Low-temperature conductivity and metal-insulator transition in compensate *n*-Ge

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The low-temperature (to ~40 mK) electric conductivity of *n*-Ge is investigated in the region of the metal-insulator transition induced by compensation. The metallic conductivity vanishes in continuous fashion near the transition. The power-law temperature dependence of the resistivity $\rho(T)$ in the critical region acquires on the insulator side at sufficiently low temperatures an activation dependence $\rho(T) \propto \exp(T_0/T)^x$, $x \approx 0.5$. The latter dependence is interpreted within the framework of the hopping conduction model with variable hop lengths in the region of the Coulomb quasigap. The critical behavior of the metallic and activated conductivities are obtained and compared with the predictions of scaling theory, as are the parameters that determine these conductivities, viz., the correlation length, the Coulomb gap, the localization radius, and the static dielectric constant.

1. INTRODUCTION

The transition of any system from a metallic into an insulator state is associated with carrier localization on a Fermi level. If the localization is induced by a disordering factor, the transition is frequently called an Anderson transition.[1] An example, in particular is the known single-electron "Anderson model." The Anderson transition is one of the most pronounced and least understood in the physics of disordered systems, because in real systems, as will be shown in the present paper, it is essentially a many-electron process. A convenient model object for the study of the transition is a strongly doped semiconductor having metallic conductivity, in which compensating impurities that capture the majority carriers are introduced in set doses. The fluctuating electrostatic potential produced in the system because of the random distribution of the impurities is in fact the disordering factor that leads to localization of the uncompensated majority carriers.[1,2]

In a strongly doped and weakly compensated semiconductor, the Fermi level E_F is situated in the region of the delocalized states of the majority or impurity band, while the energy boundary between the delocalized and localized states (the mobility threshold or the percolation level) is in the corresponding tail of the density of states. With increasing compensation this boundary and E_F shift towards each other. When they coincide the system goes over into the insulating state. This question was first considered quantitatively in the models of Mott[1] and of Shklovskiĭ and Éfros.[2] They constitute in fact a respective extrapolation, from the insulator side where the Anderson criterion for localization in the impurity band is used, and conversely, from the metal side with bands that are bent by the large-scale (poorly tunneling) fluctuating potential. Investigations[2–5] of compensated Ge gave no unequivocal answer concerning their equivalence. They showed, on the one hand, the fluctuations of the potential on the transition can tunnel quite readily at real doping levels, i.e., are not large-scale. On the other hand, they showed that, nonetheless, the uncompensated-carrier density n_c that is critical for the transition increases with the doping in accordance with the classical percolation theory.[2]

Mott[1] predicted, in the framework of quantum premises concerning the mobility threshold, a jumplike vanishing of the metallic conductivity $\sigma(0) = \lim \sigma_{T\to 0}$ at the transition, and the existence of metallic conductivity as a finite limit:

$$\sigma_M = \lim_{n \to n_c + 0} \sigma(0). \tag{1}$$

From the Anderson localization criterion and the Kubo-Greenwood formula, or with the aid of the Drude formula for the electric conductivity of a metal and the Ioffe and Regel idea that the minimum mean free path in a system cannot be smaller than the average distance between scattering, it was estimated in Ref. 1 that $\sigma_M = Ce^2/\hbar l$, where l is the average distance between potential wells or scattering centers, $C \approx 0.05$, and the remaining notation is standard.

From classical percolation considerations follows a continuous vanishing of $\sigma(0)$ at the transition, owing to the rarefaction of the percolation net up to formation in this net of one-dimensional channels in which metallic conductivity is no longer possible.[6] The conclusion that there is no break in $\sigma(0)$ follows also from the recently developing scaling theory of metal-insulator transitions (see, e.g., Refs. 7 and 8), which describes the critical behavior of the system by analogy with second-order phase transitions, in the form of power functions $\Phi(\xi)$ of the coherence length $\xi = \xi^* |1 - n/n_c|^{\nu_\xi}$, where ξ^* and ν_ξ are constants, ie.,

$$\Phi = \Phi^* |1 - n/n_c|^{\nu_\Phi}. \tag{2}$$

In particular, for metallic conductivity $\sigma(0)$, we have according to Ref. 7, $\nu_{\sigma(0)} \approx -\nu_\xi$, $\sigma^*(0) = A\sigma_M$, where $A \approx 1$. Recent experiments[9] on uncompensated Si:P have shown that $\sigma(0)$ does not become discontinuous at the transition and can be described by the scaling formula (2) with $\sigma^*(0) \approx 13\sigma_M$ (at $C = 0.05$) and $\nu_{\sigma(0)} \approx 0.55$. According to measurements[10] on Ge:Sb, introduction and increase of the compensation K lead to an increase of $\nu_{\sigma(0)}$ and to a decrease

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of A, viz., $A \approx 3.3$ and $\nu_{\sigma(0)} \approx 1$ at $K \approx 0.55$. To be sure, the error of the results of Ref. 10 is larger than in Ref. 9, since the measurements were made at higher temperatures and the samples were in a state farther from the transition. It is possible that because of these circumstances no values of $\sigma(0)$ smaller than the calculated σ_M were observed in Ref. 10.

The aforementioned Anderson-transition model are unified in a single-electron approach in which no account is taken of carrier interaction. Recently McMillan[11] and others attempted to include such polarization effects in the scaling theory of the transition. McMillan[11] used the result of Al'tsuler and Aronov[12] who, considering the interference between the electron-electron interaction and elastic scattering of electrons, reached the conclusion that a square-root singularity (pseudogap) exists in the density of states at the Fermi level on the metallic side of the transition and gives rise to anomalies of certain properties of the system. Experimental proof, obtained by tunnel spectroscopy, of the existence of such a singularity, was reported in Ref. 13. Kaveh and Mott[14] attributed to electron correlation the observed abrupt decrease of $\sigma(0)$ at the transition in uncompensated Si:P (Ref. 9). Generally speaking, the role of Coulomb interaction was recognized considerably earlier for localized carriers (see, e.g. Refs. 2, 15–17), where it leads to the appearance of the so-called Coulomb gap in the spectrum of the density of states for single-electron excitations, and according to certain calculations also for multielectron excitations of the polaron type. Such a gap should appear in the low-temperature thermodynamic and kinetic properties of the system on the insulator side of the transition. In particular, a stronger temperature dependence is expected for the conduction via hop-overs of carriers with variable (when the temperature varies) activation energy ε and with hop length r (the so-called variable range hopping, VRH), compared with the known Mott law $\ln \sigma \propto T^{-1/4}$ for noninteracting electrons, when the density of the localized states near the Fermi level is finite and constant.

