"Explosive" dynamics of the turbulence spectrum in the case of decay interactions between waves

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We show that in the initial value problem decay processes may lead to a self-accelerated (explosive) transfer of wave energy to the large wave number region up to the absorption region. Such a picture is particularly characteristic for capillary waves on a liquid surface and for electron waves in a cold magneto-active plasma. It is noteworthy that the description of the explosive transfer does not require us to go beyond the framework of the weak turbulence approximation.

1. INTRODUCTION

Induced scattering of waves by particles and the three-wave decay interaction are the main nonlinear processes in a weakly turbulent plasma. As far as induced scattering is concerned it is well known that it leads to a transfer of waves to the low frequency region (see, for instance, Ref. 1). When the frequency decreases the characteristic wavenumber usually also decreases. In the long-wavelength part of the spectrum a condensate can then be formed and it is necessary to go beyond the framework of the weak turbulence approximation to describe this. In particular, weak turbulence theory does not allow us to solve the problem of the dissipation of the energy of the condensate. This difficulty arises, for instance, when we consider the Langmuir condensate in a plasma without a magnetic field.

In conditions when decay processes dominate the nature of the energy redistribution over the spectrum is, generally speaking, not so universal as in the case of induced scattering. We show in what follows that in many cases of practical interest decay and fusion of waves lead to an energy transfer not to the long-wavelength but, to the contrary, to the short-wavelength region up to the absorption region. This transfer can quantitatively be described in the framework of weak turbulence theory. An interesting feature of the decay interaction is, as we shall see, that the evolution of the initial wave spectrum may be explosive. The energy is then transferred to the dissipation region in a time which is approximately equal to the time for the nonlinear interaction in the initial spectrum.

We turn to the kinetic equation which describes the decay and fusion of waves with a dispersion law $\omega(\mathbf{k})$:

$$\begin{aligned}\frac{\partial n_{\mathbf{k}}}{\partial t}=&\int w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}\delta(\omega-\omega_1-\omega_2)\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\\&\times[n_{\mathbf{k}_1}n_{\mathbf{k}_2}(n_{\mathbf{k}}+1)-n_{\mathbf{k}}(n_{\mathbf{k}_1}+1)(n_{\mathbf{k}_2}+1)]\frac{d\mathbf{k}_1\,d\mathbf{k}_2}{(2\pi)^6}\\&+2\int w_{\mathbf{k}_1\mathbf{k}\mathbf{k}_2}\delta(\omega_1-\omega-\omega_2)\,\delta(\mathbf{k}_1-\mathbf{k}-\mathbf{k}_2)[n_{\mathbf{k}_1}(n_{\mathbf{k}}+1)(n_{\mathbf{k}_2}+1)\\&-n_{\mathbf{k}}n_{\mathbf{k}_2}(n_{\mathbf{k}_1}+1)]\frac{d\mathbf{k}_1\,d\mathbf{k}_2}{(2\pi)^6}.\end{aligned}\tag{1}$$

It follows from this equation that the interaction leads to an increase in the entropy of the waves[2]:

$$\frac{d}{dt}\int[(n_{\mathbf{k}}+1)\ln(n_{\mathbf{k}}+1)-n_{\mathbf{k}}\ln n_{\mathbf{k}}]\,d\mathbf{k}\geqslant 0.$$

When there is no pumping or damping the only stationary solution of Eq. (1) is the Planck distribution

$$n_{\mathbf{k}}=[\exp(\hbar\omega_{\mathbf{k}}/T)-1]^{-1},\tag{2}$$

and, when we neglect spontaneous processes, the Rayleigh-Jeans distribution

$$n_{\mathbf{k}}=T/\hbar\omega_{\mathbf{k}}.\tag{3}$$

The equilibrium temperature of the waves T is determined by their initial energy density

$$W_0=\int\frac{\hbar\omega_{\mathbf{k}}}{\exp(\hbar\omega_{\mathbf{k}}/T)-1}\frac{d\mathbf{k}}{(2\pi)^3}.\tag{4}$$

In particular, when the dispersion has a power law dependence $\omega(k)=ak^\gamma$ the temperature and characteristic wavenumber $\bar{k}$ are approximately equal to

$$T\sim\hbar a(W_0/\hbar a)^{\gamma/(3+\gamma)},\quad \bar{k}\sim(W_0/\hbar a)^{1/(3+\gamma)}.$$

From these expressions it is clear that the quantity $\bar{k}$ tends to infinity when we go to the classical limit ($\hbar\to 0$). In fact, however, the increase in $\bar{k}$ is limited by dissipative effects which have not been taken into account in Eq. (1) and the role of which usually increases with decreasing wavelength. The decay interaction, by transferring waves to the large k region leads thus in final reckoning to the absorption of the energy delivered to the plasma. The presence of this absorption does not allow the spectrum (2) to be established.

