Quantum-mechanical tunneling sith dissipation. The pre-exponential factor

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On the plot of the lifetime of a metasable state vs temperature there is a point T_0 near which an abrupt transition from classical to quantum decay takes place. The vicinity of this point is investigated at arbitrary viscosity. The pre-exponential factor at arbitrary temperature is obtained in the limit of high viscosity.

1. INTRODUCTION

The influence of dissipative processes on the probability of quantum-mechanical tunneling through a potential barrier has been actively investigated in the last few years. Such effects were observed in experiments on the lifetimes of metastable states of superconducting tunnel junctions.[1,2] The phenomenological theory of this phenomenon was developed in the paper by Caldeira and Leggett.[3] The microscopic derivation of the effective action for a superconducting tunnel junction was obtained in Refs. 4 and 5. The lifetime of the metastable state is determined, with exponential accuracy, by the extremal value of the effective action. Its temperature dependence was obtained in Refs. 6 and 7. It was shown that a temperature T_0 exists above which the temperature dependence of the tunneling probability is determined by the classical formula $\Gamma \propto \exp(-U/T)$. At $T<T_0$ an important role is played by the process of quantum tunneling, and the character of the temperature dependence of Γ changes. A second-order phase transition takes place on the point T_0. It is shown in the present paper that the quantum fluctuations wash out this transition in a narrow temperature region near T_0. The transition from the quantum to the classical decay regime was considered without allowance for dissipation in Ref. 8. At temperatures $T<T_0$ the quantum fluctuations determine the pre-exponential factor in the $\Gamma(T)$ dependence. We obtain below this factor for the important particular case of high viscosity and for a potential in the form of a cubic parabola. In this limiting case the argument of the exponential is determined by the viscosity and does not depend on the mass. A large contribution to the pre-exponential factor is made by the high-frequency fluctuations, whose spectrum depends on the mass even in the limit of high viscosity. The tunneling probability increases with decreasing viscosity like m-2.

2. INFLUENCE OF QUANTUM FLUCTUATIONS ON THE TUNNELING PROBABILITY

At zero temperature the lifetime Γ^{-1} of a metastable state is determined by the value of the imaginary part of the ground-state energy

$$\Gamma=2\,\mathrm{Im}\,E. \tag{1}$$

At low temperatures $T<T_0$, when the decay of the metastable state is determined by quantum-mechanical tunneling, the lifetime is [8,9,10]

$$\Gamma=2\,\mathrm{Im}\,F, \tag{2}$$

where F is the free energy:

$$F=T\ln Z,\quad Z=Z_0+iZ_1=\int Dq(\tau)\exp\{-A[q]\}. \tag{3}$$

In (3), Z is the partition function and $A[q]$ is the effective action. Since the imaginary part Z_1 of the partition function is small compared with the real part Z_0, Eq. (2) for Γ can be represented in the form

$$\Gamma=2TZ_1/Z_0=2TZ_0^{-1}\,\mathrm{Im}\int Dq(\tau)\exp\{-A[q]\}. \tag{4}$$

The functional integral in (4) is with respect to the function $q(\tau)$ defined on the interval $[-1/2T,\ 1/2T]$ and satisfying the condition $q(-1/2T)=q(1/2T)$.

To calculate Im Z we use a method developed in Refs. 9 and 10. There exists a function $\tilde q(\tau)$ on which the action $A[q]$ assumes an extremal value. The function $\tilde q(\tau)$ is obtained from the equation

$$\delta A[q]/\delta q=0. \tag{5}$$

Near the extremal trajectory, the function $q(\tau)$ can be represented in the form

$$q(\tau)=\tilde q(\tau)+\sum_n C_n q_n(\tau), \tag{6}$$

where $q_n(\tau)$ are the normalized eigenfunctions of the operator δ^2A/δ^2q, i.e.,

$$(\delta^2A/\delta q^2)\,q_n=\Lambda_n q_n, \tag{7}$$

with periodic boundary conditions $q_n(1/2T)=q_n(-1/2T)$.