This was proved experimentally by one of us in Refs. 3, 4, and 17. It was concluded there that in compensated Ge there exists on the insulator side of the Anderson transition a parabolic quasigap $g = g_0(E - E_F)^2$ (g_0 is a constant) in the density of the localized states in the vicinity of the Fermi level E_F. The estimate obtained for the coefficient g_0 at strong compensation was in fair agreement with the prediction of the theory of Éfros and Shklovskiĭ.[2] Another important empirical result[3–5] was the observed collapse of the gap and increase of the coefficient g_0 as the transition was approached (as the localization was weakened), owing to the weakening of the Coulomb interaction. Measurements[4] of compensated Ge having another level and doped with a majority impurity of different type and chemical properties have shown that g_0 is a universal function, independent of these factors, of the ratio n/n_c, meaning also of the coherence length ξ. This indicates that the gap-collapse dynamics as well as the critical behavior of other properties of compensated Ge near the Anderson transition can be described within the framework of scaling theory. The main task of the present investigation was in fact to shed light on this question.[1)]

2. PROCEDURE AND MEASUREMENT RESULTS

To introduce the compensating Ga impurity in n-Ge in measured doses we used, as in Ref. 3, neutron doping of n-Ge:As having initial arsenic density $\sim 6 \times 10^{17}$ cm^{-3} and grown by the Czochralski method. The resultant density N of the principal impurity and the degree of compensation K were determined by the procedure described in Ref. 3, the only difference being that we used a more accurate ratio, 0.3, of the densities of the donor and acceptor states introduced in the course of the neutron doping.[19] The set of compensated n-Ge samples had the parameters $N \approx (6\text{–}7) \times 10^{17}$ cm^{-3} and $0.7 \gtrsim K \gtrsim 0$. They were cut in a plane perpendicular to the growth axis [111] and measured $\sim 0.4 \times 1 \times 7$ mm.

The electric conductivity was measured with direct current, and at $T \lesssim 1$ K also with an S-72D bridge at 237 Hz. When necessary the measurement currents and voltages were decreased to ~ 1 nA and $\sim 10\,\mu$V, to locate the operating point on the ohmic section of the current-voltage characteristic. In this case, as a rule, the dc power dissipated in the sample exceeded by more than an order of magnitude the power released in the bridge measurements. Equality of the results of both measurement methods was therefore evidence that the samples were not overheated, as was also verified by measuring the current-voltage characteristic. The temperature was determined with a KG semiconductor thermistor in the interval 77.4–4.2 K, from the saturated vapor pressure of He^4 and He^3 in the interval 4.2–0.3 K, and from the magnetic susceptibility of the paramagnetic salt cerium magnesium nitrate. Temperatures lower than 1.4 K were obtained[2)] in an He^3–He^4 dissolution refrigerator.

Figure 1 shows a family of the temperature dependences of the resistivity $\rho(T)$ for typical investigated samples.[3)] We shall be interested here exclusively in the low-temperature impurity conductance due to electronic excitations near the Fermi level. Corresponding to this conductance[3] is the temperature region $\lesssim 30$ K on the left of the inflection points of the curves in Fig. 1. The electric conductance of samples 1–5 at sufficiently low temperatures is metallic and finite, and special experiments have shown that this is not due to overheating of the samples or to surface conductance. Samples 6–10, and at certain temperatures also sample 5, show the activation temperature dependence typical of the insulator side of the transition, wherein the derivative $|\partial \log \rho / \partial \log T|$ increases with decreasing temperature. The activation energy, a measure of which is this logarithmic derivative (see below) increases with increasing distance from the transition. In samples 5–7 that were close to the transition the activation conductivity is patently transformed into power-law conductivity with rising temperature. The dashed line of Fig. 1 shows the calculated (at $C = 0.05$) value of the reciprocal metallic conductivity σ_M^{-1}. It can be seen values $\sigma(0) \ll \sigma_M$ are observed (sample No. 5) thus contradicting the Mott premise that σ_M exists and meaning that $\sigma(0)$ tends to zero continuously.

3. ANALYSIS OF THE TEMPERATURE DEPENDENCE OF THE RESISTANCE

We shall investigate the temperature dependence of the resistivity by analyzing the temperature dependence of the

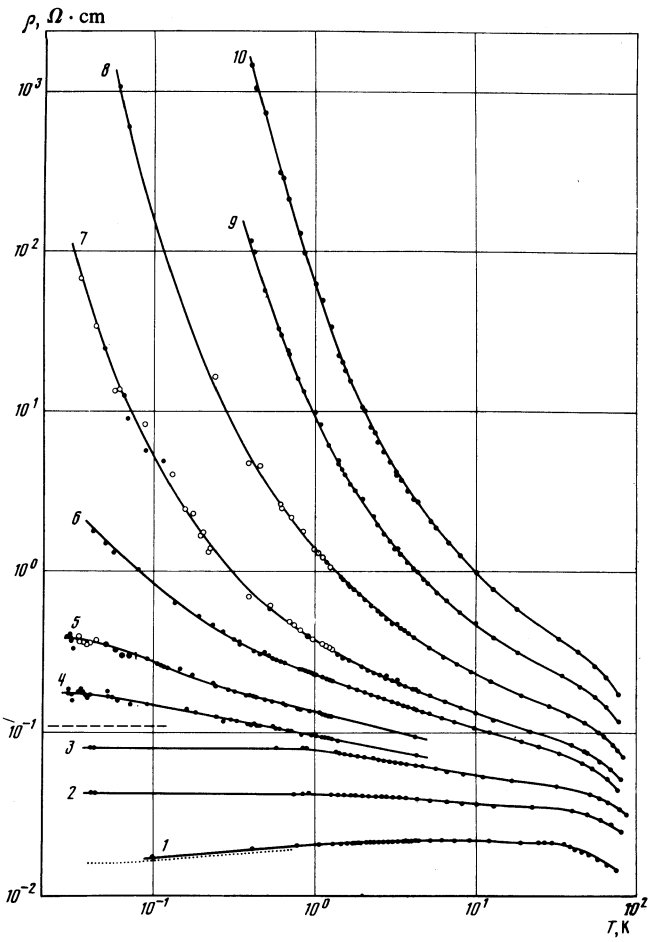

FIG. 1. The resistivity of n-Ge in the region of a metal–dielectric transition produced by compensation with direct current (●) and at 237 Hz (○). The numbers on the curves here and in the other figures correspond to the numbers of the samples. The density n, in units of 10^{17} cm^{-3} is: 1 — 5.75; 2 — 4.5; 3 — 4.15; 4 — 4.0; 5 — 3.85; 6 — 3.65; 7 — 3.3; 8 — 2.85; 9 — 2.15; 10 — 1.8. Dashed line—calculated value of σ_M^{-1}, dotted—data from Ref. 27 for Ge:Sb.

