Inverted distributions and carrier bunching in momentum space in a magnetic field crossed with an alternating electric field

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The kinetics of hot carriers in a magnetic field crossed with a linearly polarized alternating electric field is analyzed for a semiconductor in which scattering by optical phonons is predominant. The momentum-space trajectories in a coordinate system rotating at the cyclotron frequency ω_c around the direction of the magnetic field are epicycloids and hypocycloids in the cases $\omega_c > \Omega$ and $\omega_c < \Omega$, respectively, where Ω is the frequency of the microwave field. If the amplitude of the microwave field lies below a certain threshold, trapping trajectories arise in the passive region. These trapping trajectories do not intersect the boundary of the passive region, where carriers may accumulate. The carriers form a spindle-shaped swarm or bunch there, which executes a periodic motion within the passive region. The distinctive features of the absorption of the microwave field are analyzed. The cyclotron resonance line is shown to be asymmetric and broadened on the weak-field side. The possibility of producing an energy-inverted carrier distribution in a semiconductor with degenerate bands, of the p-Ge type, is analyzed. The most favorable conditions for changing the population of a light-hole band are determined.

INTRODUCTION

After a hiatus of nearly three decades since the work by Vosilyus *et al.*,[1–4] there has been a revival of experimental and theoretical research on kinetic effects in semiconductors in which the inelastic scattering of current carriers by optical phonons is important. The revival followed the experiments of Komiyama *et al.*,[5] who observed that in crossed fields free electrons accumulate on momentum-space trajectories which are closed within a so-called passive region (P region), where an electron cannot emit an optical phonon. That electrons might accumulate on such trajectories in a static electric field crossed with a magnetic field had been suggested by Maeda and Kurosawa[6] and has also been confirmed by Monte-Carlo calculations for the particular case of p-Ge.

In degenerate-band semiconductors of the p–Ge type, the accumulation of light holes on such trajectories can cause a deviation from the equilibrium population of the light-hole band[3] and can produce an energy-inverted distribution between the light-and heavy-hole bands. As Andronov *et al.*[7] have pointed out, this effect might be exploited to generate far-IR radiation, and this radiation has been observed in several experiments.[8–11]

In the most recent of these experimental studies, Vorob'ev *et al.*[11] observed a stimulated emission at the wavelength $\lambda = 100\ \mu$m from a p-Ge sample in a resonator; this emission was 1.5–2 orders of magnitude more intense than the spontaneous emission.

The trapping trajectories in momentum space on which carriers may accumulate also exist in an alternating electric field.[12,13] It would be quite tempting to make use of the existence of such traps to develop a contactless method for producing an energy-inverted carrier population in degenerate-band semiconductors of the p-Ge type and, ultimately, to excite infrared radiation. This cannot be done in an alternating electric field by itself, however, since in this case if the trap lies in the light-hole band it must necessarily also be present in the heavy-hole band.

It turns out that the situation can be "corrected" by a static magnetic field $\mathbf{H}$ directed perpendicular to the alternating electric field $\mathbf{E}(t)$. If the alternating electric field is circularly polarized in the plane perpendicular to $\mathbf{H}$, then in the isotropic model the problem reduces to a static transformation in momentum space to a coordinate system which is rotating with the field $\mathbf{E}$ (Ref. 2). An inverted carrier distribution on trajectories in momentum space under these conditions has been studied elsewhere.[14,15] In the present paper we are interested in the case in which the alternating electric field applied to the semiconductor, in the direction perpendicular to the magnetic field, is linearly polarized.