2. CAPILLARY WAVES

The simplest example which enables us to follow the dynamics of the spectrum when decay interactions are important is provided by capillary waves on a liquid surface. For these waves

$$\omega(k)=(\alpha k^3)^{1/2},\tag{5}$$

and the decay probability is equal to[1)] (Ref. 3)

$$\begin{aligned}w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}=\pi^5(\alpha/4)^{1/2}\{&(kk_1/k_2)^{1/4}[(k-k_1)^2-k_2^2]\\&+[(k-k_2)^2-k_1^2](kk_2/k_1)^{1/4}\\&-[(k_1-k_2)^2-k^2](k_1k_2/k)^{1/4}\}^2.\end{aligned}\tag{6}$$

Here $\mathbf{k},\mathbf{k}_1,\mathbf{k}_2$ are two-dimensional wavevectors and α is the ratio of the surface tension coefficient to the liquid density. If

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we neglect spontaneous processes the kinetic equation for the waves has the form

$$\frac{\partial n_{\mathbf{k}}}{\partial t}=\int w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\delta(\omega-\omega_1-\omega_2)\,(n_{\mathbf{k}_1}n_{\mathbf{k}_2}-n_{\mathbf{k}}n_{\mathbf{k}_1}-n_{\mathbf{k}}n_{\mathbf{k}_2})\frac{d\mathbf{k}_1\,d\mathbf{k}_2}{(2\pi)^6}+2\int w_{\mathbf{k}_1\mathbf{k}\mathbf{k}_2}\delta(\mathbf{k}_1-\mathbf{k}-\mathbf{k}_2)\,\delta(\omega_1-\omega-\omega_2)\,(n_{\mathbf{k}}n_{\mathbf{k}_1}+n_{\mathbf{k}_1}n_{\mathbf{k}_2}-n_{\mathbf{k}}n_{\mathbf{k}_2})\frac{d\mathbf{k}_1\,d\mathbf{k}_2}{(2\pi)^6}. \tag{7}$$

It follows from Eqs. (6) and (7) that the nonlinear interaction time τ is inversely proportional to the energy density and to the characteristic frequency of the waves:

$$\tau\propto(\bar{k}^{3/2}W_0)^{-1}. \tag{8}$$

Since the energy density is conserved in the initial-value problem, Eq. (8) shows that the time for spectral transfer decreases with increasing characteristic wavenumber $\bar{k}$. In other words, the process of energy transfer to the short-wavelength region is self-accelerating. We note that the decrease of the characteristic interaction time with increasing average wavenumber does not violate the conditions for the applicability of the random phase approximation as for a sufficiently broad spectrum ($\delta\omega\sim\omega$) the product $\tau\delta\omega$ remains constant [see Eq. (8)]. It is thus sufficient to require that the condition for fast phase mixing $\tau\delta\omega\gg 1$ be satisfied initially. The estimate (8) is a very rough one since, as we shall see in what follows, in the initial stage of the evolution of the spectrum not the whole energy of the waves is involved in the self-accelerated transfer process. Nonetheless the qualitative conclusion that the larger part of the wave energy lies after a finite time in the region of arbitrarily large wavenumbers remains correct also when we give a more correct discussion.

It is not possible to find the complete solution of the problem of the evolution of an initial wave distribution; one can, however, find the short-wavelength asymptotic behavior of the solution which is of interest to us and describe qualitatively the time-dependence of the spectrum. We assume that the initial spectrum is isotropic and that the main part of the energy in it corresponds to waves with wavenumbers of the order k_0. We estimate the characteristic times for the rearrangement of various segments of the spectrum. The interaction time for waves of the main scale k_0 with one another is approximately equal to

$$\tau_0\sim 1/k_0^5 n(k_0). \tag{9}$$

We now split off in the spectrum the group of short wavelength waves with $k_1\gg k_0$. The time for interaction with one another for them is

