Interaction between acoustic pulses and electrons in metals

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The interaction between longitudinal acoustic pulses and electrons is studied under conditions in which the time of flight through the pulse region by resonant particles exceeds the relaxation time. The system is described by the equations of elasticity theory and the kinetic equations. The electron distribution functions and the resonant particle densities are found in the compression region (potential well) and in the rarefaction region (potential barrier), the deformation-potential constant being assumed to be positive. The elasticity-theory equation can be reduced by the method of slowly varying profile to a first-order partial differential equation [see (11)]. The latter equation is employed to investigate the evolution of a simple-shape pulse analytically in the case of short times. It is shown that the deformation amplitude varies according to a power law under nonlinear conditions, when the condition (1) is satisfied. This is in accordance with Eq. (31). The deformation does not vary in the region of the potential well. The nonlinear attenuation of a pulse that is a combination of compression and rarefaction regions is studied numerically. The different nature of the evolution of the forward and back fronts of the $u(x,t)$ lattice displacement pulse can be predicted from the results of analytic and numerical calculations. In particular, the sign of the deformation-potential constant can be determined by studying experimentally the attenuation of acoustic pulses under nonlinear conditions.

§1. INTRODUCTION

The evolution of acoustic pulses in metals is determined by their interaction with the conduction electrons. Depending on the pulse amplitude, two different regimes of this interaction can be distinguished. If the time of flight of the resonance electrons in the region of the pulse L is greater than the electron relaxation time τ_p and the electron velocity changes insignificantly in the time between collisions, a linear regime is realized. In this case, when the electron interacting with the pulse does not scatter within the time of interaction and changes its velocity significantly, a nonlinear regime exists.[1)] The condition for the realization of the nonlinear regime of interaction can be written in the form

$$a=L/\tilde{v}\tau_p\ll 1, \qquad \tilde{v}=(V_0/m)^{1/2}, \tag{1}$$

where $\tilde{v}$ is the characteristic velocity of the particle in the region of the pulse (in a system of coordinates that moves with the pulse), v_0 is the amplitude of the potential energy of the electron, m is the mass of the electron. If the acoustic pulse forms a potential well, then, upon satisfaction of condition (1), the resonance particles are divided in natural fashion into trapped and untrapped, and in the case of a potential barrier, between reflected and untrapped.

We now consider qualitatively the picture of the interaction of an acoustic pulse with resonant particles. First, we shall discuss the character of the electron distribution function. We shall assume that, in scales of the order of the dimensions of the pulse L, collisions are unimportant and the distribution function of the traveling particles is an equilibrium one. In the case of a potential barrier, the reflected particles have a velocity in the range $(w-\tilde{v}, w+\tilde{v})$, where w is the velocity of sound. In collisions with the pulse, the velocity of the electron in a system of coordinates moving with the velocity w, the sign changes at distances of the order of L from the center of the pulse. Here the particle distribution function $f(v_x', v_\perp)$ is equal to $f_0(-v_x', v_\perp)$, where $v_x' = v_x - w$ and $f_0(v_x', v_\perp)$ is the equilibrium distribution function. This fact is illustrated in Figs. 1c,d, on which are drawn the distribution functions of particles incident on the potential barrier from the right and from the left. Since the number of electrons moving counter to the pulse is greater than the number of electrons overtaking it, an inevitable concentration of resonant particles is formed in front of the potential well, while

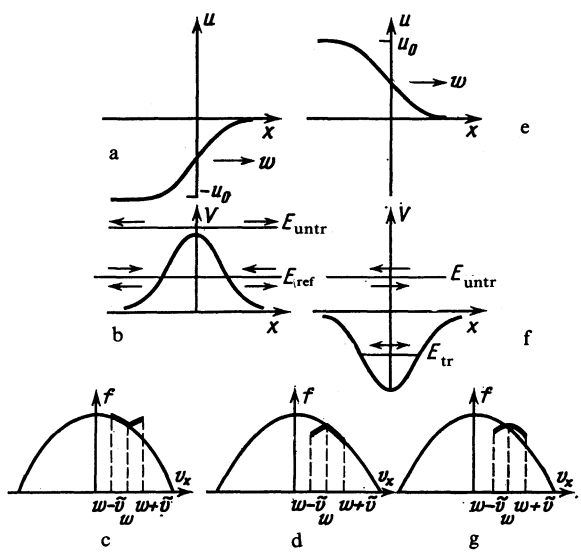

FIG. 1. The lattice displacement $u(x)$ (a), the potential energy $V(x)$ (b) and the electron distribution function, integrated over the transverse velocity, in front of the pulse (c) and behind it (d) for a rarefaction pulse. In the drawings *e*, *f*, *g*, the displacement $u(x)$, the potential energy $V(x)$ and the distribution function, respectively, are shown for a compression pulse (Λ_{xx} is assumed to be positive).

