) that can be solved once we have found a way of computing the mass operators $\Sigma^{\pm}$ and $\Sigma^{\gtrless}$. We use for this purpose the diagrammatic technique developed in the theory of many-particle systems (see, for example, Refs. 8–10). In this case only two of the four Green functions in Eqs. (14) and (16) are linearly independent, since the functions $g^{\pm}$ and $g^{\gtrless}$ are connected by the relation

$$\begin{aligned}g^{\pm}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = \pm\eta_{\pm}(t_1 - t_2)\,[&g^{>}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) \\ &- g^{<}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2)],\end{aligned} \tag{18}$$

where $\eta_{\pm}(t) = 1 \pm t/|t|$. The single-particle character of the initial conditions allows us to justify the applicability of the diagrammatic technique to the computation of the mass operators, since in this case, to compute Σ, it is sufficient to know only the single-particle Green functions, or, in other words, the higher-order Green functions can always be determined in terms of the single-particle functions. Notice that the single-particle character of the initial conditions arises in our case as a result of the separation of the mean field $U(\mathbf{r})$, which is equivalent to the use of the single-particle model to describe the state of the nucleus.

Since we assume that the mean field $U(\mathbf{r})$ of the nucleus does not depend on the time, i.e., since we neglect the reconstruction of the nucleus in the course of the reaction, this implies that the problem being solved by us is a stationary one, i.e., the Green functions and the mass operators depend only on $t_1 - t_2$, and not on $t_1 + t_2$. In this case the cross sections for nuclear reactions can be determined in terms of the asymptotic values of the distribution function (7) for $R \to \infty$ (see, for example, Ref. 12), which is in accord with the experimentally observed cross sections.

3. THE EQUATIONS IN THE QUASICLASSICAL APPROXIMATION AND THE CONDITIONS OF THEIR APPLICABILITY TO THE DESCRIPTION OF NUCLEAR REACTIONS

The use and interpretation of the Dyson equations (14) and (16) in the description of nuclear reactions are a complicated problem. But they can, in the quasiclassical approximation, be related to the kinetic approach, and are therefore a constructive method of describing nuclear reactions. For the kinetic equation to be derivable, it is necessary that the

single-particle operators $(g_0^-)^{-1}$ and $(g_0^+)^{-1}$ exist,[10] a condition which is fulfilled in our case. The scheme for deriving the kinetic equation is the traditional one (see, for example, Refs. 8 and 10):

1) we let Eq. (14) be acted upon first by the operator $(g_0^+)^{-1}$ and then by $(g_0^-)^{-1}$ and use the relations (16);

2) we take the difference between the resulting equations;

3) we go over to the coordinates $\mathbf{R},\mathbf{r},t$ (5);

4) we carry out the quasiclassical series expansions about the point $\mathbf{R}$ of all the quantities entering into the equation obtained, and perform Fourier transformations.

The use of the quasiclassical approximation is equivalent to the following estimate:

$$\int e^{-i\mathbf{pr}/\hbar} a\left(\varepsilon, \mathbf{r}-\mathbf{r}', \mathbf{R}+\frac{\mathbf{r}}{2}\right) b\left(\varepsilon, \mathbf{r}', \mathbf{R}-\frac{\mathbf{r}-\mathbf{r}'}{2}\right) d\mathbf{r}'\, d\mathbf{r}$$
$$\approx a(\varepsilon, \mathbf{p}, \mathbf{R})\, b(\varepsilon, \mathbf{p}, \mathbf{R}) + {}^1/_2 i\hbar \{a(\varepsilon, \mathbf{p}, \mathbf{R}), b(\varepsilon, \mathbf{p}, \mathbf{R})\}, \quad (19)$$

where

$$\{a, b\} = (\nabla_{\mathbf{R}} a)(\nabla_{\mathbf{p}} b) - (\nabla_{\mathbf{p}} a)(\nabla_{\mathbf{R}} b) \quad (20)$$

is the classical Poisson bracket (in the expansion (19) we have explicitly written out the $\hbar$ dependence, so as to emphasize its quasiclassical character, as an expansion in powers of $\hbar$).

In this case we obtain equations that are a self-consistent generalization of the classical Boltzmann equation:

$$\{\varepsilon - p^2/2m - U(\mathbf{R}),\ g^<(\varepsilon, \mathbf{p}, \mathbf{R})\} - \{\mathrm{Re}\,\Sigma(\varepsilon, \mathbf{p}, \mathbf{R}),$$
$$g^<(\varepsilon, \mathbf{p}, \mathbf{R})\}$$
$$+\{\mathrm{Re}\, g(\varepsilon, \mathbf{p}, \mathbf{R}),\ \Sigma^<(\varepsilon, \mathbf{p}, \mathbf{R})\} = -\Sigma^>(\varepsilon, \mathbf{p}, \mathbf{R}) g^<(\varepsilon, \mathbf{p}, \mathbf{R})$$
$$+\Sigma^<(\varepsilon, \mathbf{p}, \mathbf{R}) g^>(\varepsilon, \mathbf{p}, \mathbf{R}), \quad (21a)$$
$$\pm\{\varepsilon - p^2/2m - U(\mathbf{R}),\ g^>(\varepsilon, \mathbf{p}, \mathbf{R})\} \mp \{\mathrm{Re}\,\Sigma(\varepsilon, \mathbf{p}, \mathbf{R}),$$
$$g^>(\varepsilon, \mathbf{p}, \mathbf{R})\}$$
$$\pm\{\mathrm{Re}\, g(\varepsilon, \mathbf{p}, \mathbf{R}),$$
$$\Sigma^>(\varepsilon, \mathbf{p}, \mathbf{R})\} = -\Sigma^>(\varepsilon, \mathbf{p}, \mathbf{R}) g^<(\varepsilon, \mathbf{p}, \mathbf{R})$$
$$+\Sigma^<(\varepsilon, \mathbf{p}, \mathbf{R}) g^>(\varepsilon, \mathbf{p}, \mathbf{R}), \quad (21b)$$

where

$$2\mathrm{Re}\,\Sigma = \Sigma^+ + \Sigma^-, \qquad 2\mathrm{Re}\, g = g^+ + g^-. \quad (22)$$