1. CLASSIFICATION OF TRAJECTORIES

The solution of the equation of motion for a current carrier in an alternating electric field $\mathbf{E}(t) = (0, E_0 \cos\Omega t, 0)$ and a static magnetic field $\mathbf{H} = (0,0,H)$ in the case of a quadratic and isotropic dispersion law $\varepsilon(\mathbf{p}) = p^2/2m$, where $\mathbf{p}$ is the momentum and m the effective mass, can be written

$$\xi = p_x + ip_y = \alpha \exp(i\Omega t) + \beta \exp(-i\Omega t) + \gamma_0 \exp(-i\omega_c t),$$
$$p_z = p_{0z}, \tag{1}$$
$$\alpha = eE_0/2(\omega_c+\Omega), \qquad \beta = eE_0/2(\omega_c-\Omega), \tag{2}$$
$$\gamma_0 = \xi_0 \exp(i\omega_c t_0) - \alpha \exp[i(\omega_c+\Omega)t_0] - \beta \exp[i(\omega_c-\Omega)t_0]. \tag{3}$$

Here e is the charge, $\omega_c = eH/mc$ is the cyclotron frequency (for definiteness, e, $H > 0$), $\xi_0 = p_{0x} + ip_{0y}$, and $\mathbf{p}_0 = (p_{0x}, p_{0y}, p_{0z})$ is the value of the momentum at the time t_0.

The sum of the first two terms in (1) gives us a vector whose tip traces out an ellipse in the $p_z = p_{0z}$ plane with an angular frequency Ω and semiaxes

$$a = |\alpha+\beta| = eE_0\omega_c/|\omega_c^2-\Omega^2|, \quad b = |\alpha-\beta| = eE_0\Omega/|\omega_c^2-\Omega^2|. \tag{4}$$

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The ratio of semiaxes of this ellipse is $a/b = \omega_c/\Omega$.

The third term in (1) describes a vector whose tip traces out a circle of radius $|\gamma_0|$ with an angular frequency ω_c in the $p_z = p_{0z}$ plane. The magnitude $|\gamma_0|$ is arbitrary and depends on the initial conditions. As a result, the vector ξ traces out some complicated trajectory which is closed only if the frequency ratio ω_c/Ω is rational.

a. If the frequency ratio ω_c/Ω is irrational, and if these frequencies are furthermore very different, $\Omega \gg \omega_c$ or $\Omega \ll \omega_c$, the carrier trajectory densely[1)] fills a strip of width $2|\gamma_0|$ in the $p_z = p_{0z}$ plane. This strip lies between two concentric ellipses with respective semiaxes $a - |\gamma_0|$, $b - |\gamma_0|$ and $a + |\gamma_0|$, $b + |\gamma_0|$.

The strip becomes filled in a characteristic time $\tau_c = \max\{\Omega^{-1}, \omega_c^{-1}\}$ if the frequencies ω_c and Ω are very different.

b. We now consider the case of a rational frequency ratio. Specifically, we assume $\Omega/\omega_c = m/n$, where m and n are integers. In this case the carrier trajectories in momentum space are closed. These trajectories are conveniently classified in the coordinate system p_x', p_y', p_z', which is rotating in momentum space around the p_z axis with angular frequency ω_c (Ref. 16)[2)]:

$$\xi' = p_x' + ip_y' = \xi \exp(i\omega_c t)$$
$$= \gamma_0 + \alpha \exp[i(\omega_c + \Omega)t] + \beta \exp[i(\omega_c - \Omega)t],$$
$$p_z' = p_z = p_{0z}. \qquad (5)$$

It follows that in this coordinate system the shape of the trajectories does not depend on the initial conditions; it depends on only a single parameter—the frequency ratio ω_c/Ω. If $\omega_c > \Omega$, then the trajectory of a carrier in the rotating coordinate system is an epicycloid in the $p_z' = p_{0z}$ plane, described by a point on a circle of radius $r_1 = \alpha$ (the generating circle) when the circle is rolled without slippage along the outer side of a fixed circle of radius $r_2 = b$ (the guiding circle; Fig. 1). The center of the guiding circle lies at the point (γ_0, p_{0z}).

The ratio of radii is the rational number

$$\eta = \frac{r_2}{r_1} = \frac{2}{|\omega_c/\Omega - 1|} = \frac{2m}{|m-n|} = \frac{s}{q}, \qquad (6)$$

where s and q are mutually simple integers. The number s gives the number of turning points (A_1, A_2, and A_3 in Fig. 1b).