One eigenvalue Λ_0 is negative. The contour of integration over C_0 must be shifted to the imaginary axis, so that an imaginary part appears in the partition function.

At $T<T_0$ the function $\tilde q(\tau)$ differs from a constant, and at an arbitrary τ_0 the periodic (with period $1/T$) function $\tilde q(\tau-\tau_0)$ is also a solution of Eq. (5). It follows therefore that the function $\partial\tilde q/\partial\tau$ satisfies Eq. (7) with zero eigenvalue. For each function $q(\tau)$ we choose τ_0 such as to approximate as close as possible $q(\tau)$ by the function $\tilde q(\tau-\tau_0)$, i.e, we determine from the condition for the minimum of the functional[11]:

$$D(\tau'|q)=\int_{-1/2T}^{1/2T} d\tau[q(\tau)-\tilde q(\tau-\tau')]^2;\qquad \left.\frac{\partial D}{\partial\tau'}\right|_{\tau'=\tau_0}=0. \tag{8}$$

The quantity Z_1 can be written in the form

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$$Z_1=\operatorname{Im}\int_{-1/2T}^{1/2T}d\tau'\int Dq(\tau)\exp\{-A[q]\}\delta(\tau'-\tau_0[q(\tau)])$$

$$=\operatorname{Im}\int_{-1/2T}^{1/2T}d\tau'\int Dq(\tau)\exp\{-A[q]\}\delta\left(\frac{\partial D(\tau'|q)}{\partial\tau'}\right)\left|\frac{\partial^2 D(\tau'|q)}{\partial\tau'^2}\right|. \quad (9)$$

It follows from (6) and (8) that

$$\frac{\partial D(\tau'|q)}{\partial\tau'}=2C_1\left[\int_{-1/2T}^{1/2T}d\tau\left(\frac{\partial\tilde{q}}{\partial\tau}\right)^2\right]^{1/2},$$

$$\frac{\partial^2 D(\tau'|q)}{\partial\tau'}=2\int_{-1/2T}^{1/2T}d\tau\left(\frac{\partial\tilde{q}}{\partial\tau}\right)^2. \quad (10)$$

Substituting expressions (10) in (9) we get

$$Z_1=\left[\int_{-1/2T}^{1/2T}d\tau\left(\frac{\partial\tilde{q}}{\partial\tau}\right)^2\right]^{1/2}\int_{-1/2T}^{1/2T}d\tau'\int_0^\infty\frac{dC_0}{(2\pi)^{1/2}}\exp\left(-\frac{C_0^2}{2}|\Lambda_0|\right)$$

$$\times\prod_{n\neq0}\int_{-\infty}^{\infty}\frac{dC_n}{(2\pi)^{1/2}}\exp\left\{-\frac{\Lambda_n}{2}C_n^2\right\}\delta(C_1)\exp\{-A[\tilde{q}]\}. \quad (11)$$

The quantity Z_0 can be analogously written in the form of a Gaussian integral over the region of the values of $q(\tau)$ near the minimum of the effective action. As a result we obtain for the lifetime Γ^{-1} of the metastable state the expression

$$\Gamma=\mathscr{B}\exp(-A), \quad (12)$$

where

$$A=A[\tilde{q}]-A[q_{min}];$$

$$\mathscr{B}=\frac{1}{(2\pi)^{1/2}}\left[\int_{-1/2T}^{1/2T}d\tau\left(\frac{\partial\tilde{q}}{\partial\tau}\right)^2\right]^{1/2}$$

$$\times\left|\operatorname{Det}'\left(\frac{\delta^2A}{\delta q^2}\right)_{q=\tilde{q}}\right|^{-1/2}\left|\operatorname{Det}\left(\frac{\delta^2A}{\delta q^2}\right)_{q=q_{\min}}\right|^{1/2}. \quad (13)$$

The prime on the determinant symbol in (13) means that the zero eigenvalue has been left out.