reduced activation energy of the conductivity $w \equiv \varepsilon / kT = T^{-1}\partial \ln \rho / \partial T^{-1}$, using a method developed by one of us.[20,3,17] The method makes use of the circumstance that most known laws governing the variation of $\rho(T)$ can be obtained as particular cases of the general expression

$$\rho(T) = BT^{-m} \exp\,(T_0/T)^x, \tag{3}$$

where B, m, T_0, and x are constants. In particular, for the VRH regime in the region of the quasigap $g(E) = g_0(E - E_F)^n$ for localized states in the vicinity of the Fermi level we have $x = (n+1)/(n+d+1)$, where d is the dimensionality of the space.[17] The essence of the method is to investigate in place of $\rho(T)$ the quantity

$$w(T) = m + x(T_0/T)^x. \tag{4}$$

To proceed now to the analysis of the doubly logarithmic $\rho(T)$ curves of Fig. 1 it must be recognized that $w = -\partial \log \rho / \partial \log T$. For an activation (exponential) $\rho(T)$ dependence, when the second term in the right-hand side of (4) is much larger than the first, we have $\log(w) \approx \log(xT_0^x) - x \log T$ so that by plotting $\log w$ vs $\log T$ we can easily find the value of x. The activation energy $\varepsilon(T)$ characteristic of the VRH regime is constant if the result is $x = 1$, and variable at $0 < x < 1$ (it decreases with temperature). It is then easy, in accord with (4), to determine the value of T_0 from the equation $T_0 = (w/x)^{1/x}$ at $T = 1$ K or from the equation $T_0 = (1/x)^{1/x} T$ at $w = 1$ (at $x = 1$ both determine the usual constant activation energy). In the case of a power law $\rho(T)$ dependence, however, $w = m$, i.e., we get the exponent directly.

Figure 2 shows the temperature dependences of $w(T)$, obtained by graphic differentiation of the curves of Fig. 1. Those with primed numbers were taken from Ref. 3. They correspond to higher compensations and are needed to single out distinctly the characteristic region. Curve a of Fig. 2 shows the high-temperature limit where the conductance is via low-energy electronic excitations in the vicinity of the Fermi level. At higher temperatures the energy of the current excitations increases quite steeply.

If the temperature is low enough ($T < T_v$, to the left of curve c), a region of linear dependence of $\log w$ on $\log T$ exists on the insulator side of the transition and corresponds to an exponential variation of $\rho(T)$ with a variable activation energy (3) with $x \approx 0.5$:

$$\rho(T) = \rho_0 \exp\,(T_0/T)^{1/2}. \tag{5}$$

The values of x in the region where the curves of Fig. 2 are linear were obtained also independently with a computer, using the experimental data (i.e., the points, and not the curves as above) described by the approximating formula (3) and using least squares. The results are given in parentheses in the caption of Fig. 2. It can be seen that they agree well with those obtained from Fig. 2. Near the transition (sample 5) or, conversely, far from it (sample 7), the largest attainable values of x are somewhat smaller than 0.5. We note in this connection that the error in the determination of x is larger the narrower the temperature variation interval in the region of validity of (5), i.e., the error is a maximum at the highest compensations and in the vicinity of the transition. For the

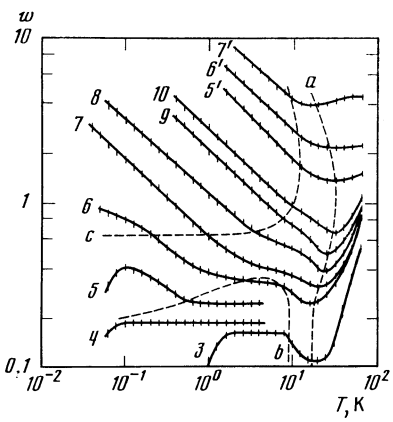

FIG. 2. Relative activation energy for the curves of Fig. 1. The primes mark the data of Ref. 3 with the following values of n [in units of 10^{17} cm^{-1}]: 5—1.3; 6′—0.9; 7′—0.14. The values of x are: 6 — 0.44(0.49); 7 — 0.47(0.47); 8 — 0.46(0.45); 9 — 0.48(0.49); 10 — 0.48(0.47); 5′ — 0.50; 6′ — 0.47; 7′ — 0.44.

intermediate samples Nos. 7–10, however, the relation (5) is valid for a temperature variation exceeding an order of magnitude. Figure 1 of Ref. 18 shows that those sections of the $\rho(T)$ curves which satisfy the condition $w(T) \propto T^{-1/2}$ are indeed well rectified when plotted in the coordinates $\log \rho$ and $T^{-1/2}$ that are characteristic of the insulator side of the transition, thus verifying the obtained $x \approx 0.5$.

Just as in Refs. 3 and 17, the exponential relation (5) observed at $d = 3$ and $T < T_c$ will be interpreted in the framework of the VRH model in the region of the parabolic semigap, when

$$g = g_0 (E - E_F)^2, \quad T_0 = \beta / g_0^{1/3} ak, \tag{6}$$

where β is a constant ($\beta \approx 2.8$ according to Ref. 21), a is the localization radius, and k is Boltzmann's constant. We have already noted that such a spectrum results from Coulomb interaction of the localized carriers[2,16,22] and that this viewpoint is confirmed by the fact that the experimentally obtained[3,4] value of the coefficient g_0 for strong compensation is of the same order as its value in the theory of Éfros and Shklovskiĭ.[2,21] According to this theory we have from strong localization

$$g_0 \approx \varkappa^3 / e^6, \tag{7}$$

where $\varkappa$ is the static dielectric constant, which tends on the insulator side, with increasing distance from the transition, to its limiting value $\chi_0 = 16$ (in Ge).