Let us enumerate the conditions of applicability of the equations (21).

a) The quasiclassical behavior of the mean field:

$$|dU/dR| \gg |d^2U/dR^2|\, \lambda\!\!\!^{-2}; \quad (23)$$

b) the expansion (19) is applicable if the nonlocal term is significantly smaller than the local term, i.e., if

$$\Sigma G \gg (\nabla_{\mathbf{p}}\Sigma)(\nabla_{\mathbf{R}} G), \quad (24)$$

or

$$\frac{\nabla_{\mathbf{p}}\Sigma}{\Sigma}\frac{\nabla_{\mathbf{R}} G}{G} \ll 1.$$

The normal estimates for the factors on the left-hand side of this inequality are as follows:

$$\frac{\nabla_{\mathbf{R}} G}{G} \sim l^{-1}, \quad \frac{\nabla_{\mathbf{p}}\Sigma}{\Sigma} \sim \frac{1}{\Delta p} \sim \lambda\!\!\!^{-},$$

where l is the mean free path of the particle and $\lambda\!\!\!^{-}$ is its wavelength. Then the condition (24) assumes the form

$$l \gg \lambda\!\!\!^{-}.$$

Strictly speaking, the expansion (19) and, accordingly, the equations (21) contain the first terms of the power series expansion in this parameter, i.e., the accuracy of the quasiclassical approximation under consideration together with the condition (23) is determined by the parameter

$$(\lambda\!\!\!^{-}/l)^2 \ll 1. \quad (25)$$

As has already been noted in Sec. 2, the function $g^<(\varepsilon,\mathbf{p},\mathbf{R})$ characterizes the density of the particles with energy ε and momentum $\mathbf{p}$ at the point $\mathbf{R}$, while the function $g^>(\varepsilon,\mathbf{p},\mathbf{R})$ characterizes the density of admissible $(\epsilon,\mathbf{p})$ states at the point $\mathbf{R}$. The quantitites U and $\mathrm{Re}\,\Sigma$ are respectively the energy-independent and energy-dependent real parts of the mass operator characterizing the interaction of the particle with nucleus, with $\mathrm{Re}\,\Sigma$ characterizing the polarization of the nuclear matter during the propagation of the particle in it. The right-hand sides in the equations (21) result from the incoherent collision processes involving the particle and the nucleons of the nucleus. The quantity $\Sigma^>(\varepsilon,\mathbf{p},\mathbf{R})$ characterizes the collision rate for a particle with energy ε and momentum $\mathbf{p}$ at the point $\mathbf{R}$, while $\Sigma^<(\varepsilon,\mathbf{p},\mathbf{R})$ characterizes the probability of transition into the $(\varepsilon,\mathbf{p})$ state, provided it is not occupied.

Since we shall use the diagrammatic technique in $(\mathbf{R},\mathbf{p})$ space to compute the mass operators $\Sigma^>$, $\Sigma^<$, and $\Sigma^\pm$ (Refs. 9 and 10), it follows from the condition (18) that

$$g^+(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) - g^-(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = g^>(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2)$$
$$- g^<(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2). \quad (26)$$

The use of Eqs. (14) and (16) leads to the relation

$$\Sigma^+(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) - \Sigma^-(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = \Sigma^>(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2)$$
$$- \Sigma^<(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2), \quad (27)$$

which, in the Wigner representation (6), is equivalent to

$$2\mathrm{Im}\, g(\varepsilon, \mathbf{p}, \mathbf{R}) = g^>(\varepsilon, \mathbf{p}, \mathbf{R}) \pm g^<(\varepsilon, \mathbf{p}, \mathbf{R}), \quad (28)$$
$$2\mathrm{Im}\,\Sigma(\varepsilon, \mathbf{p}, \mathbf{R}) = \Sigma^>(\varepsilon, \mathbf{p}, \mathbf{R}) \pm \Sigma^<(\varepsilon, \mathbf{p}, \mathbf{R}), \quad (29)$$

while the quantity $\mathrm{Re}\,\Sigma$ is given by the dispersion relation[8]

$$\mathrm{Re}\,\Sigma(\varepsilon, \mathbf{p}, \mathbf{R}) = \int \frac{d\varepsilon'}{2\pi} \frac{2\,\mathrm{Im}\,\Sigma(\varepsilon', \mathbf{p}, \mathbf{R})}{\varepsilon - \varepsilon'}, \quad (30)$$

where the symbol $\int$ denotes the principal value of the integral.

On account of the relation (7), we shall seek the solution to the equations (21) in the form

$$g^<(\varepsilon, \mathbf{p}, \mathbf{R}) = a(\varepsilon, \mathbf{p}, \mathbf{R}) f(\mathbf{p}, \mathbf{R}),$$
$$g^>(\varepsilon, \mathbf{p}, \mathbf{R}) = a(\varepsilon, \mathbf{p}, \mathbf{R})\,[1 \pm f(\mathbf{p}, \mathbf{R})]. \quad (31)$$

Subtracting Eq. (21b) from Eq. (21a), we obtain

$$\{\varepsilon - p^2/2m - U - \mathrm{Re}\,\Sigma,\ a\} + \{\mathrm{Re}\, g,\ 2\mathrm{Im}\,\Sigma\} = 0 \quad (32)$$

The integration of Eq. (32) yields

$$a(\varepsilon, \mathbf{p}, \mathbf{R}) = \frac{2\,\mathrm{Im}\,\Sigma}{(\varepsilon - p^2/2m - U - \mathrm{Re}\,\Sigma)^2 + (\mathrm{Im}\,\Sigma)^2}, \quad (33)$$

$$\mathrm{Re}\, g(\varepsilon, \mathbf{p}, \mathbf{R}) = \frac{\varepsilon - p^2/2m - U - \mathrm{Re}\,\Sigma}{(\varepsilon - p^2/2m - U - \mathrm{Re}\,\Sigma)^2 + (\mathrm{Im}\,\Sigma)^2}. \quad (34)$$