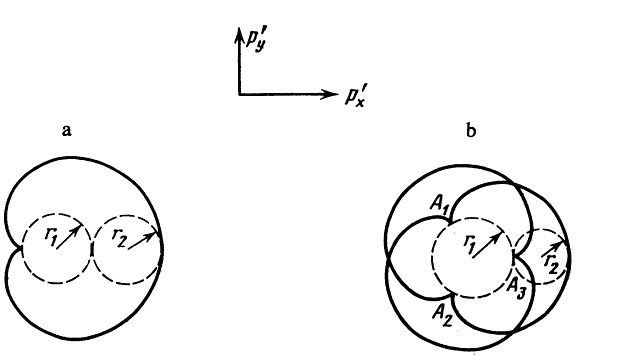

FIG. 1. Current-carrier trajectories in momentum space in a rotating coordinate system for the case $\omega_c > \Omega$. a—Cardioid, $\omega_c = 3\Omega$; b—$\omega_c = 5/3\ \Omega$.

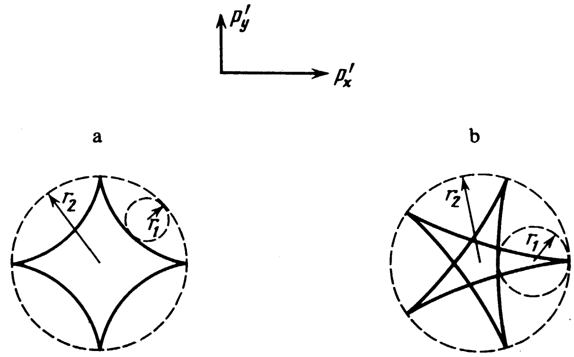

FIG. 2. Carrier trajectories in momentum space in a rotating coordinate system for the case $\omega_c < \Omega$. a—Astroid, $\Omega = 2\omega_c$; b—$\Omega = 5\omega_c$.

This number also determines the symmetry of the epicycloid, which has an s-fold axis passing through the center of the guiding circle, perpendicular to its plane, and s reflection planes passing through this axis (one of these reflection planes coincides with the $p_z'p_x'$ plane). The symmetry point group of an epicycloid with this value of s is thus C_{sv}. The number q is the number of revolutions executed in the course of its motion by the radius vector connecting the center of the guiding circle to a point on the trajectory until the original point is reached again. The number of self-intersections of the epicycloid is $k = s(q - 1)$. The period of the revolution along the trajectory is

$$T = 2\pi q / |\omega_c - \Omega| \qquad (7)$$

and increases toward the resonance.

If $\omega_c < \Omega$, then the trajectory in the rotating coordinate system is a hypocycloid in the $p_z' = p_{0z}$ plane. The hypocycloid is described by a point on a generating circle of radius $r_1 = \alpha$ which is rolling without slippage along the interior of a guiding circle of radius $r_2 = b$ [centered at the point (γ_0, p_{0z})] (Fig. 2). The symmetry of the hypocycloid is, as in the case of the epicycloid, determined by the point group C_{sv}. Since we have $\eta > 2$ if $\omega_c < \Omega$ [see (6)], the trajectories can only be hypocycloids with $s \geqslant 3$ and $q < s/2$. The trajectories are thus quite symmetric in the case $\omega_c < \Omega$. The period of the motion along the trajectory is given by (7). The number of turning points, the number of times the trajectory revolves around the center of the guiding circle, and the number of self-intersections are given by the same expressions as in the case of the epicycloid.

Finally, at cyclotron resonance, $\omega_c = \Omega$, the trajectory in the rotating coordinate system is a cycloid, which is traced out by an arbitrary point on a circle of radius $r_1 = \alpha$ when it rolls without slippage along a straight line. The time evolution of the momentum $\mathbf{p}'$ in this case is described by

$$\xi' = \xi_0 e^{i\Omega t_0} - \frac{eE_0}{4\Omega}(e^{2i\Omega t_0} + 2i\Omega t_0) + \frac{eE_0}{4\Omega}(e^{2i\Omega t} + 2i\Omega t).$$

The time required for the point to traverse one arc of the cycloid is π/Ω. If a carrier is initially at the center of the P region, then after a time on the order of p_P/eE_0 it will reach the boundary of the P region, i.e., will acquire an energy on the order of the energy of an optical phonon.