We present one more argument in favor of the Coulomb character of the quasigap, by comparing the theoretical[2] and experimental values of the parameter T_0 for the case of strong compensation. The necessary estimate of the localization radius will be made in the effective mass approximation: $a = a_{eff} (E_i / \varepsilon_1)^{1/2}$, where $E_i = 12.7$ meV is the ionization energy of As in Ge, ε_1 is the distance from the Fermi level to the percolation level, $a_{eff} = 1.42 (a_1^2 a_\parallel)^{1/3} = 53$ Å is the effective Bohr radius of the electron on the As impurity with account taken of the complicated four-ellipsoid spectrum,[23] and $a_{\perp,\parallel} = \hbar (2 m_{\perp,\parallel} E_i)^{-1/2}$. Substituting in (6) the value of a for the most compensated sample (No. 7′), calculated from the value $\varepsilon_1 \approx 25$ meV obtained from the high-temperature Hall effect,[3] we get the theoretical estimate $T_0 \approx 67$ meV. Experiment, on the other hand, yields 55 meV. We see that the agreement is fair if allowance is made for the possible error of the estimate and for the fact that possibly even in sample No. 7′ the localization has not reached the strong limit at which the wave-function overlap can be completely neglected and for which Eq. (7) was deduced on the basis of computer simulation. Simulation shows also that at strong compensations the only parabolic energy region is near EF and is much less than the gap width. An appreciable part of the gap, on the other hand, is not parabolic, and has more readily a linear or even a square-root variation of the density of states. This explains a certain decrease of x in the most compensated samples.

It can thus be concluded that the theory of Éfros and Shklovskiĭ[2] describes adequately the VRH regime far from the transition, on its insulator side. Our experiments, however, just as those of Refs. 3 and 4, show that the approximation (5) holds well for the entire insulator side of the transition, except in its immediate vicinity, as $T < T_v$. The observed decrease of T_0 on approaching the transition is explained in accordance with Refs. 2, 3 and 25 by the growth of the values of a and g as a result of the weakening of the localization. The x values somewhat lower than 0.5, observed near the transition, may result from the fact that at the very narrow Coulomb gap the experimental temperature was still not low enough to completely exclude the effect of thermal "washout," or from the fact that as the transition is approached the parabolic quasigap is continuously transformed into the square-root quasigap predicted in Ref. 12.

We consider now the temperature T_v at which the relation (5) is "suppressed." It can be seen from Fig. 2 that T_v has a maximum at moderate compensations corresponding to sample No. 5′, and decrease both with increasing distance from the transition and when the latter is approached in the critical region. The nature of the suppression, in our opinion, is different on the two sides of the maximum T_c. For strong compensations, according to Ref. 5, the onset of the law (5) from the high-temperature side must be preceded by a temperature-dependent tunneling through the fluctuation barriers with a rapidly decreasing activation energy. This process is made more difficult the higher the compensation, and thus lowers the value of T_v compared with the maximum. In this case the condition for T_v is that in that vicinity of the Fermi level in which the tunnel transition takes place the energy band that decreases with temperature become comparable with the width of the Coulomb gap.[3] In the critical region, however, where the VRH regime is preceded by nonactivated conduction, T_c is determined by the gap washout temperature, which is of the order of the gap width. The decrease of T_c as the transition is approached is in this case a consequence of the collapse of the gap. Of course, to observe the VRH regime in the quasigap region it is necessary at any rate that in the Fermi-level vicinity in which hops of the localized carriers take place the largest energy-band width, which is equal to $\sim 0.5\, k (T_0 T_v)^{1/2}$, not exceed the gap width.

As the transition is approached and T_v is decreased, a region appears, bounded by curve b of Fig. 2 and having a power-law dependence $\rho(T) \propto T^{-m}$. The exponent m decreases continuously from values ~ 0.4 (No. 7) to ~ 0.25 (No. 5). The effective power-law region increases then towards lower temperatures and reaches a maximum at the transition, when $m = 0.22 \pm 0.03$. On the metallic side, with increasing distance from the transition, the sphere of action of the power law with positive m is cancelled out on the low-temperature side, where an extensive region with a very weak $\rho(T)$ dependence appears. Here w falls off abruptly with temperature (No. 5-2). The weakening of $w(T)$ can be noted in sample No. 6 at $T \lesssim 0.1$ K. With increasing distance from the transition on the metallic side, the exponent m decreases to zero and reverses sign: $m > 0$ for No. 2 and $m < 0$ for No. 1. For the initial sample No. 1, a positive resistance temperature coefficient ($m < 0$) is preserved to the lowest working temperatures. We note that the change of the sign of the resistance temperature coefficient at low temperatures in the region of the metal-insulator transition in uncompensated

Ge is a known fact.[26,27] The distinguishing feature of our data is that introduction of compensation shifts this sign reversal from the transition towards the metallic side.

Consider the initial sample No. 1. Figure 1 shows for comparison a curve from Ref. 26 (shown dotted) for uncompensated Ge:Sb with Sb density $\sim 3.3\times 10^{17}$ cm^{-3}, having a value of $\sigma(0)$ close to that of No. 1. It can be seen that their $\sigma(T)$ dependences are close. We can therefore use the conclusions of Ref. 27 that such a $\sigma(T)$ dependence is determined mainly by the so-called correlation correction[12] to $\sigma(0)$, namely: $\Delta\sigma \propto -T^{1/2}$. When localization effects appear, the dominant role is gradually assumed by "localization" corrections $\Delta\sigma_l$ of opposite sign. From the observed monotonicity of the resultant $\sigma(T)$ plot it follows that the exponent α of the correction $\Delta\sigma_l \propto T^{\alpha}$ does not differ strongly from the exponent for $\Delta\sigma_c$ ($\alpha = 1$ in Refs. 14 and 27). Allowance for both corrections explains the reversal of the sign of m and its subsequent increase with increasing distance from the transition in the insulator direction. We call attention to the fact that in contrast to the metal-insulator transition in an uncompensated material, when the localization effects evolve near the transition itself, in our case these effects are initiated by introduction of a disordering factor—compensation—much earlier on the metallic side. The sign of the resistance temperature coefficient is correspondingly reversed earlier. One should think that this difference is typical in any case for the Anderson transition, and the distance from the region where m reverses sign to the transition should be larger the larger the disorder needed to stop the metallic conductance, e.g., the wider the initial impurity band or the higher the doping level. It is also understandable that it is incorrect to determine the Anderson transition from the reversal of the sign of the resistance temperature coefficient, as was done in several papers: the transition sets in fact at the larger disorder.