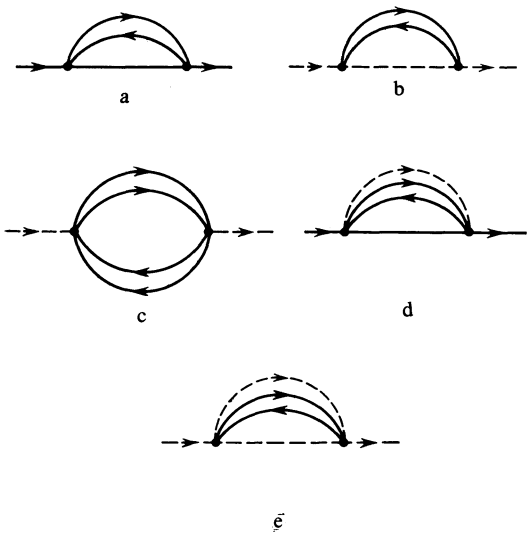

FIG. 1. Diagrams considered in the computation of Σ. a) and b) correspond respectively to elastic nucleon-nucleon and pion-nucleon scattering; c) pion absorption; d) and e) pion production in nucleon-nucleon and pion-nucleon interactions.

The solutions (33) and (34) satisfy the boundary conditions, since $U = \Sigma = 0$ outside the interaction region and the particle is located on the mass surface

$$g^<(\varepsilon, \mathbf{p}, \mathbf{R}) = 2\pi\delta(\varepsilon - p^2/2m) f(\mathbf{p}, \mathbf{R}), \tag{35}$$

and correctly reproduce the ground state of the nucleus within the framework of the single-particle model.

We should, in computing the quantities $\Sigma^{\gtrless}$ in the lowest order in the interactions V_R governing the internucleon collisions, consider a diagram of the type shown in Fig. 1a, in which the lines are associated with the Green functions and the points are associated with the interactions. Then we obtain

$$\begin{aligned}\Sigma^{\gtrless}(\varepsilon, \mathbf{p}, \mathbf{R}) = \int \frac{d\varepsilon'\, d\mathbf{p}'\, d\varepsilon_q\, d\mathbf{q}\, d\varepsilon_q'\, d\mathbf{q}'}{(2\pi)^4 \cdot (2\pi)^4 \cdot (2\pi)^4} \\ \times (2\pi)^4 \cdot \delta(\mathbf{p} + \mathbf{q} - \mathbf{p}' - \mathbf{q}') \\ \times \delta(\varepsilon + \varepsilon_q - \varepsilon' - \varepsilon_q') |V_R(\mathbf{p} - \mathbf{q})|^2 g^{\lessgtr}(\varepsilon_q, \mathbf{q}, \mathbf{R}) \\ \times g^{\gtrless}(\varepsilon', \mathbf{p}', \mathbf{R}) g^{\gtrless}(\varepsilon_q', \mathbf{q}', \mathbf{R}).\end{aligned} \tag{36}$$

If we neglect in the equations (21) the nonlocal terms $\{\mathrm{Re}\,\Sigma, g^{\gtrless}\}$ and $\{\mathrm{Re}\, g, \Sigma\}$, which are due to the polarization of the nuclear matter, then

$$g^<(\varepsilon, \mathbf{p}, \mathbf{R}) = 2\pi\delta(\varepsilon - p^2/2m - U) f(\mathbf{p}, \mathbf{R}). \tag{37}$$

Performing the integration over energy in (21), and using the formulas (36) and (37), we obtain the usual Boltzman equation

$$\begin{aligned}\left\{\frac{p^2}{2m} + U, f(\mathbf{p}, \mathbf{R})\right\} = \int \frac{d\mathbf{p}'\, d\mathbf{q}\, d\mathbf{q}'}{(2\pi)^3 \cdot (2\pi)^3 \cdot (2\pi)^3} \cdot (2\pi)^3 \\ \times \delta(\mathbf{p} + \mathbf{q} - \mathbf{p}' - \mathbf{q}') |V_R(\mathbf{p} - \mathbf{q})|^2 [f(\mathbf{p}, \mathbf{R}) f(\mathbf{q}, \mathbf{R}) \\ \times (1 \pm f(\mathbf{p}', \mathbf{R}))(1 \pm f(\mathbf{q}', \mathbf{R})) \\ - (1 \pm f(\mathbf{p}, \mathbf{R})) f(\mathbf{p}', \mathbf{R})(1 \pm f(\mathbf{q}, \mathbf{R})) f(\mathbf{q}', \mathbf{R})].\end{aligned} \tag{38}$$

A comparison of the generalized Boltzmann equations (21) with the classical equation (38) allows us to better understand their physical meaning. As in the Boltzmann equation, the term $\{U, g^<\}$ describes the motion of the particle along a classical trajectory under the action of the force $(-\nabla_{\mathbf{R}} U(R))$. The term $(-\nabla_{\mathbf{R}} \mathrm{Re}\,\Sigma)(\nabla_{\mathbf{p}} g^<)$ gives rise to an additional force due to the polarization of the medium. The presence of the term $(\nabla_{\mathbf{p}} \mathrm{Re}\,\Sigma)(\nabla_{\mathbf{R}} g^<)$ leads to a change in the coordinate of the particle as a result of the nonlocal character of the interaction.[1)]

The quantum mechanics we cannot uniquely determine the trajectory of the particle; therefore, the propagation of the particle inside the nucleus corresponds to a multiplicity of trajectories. This effect also occurs in the equations (21). The terms $\{\mathrm{Re}\, g, \Sigma^{\gtrless}\}$ lead to the departure of the particle from a given trajectory and the appearance of it on other trajectories (i.e., the coordinate and momentum of the particle change), these trajectories occupying in terms of the coordinates a phase volume characterized by the wavelength $\lambda\!\!\!^{-}$. Naturally, we can speak of a particle trajectory between collisions only in the case when the mean free path is significantly longer than the wavelength, as required by the condition (25).