2. DETERMINATION OF THE THRESHOLD FIELD E_{c1}

We now seek the conditions under which trajectories exist in the passive region which do not intersect the boundary of this region.

a. If the frequency ratio ω_c/Ω is irrational, and if these frequencies are furthermore greatly different ($\Omega \gg \omega_c$ or $\Omega \ll \omega_c$), then the trajectory traced out by a carrier lies entirely in the passive region and has the following form, according to Subsection 1a:

$$|\gamma_0| + \max\{a, b\} \leqslant (p_P^2 - p_{0z}^2)^{1/2}, \tag{8}$$

where $p_P = (2m\hbar\omega_0)^{1/2}$ is the radius of the passive region, and ω_0 is the frequency of an optical phonon. Condition (8) can be rewritten as

$$|\xi_0 - \alpha e^{i\Omega t_0} - \beta e^{-i\Omega t_0}| \leqslant R(p_{0z}), \tag{9}$$

$$R(p_{0z}) = (p_P^2 - p_{0z}^2)^{1/2} - eE_0\Omega_M / |\omega_c^2 - \Omega^2|, \tag{10}$$

where $\Omega_M = \max\{\omega_c, \Omega\}$. The right side of inequality (9) must be real and positive. It follows that the necessary trajectories exist if the amplitude E_0 does not exceed the threshold value

$$E_{c1} = p_P |\omega_c^2 - \Omega^2| / e\Omega_M. \tag{11}$$

If $E_0 < E_{c1}$, then (9) determines a region $\{\xi_0, p_{0z}\}$ in momentum space such that a carrier which reaches this region at the time $t = t_0$ turns out to lie on a trajectory which lies entirely in the P region. We call this region a "trap." Its intersection with the $p_z = p_{0z}$ plane is a circle of radius $R(p_{0z})$ whose center moves in the p_{0z} plane along an ellipse with semiaxes a and b at an angular frequency Ω. This region is spindle-shaped, as in the case of static crossed **E** and **H** fields.[5] The spindle moves as a whole along this ellipse at a frequency Ω (Fig. 3).

b. To determine the threshold field E_{c1} in the case in which the ratio of frequencies is rational, we note that the threshold field amplitude is found by equating the radius of the passive region, p_P, to the radius, $R_{\min}$, of the smallest circle which can circumscribe trajectory (5). The value of $R_{\min}$ depends on the field amplitude E_0. If $p_P > R_{\min}$, then there are trajectories in the passive region which do not intersect the boundary of this region. If $p_P < R_{\min}$, on the other hand, there are no such trajectories. We will determine $R_{\min}$ for the case $\omega_c > \Omega$, in which a trajectory is an epicycloid in the rating coordinate system. If the epicycloid has a symmetry axis, i.e., if $s \geqslant 2$ [see (6)], then the center of the circumscribed circle lies on this axis in the plane of the figure; i.e., it coincides with the center of the guiding circle. The value of $R_{\min}$ in this case is $r_2 + 2r_1$. It is not difficult to see that the threshold field in this case is the same as that in (11). If there is no symmetry axis, i.e., if $\eta = 1/q$, where $q = 1, 2, 3, \ldots$ [see (6)], then it can be shown that

$$R_{min} = (r_2 + 2r_1)\cos[\pi/2(2q+1)]. \tag{12}$$

In this case the center of the circumscribed circle does not coincide with the center of the guiding circle and is instead displaced from it by a distance

$$\Delta_0 = r_2 \sin[\pi/2(2q+1)] \tag{13}$$

along the twofold symmetry axis p_x'. The threshold field E_{c1} is stronger by a factor of $\chi = \cos^{-1}[\pi/2(2q+1)]$ than that given by (11). For $q = 1$ ($\omega_c = 3\Omega$), for example, we have $\chi = 2/\sqrt{3} \approx 1.16$; for $q = 2$ ($\omega_c = 5\Omega$) we find $\chi = 1.05$; etc., (we move progressively closer to unity).