4. CRITICAL BEHAVIOR OF THE CONDUCTIVITY

In principle, by measuring the conductivity one can spot the transition in two ways: from the condition that the insulator state vanish, i.e., that the activation energy vanish at low temperatures, or from the condition that the metallic conductivity $\sigma(0)$ vanish. The latter way is based on an *a priori* assumptions that the scaling approximation is valid and that the material is homogeneous. We used therefore the first method and obtained for the transition the critical density nc of the uncompensated carriers from the condition

$$\lim_{n\to n_c-0} T_0(n)=0. \tag{8}$$

To reduce to a minimum the possible influence of the inhomogeneity of the initial samples, we corrected the values obtained from Hall measurements at $T = 300$ K using the $w(n)$ dependence at $T = 2$ K averaged for all the available samples. This is equivalent to using w as the argument of the studied functions, followed by a transition from w to n.

The procedure for determining n_c is illustrated by the inset of Fig. 3, where data near the transition are used for all the samples. It was found that $n_c = 3.95\times 10^{17}$ cm^{-3} and

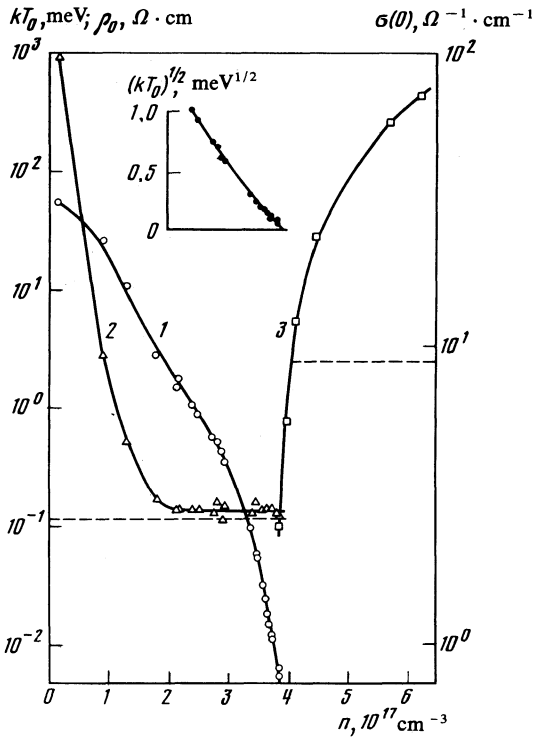

FIG. 3. Critical behavior of the conductivity: $1 - T_0$, $2 - \rho_0$, $3 - \sigma(0)$. Dashed: σ_M^{-1} (on the left) and σ_M (on the right).

accordingly $K_c = 1 - n_c/N \approx 0.3$. The possible relative error of n_c, just as the error of the corrected values of n, is of the order of several percent in the critical region. Understandably, the inaccuracy of the scaling parameter $|1 - n/n_c|$ increases as the transition is approached, thus bounding from below its more or less reliable values at a level of several percent. It also determines to a considerable degree the error in the critical exponents which are determined below.

We turn first to the insulator side of the transition, where (5) holds at sufficiently low temperatures $T < T_c$. In this case the critical behavior of the conductivity as a function of the density n is determined by the corresponding dependences of the coefficients T_0 and ρ_0.

A plot of $T_0(n)$ is shown in Fig. 3 (the values of T_0 for the samples of type No. 5 which are closes to the transition were obtained by extrapolating $w(T)$ to the boundary c in Fig. 2). In the critical region (at K = 0.3–0.5) the approximation of the evolution of $T_0(n)$ by the scaling-type equation (2) leads to a coefficient $T^*_0 \approx 40$ K and to a critical exponent $\nu_{T_0} = 2.1 \pm 0.1$. In accordance with (6), the $T_0(n)$ dependence is determined by the critical behavior of the Coulomb gap and of the localization radius. In particular, the following relation holds

$$\nu_{T_0} = -\nu_{g_0}/3 - \nu_a. \tag{9}$$

Questions concerning the dynamics of the Coulomb gap and the localization radius will be discussed later.

The factor ρ_0 of the exponential was determined by extrapolating the low-temperature ($T < T_v$) straight-line sections of the $\log\rho - T^{-1/2}$ curves (see Fig. 1 of Ref. 18) to infinite temperature. Its plot is also shown in Fig. 3. Values of ρ_0

and T_0 close to those shown in Fig. 3 were obtained by least-squares computer reduction of the data. The large value of ρ_0 at large compensations is the result of tunneling of large-scale potential fluctuations of the order of the screening radius R; this tunneling precedes the VRH regime from the side of high temperatures $T > T_v$.[3] The tunneling probability of such fluctuations is contained as a factor in ρ_0^{-1}, and the necessary condition for the realization of the VRH regime is a hop length $r(T_v) > R$. With increasing tunnel transparency of the potential fluctuations, the value of ρ_0 falls off steeply and when the fluctuations become transparent ρ_0 practically ceases to depend on the compensation. When the transition is approached, as can be seen from Fig. 3, ρ_0 has no critical behavior, so that the activation conductivity in the system has an upper bound:

$$\lim_{n \to n_c - 0} \rho_0 \equiv \rho_0^{min} \equiv (\sigma_a^{max})^{-1}, \tag{10}$$

where $\sigma_a^{max} \approx 8\,\Omega^{-1} \cdot \mathrm{cm}^{-1}$. According to the definition (10), σ_a^{max} is a metallic conductivity, i.e., independent of temperature, since $\lim_{n \to n_c - 0} T_0 = 0$. It is interesting that it is close to the calculated minimum Mott metallic conductivity $\sigma_M \approx 0.05(e^2/\hbar) n_c^{1/3} \approx 9\,\Omega^{-1} \cdot \mathrm{cm}^{-1}$, shown dashed in Fig. 3. Such an agreement was frequently regarded as evidence of the adequacy of the concept of minimum metallic conductivity. This conclusion is incorrect, at least in the system investigated here, where finite values $\sigma(0) \ll \sigma_M$ are observed for samples close to the transition on the insulator side.