It should be noted that since in Eq. (21) the terms proportional to $(\lambda\!\!\!^{-}/l)^2$ are not considered, it is sufficient, in computing Σ, to restrict ourselves to the consideration of the diagrams of the type shown in Fig. 1, and treat the interactions described by the interparticle collisions as free interactions. Allowance for the higher-order diagrams is equivalent to the analysis of the corrections of the next order in the parameter $\lambda\!\!\!^{-}/l$, and would mean our going beyond the degree of accuracy of the equations (21).

In Ref. 13 an algorithm is formulated for solving the generalized Boltzmann equations describing the incoherent interactions of a particle with the nucleons of a nucleus. This algorithm contains quantum-mechanical effects that play an important role in the description of the process of propagation of the particle inside the nucleus, and allows us to take into account the nonlocal properties, i.e., the dependence on the energy and momentum of the mass operator characterizing the interaction of the particle with the nucleons of the nucleus. The proposed algorithm uses the method of random trials, and possesses the same property of universality as the model of intranuclear cascades. This allows us to describe diverse many-particle reactions within its framework, compute the excitation energy brought into the nucleus in the forward phase of the reaction, and, consequently, combine the proposed method with the models describing the decay of the excited nucleus.

The mass operators are computed by the method of successive approximations. Let us set $\mathrm{Re}\,\Sigma = \mathrm{Im}\,\Sigma = 0$ in the expression (33), and compute $\Sigma^{\gtrless}$, approximating the interactions $|V(\mathbf{p} - \mathbf{q})|^2$ with the aid of the cross sections for interaction between free particles (in the spirit of Ref. 10). Using the relations (29) and (30), we find $\mathrm{Im}\,\Sigma$ and $\mathrm{Re}\,\Sigma$. Then substituting them into (36), we compute new values for $\Sigma^{\gtrless}$ and, consequently, for $\mathrm{Im}\,\Sigma$ and $\mathrm{Re}\,\Sigma$. Such a procedure converges after four-to-five iterations. The multiple integrals in the formula (36) were evaluated by the Monte-Carlo method, and the computational error was not higher than 5%. Notice that, since, in using the cross sections for interaction between free particles, we consider the relativistic

kinematics, we must also use in place of the expression (33) the relativistic formula

$$a=2\ \mathrm{Im}\ \Sigma[(\varepsilon-m-(p^2+m^2)^{1/2}-U-\mathrm{Re}\ \Sigma)^2+(\mathrm{Im}\ \Sigma)^2]^{-1}. \quad (39)$$

4. COMPUTATION OF THE NUCLEON AND PION MEAN FREE PATHS IN NUCLEI. ACCURACY OF THE METHOD

The dependence on the energy and momentum of the mass operator characterizing the interaction of the particle with the nucleus leads to the renormalizaton of the nucleon mass and to a change in the mean free path (MFP) of the particle inside the nucleus, as compared with the usual estimate used in the solution of the Boltzmann equation[14]:

$$l=P/\rho\sigma; \quad (40)$$

here ρ is the nuclear matter density, σ is the total cross section for interaction of the particle with the nucleons of the nucleus, and P is a factor taking account of the Pauli principle. Since the approach expounded here allows us to take these effects into account, it is of interest to carry out the computation of the values of the MFP within the framework of the quasiclassical approximation in a self-consistent fashion, and compare it with the values extracted from the data on the elastic scattering of protons by nuclei. For such a comparison, we must compute the MFP values corresponding to the propagation of a particle in a nucleus whose distribution density is characterized by the function $f(\mathbf{p},\mathbf{R})$. Since in the classical limit the function $f(\mathbf{p},\mathbf{R})$, given by the expression (7), goes over into the classical distribution function, the MFP $l(\mathbf{p},\mathbf{R})$ of the particle can be determined from the relation

$$\frac{v}{l(\mathbf{p},\mathbf{R})}=\int\frac{d\varepsilon}{2\pi}a(\varepsilon,\mathbf{p},\mathbf{R})\Sigma^{>}(\varepsilon,\mathbf{p},\mathbf{R}), \quad v=\frac{p}{(p^2+m^2)^{1/2}}. \quad (41)$$

In the region of intermediate energies (from hundreds of MeV to several GeV), it is sufficient to restrict ourselves to a simultaneous description of nucleons and pions. Therefore, in computing the mass operators and, consequently, the MFP, we took the following processes into account: the elastic nucleon-nucleon and pion-nucleon scattering and pion production and absorption by a pair of nucleons. The corresponding diagrams are shown in Fig. 1. The contribution of the pion absorption process is taken into consideration by approximating the interaction with the aid of the cross section for pion absorption by the deuteron, and the probability for finding a pair correlation inside the nucleus is related to the cross section for absorption in γ quanta by nuclei.[14] In computing the diagrams that take account of the pion rescattering (Fig. 1b), we also take the charge-exchange process involving pions into consideration.

The results of the MFP calculations are shown in Figs. 2 and 3. It can be seen that the MFP values obtained in the self-consistent calculation in the central regions of the nucleus are two-to-three times higher than the estimates obtained from the formula (40). Figure 3 shows the nucleon-density dependence of the coefficient characterizing the increase in the MFP given by the expression (41) in comparison with the estimate (40), which corresponds to the use for Σ in (41) of the zeroth approximation in the method of computing the mass operators. It follows from Fig. 3 that no significant increase

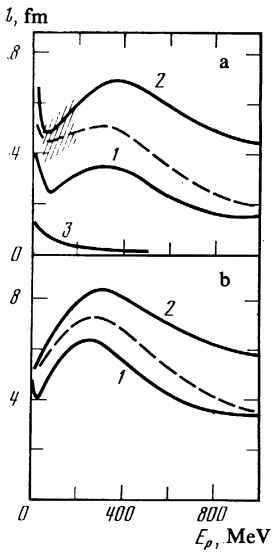

FIG. 2. Dependence of the mean free path (MFP) of protons in the central region of the nucleus (a) and in the case when the density $\rho=\rho_0/2$ (b) on their energy. The hatched region corresponds to data obtained in an experimental analysis.[15] 1) Classical MFP estimates made with the use of the free cross section and with allowance for the Pauli principle (i.e., they correspond to the zeroth approximation); 2) result of the self-consistent MFP calculation; 3) the de Broglie wavelength; the dashed curves indicate the MFP values corresponding to the quasiclassical approximation to the optical model.

in the MFP occurs at the surface of the nucleus.