For a particle moving in a potential field and at zero temperature we have

$$\int_{-1/2T}^{1/2T}d\tau\left(\frac{\partial\tilde{q}}{\partial\tau}\right)^2=A/m. \quad (14)$$

In the presence of viscosity, however, relation (14) is incorrect even at zero temperature.

We have derived here the known formula (13) to demonstrate that it is valid for arbitrary $T<T_0$ and for any type of effective action, including in the presence of dissipation.

3. HIGH VISCOSITY

We consider the important case when the effective action is of the form

$$A[q]=\int_{-1/2T}^{1/2T}d\tau\left\{\frac{m}{2}\left(\frac{\partial q}{\partial\tau}\right)^2+V(q)+\frac{\eta}{4\pi}\int_{-\infty}^{\infty}d\tau_1\left(\frac{q(\tau)-q(\tau_1)}{\tau_1-\tau}\right)^2\right\};$$

$$V(q)=3V_0(q/q_0)^2\left[1-\frac{2q}{3q_0}\right]. \quad (15)$$

Here $q(\tau)$ is a periodic function with period $1/T$: $q(\tau+1/T)=q(\tau)$. The effective action has this form in superconducting tunnel junctions at a current J close to the critical J_c. In this case

$$q_0=[1-(J/J_c)^2]^{1/2},\quad V_0=\frac{J_c}{3e}[1-(J/J_c)^2]^{1/2}, \quad (16)$$

$$m=C^*/e^2,\quad \eta=1/R_{sh}e^2,$$

where C^* is the effective capacitance of the junction[5] and R_{sh} is the shunting resistance.

At high viscosity ($\eta^2q_0^2\gg6mV_0$) the extremal value of the action $A[q]$ reached on a function $\tilde{q}(\tau)$ equal to[6]

$$\tilde{q}(\tau)=\frac{q_0^3}{3V_0}\pi T\eta\sum_{n=-\infty}^{\infty}\exp\{-b|n|+2i\pi T\tau\}, \quad (17)$$

where $\tanh b=T/T^*$, $T^*=3V_0/\pi\eta q_0^2$.

In this case $A[\tilde{q}]$ is equal to

$$A=\frac{V_0}{T_0}\left[\frac{3}{2}-\frac{1}{2}\left(\frac{T}{T_0}\right)^2\right],\quad T_0=T^*[1-6m^*V_0/\eta^2q_0^2]. \quad (18)$$

Substituting expression (17) for $\tilde{q}(\tau)$ in (7) we get

$$-m^*\frac{\partial^2q}{\partial\tau^2}+\frac{6V_0}{q_0^2}\left(1-\frac{2\tilde{q}(\tau)}{q_0}\right)q$$

$$-\frac{\eta}{2\pi}\int_{-\infty}^{\infty}d\tau_1\frac{\partial q(\tau_1)}{\partial\tau_1}\left(\frac{1}{\tau_1-\tau+i\nu}+\frac{1}{\tau_1-\tau-i\nu}\right)=\Lambda q. \quad (19)$$

We have retained in this equation the term with m^*, which becomes significant for eigenvalues with large numbers.

We seek the solution of $q(\tau)$ in the form of the Fourier series:

$$q(\tau)=\sum_{n=-\infty}^{\infty}C_n\exp(i2\pi nT\tau). \quad (20)$$

For the Fourier coefficients C_n we obtain the equation

$$C_n[1+|n|\operatorname{th}b]-2\operatorname{th}b\sum_{n=-\infty}^{\infty}C_{n_1}\exp(-b|n-n_1|)$$

$$+\frac{m^*q_0^2}{6V_0}(2\pi Tn)^2C_n=\Lambda\frac{q_0^2}{6V_0}C_n. \quad (21)$$