We proceed now to the metallic conductivity $\sigma(0)$. Its value was taken to be the conductivity at $T \approx 40$ mK, which depends little on the temperature. Figure 3 shows a plot of $\sigma(0)$ for the available samples. Its approximation by Eq. (2) yields a coefficient $\sigma^*(0) = A\sigma_M$, where $A \approx 13$, and a critical exponent $\nu_{\sigma(0)} = 0.8 \pm 0.1$. The latter agrees with the data of Ref. 10 for compensated Ge:Sb, whereas the value of A is several times larger and agrees with its value in uncompensated Si:P (Ref. 9). We point out that in the vicinity of the transition deviations occur from the scaling formula and finite values of $\sigma(0)$ are observed on the insulator side of the transition. A similar singularity was observed recently[28] in uncompensated Si:P, where the passage through the transition was in small steps from the insulator side under uniaxial deformation. This behavior can in principle be due to macroinhomogeneities of the material, which "smear out" the transition point n_c over a certain region $n_c \pm \Delta n$. In our case, when the compensation was effected homogeneously during the course of the neutron doping, the inhomogeneities present are due to technological imperfections of the initial n-Ge:As grown by the Czochralski method.

5. CRITICAL BEHAVIOR OF COULOMB GAP

Fundamental to the understanding of the critical behavior of the system on the insulator side of the transition is the question of the dynamics of the Coulomb gap. Given its form (say, parabolic, $g(E) = g_0(E - E_F)^2$, as in our case) the gap is determined by two of three parameter: depth, width, and the coefficient g_0.

The gap depth δ is determined by the density of states near the Fermi level in the impurity band that is not perturbed by Coulomb interaction, or near its tail at weak and strong compensation, respectively. It is important that in the region of compensations that are critical for the investigated system the value of δ changes little with K, since these compensations correspond to the maximum of the density of states in the unperturbed band. The value of δ can be estimated from the empirical relation[3] $\delta = n/\gamma$, where γ is the width of the impurity band at moderate compensation, and the fall-off decrement of the density of state at strong compensations. The decrement, generally speaking, does not increase strongly with decreasing density n of the uncompensated carriers. As a measure of γ one can take from Ref. 3 the high-temperature activation energy of the Hall coefficient ε_1. The variation of δ obtained in this manner is shown in Fig. 1 (curve 1). Values of the same order are obtained by using the results of computer simulation of the gap in the impurity band,[4] which were partly published in Ref. 24 (curve 2 of Fig. 4.). The error of these data, including the case of very strong compensations, is $\sim 10\%$.

We estimate the gap width from experiment, using the condition for its thermal "washout":

$$\Delta = DkT_v, \tag{11}$$

where D is a quantity of the order of unity. This quantity should be assumed to decrease slowly with compensation, in view of the decrease of the density of states at the gap boundaries, i.e., of the value of δ. According to numerical calculations,[22] $D \approx 3$ (the less accurate estimate $D \approx 1$ was used in Ref. 18).

The behavior of Δ obtained from (11) at $D = 3$ is illustrated in Fig. 4. It turns out that $\Delta^* = 5$ meV and $\nu_\Delta = 1.9 \pm 0.2$. The absolute value of Δ is determined accurate to a factor on the order of two while the relative error in the critical region, which sets the error of ν_Δ, is considerably lower and is determined mainly by the accuracy of T_v. It can be seen that the exponent ν_Δ is close to the previously obtained ν_{T_0}, meaning that the boundary c is "horizontal" in the critical region. In fact, this horizontality means that the high-temperature suppression of the VRH regime occurs at slowly varying values $w(T_v) \propto (T_0/T_v)^{1/2}$, on the order of unity, from which it follows that $T_v \propto T_0$. We note that Δ^*

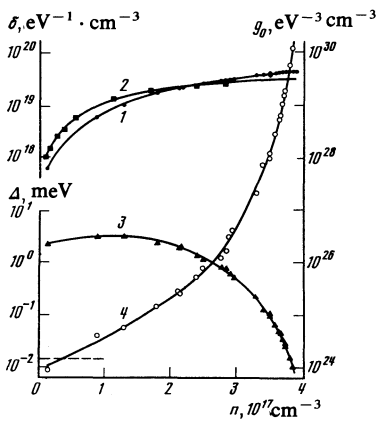

FIG. 4. Behavior of the Coulomb-gap parameters: $1, 2 - \delta$, $3 - \Delta$, $4 - g_0$. The dashed line marks the insulator limit $g_0 = \varkappa^3/e^6$.

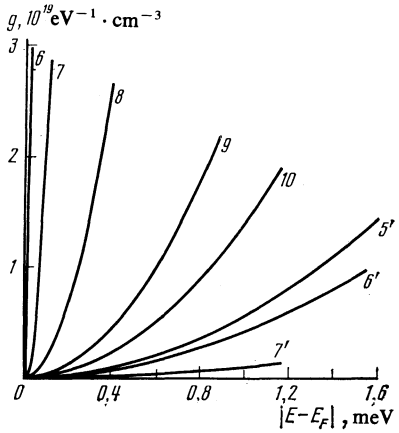

FIG. 5. Dynamics of Coulomb-gap collapse on the insulator side of the transition. The curves are terminated on the gap boundary at $|E - E_F| = \Delta/2$.

for weak compensation has the reasonable value of the order of the width of the impurity band, namely $\sim e^2 N^{1/3} \varkappa_0^{-1} \approx 8$ meV, and that if (11) is satisfied the condition $\Delta \gtrsim 0.5\, k\,(T_0 T_v)^{1/2}$ is also satisfied up to the highest compensation. The last circumstance allows us to use (11) also outside the critical region.