$$\tau_1\sim 1/k_1^5 n(k_1), \tag{10}$$

while the interaction time with the main spectrum is

$$\tau_{01}\sim(k_1/k_0)^{3/2}\tau_0. \tag{11}$$

It is clear from the estimates (9) to (11) that in the case when the spectral function decreases with increasing k more slowly than k^{-5} the short-wavelength part of the spectrum changes faster than its main part and this change is caused primarily by the interaction of the short-wavelength waves with one another. It is thus natural to assume that the solution of Eq. (7) in the range $k\gg k_0$ is independent of the details of the initial distribution and is self-similar. Equation (7) admits the following self-similar substitutions:

$$n_k(t)=t^{5b-1}f(kt^b),\quad t>0; \tag{12}$$

$$n_k(t)=(t_0-t)^{5b-1}f[k(t_0-t)^b],\quad 0<t<t_0, \tag{13}$$

where b and t_0 are arbitrary constants. The Rayleigh-Jeans distribution

$$n_k=T/k^{3/2} \tag{14}$$

in particular, belongs to the number of solutions of the form (12) and (13) and also the solution found in Ref. 3 with a constant energy flux P along the spectrum:

$$n_k=P^{1/2}/k^{17/4}. \tag{15}$$

Apart from these two solutions there is yet one other exact power-law solution of Eq. (7):

$$n_k(t)=C/k^5(t_0-t). \tag{16}$$

It corresponds to the substitution (13) and has an "explosive" time-dependence. The fact that there is no such solution for the substitution (12) is connected with the requirement that $n_k(t)$ be positive. We show that solution (16) indeed satisfies Eq. (7). To do that we substitute the spectrum (16) into Eq. (7) and integrate over the angles, after which the right-hand side of Eq. (7) takes the form

$$\frac{C^2}{64\pi k^5(t_0-t)^2}\int\frac{(xy)^{3/2}\delta(1-x^{3/2}-y^{3/2})}{[(2xy)^2-(1-x^2-y^2)^2]^{1/2}}\times\left\{\frac{(1-x)^2}{y^{1/2}}+\frac{(1-y)^2}{x^{1/2}}-(x-y)^2\right\}^2\left(\frac{1}{x^5y^5}-\frac{1}{x^5}-\frac{1}{y^5}\right)dx\,dy+\frac{C^2}{32\pi k^5(t_0-t)^2}\int\frac{(xy)^{3/2}\delta(x^{3/2}-1-y^{3/2})}{[(2y)^2-(x^2-1-y^2)^2]^{1/2}}\left\{\frac{(1-x)^2}{y^{1/2}}+(x-y)^2-\frac{(1-y)^2}{x^{1/2}}\right\}^2\left(\frac{1}{x^5y^5}+\frac{1}{x^5}-\frac{1}{y^5}\right)dx\,dy. \tag{17}$$

We have here introduced the dimensionless integration variables $x=k_1/k$ and $y=k_2/k$ instead of the wavenumbers k_1 and k_2. Moreover, we have dropped in the expression for the probability w terms which vanish when the conditions for decay are satisfied. Making in the second integral in (17) the variables changes $x=1/q$ and $y=p/q$ we can obtain from Eq. (7) the following expression for the constant C:

$$C=64\pi\left\{\int_0^\infty\int_0^\infty(1-2q^3)(1-p^5-q^5)U(p,q)\,dp\,dq\right\}^{-1}, \tag{18}$$

where

$$U(p,q)=\frac{\delta(1-p^{3/2}-q^{3/2})}{(pq)^{7/2}[(2pq)^2-(1-p^2-q^2)^2]^{1/2}}\left\{\frac{(1-p)^2}{q^{1/2}}+\frac{(1-q)^2}{p^{1/2}}-(p-q)^2\right\}^2.$$

In order that the solution (16) make sense the constant C must be positive and finite. We show that the integral (18) is indeed positive definite. Using the symmetry of the function

U(p,q) and the decay condition

$$1-p^{3/2}-q^{3/2}=0, \tag{19}$$

we can rewrite the integrand in the form

$$(1-2q^3)(1-p^5-q^5)U(p,\ q)=(1-p^3-q^3)(1-p^5-q^5)U(p,\ q)$$
$$=2p^{3/2}q^{3/2}(1-p^5-q^5)U(p,\ q).$$

Under condition (19) the quantity $1-p^5-q^5$ is positive. The constant C therefore also turns out to be positive. One can also easily check that the integral (18) is bounded from above.