For the first eigenvalues, the significant values of the index n are of the order of unity, and at high viscosity we can leave out the last term in the left-hand side of (21). We seek the eigenfunctions in the form

$$C_n=(|n|+C)\exp(-b|n|). \quad (22)$$

Functions of the type (22) satisfy the system (21) if the following relations hold:

$$C \operatorname{th} b = -\Lambda q_0^2/6V_0, \quad C^2 \operatorname{th} b = C + 2/\operatorname{sh}(2b). \tag{23}$$

From these equations we obtain the first two eigenvalues:

$$\Lambda_{0,1} = -\frac{3V_0}{q_0^2}\left[1 \pm \left(1 + \frac{4}{\operatorname{ch}^2 b}\right)^{1/2}\right]. \tag{24}$$

To find the remaining eigenvalues, we note that when n is replaced by $-n$ the system (21) goes over into itself. We can therefore seek the solutions C_n in the form of even and odd functions of the number n. The solution of even type, corresponding to the eigenvalue Λ_{N+2}, we seek in the form

$$C_n = (|n| + C)\exp(-b|n|) + d_n, \tag{25}$$

where $d_n = d_{-n}$ and is different from zero only at $|n| \leqslant N$.

The equation for $n = N$ with allowance for the equations for $n > N$ is

$$(6V_0/q_0^2 + N 2\pi T\eta) d_N = \Lambda_{N+2} d_N. \tag{26}$$

Since $d_N \neq 0$, we have

$$\Lambda_{N+2} = 6V_0/q_0^2 + 2\pi T\eta N, \quad N = 0, 1, 2, 3\ldots \tag{27}$$

The odd eigenfunctions with $n \geqslant 1$ are of the form (25) and yield the set of eigenvalues

$$\Lambda_{-1} = 0, \quad \Lambda_{-(N+2)} = \Lambda_{N+2}, \quad N = 0, 1, 2\ldots \tag{28}$$

For $|N| \gg 1$ we must take into account the last term in the left-hand side of (21). The second term of (21) can then be obtained by perturbation theory. In the zeroth approximation, the eigenfunction corresponding to the eigenvalue Λ_N is equal to $C_n = \delta_{n,N}$. Calculating the eigenvalues Λ_N in first-order perturbation theory, we obtain

$$\Lambda_N = 6V_0/q_0^2 + (|N| - 2) 2\pi T\eta + m^* (2\pi T N)^2. \tag{29}$$

Since the last term of (29) is small at $|N| \sim 1$ and (28) goes over into (27), we can assume that (29) is the correct expression for the spectrum at $|n| \geqslant 2$.

As T tends to T_0, the parameter b tends to infinity. The system (21) is then diagonalized and all the eigenvalues are easily found. Comparing the eigenvalue obtained in this manner with those obtained above, we verify that Eqs. (24), (28), and (29) yield the complete set of eigenvalues. The product of these eigenvalues determines the value of Z_1. The value of Z_0 is calculated in the same manner.

The minimum value of the action $A[q]$ is reached at $q(\tau) = 0$. The eigenfunction equation for this value of $q(\tau)$ is the same as the system (21) without the second term. The eigenvalues of such a system can be easily found and are equal to

$$\lambda_N = 6V_0/q_0^2 + 2\pi T\eta|N| + m^*(2\pi T N)^2, \quad N = 0, \pm 1, \pm 2\ldots. \tag{30}$$

Using Eq. (17) for the function $q(\tau)$ we obtain the normalization coefficient that enters in the equation for the lifetime Γ^{-1}:

$$\int_{-1/2T}^{1/2T} d\tau \left(\frac{\partial \tilde{q}}{\partial \tau}\right)^2 = \frac{6\pi V_0}{\eta}\left[1 - \left(\frac{\pi T\eta q_0^2}{3V_0}\right)^2\right]. \tag{31}$$