In the case $\omega_c < \Omega$ the trajectory in the rotating coordinate system is a hypocycloid, which always has at least a three-fold symmetry axis. The center of the circumscribed circle in this case coincides with the center of the guiding circle; $R_{\min} = r_2$; and the threshold field E_{c1} is described by (11).

When the field amplitude E_0 is equal to the threshold value, we thus have only a single trajectory which does not intersect the boundary of the passive region (but does touch it). The center of the guiding circle which determines the position of the trajectory is at the point O′, with the coordinates $(\gamma_0, 0)$. For an epi-hypocycloid with a symmetry axis we have $\gamma_0 = 0$, while for an epicycloid without a symmetry axis ($s = 1$) we have $\gamma_0 = -\Delta_0$. If E_0 is below the threshold value, the centers of the guiding circles for trajectories which do not intersect the boundary of the passive region fill some region $\{\gamma_0, p_{0z}\} \equiv L'$ around the point O′ in the **P**′ momentum space. The symmetry of the region L' is described by the point group D_{sh}, as is easily shown. The s-fold symmetry axis coincides with the p_z' axis, and the horizontal symmetry plane coincides with the $p_z' = 0$ plane.

We are interested in the locus of points $\{\xi_0, p_{0z}\} = L$ such that a carrier reaching such a point at the time t_0 turns out to lie on a trajectory which is entirely within the P region. From (3) we find

$$\xi_0 = \gamma_0 \exp(-i\omega_c t_0) + \alpha \exp(i\Omega t_0) + \beta \exp(-i\Omega t_0). \tag{14}$$

The region L can thus be found from L' by rotating the latter counterclockwise an angle $\omega_c t_0$ around the p_z' axis and displacing it as a whole along the direction of the vector $\alpha \exp(i\Omega t_0) + \beta \exp(-i\Omega t_0)$. The trap in which the current carriers can accumulate thus executes a rather complicated motion in the passive region. It rotates as a whole around some axis (the symmetry axis of the trap if the latter exists) at an angular frequency ω_c and also undergoes a translational motion along an ellipse at an angular frequency Ω. The shape of the trap is the same as the shape of the region L' in this case.

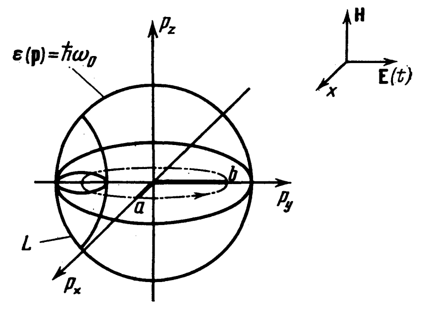

FIG. 3. Spindle-shaped trap L in which carriers can accumulate. The dot-dashed curve in the trajectory of the center of the trap (an ellipse with semiaxes a and b). The trap moves along this trajectory at an angular frequency Ω.

3. ACCUMULATION CONDITIONS

Current carriers accumulate on trajectories in the passive region when the field E_0 is below the threshold field under the following conditions:

1) The very concept of a trajectory in the P region is valid, i.e., the carrier dynamics is clearly expressed in the P region.

2) The carrier scattering probability in the active region, $1/\tau^+$ (the frequency of the spontaneous emission of an optical phonon), is considerably higher than the scattering probability, $1/\tau^-$, in the P region (scattering by an impurity or an acoustic phonon, for example). Here the temperature must be low enough to satisfy the condition $kT \ll \hbar\omega_0$.

The accumulation regions undergo a periodic motion at the field frequency in the passive region. The reason for this periodic motion is that a given point in momentum space may belong at different times to completely different trajectories: both trajectories which lie completely in the passive region and trajectories which intersect its boundary. Consequently, we do not have simply a random accumulation of carriers on trajectories in the p region but instead a bunching (due to scattering by optical phonons) and a formation of carrier bunches which fill the trap and which undergo a periodic motion along with the trap. The lifetime of the carriers in the trap is determined by τ^-, the characteristic time between scattering events in the passive region.