Apart from the power-law solutions (14) to (16) the equation has exponentially decreasing asymptotic solutions:

$$n_k(t) \propto \frac{t^{5b-1}}{z^{7/2}} e^{-z^{3/2}}, \qquad z=kt^b \gg 1, \qquad b<0; \tag{20}$$

$$n_k(t) \propto \frac{(t_0-t)^{5b-1}}{z^{7/2}} e^{-z^{3/2}}, \qquad z=k(t_0-t)^b \gg 1, \qquad b>0. \tag{21}$$

An attempt to join the asymptotic forms (16), (20), (21) directly to the energy-containing part of the spectrum turns out to be unsuccessful. Such joining should take place at a point $k \sim k_0$ where, owing to the slow change in the main spectrum with time, the value of the spectral function $n(k_0)$ can be assumed to be constant. None of the asymptotic forms (16), (20), (21) satisfies this requirement. The impossibility to join the asymptotic forms (16), (20), (21) with the energy-containing part of the spectrum can be explained by the fact that the lower limit of applicability of the asymptotic solutions with increasing time shifts rapidly in the direction of large k. One may expect that in the region between this limit and the main spectrum there is formed a universal distribution of waves with a lifetime much larger than the time for shifting the limit. We shall see that the solution with constant energy flux along the spectrum has this property. We note that in the intermediate range the Rayleigh-Jeans spectrum cannot be formed as the shift of its upper boundary to the short-wavelength region would mean that the total energy of the waves would grow without limit with time.

The equation of motion for the short-wavelength limit of the distribution (15) $k_*(t)$ can be obtained from the following considerations. We assume that after a time Δt the limit is shifted by Δk to the short-wavelength region. The energy density of the short-wavelength waves then increases by an amount

$$\Delta W \sim P^{1/2}\Delta k/k^{7/4}.$$

On the other hand, the increase in W is given by the energy flux of the waves

$$\Delta W = P\Delta t.$$

Hence it follows that

$$k_*(t) \propto (t_0-t)^{-4/3}, \tag{22}$$

where the time t_0 is determined by the magnitude of the energy flux and the initial position of the boundary. This relation enables us to choose two asymptotic forms which allow a joining with the spectrum (15), namely, the solutions (16) and (21) with self-similarity exponent $b = 4/3$. The presence of two self-similar short-wavelength asymptotic forms indicates that the actual form of the solution to the right of the boundary (22) may depend strongly on the initial conditions. This fact, however, does not affect the equation of motion of the boundary itself which after a finite time gets into the region of arbitrarily large k.

In the initial stage of the evolution of the original distribution of the waves there is thus split off from the spectrum a short-wavelength "tail" which soon reaches the absorption region. After the spectrum (15) is established in the whole range from k_0 to the absorption region the flux P (as function of time) starts to decrease due to a decrease in the total wave energy. As a result the larger part of the energy of the original spectrum reaches the absorption region after a time of the order of magnitude of τ_0.

3. WAVES IN A PLASMA

The self-accelerating transfer of energy to the large wavenumber region is not an exclusive feature of capillary waves. This effect may also occur for other kinds of waves. We consider here three other examples of practical interest: helicons, electron cyclotron waves and Langmuir waves in a plasma in a strong magnetic field.

The helicon dispersion law has the form

$$\omega(\mathbf{k}) = |\omega_H x| k^2c^2/\omega_p^2, \tag{23}$$

where ω_p and ω_H are the electron plasma and cyclotron frequencies and x is the cosine of the angle between the wavevector and the external magnetic field. The probability for the decay interaction of these waves was calculated in Ref. 4:

$$w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2} = \frac{\pi^4}{mn_0}\frac{|\omega_H|}{\omega_p^4}\frac{c^4}{k^2}\left|\frac{x_1x_2}{x}\right|$$
$$\times[\mathbf{k}_1\mathbf{k}_2]^2\left(\frac{x_1}{|x_1|}k_1-\frac{x_2}{|x_2|}k_2\right)^2\left(\frac{x}{|x|}k+\frac{x_1}{|x_1|}k_1+\frac{x_2}{|x_2|}k_2\right)^2 \tag{24}$$

(n_0 is the plasma density). It follows from the kinetic Eq. (7) that the time for spectral transfer of helicons is inversely proportional to their energy density and to the square of the average wavenumber:

$$\tau \propto (\bar{k}^2 W_0)^{-1}. \tag{25}$$

As in the case of capillary waves the energy transfer to the short-wavelength region does not violate in this case the applicability of the random phase approximation. We note that, in contrast to the capillary waves, helicons can reach the large wavenumber region without appreciably increasing their frequency as for them the constant frequency lines in **k**-space are unbounded. When the characteristic value of the wavenumber of the helicons reaches the magnitude ω_p/c the dispersion law (23) changes and becomes the cyclotron wave dispersion law.