Substituting the obtained value of the spectrum (24), (29), (30) and of the normalization coefficient (32) in Eq. (13) we obtain

$$\Gamma = \frac{q_0}{(2\eta)^{1/2}} \exp(-A) \frac{\Pi_1}{\Pi_2},$$

where

$$\Pi_1 = \prod_{n=1}^{N}\left[\frac{6V_0}{q_0^2} + 2\pi T\eta n + m^*(2\pi T n)^2\right],$$

$$\Pi_2 = \prod_{n=2}^{N}\left[\frac{6V_0}{q_0^2} + 2\pi T\eta(n-2) + m^*(2\pi T n)^2\right]; \quad N \to \infty. \tag{32}$$

The products in (32) are expressed in terms of the Euler function $\Gamma(x)$, and in the high-viscosity limit we get

$$\Gamma = \frac{\eta^{7/2} q_0^3}{6 \cdot 2^{1/2} V_0 m^{*3}} \exp(-A). \tag{33}$$

We note that the pre-exponential factor in Eq. (33) for the lifetime does not depend on temperature at $T < T_0$. The temperature dependence of the normalization factor (31) was cancelled out by the temperature dependence of eigenvalues $\Lambda_{0,1}$ [Eq. (24)]. In the limit of high viscosity the argument of the exponential in (33) depends little on the mass. The pre-exponential factor, however, is determined by the eigenvalues with large number N and is inversely proportional to the square of the mass.

The dependence of the pre-exponential factor on the high-frequency fluctuations can make the frequency dispersion of the effective mass substantial. Equation (33) for the lifetime Γ^{-1} acquires then an additional factor

$$G = \exp\left\{2\int_0^\infty d\omega\left[\frac{1}{\omega + \omega^2 m(\omega)/\eta} - \frac{1}{\omega + \omega^2 m^*/\eta}\right]\right\}, \tag{34}$$

where $m^* = m(\omega = 0)$. In the case of a superconducting tunnel junction we have for $m(\omega)$ (Ref. 7)

$$m(\omega) = \frac{C}{e^2} - \frac{1}{4R_N e^2 \omega^2}\int_{-\infty}^{\infty} d\omega_1 g_l(\omega_1)[2g_R(\omega_1) - g_R(\omega_1 + \omega) - g_R(\omega_1 - \omega)], \tag{35}$$

where C is the capacitance of the junction. At frequencies $|\omega| < \Delta$ the second term in (35) renormalizes the capacitance and $m(\omega) = m^*$. For frequencies $|\omega| > \Delta$ the second term is proportional to ω^{-1} and leads to renormalization of the viscosity. Accurate to a number of the order of unity we have in this case

$$G = \exp\left\{\frac{2\eta}{\eta^*}\ln\left(1 + \frac{\eta^* e^2}{\Delta C}\right) - 2\ln\left(1 + \frac{\eta}{m^*\Delta}\right)\right\}, \tag{36}$$

where η^* is the effective viscosity at high frequency:

$$\eta^* = \frac{1}{e^2}\left(\frac{1}{R_{\text{sh}}} + \frac{1}{R_N}\right).$$

4. TEMPERATURE CLOSE TO T_0

An expression for the transition probability near the transition temperature can be obtained at an arbitrary form of the potential $V(q)$ and at any ratio of mass to viscosity. The reason is that the significant values of $q(\tau)$ are close to the extremal value of q_0. Therefore the effective action can be expanded in powers of $q(\tau) - q_0$.