Since $\delta = g_0(\Delta/2)^2$ for a truly parabolic gap, it follows that knowing its depth and width we can calculate the coefficient g_0:

$$g_0 = 4\delta/\Delta^2. \tag{12}$$

A plot of $g_0(n)$ is shown in Fig. 4. The critical divergence g_0 is described by the coefficient $g_0 \approx 3 \times 10^{24}$ eV^{-3} · cm^{-3} with exponent $\nu_{g_0} = -(3.9 \pm 0.4)$. The absolute value of the latter, by virtue of the weak density dependence of $\delta(n)$ and the close values of the exponents $\nu_\Delta = \nu_{T_v} \approx \nu_{T_0}$ is approximately equal to $2\nu_{T_0}$, and its error is governed by the error of ν_{T_0}. In the limit of strong compensations, g_0 approaches the theoretical limit[2] $\sim \varkappa_0^3/e^6 \approx 1.4 \times 10^{24}$ eV^{-3} · cm^{-3}, shown by the dashed line.

From the obtained parameters of the Coulomb gap we can reconstruct the absolute variation of the density of the localized states in the gap at energies $E \lesssim \Delta$. By way of illustration of the indicated which can arbitrarily be called "VRH spectroscopy," Fig. 5 shows the dynamics of the collapse of the Coulomb gap in the investigated system.

6. CRITICAL DIVERGENCE OF THE LOCALIZATION RADIUS AND OF THE STATIC DIELECTRIC CONSTANT

The localization radius a can be calculated from Eq. (6) for the VRH regime in the region of a parabolic quasigap:

$$a = \beta/g_0^{1/6}\, kT_0. \tag{13}$$

The value $\beta = 2.8$ was used in the calculation.[21] The results are shown in Fig. 6. In the critical region they are described by the parameters $a^* \approx 8a_H \approx 400$ Å and $\nu_a = -(0.8 \pm 0.2)$. It is seen from Fig. 6 that outside the critical region the decrease of the localization becomes faster with increasing compensation, a fact we attribute to a deep-

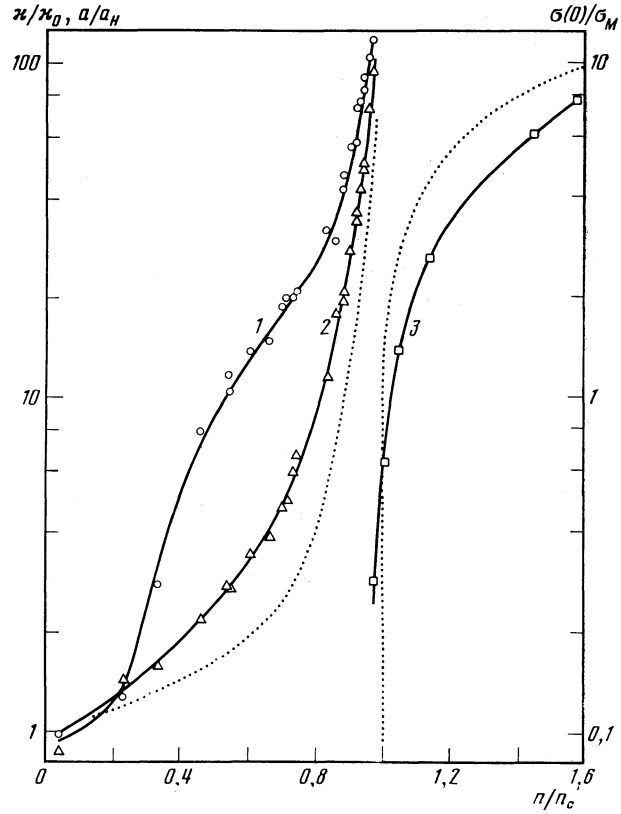

FIG. 6. Critical behavior of 1 — a/a_H, 2 — $\varkappa/\varkappa_0$, 3 — $\sigma(0)/\sigma_M$, dashed—data from Refs. 9 and 29 for Si:P.

ening of the Fermi level. The radius in the most compensated sample (No. 7') is $a \approx a_H$, i.e., comparable with its isolated-donor value that is typical of weak doping at low compensation. Clearly, this value is not the limit, and the localization radius should be smaller in a more accurately compensated material.

We have found above (see also Ref. 3) that, on the one hand, in the limit of large compensation we have in accord with the prediction[2,21]

$$\lim_{\kappa \to 1} g_0 \approx \varkappa_0^3/e^6,$$

and on the other hand g_0 diverges as the Anderson transition is approached (Fig. 4). This divergence is due mainly to the weakening of the Coulomb interaction, i.e., to the divergence of $\varkappa$, and can be understood from relation (7) if it is assumed that this relation is valid not only in the region of sufficiently strong localization, as corroborated in Refs. 2 and 21, but also in the case of weak localization, i.e., in the critical region. Under this assumption we can express the critical behavior of $\varkappa$ in terms of the coefficient g_0 obtained above:

$$\varkappa/\varkappa_0 \approx e^2 g_0^{1/6}/\varkappa_0. \tag{14}$$

The variation of the dimensionless parameter $\varkappa/\varkappa_0$ on the insulator side of the transition, obtained in accord with (14), is shown in Fig. 6. As the transition is approached one can track the behavior of over almost two decades compared with $\varkappa_0$. The divergence of $\varkappa$ can be described by the scaling-

type formula (2) with $\varkappa^* \approx 1.3\,\varkappa_0$, $\nu_\varkappa = -(1.3 \pm 0.15)$. The result $\varkappa^* \approx \varkappa_0$ seems quite reasonable, since this is the natural insulating limit. The exponent $\nu_\varkappa$ exceeds slightly the value obtained for it in Ref. 29 from direct measurements 29, 30 of uncompensated Si:P. The ratio $\nu_\varkappa / \nu_{\sigma(0)} \approx 1.6$ in our case and ≈ 2.1 in Ref. 29. The correspondence between our present results and those in Ref. 29 is evidence in favor of the assumption made above that relations (7), (13), and (14) are valid on the entire insulator side of the Anderson transition, including the critical region.

7. CORRELATION LENGTH

According to scaling theory, the critical behavior of the metallic conductance and of the localization radius are determined respectively by the values of ξ^{-1} and ξ. This enables us to reconstruct ξ from both sides of the transition independently of the analysis of the variation of $\sigma(0)$ and a, to ascertain the degree of its "symmetry," and, next, check on the validity of the approach used to estimate the localization radius and on the adequacy of the very scaling theory that suggest this symmetry.