The calculations carried out allow us to estimate the accuracy, determined by the condition (24), of the proposed method of describing nuclear reactions. Since $\lambda\!\!\!^{-}/l\approx 0.25$ in the worst case for pions in the (3,3) resonance region, the equations (21) are accurate to within 10%. For nucleons and pions with energy higher than 50 MeV, the error connected with the condition (23) does not exceed 10–20%. Thus, in the intermediate nucleon- and pion-energy region the proposed approach to the description of nuclear reaction is accurate to within 20–30%; the remaining errors here should be ex-

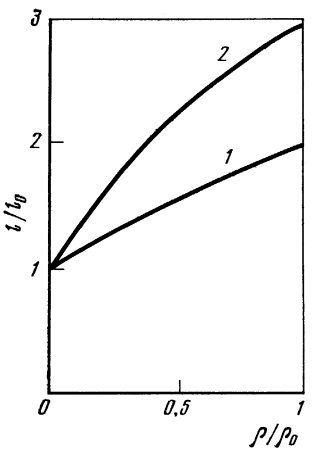

FIG. 3. Factor characterizing the excess of the MFP computed in a self-consistent manner in the quasiclassical approximation over the classical MFP value as a function of the nucleon density in the nucleus for nucleons (1) and pions (2) with energy 200 MeV.

plained by deficiencies in the nuclear model, or in the reaction mechanism. As to the accuracy of the Boltzmann equation, it is characterized by a factor of 2.

5. DESCRIPTION OF THE INELASTIC PION-NUCLEUS INTERACTIONS

Analysis of the data on elastic pion scattering by nuclei convinces us that, first, the effects contained in the terms $\{\mathrm{Re}\, g, \Sigma^{\geq}\}$ and $\mathrm{Re}\, \Sigma, g^{\geq}\}$ in the generalized Boltzmann equation (GBE), (21), should appear in the description of the propagation of pions in a nucleus, i.e., should manifest themselves in pion-nucleus interactions, and, secondly, it is precisely these reactions that are the most critical for the verification of the approach, proposed in the present paper, to the description of nuclear reactions.

We shall focus our attention on the description of the inclusive processes of inelastic scattering $\pi + {}^{z}A \to \pi' + X$, or pion absorption $\pi + {}^{z}A \to$ absence of $\pi + X$, as well as the description of exclusive reactions of the type $(\pi,\pi' N)$. We took the diagrams shown in Fig. 1 into account when computing the mass operators. Notice that, since our computations are based on the use of the Monte-Carlo method, they contain statistical errors. In those cases where they are not indicated in the comparison with the experimental data, it means that the errors do not exceed 5%. No free parameters were used in the calculations, and all the cross sections are given in absolute units. The calculations are presented in order of increasing degree of detailedness of the experimental data: the analysis begins with the description of the total cross sections for inelastic interaction of pions with nuclei (reaction cross sections) and ends with the description of the double differential characteristics.

Figure 4 shows the experimental and computed energy dependences of the cross sections for inelastic interaction of pions with the ^{12}C nucleus. It can be seen that the calculation performed within the framework of our approach (i.e., of the GBE) reproduces both the magnitude of the cross sections and the position of the peak. This means that the proposed approach allows us to reproduce correctly both the imaginary and the real parts of the mass operator Σ. The calculation performed in the solution of the Boltzmann equation within the framework of the method of intranuclear cascades (MIC), i.e., without allowance for the terms in the GBE that take account of the polarization of the nuclear matter in the presence of pions, does not reproduce the peak's position, which in this approach coincides with the position of the peak in the pion-nucleon interaction, this peak occurring at an energy of 180 MeV. This circumstance has made necessary the introduction in MIC calculations of an energy-independent pion-nuclear potential in the form of a square well of depth 25 MeV (Refs. 14 and 17). It is clear that the introduction of such a free parameter, which is inconsistent with the experimental data on elastic scattering, is incorrect, if only because the result obtained in the MIC without the introduction of this parameter is in agreement with experiment if we taken into consideration the accuracy, discussed in the preceding section, of the quasiclassical approximation when it is used to describe this type of reaction.

Thus, we verify in the particular case of the most integrated characteristic of the pion-nucleus interactions the necessity of a self-consistent computation of the real and imaginary parts of the mass operator and the fact that it is incorrect to neglect any terms in Eq. (21). Let us note that our approach allows us to reproduce as well the cross sections for pion absorption on the basis of the quasideuteron mechanism.

The most critical for the verification of the theory are data in which an important role is played by the multiple interaction of the pion with the nucleons of the nucleus, and in which the correct description of the scattering in the central regon of the nucleus, where the effect of the polarization of the nuclear matter is strongest, is important. An example of such a process is the double charge-exchange process involving the π meson. Unfortunately, this process has been experimentally studied only on the nuclei of photoemulsions, where it is difficult to uniquely establish the identity of the target nucleus, but the trend of the energy dependence of the double charge exchange cross sections can be verified.