It is convenient to expand the function $q(\tau)$ in a Fourier series

$$q(\tau)=q_0+T^{1/2}\sum_{n=-\infty}^{\infty} C_n \exp(2i\pi T\tau n), \quad C_n=C_{-n}^*. \tag{37}$$

Substituting Eq. (37) for $q(\tau)$ in Eq. (15) for the effective action, we get

$$A[q]=\frac{V(q_0)}{T}+\frac{1}{2}\sum_{n=-\infty}^{\infty}\Lambda_n|C_n|^2+\frac{1}{2}V'''(q_0)T^{1/2}[2C_0|C_1|^2+C_2C_{-1}^2+C_{-2}C_1^2]+\frac{1}{4}V^{(IV)}(q_0)T|C_1|^4;$$
$$\Lambda_n=m^*(2\pi Tn)^2+V''(q_0)+2\pi T\eta|n|. \tag{38}$$

The effective action was expanded in terms quadratic in $C_{|n|\neq 1}$ and the small terms of fourth order in $C_{\pm 1}$ were retained, since the coefficients $\Lambda_{\pm 1}$ vanish at the transition point

$$T_0=\frac{1}{2\pi m^*}\left[-\frac{\eta}{2}+\left(\frac{\eta^2}{4}-m^*V''(q_0)\right)^{1/2}\right]. \tag{39}$$

As before, by shifting the contour of integration with respect to C_0 to the complex plane and integrating over all $C_{|n|\neq 1}$, we get

$$Z_1=\frac{\exp(-V(q_0)/T)}{2[-V''(q_0)]^{1/2}}\prod_{n=2}^{\infty}\frac{1}{2\Lambda_n}\int_{-\infty}^{\infty}\frac{d^2C_1}{2\pi}\exp\{-\Lambda_1|C_1|^2-B|C_1|^4\}, \tag{40}$$

where

$$B=T\left\{\frac{V^{(IV)}(q_0)}{4}-\frac{[V'''(q_0)]^2}{2V''(q_0)}\left[1+\frac{V''(q_0)}{2\Lambda_2}\right]\right\}. \tag{41}$$

The partition function Z_0 [Eq. (31)] is determined by a Gaussian functional integral near the minimum of the action:

$$Z_0=\frac{1}{[V_0'']^{1/2}}\prod_{n=1}^{\infty}\{2[m^*(2\pi Tn)^2+V''(0)+2\pi T\eta n]\}^{-1}. \tag{42}$$

The integral in (40) is expressed in terms of the error integral $\Phi(x)$, so that

$$\Gamma=\frac{1}{2}\mathscr{B}[1-\Phi(x)]\exp\left\{x^2-\frac{V(q_0)}{T}\right\}, \tag{43}$$

where

$$\mathscr{B}=4m^*\pi^2T_0^3\left[-\frac{\pi V''(0)}{BV''(q_0)}\right]^{1/2}\frac{\Gamma(3+\eta/2\pi Tm^*)}{\Gamma(1-n_1)\Gamma(1-n_2)},$$

$$x=\frac{\Lambda_1}{2B^{1/2}};$$

$$\Phi(x)=\frac{2}{\pi^{1/2}}\int_0^x dt\exp(-t^2),$$

$$n_{1,2}=\frac{1}{2\pi Tm^*}\left[-\frac{\eta}{2}\pm\left(\frac{\eta^2}{4}-m^*V''(0)\right)^{1/2}\right].$$

For potential motion (viscosity coefficient $\eta = 0$) Eq. (43) coincides with the result of Ref. 8. In the high-viscosity limit we obtain from (43)

$$\mathscr{B}=\frac{4\eta(\pi T_0)^{3/2}}{\Gamma(\alpha)}\left(\frac{\eta}{2\pi Tm^*}\right)^{\alpha}\times\left[-\frac{V''(0)}{V^{(IV)}(q_0)V''(q_0)-(V'''(q_0))^2}\right]^{1/2}, \tag{44}$$

where

$$\alpha=1-\frac{V''(0)}{V''(q_0)};$$

$$x=\frac{2\pi\eta(T-T_0)}{T_0^{1/2}(V^{(IV)}(q_0)-(V'''(q_0))^2/V''(q_0))^{1/2}}.$$

For a potential $V(q)$ in the form of a cubic parabola we have $V''(0) = -V''(q_0)$ and Eq. (44) goes over into Eq. (33) at $T < T_0$. For a potential of different form, the pre-exponential factor has a power-law dependence on the mass and the argument of the exponential is determined by the form of the potential.