The frequency of the electron cyclotron waves depends only on the angle of their propagation relative to the external magnetic field:

$$\omega(\mathbf{k}) = |\omega_H x|, \tag{26}$$

and the decay probability equals

$$w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}=\frac{\pi^4}{mn_0}\frac{|\omega_H|}{k^2}\left|\frac{x_1x_2}{x}\right|\frac{[\mathbf{k}_1\mathbf{k}_2]^2}{k_1^2k_2^2}$$

$$\times\left(\frac{x_1}{|x_1|}k_1-\frac{x_2}{|x_2|}k_2\right)^2\left(\frac{x}{|x|}k+\frac{x_1}{|x_1|}k_1+\frac{x_2}{|x_2|}k_2\right)^2. \quad (27)$$

The characteristic time for spectral transfer (as in the case of helicons) decreases when the wavenumber in the spectrum increases:

$$\tau\propto(\bar{k}^2W_0)^{-1}. \quad (28)$$

The conclusion about the self-accelerating nature of the transfer remains valid also for cyclotron waves.

We note that Eqs. (23) and (26) are particular cases of the more general dispersion law

$$\omega(\mathbf{k})=|\omega_Hx|(1+\omega_p^2/k^2c^2)^{-1},$$

which is valid the whole range of k values. It is easy to find also the corresponding general formula for the probability $\omega_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}$:

$$w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}=\frac{\pi^4}{mn_0}\frac{|\omega_H|c^4}{k^2\omega_p^4}\left|\frac{x_1x_2}{x}\right|\left(1-\frac{k_1k_2x_1x_2c^2}{\omega_p^2|x_1x_2|}\right)^2$$

$$\times\frac{[\mathbf{k}_1\mathbf{k}_2]^2(k_1x_1/|x_1|-k_2x_2/|x_2|)^2}{(1+k_1^2c^2/\omega_p^2)^2(1+k_2^2c^2/\omega_p^2)^2}\left(\frac{x}{|x|}k+\frac{x_1}{|x_1|}k_1+\frac{x_2}{|x_2|}k_2\right)^2.$$

In the limiting cases $k\ll\omega_p/c$ and $k\gg\omega_p/c$ this expression reduces to (24) and (27).

We finally consider the potential oscillations of a magnetized ($\omega_H\gg\omega_p$) plasma for which

$$\omega(\mathbf{k})=\omega_p|x|, \quad (29)$$

and the decay probability is given by the following expression[5]:

$$w_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}=\frac{\pi^4}{mn_0}\omega_p^3|xx_1x_2|\left(\frac{kx}{\omega}+\frac{k_1x_1}{\omega_1}+\frac{k_2x_2}{\omega_2}\right)^2. \quad (30)$$

An estimate for the characteristic interaction time for the waves is

$$\tau\propto(\bar{k}^2W_0)^{-1} \quad (31)$$

and in this case it indicates the explosive nature of the energy transfer to the large wavenumber region.

In contrast to capillary waves and helicons, the increase in average wavenumber in the spectrum can in the last two cases lead to a violation of the condition for the applicability of the random-phase approximation. Indeed, the width of the spectrum in frequency cannot exceed the values of ω_H and ω_p for the electron cyclotron and magnetized Langmuir waves, whereas the characteristic time for the energy transfer in both cases decreases unrestrictedly with increasing $\bar{k}$. The description of the dynamics of the spectrum by means of the kinetic Eq. (7) is thus correct only for not too large values of $\bar{k}$:

$$\bar{k}\ll\frac{|\omega_H|}{c}\left(\frac{n_0mc^2}{W_0}\right)^{1/2}$$

for cyclotron waves and

$$\bar{k}\ll\frac{\omega_p}{c}\left(\frac{n_0mc^2}{W_0}\right)^{1/2}$$

for Langmuir waves in a magnetized plasma.