In determining the threshold field E_{c1} above, we showed that in most cases its dependence on the frequency Ω and the magnetic field H is given by (11). Exceptional cases are those with $\omega_c = (2q+1)\Omega$, where $q = 1, 2, 3, \ldots$. The threshold field turns out to be larger than that found from (11) by a factor of $\chi = \cos^{-1}[\pi/2(2q+1)]$. The greatest difference between the threshold field and (11)—on the order of 16%—is found at $\omega_c = 3\Omega$. The dependence of the threshold field on the magnetic field is thus extremely unusual: It is described by a discontinuous curve with finite jumps at the discrete points $H_q = mc\Omega\,(2q+1)/e$.

How would be this behavior be seen experimentally? The threshold field could be measured quite accurately by taking the following approach (cf. Ref. 5): We apply to a semiconductor, along with the magnetic and alternating electric fields, a weak static electric field $\mathbf{E}_\parallel$ directed parallel to the magnetic field $\mathbf{H}$. This weak field drives a current $j_\parallel$ along the direction of the magnetic field. We measure the dependence of this current on the amplitude of the alternating electric field, $j_\parallel(E_0)$, in various magnetic fields. As long as the field E_0 is below the threshold value, the dependence $j_\parallel(E_0)$ will be smooth, and the magnitude of $j_\parallel$ will be determined by the number of carriers trapped on a trajectory in the passive region and by the mobility of these carriers along the magnetic field, which is proportional to the time τ^- and is large. As E_0 is increased and goes through the threshold value, the current $j_\parallel$ should drop sharply, since there will no longer be any carriers with a high mobility along the magnetic field: All the carriers will be on trajectories which pass through the active region, where they are strongly scattered by optical phonons. The decrease in $j_\parallel$ when the field E_0 crosses the threshold value will be quite sharp because the carriers on the trajectories in the passive region, even if few in number in comparison with the total number of carriers, will be primarily responsible for the current $j_\parallel$ because of their high mobility along the magnetic field.

We now assume $\omega_c/\Omega = 100/33$, i.e., a ratio very nearly equal to 3. The ratio of the radii of the guiding and generating circles, (6), is $\eta = 66/67$. In this case the trajectory is a very symmetric epicycloid with a 66-fold symmetry axis. The period at which this curve is traversed, (7), is $T = 100\cdot 2\pi/\omega_c$, and the threshold field is the same as (11), according to our definition.

Since ω_c/Ω is very nearly equal to 3, the motion along this complicated trajectory can be described as a rapid motion along a cardioid (Fig. 1a) which is slowly precessing around the center of the guiding circle. In our experiment the position of the jump on the $j_\parallel(E_0)$ dependence may be completely different from (11). It will be if the carrier lifetime on the trajectory, τ^-, is considerably longer than the time required for motion along the cardioid, $3\pi/\omega_c$, but considerably smaller than the time required for a complete rotation of the cardioid through 2π, i.e., $200\pi/\omega_c$. The jump will then occur near the threshold field corresponding to the cardioid, which is higher than (11); i.e., the threshold field measured in the same experiment as a function of the magnetic field can have a maximum at $H = 3mc\Omega/e$. The width of the maximum is determined from the condition $\Delta H = mc/e\tau^-$; this condition is the condition that τ^- be equal to the period T in (7).

4. TOPOLOGY OF THE MAIN TRAJECTORY. THE CRITICAL FIELDS E_{c2} and E_{c3}

We have been discussing conditions such that trajectories appear in the P region which do not cross its boundary. The kinetic coefficients may also acquire some distinctive features upon a change in the topology of the main trajectory, i.e., of the trajectory passing through the center $\mathbf{p} = 0$ (Refs. 1, 2, and 17). Since the carrier trajectories are not circles in our case, as in crossed static fields $\mathbf{E}$ and $\mathbf{H}$, we find not one but two critical fields E_{c2} and E_{c3}. The first, E_{c2}, is the field at which, upon a decrease in the amplitude E_0, the first main trajectory, which lies entirely in the P region, appears. The field E_{c3} is the amplitude at which all the main trajectories lie entirely in the passive region. The first main trajectory in the P region evidently appears when, as the field amplitude E_0 is decreased, the trap begins to touch the center of the passive region in the course of its motion. With a further decrease in the amplitude E_0 and an increase in the dimensions of the trap, we eventually reach a time at which the center of the passive region is in the trap at all times. It is this value of E_0 which corresponds to the critical field E_{c3}.