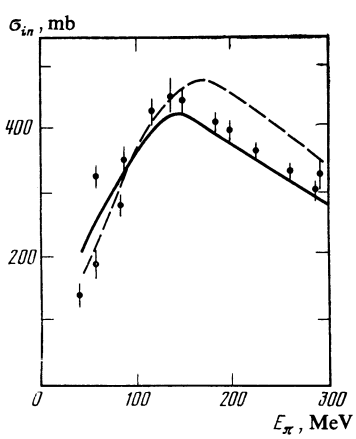

FIG. 4. Dependence of the cross section for inelastic interaction of pions with the ^{12}C nucleus on the energy of the initial particle. The points represent experimental data[16]; the continuous curve is the result of a GBE calculation; the dashed curve, the result of an MIC calculation.

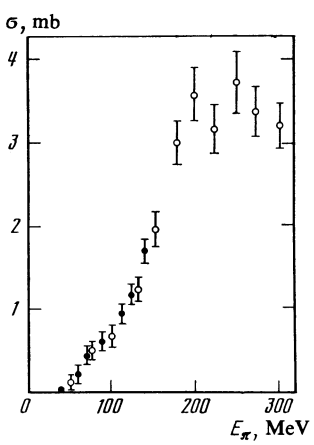

FIG. 5. Dependence of the cross sections for the double charge transfer (π^+,π^-) on the π^+-meson energy. The closed circles represent experimental data for emulsion nuclei[18a]; the open circles, the results of a GBE calculation.

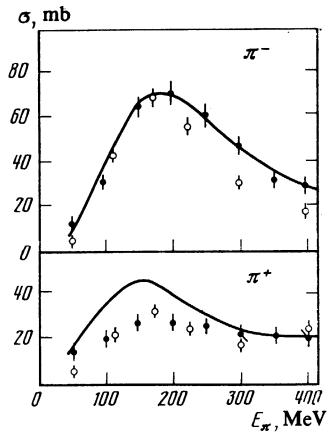

FIG. 6. Dependence of cross section for the reaction $^{12}C(\pi^+,\pi'N)^{11}C$, involving the quasielastic knocking out of a nucleon, on the energy of the initial pion. The curve is a plot of experimental data taken from Ref. 19a; the closed circles are the result of a GBE calculation; the open circles, the result of an MIC calculation (a version of the calculation performed in Ref. 17a).

Figure 5 shows the computed and experimental cross sections for the process (π^+,π^-). The calculation was performed for the ^{12}C nuclei. It can be seen that the calculation reproduces well the growth of the cross section for this process as the energy of the initial pion increases, and predicts the saturation of this dependence at an initial energy of 200 MeV. Mischke *et al.*[18b] have measured the cross section for double charge exchange in the reaction $^{16}O(\pi^+,\pi^-)$ with $E_{\pi^+} = 240$ MeV. The experimental cross section ($\sigma_{exp} = 5.8 \pm 0.9$ mb) for this process is also fairly well reproduced in the calculation performed within the framework of the solution of the GBE ($\sigma_{calc} = 4.8 \pm 0.6$ mb).

Figure 6 shows the yield of the ^{11}C isotope in the reaction $^{12}C(\pi^+,\pi'N)^{11}C$ as a function of the energy of the initial pion. In computing the cross section for this process together with the forward reactions, we must also take into consideration the excited-nucleus decay process.[14] We see that the calculation with the use of the GBE for the description of the forward phase reproduces not only the magnitude of the cross sections in the case of the interaction of the π^- mesons, but also the curve width, which is not reproduced in cascade-evaporation model calculations (the results for this model calculation were taken from Ref. 19a, and correspond to that version of the MIC which is described in Ref. 17a). In Ref. 19a this circumstance is related to the more important role played by the central regions of the nucleus in processes of this sort as compared with the role realized in cascade calculations, since the higher Fermi momenta corresponding to the higher nucleon densities lead to a greater degree of smearing of the energy dependence of the πN-interaction cross section. Indeed, as we saw in Sec. 4, the self-consistent procedure for computing pion mean free paths in a nucleus yields larger values in comparison with the values used in cascade calculations, which leads to a situation in which the central regions of the nucleus play the major role.

The discrepancy between the calculations and experiment in all the approaches to the description of the reaction with π^+ mesons is apparently due to the major role played by the coherent processes of charge transfer from a proton to a neutron. The coherence is attested by the good description of the analogous reaction on gold nuclei for both π^+ and π^- mesons (Table I). At the same time, let us note that our approach also describes well the angular distributions of the π^+ mesons in the reaction (π^+,π^+p) (Fig. 7), which indicates a correct description of the initial phase of the pion-nucleus interaction process. Analysis of the reaction $(\pi,\pi p)$ within the framework of the dispersion approach[19c] shows that the greatest contribution to its cross section is made by the pole mechanism and the mechanism of decay of the excited nucleus following the inelastic scattering of the pion. Both of these mechanisms are contained in our approach, which also takes into account in a natural way the so-called high-momentum component "cutoff" effect occurring in the spectrum of the outgoing pions, and connected with the necessity of the consideration of the proton binding energy in the nucleus.

A comparison of the computed double differential distributions of the pions produced in the charge-transfer reaction $^{16}O(\pi^+,\pi^0)X$ with experiment shows that the GBE describes such a process well, whereas the MIC provides a significantly softer energy distribution for the pions (Fig. 8), which indicates a greater role for the rescattering process in the latter approach.

Thus, the analysis performed shows that the approach proposed in the paper allows us to describe the experimental data without the introduction of adjustable parameters, this description being better than the description within the framework of the approaches based on the solution of the Boltzmann equation. It is important to emphasize that, on

TABLE I. Cross sections for the reaction $^{197}Au(\pi^\pm,\pi'N)^{196}Au$, involving the quasielastic knocking out of a nucleon, for different initial-pion energies. The experimental data and the results of the computations performed within the framework of the MIC (a version of Ref. 17a) were taken from Ref. 19b). The cross sections are given in mb. The measurement and computational errors are equal to 10%.