Just as in the case of temperatures not close to T_0, the pre-exponential factor in (44) depends on the high-frequency fluctuations. An important role can therefore be assumed by the frequency dispersion of the effective mass, and in this case an additional factor in the form of (34) or (36) will appear in the right-hand side of (44).

The temperature dependence of the lifetime near T_0 is determined by an integral with respect to the complex parameter $C_1 = C^*_{-1}$. The integrand does not depend on the phase of C_1; this is equivalent to separating the zeroth mode at $T < T_0$. Integration with respect to the modulus of C_1 yields the error function $\Phi(x)$.

At high values of the argument, below the transition point, the difference between the error function $\Phi(x)$ and -1 is exponentially small. Therefore the proximity to the transition point manifests itself only in the exponentially small terms. The situation is reversed above the transition point: the argument of the exponential does not depend on the proximity of T to T_0, and the pre-exponential factor has a power-law singularity. This singularity becomes smeared out in a narrow region near a point T_0 whose width, as follows from (43), is of the order of

$$\Lambda_1\sim 2B^{1/2} \quad\text{or}\quad \delta T/T\sim(T/V)^{1/2}. \tag{45}$$

It must be noted that when the temperature is raised Eq. (2) no longer holds, since it does not describe correctly the passage of the above-barrier excitations. In this temperature region we have[9]

$$\Gamma=2T_0Z_1/Z_0. \tag{46}$$

Near T_0, Eq. (46) coincides with expression (3) for Γ. At $T-T_0\gg T_0(T/V)^{1/2}$ we get from (46)

$$\Gamma=T_0\left[-\frac{V''(0)}{V''(q_0)}\right]^{1/2}\frac{\Gamma(1-\chi_1)\Gamma(1-\chi_2)}{\Gamma(1-n_1)\Gamma(1-n_2)}\exp\left(-\frac{U}{T}\right), \tag{47}$$

where

$$
\begin{aligned}
\chi_{1,2}&=\frac{1}{2\pi T m^{*}}\left[-\frac{\eta}{2}\pm\left(\frac{\eta^{2}}{4}-m^{*}V''(q_0)\right)^{1/2}\right],\\
n_{1,2}&=\frac{1}{2\pi T m^{*}}\left[-\frac{\eta}{2}\pm\left(\frac{\eta^{2}}{4}-m^{*}V''(0)\right)^{1/2}\right],
\end{aligned}
\qquad (48)
$$

$U = V(q_0) - V(0)$ is the height of the potential barrier. Equation (47) agrees with the result of Ref. 12.

We have used above for the action expression (15) which, in particular, yields the effective action of a superconducting tunnel junction at currents close to critical. At currents not close to critical, the effective action has a more complicated form.[4,7] In particular, the damping is nonlinear in this case. Near the transition temperature, however, expansion (38) for the action remains valid, but the connection between the coefficients of this expansion and the physical parameters becomes more complicated. Therefore the qualitative picture of the dependence of the lifetime Γ^{-1} on the temperature remains the same.

5. CONCLUSION

The plot of the lifetime of the metastable state vs temperature has a point T_0 in the vicinity of which the decay regime changes from classical at $T > T_0$ to quantum at $T < T_0$. The transition temperature T_0 is defined by Eqs. (18) and (39) and decreases with increasing viscosity. The width δT of the transition region is small, $\delta T/T \sim A^{-1/2}$, where A is the argument of the exponential in the decay probability of the metasstable state. The temperature dependence of the lifetime near T_0 is determined by the presence of two soft modes.

In superconducting tunnel junctions at a current close to critical, the potential energy takes the form of a cubic parabola. In such a potential, the pre-exponential factor is independent of temperature at $T < T_0$.

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