In all examples given above we considered decay and fusion of one kind of waves. If, however, several kinds of waves participate it is impossible to reach a universal conclusion about the direction of spectral transfer. In that case energy transfer to the long-wavelength region rather than to the short-wavelength region is possible. A buildup of waves in the small wavenumber region occurs, in particular, when Langmuir waves decay into Langmuir and ion-sound waves. We explain this in more detail. We assume that initially a Langmuir wave spectrum is given with characteristic wavenumber k_0 and energy density W_0. The number density of quanta in that spectrum is approximately equal to

$$N_0\sim W_0/\hbar\omega_p. \quad (32)$$

The decay interaction of the form $l\rightarrow l'+s$ leads to the establishment of equilibrium distributions for the plasmons and the ion-sound waves:

$$n_k^l=(e^{(\hbar\beta k^2+\nu)/T}-1)^{-1}, \quad (33)$$

$$n_q^s=(e^{\hbar c_sq/T}-1)^{-1}. \quad (34)$$

We have introduced in Eqs. (34), (34) the following notation: $\beta=3/2\omega_pr_D^2$; c_s is the ion sound speed; $\nu=\hbar\omega_p-\mu>0$, where μ is the plasmon chemical potential. The plasmon number density is conserved when decay takes place and the energy of the initial spectrum is redistributed between the Langmuir and ion-sound waves. Reckoning the energy density for the quantity $\hbar\omega_pN_0$ we can write the energy conservation law in the form

$$\int\frac{\hbar\beta k^2\,d\mathbf{k}}{e^{(\hbar\beta k^2+\nu)/T}-1}+\int\frac{\hbar c_sq\,d\mathbf{q}}{e^{\hbar c_sq/T}-1}=\varepsilon W_0, \quad (35)$$

where ε is a quantity of the order $k_0^2r_D^2$. The integrals occurring in this relation are approximately equal to $T(T/\hbar\beta)^{3/2}\exp(-\nu/T)$ and $T(T/\hbar c_s)^3$. One easily checks that in equilibrium the energy of the plasmons with $k\neq0$ is small compared to the energy of the ion-sound waves. Indeed, it follows from Eq. (35) in that case that the temperature of the waves is of the order of magnitude of

$$T\sim[\varepsilon W_0(\hbar c_s)^3]^{1/4}, \quad (36)$$

while the energy of plasmons with $k>0$ is proportional to $\hbar^{3/8}$ and in the classical limit tends to zero. The equilibrium number density of Langmuir quanta with $k>0$ is equal to

$$N(k>0)=\int\frac{1}{e^{(\hbar\beta k^2+\nu)/T}-1}\frac{d\mathbf{k}}{(2\pi)^3}\sim\left(\frac{T}{\hbar\beta}\right)^{3/2}e^{-\nu/T}, \quad (37)$$

and turns out to be negligibly small compared to the original density [see Eq. (32)]. All the Langmuir waves therefore go into the lowest energy state with $k=0$, forming the condensate and only a small fraction of the energy ($\sim\varepsilon W_0$) is transferred to the ion-sound waves.

In conclusion we note that the possibility of a self-accelerating energy transfer to the short-wavelength region of the spectrum due to decay processes appears completely natural if we bear in mind that all waves considered by us are hydrodynamic in nature. In the original (not phase-averaged) dynamical equations such a transfer must appear as the steepening of the wave profile up to the formation of discontinuities.

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[1] The difference between the probability (6) and the corresponding expression given in Ref. 3 is explained by the fact that an error has slipped in: one must write the moduli instead of the vectors $\mathbf{k}$, $\mathbf{k}_1$, $\mathbf{k}_2$ themselves.

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[1] V. N. Tsytovich, Nelineĭnye effekty v plazme (Non-linear effects in a plasma) Nauka, Moscow, 1967, p. 187 [English translation published by Plenum Press].

[2] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Elektrodinamika plazmy (Plasma electrodynamics) Nauka, Moscow, 1974, p. 506 [English translation published by Pergamon Press].

[3] V. E. Zakharov and N. N. Filonenko, Prikl. Mekh. Tekh. Fiz. No. 5, 62 (1967).

[4] M. A. Lifshits and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **62**, 606 (1972) [Sov. Phys. JETP **35**, 321 (1972)].

[5] V. V. Pustovalov and V. P. Silin, Trudy Fiz. Inst. Akad. Nauk SSSR **61**, 42 (1972).

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