E_π, MeV	$\sigma(\pi+,\pi'N)$			$\sigma(\pi-,\pi'N)$		
	exp.	MIC	GBE	exp.	MIC	GBE
100	68	75	66	130	210	149
180	79	65	73	181	170	188
300	61	55	63	107	80	97

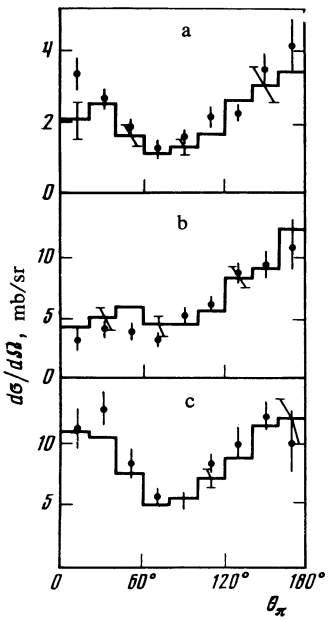

FIG. 7. Angular distributions of the secondary pions produced in the quasielastic knocking-out reaction $^{12}C(\pi^+,\pi^+p)$ for different initial-particle energies E_π: a) $E_\pi = 60$ MeV; b) $E_\pi = 112$ MeV; and c) $E_\pi = 170$ MeV. The points represent the experimental data given in Ref. 19c; the histograms are the results of GBE calculations.

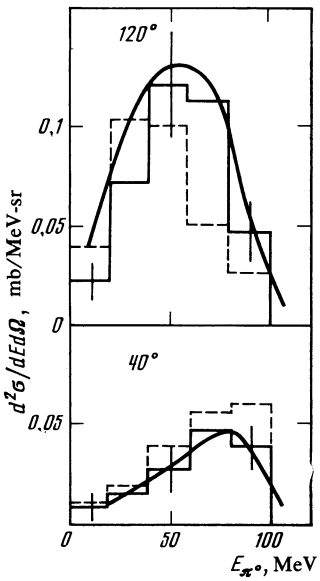

FIG. 8. Double differential distributions of the π^0 mesons produced in the charge-transfer reaction $^{16}O(\pi^+,\pi^0)X$ in the case of an initial-pion energy of $E_{\pi^+} = 100$ MeV. The curves are experimental curves[20]; the histograms are the results of GBE (continuous histograms) and MIC (dashed ones) calculations.

the whole, both approaches reproduce the experimental data within the limits of the accuracies determined by the conditions of their applicability (see Sec. 4).

6. DESCRIPTION OF PION AND PROTON PRODUCTION ON NUCLEI BY PROTONS OF INTERMEDIATE ENERGIES

An important criterion for the validity of a given approach to the description of nuclear reactions with nucleons of intermediate energies is the successful description of the processes of pion production on nuclei. In this case we must correctly take into account both the pion production in the nucleon-nucleon collisions and the propagation of the pions in the nuclear matter. In this case a pion may be created in a state lying outside the mass shell, and the correct description of its propagation in the nucleus and all possible rescatterings by the nucleons of the nucleus requires a quantum-mechanical treatment.

The results of the application of the generalized Boltzmann equation to the description of the process of pion production in proton-nucleus collisions are presented in Table II. The double differential distributions of secondary pions (Fig. 9) are also well reproduced by our calculations. A comparison with the MIC calculations performed with the same parameters used in the solution of the GBE (21), i.e., without allowance for the nonlocal effects in the mass operator and its dependence on energy, shows that the MIC yields lower values for the energy of the secondary pions, as compared with the values given by the GBE; this is especially noticeable in the case of π^- mesons. It is important to emphasize that both the GBE and the MIC reproduce the experimental characteristics with an accuracies equal to the accuracies determined by the conditions of applicability of these quasiclassical approximations (up to a factor of 2 for the MIC and to within 30% for the GBE).

In conclusion of this section, let us consider within the

TABLE II. Cross sections (in mb) for pion production in proton-nucleus interactions for different proton energies E_p.

E_p, MeV			Target			
			C	Al	Cu	Pb
585	π^+	exp.[21a]	28.5±3.5	43.8±5.4	60±7.4	86±11
		theory	31.2±0.6	48.0±1.1	68±3.1	98±2.5
	π^-	exp.[21a]	4.72±0.58	9.8±1.2	18±2.2	41.5±5.1
		theory	5.3±0.2	9.0±0.6	21.4±1.5	47.4±1.7
660	π^0	exp.[21b]	27.3±1.5	45.9±2.6	73.4±4.2	143±8.0
		theory	26.0±1.5	44±2	74±4	155±7
730	π^+	exp.[21c]	35.0±1.8	53.1±2.9	77.3±4.3	104.2±5.8
		theory	40.1±2.1	59±3	86±4.5	118±6
	π^-	exp.[21c]	6.6±0.4	13.2±0.09	25.2±2.0	53.7±4.9
		theory	7.8±0.9	16.1±1.6	28±2.5	58±6

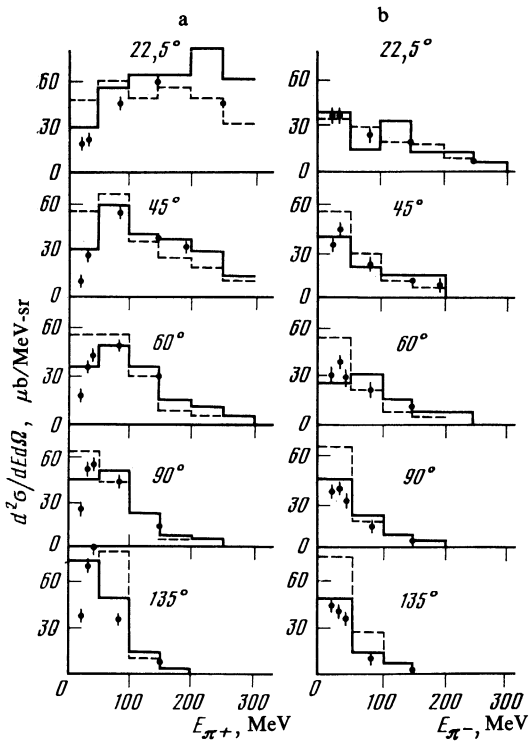

FIG. 9. Double differential distributions of the $\pi^{\pm}$ mesons produced in the interaction of 585-MeV protons with lead nuclei. The points represent the experimental data given in Ref. 21a; the histograms are the results of GBE (the continuous histograms) and MIC (the dashed ones) calculations.