In determining the critical field E_{c1} above we showed that it is given in essentially all cases by expression (11), derived for a spindle-shaped trap. The reason is that the actual shape of the trap is in fact very nearly a spindle. When we also take into account its rotation around its own axis, we are no longer surprised by this fact. To determine the critical fields E_{c2} and E_{c3} we will thus adopt the approximation of a spindle-shaped trap. It is then a simple matter to show that

$$E_{c2}=(p_P/e)\,|\omega_c-\Omega|,\qquad E_{c3}=(p_P/2e\Omega_M)\,|\omega_c^2-\Omega^2|. \quad (15)$$

The singularities at the fields E_{c2} and E_{c3}—in the absorption coefficient of a strong alternating electric field, for example—can occur at quite small values of the time τ^+, i.e., when the emission of an optical phonon occurs essentially instantaneously, and a large fraction of the carriers lie on main trajectories. As the amplitude E_0 is decreased, the first main trajectory which lies entirely in the passive region then appears at $E_0 = E_{c2}$, and the absorption of energy from the microwave field begins to decrease. This decrease occurs in the amplitude interval $E_{c3} < E_0 < E_{c2}$, i.e., quite smoothly (even in the limit $\tau^+ = 0$), since in this interval there are both main trajectories which lie entirely in the passive region (a carrier moving along such a trajectory weakly absorbs energy of the alternating field) and main trajectories which cross the boundary of the passive region. Just which of the main trajectories the carrier will move along depends on the phase of the electric field at the time in which the optical phonon is emitted. The situation is very similar to that during the absorption of a strong, linearly polarized microwave signal in the absence of a magnetic field.[4] In contrast, in the case of a circular polarization all the main trajectories are closed in the passive region at a common field amplitude E_0, so that the singularity in the absorption is sharper in the case of circular polarization.[2]

If, on the other hand, the time τ^+ is not too short, and the carriers tend to accumulate in the trap, then the absorption coefficient will already be beginning to decrease at $E_0 = E_{c1}$, when carriers begin to appear in the trap, where they weakly absorb energy from the alternating field.[13] This decrease will become sharper with increasing carrier lifetime in the trap, τ^-. In the limit $\tau^- \to \infty$, all the carriers accumulate in a trap, and the absorption coefficient abruptly drops to zero at $E_0 = E_{c1}$ (cf. Ref. 13).

5. WIDTH OF THE CYCLOTRON RESONANCE REGION

Experimentally, the absorption coefficient for a strong *rf* field is usually studied as a function of the magnetic field in the cyclotron resonance region.[18] Using (11) and (15) for the critical fields $E_{c1,2,3}$, we can easily determine the magnetic-field intervals $\Delta_1 H$, $\Delta_2 H$, and $\Delta_3 H$ in which, respectively, there is no trap (so that the absorption is high), there are no main trajectories in the passive region, and, finally, all the main trajectories lie in the passive region:

$$\Delta_1 H:\quad \Omega\frac{mc}{e}\left(1-\frac{eE_0}{\Omega p_P}\right)^{1/2}<H<\Omega\frac{mc}{e}\left\{\frac{eE_0}{2\Omega p_P}+\left[\left(\frac{eE_0}{2\Omega p_P}\right)^2+1\right]^{1/2}\right\}, \quad (16a)$$

$$\Delta_2 H:\quad \Omega\frac{mc}{e}\left(1-\frac{eE_0}{p_P\Omega}\right)<H<\Omega\frac{mc}{e}\left(1+\frac{eE_0}{\Omega p_P}\right), \quad (16b)$$

$$\Delta_3 H:\quad \Omega\frac{mc}{e}\left(1-\frac{2eE_0}{\Omega p_P}\right)^{1/2}<H<\Omega\frac{mc}{e}\left\{\frac{eE_0}{\Omega p_P}+\left[\left(\frac{eE_0}{\Omega p_P}\right)^2+1\right]^{1/2}\right\}. \quad (16c)$$