On the insulator side the theory yields simply $\xi \approx a$. This leads to $\xi_a^* \approx a^* \approx 400$ Å and $\nu_{\xi_a} \approx \nu_a = -(0.8 \pm 0.2)$. On the metallic side $\sigma(0) \approx Ge^2/\hbar\xi$, where G is a certain constant. It follows hence that $\xi_\sigma^* = Ge^2/\hbar\sigma^*(0) \approx 200\,G$ Å, $\nu_{\xi_\sigma} \approx -\nu_{\sigma(0)} = -(0.80 \pm 0.15)$. We see that the exponents are in good agreement: $\nu_{\xi_a} = \nu_{\xi_\sigma}$. Matters are worse with the absolute value of ξ, which is determined by the coefficient ξ^*. The point is that the available theoretical estimates[11] lead to a value $G \sim 0.1$. We find thus that ξ_a^* is approximately 20 times larger than $\xi_\sigma^* \approx 20$ Å. Leaving aside the question of the accuracy with which the constants β and G were determined in Refs. 21 and 11, respectively, we wish to call attention to one physical cause of the symmetry of ξ on both sides of the transition in the system investigated here. Namely, the observed[10] dependence of the critical behavior of $\sigma(0)$ on the compensation leads, in the scaling approach, to a dependence of ξ on K, namely, ξ^* and $|\nu_\xi|$ increase with increasing compensation. The specific feature of the Anderson transition investigated by us, however, is that this transition is implemented by varying the compensation, while the critical region, where all the coefficients and exponents are defined, corresponds to $K \approx 0.4$ on the insulator side and $K \approx 0.2$ on the metallic side. The value of ξ determined from the metallic region is expected therefore to be several times smaller than that from the insulator side. This explains, incidentally, why our coefficient $\sigma^*(0)$ is close to the one observed in uncompensated Si:P.

8. CONCLUSION

We shall dwell here on the main conclusion of the work and on the problems.

The principal qualitative result of the conductivity measurements on the metallic side of the transition is that $\sigma(0)$ vanishes continuously in the compensation-induced Anderson transition. The critical densities corresponding to the vanishing of the low-energy activation energy and of the metallic conductivity differ somewhat. The transition point is therefore transformed into some region where at the very lowest temperature a finite value of $\sigma(0)$ exists, but at higher temperatures one can distinguish a temperature interval with activated conductivity.

The analysis of the experimental data for the insulator side of the transition is based on an *a priori* assumption that the observed conductance with variable activation energy corresponds to a Mott hopping conductance with variable hop range (VRH). We deem it necessary to call special attention to this, even though the VRH premise has become universally accepted in the last few years and there is no alternate explanation. Starting from this premise, it is stated that a parabolic Coulomb quasigap exists in the vicinity of the Fermi level. The critical behavior of the gap, especially the result that it collapses at the transition, is deduced from the empirical fact that the temperature T_v decreases if the condition for thermal washout of the gap is resorted to.[11] The basis of the continuation of the analysis is the assumption that gap preserves its parabolic shape all the way to its boundaries. This assumption yields the coefficient g_0 (12) that determines the variation of the density of states in the gap and describes next with the aid of (13) and (14) the critical divergence of the localization radius and of the static dielectric constant.

The critical behavior of the parameters of the system agrees both with the known experimental data and with the predictions of the scaling theory. The problem here is only the asymmetry of the absolute value of the correlation length on the two sides of the transition.

Recognizing the important role, established in this paper, of the electron-electron interaction in the region of the transition, it is clear that an adequate scaling theory must take this interaction into account. Let us compare our data with the predictions of McMillan's theory.[11] It makes use of a certain indeterminate parameter $1 \leqslant \eta \leqslant 3$ whose value depends on the role of the electron-electron interactions, with $\eta = 3$ corresponding to absence of these interactions. Within the framework of Ref. 11 the quantity η can be expressed in terms of the critical exponents in two ways, by the relation $\nu_\varkappa = \nu_\xi(\eta - 1)$ or $|\nu_\xi|\eta = \nu_\Delta$. For the system investigated they lead respectively to $\eta = 2.6$ and 2.4 [it turns out[29] that for the Si:P system $\eta > 3$(!)]. According to Ref. 11 the $\rho(T)$ dependence near the transition is reminiscent of a "tree,"[5] whose trunk is the critical relation $\rho(T) \propto T^{-m}$, where $m \approx \eta^{-1}$. On the two sides of the trunk are located respectively the "metallic" and "insulator" branches. From our experiment, however, it follows that in the critical regime $m \approx 0.22 \pm 0.03$, or approximately half the value of η^{-1}. Moreover, the three is replaced by a "bush" of diverging curves, whose exponent m continuously decreases on going through the transition from the insulator side, and at sufficiently low compensation reverses sign on the metallic side.

Finally, we wish to call attention to certain peculiarities observed by us and due to the fact that the transition takes place in a compensated material. We have replotted in Fig. 6, for comparison, the data of Refs. 9 and 29 on the critical behavior of $\sigma(0)$ and $\varkappa$ in uncompensated Si:P (the divergence of a and the collapse of the Coulomb gap are investigated here for the first time ever). It is seen that the compen-

sation smears out the metal-insulator transition and makes the critical dependences less abrupt. This manifests itself quantitatively in a certain decrease of the critical exponents (an increase of $\nu_{\sigma(0)}$ with increasing compensation was reported earlier in Ref. 10). The smearing effect is due to introduction into the system of a disordering factor, namely the fluctuating electrostatic potential of the charged impurities. The disorder due to the compensation causes also the resistance temperature coefficient to reverse sign on the metallic side farther from the transition than in the absence of the disorder.

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[1)]Preliminary reports of certain results were published by us in Ref. 18.
[2)]The corresponding measurements were made at the Institute of Physics Problems of the USSR Academy of Sciences.
[3)]The choice of a doubly logarithmic scale rather than the Arrhenius scale customarily used for semiconductors is due to the larger range of temperature variation, and also to the fact that this choice involves no *a priori* assumptions concerning the $\rho(T)$ dependence and permits as we shall see below, the necessary analysis.
[4)]These data were kindly provided by B. I. Shklovskiĭ and A. L. Éfros.
[5)]The term "tree" is due to B. I. Shklovskiĭ.

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