framework of the proposed method one of the problems that are of interest for nuclear physics. There occurs in nuclear reactions with particles of intermediate energy intensive production of fast particles, called cumulative particles, that fly out into the region kinematically forbidden in the case of scattering on a free nucleon. A comparison of the results of the calculations with experiment shows that allowance for the multiple particle-rescattering processes, including the pion production processes, enables us to satisfactorily explain the main inclusive characteristics of the spectra of the secondary nucleons produced during the interaction of protons of energy 1 GeV with nuclei[22] (Fig. 10). It should be

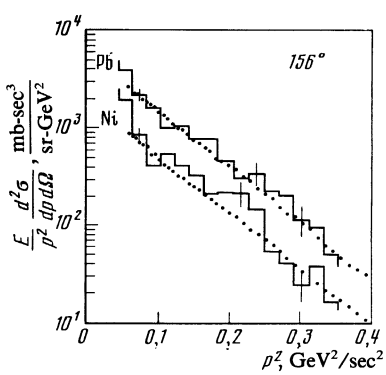

FIG. 10. Inclusive distributions of the cumulative protons produced in the interaction of protons of energy 1 GeV with nuclei. The points represent experimental data[22a]; the histograms, the results of computations.

noted that this simple mechanism gives a good account of so important a characteristic of the cumulative-nucleon production process as the ratio of the neutron yield to the proton yield (let us make a comparison for the carbon nucleus[22b]:

$$W_n/W_p|_{\text{theor}}=0.66\pm0.06;\ W_n/W_p|_{\text{exp}}=0.71\pm0.18).$$

Furthermore, the experimentally observed irregularities in the cumulative-proton spectra for light nuclei (of the Li, Be, or C type) are also reproduced in a calculation.[22] Notice that in our approach these characteristics are connected with the resonance character of the production of a pion in a nucleon-nucleon collision and its subsequent absorption by the nucleus. It is possible that this mechanism is universal also for deep-inelastic processes involving higher initial-particle energies.[23]

7. CONCLUSION

Let us enumerate the main results of the paper. We have formulated for the description of nuclear reactions an approach based on the solution of the Dyson equations for the single-particle double-time Green functions and the separation of the particle-nucleus interaction process into coherent and incoherent parts. It is shown that in the quasiclassical approximation, i.e., in the case when the man free path l is significantly longer than the particle wavelength $\lambda\!\!\!^{-}$, this approach admits of a kinetic interpretation, and allows us to analyze the quantum effects as corrections in the quasiclassical parameter $\lambda\!\!\!^{-}/l$. In this case, to describe nuclear reactions, we can use generalized Boltzmann equations for the Green functions, these equations being a self-consistent generalization of the classical kinetic equation for the distribution function.

Two important circumstances should be noted. First, the neglect of the $\{\mathrm{Re}\, g, \Sigma^{<}\}$ terms in Eq. (21), while the $\{\mathrm{Re}\, \Sigma, g^{<}\}$ terms are retained, leads to the violation of the energy conservation law.[9] Therefore, the description of nuclear reactions in the quasiclassical approaches that use the energy dependence of the real part of the optical potential, but neglect the imaginary-part-related effects of the descent from the mass surface in the spectral function $a(\varepsilon,\mathbf{p},\mathbf{R})$ (as is done in Ref. 17), though leads to an improvement in the accuracy of the Boltzmann equation, can be admissible only in the description of inclusive reactions of the type (N,N') and (π,π') (although it can lead to errors in the determination of such important characteristics of nuclear reactions as the excitation energies of the nuclei), or in the description of exclusive processes. Secondly, in the description of surface nuclear reactions or reactions with nucleons with energy $\gtrsim 500$ MeV, the lowest quasiclassical approximation, namely, the Boltzmann equation, is also a fairly good approximation.

We have seen that an important role is played in the description of the pion-nucleus interactions, as well as of the processes of pion production by nucleons of intermediate energies, by the dependence on the energy and momentum of the mass operator characterizing the particle-nucleus interaction. The proposed quasiclassical approach, which takes these effects into account, allowed us to correctly reproduce

so fundamental a characteristic of the propagation of a particle in a nucleus as the particle's mean free path, and to establish the accuracies of the various quasiclassical approaches (we have in mind the approximations of the lowest and next order in the parameter $\lambda\!\!\!^{-}/l$ [i.e., the Boltzmann equation and Eq. (21) respectively)]. A comparison of the calculations with the experimental pion-nucleus interaction cross sections and cross sections for pion production in proton-nucleus interactions shows that, on the whole, within the limits of the indicated accuracy, the two methods correctly describe the entire set of experimental data. This circumstance attests the validity of the quasiclassical approximation to the Dyson equations in the description of nuclear reactions. Thus, we have a method of describing the inelastic pion- and nucleon-nucleus interactions that does not contain any free parameters. An approach based on such a method is universal enough for the analysis of the role of the various mechanisms underlying nuclear reactions. In particular, it allowed us to demonstrate the important role played by the polarization of the nuclear matter in the interaction of pions with nuclei.

Naturally, the proposed approach can also be used to describe the interaction with nuclei of other particles possessing sufficiently long mean free paths in the nuclei. Thus, we can, for example, hope for a successful description of the interactions of K mesons with nuclei within the framework of such an approach.

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[1)]Let us note that, since the Green function is not a particle distribution function, here the term "particle," like the statements regarding its propagation and collisions inside the nucleus, should be understood conditionally. This terminology is convenient, since, owing to the condition (37), the transition from the Green function to the distribution function is trivial outside the interaction region.

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