Each interval incorporates the preceding interval. If the left sides of these inequalities are purely imaginary or negative they should be replaced by zero. We may conclude from these inequalities that the cyclotron resonance curve is generally asymmetric, broader on the side of weak magnetic fields. This tendency was clearly seen in the experiments of Ref. 18. This circumstance also distinguishes the cyclotron resonance in the field of a linearly polarized wave from that in the field of a circularly polarized wave, where the absorption curve is symmetric.[2]

6. INVERTED DISTRIBUTION IN THE CASE OF DEGENERATE BANDS

Another important field of application of this theory is a semiconductor with degenerate light-hole and heavy-hole bands, such as *p*-Ge. Under certain conditions, a situation may arise in such a semiconductor such that a trap in which carriers can accumulate occurs in the light-hole band but not in the heavy-hole band. This circumstance can cause a change in the population of the light-hole band and, ultimately, the generation of infrared light.[16]

It follows from the analysis above that a necessary condition for this situation is

$$E_{c1}^H<E_0<E_{c1}^L, \quad (17)$$

where E_{c1}^L and E_{c1}^H are the threshold fields for the light and heavy holes, respectively. Using (11) for the threshold field, we find that the condition $E_{c1}^H < E_{c1}^L$ and, correspondingly, (17) can hold only if

$$\Omega<\Omega^*=(\omega_c^L\omega_c^H)^{1/2}, \quad (18)$$

where ω_c^L and ω_c^H are the cyclotron frequencies of the light and heavy holes. Since $\omega_c^H < \Omega^* < \omega_c^L$, an energy-inverted distribution can also be produced in a degenerate-band semiconductor at $\Omega = \omega_c^H$, for example, i.e., under conditions corresponding to a heavy-hole cyclotron resonance. The possibility of this occurrence in a magnetic field crossed with a circularly polarized rf electric field was discussed in Ref. 14.

We believe that the most favorable situation for producing a repopulation of the light–hole band is that in which the trap in the light-hole band contains the center of the passive region at all times, while there is no trap in the heavy-hole band. A necessary condition here is

$$E_{c1}^H<E_0<E_{c3}^L, \quad (19)$$

i.e., $E_{c1}^H < E_{c3}^L$. Analysis with the help of (11) and (15) shows that the latter inequality holds if

$$\omega_c^L\left(\frac{2-\varkappa^{1/2}}{2\varkappa^2-\varkappa^{1/2}}\right)^{1/2}<\Omega<\omega_c^L f(\varkappa), \quad (20)$$

where $\varkappa = m_H/m_L$ is the ratio of the masses of the heavy and light holes, and $f(\varkappa)$ is the positive root of the equation

$$x^3+2\varkappa x^2-\varkappa x-2=0. \quad (21)$$

For *p*-Ge we have $\varkappa = 7.9$ and $f(\varkappa) = 0.24$. It follows from (20) that at $\varkappa > 4$ the frequency Ω has only an upper bound [in this case the left side of inequality (20) is taken to be zero], while at $\varkappa < 4$ it has both upper and lower bounds. To produce an inverted distribution in a semiconductor with $m_H/m_L < 4$ (*p*-Si), therefore, we should use an alternating electric field, since in crossed static fields, without a trap in the heavy-hole band, the trap in the light-hole band may not be large enough to contain the center of the passive region.

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[1)]If the frequency ratio ω_c/Ω is irrational, there are no gaps in the filling. If the ratio ω_c/Ω is rational, there are gaps in the filling, although the filling is very "dense" if ω_c and Ω are greatly different.

[2)]Trajectories with irrational ratios ω_c/Ω could be classified in the same way.

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