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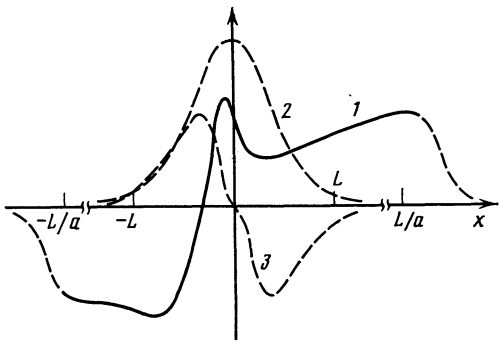

FIG. 2. Concentration of the resonant electrons in the field of the pulse $V(x) = -\Lambda_{xx}(x|x|/L^2)e^{-|x|/L}$ (curve 1). At distances of the order L/a, the concentration falls off as a consequence of electron collisions. The dashed curves show the shift of the lattice u (curve 2) and the potential energy $V = \Lambda_{xx}(\partial u/\partial x)$ (curve 3).

the concentration behind the pulse is below the equilibrium value. The characteristic profile of the concentration of resonant particles is shown in Fig. 2. The electrical neutrality of the metal here is assured by the nonresonant particles. It is not difficult to establish the fact that in this case, too, the total energy of the particles scattered by the barrier increases. Therefore, in the case of a pulse that forms the potential barrier (Figs. 1 a,b), exchange of energy between the reflected particles and the lattice leads to a finite damping even in a extremely nonlinear regime ($a = 0$).

At distances of the order $L/a = \bar{v}\tau_p$, where electron collisions become important, the distribution function relaxes to local equilibrium, while the concentration of resonant particles falls off exponentially, as is shown in Fig. 2. Here the momentum of the electron system is transferred to the lattice and an addition deformation appears on the scale of order $L/a \gg L$.

The distribution function of the resonant electrons in the field of the potential well, which is integrated over the transverse velocity, just as in the case of a monochromatic wave (see Ref. 2), has the form of a dome in the zeroth approximation in a (Fig. 1g). The concentration of resonant electrons is proportional to the small parameter a. Therefore, in the field of the potential well (Figs. 1e,f), upon satisfaction of condition (1), the exchange of energy between electrons and lattice takes place more slowly by a factor a than in the linear regime, and the damping is small. The same situation occurs in the case of a monochromatic wave.

In the present work, we have investigated the evolution of a pulse of longitudinal sound. A quantitative study is carried out with the help of a set of equations that includes the equation of elasticity theory and the kinetic equation. In the second section, we find the electron distribution functions in the fields of a potential barrier and of a potential well in an approximation of zeroth order in a, and also the concentration of the electrons, which determines the force acting on the lattice. The equation of elasticity theory is reduced, by the method of slowly changing profile, to a partial differential equation of first order. The evolution of the pulse is investigated analytically with the aid of this equation for times satisfying the condition $t < L/C_0$, where $C_0 \sim w(w/v_F)$, and v_F is the Fermi velocity. It is shown that at short times, the amplitude decay follows a power law with an exponent determined by the behavior of the initial profile of the pulse near the maximum [see Eqs. (28) and (30)]. If the pulse is asymmetric at $t = 0$, then the point of the maximum (in the system of coordinates connected with the pulse) shifts in the direction of a more gentle slope [see (27)]. It must be noted that the behavior of the amplitude in the nonlinear regime differs from the decay law in the linear regime, when, according to (35) and (36), the amplitude falls off according to a linear law. The latter assertion is valid for pulses described by sufficiently smooth functions. The equation that describes the evolution of the pulse has also been solved numerically. The corresponding graphs (Fig. 3) and discussion are given in Sec. 4.

2. BASIC EQUATIONS. ELECTRON DISTRIBUTION FUNCTION AND THE CONCENTRATION OF RESONANT PARTICLES

The complete set of equations describing the propagation of an acoustic pulse in metals consist of the equation of elasticity theory, the kinetic equation, and the condition of electric neutrality:

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k} + \frac{\partial}{\partial x_k}\int \frac{2d\mathbf{p}}{(2\pi\hbar)^3}\Lambda_{ik}(\mathbf{p})g(\mathbf{p},\mathbf{r},t), \tag{2a}$$

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \varepsilon}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{p}} = -\frac{f - f_0(\varepsilon_0(\mathbf{p}) + \delta\varepsilon)}{\tau_p}, \tag{2b}$$

$$Q + e\int \frac{2d\mathbf{p}}{(2\pi\hbar)^3} f(\mathbf{p},\mathbf{r},t) = 0. \tag{2c}$$

We have introduced the following notation here: u_i is the projection of the lattice displacement vector, σ_{ik} is the stress tensor, u_{ik} is the strain tensor, ρ is the density, e is the charge on the electron. The total energy of the electron in the deformed crystal is

$$\varepsilon(\mathbf{p},\mathbf{r},t) = \varepsilon_0(\mathbf{p}) + \delta\varepsilon, \quad \delta\varepsilon = (\lambda_{ik} + p_i v_k)u_{ik} - \bar{\lambda}_{ik}u_{ik},$$

where $\varepsilon_0(p)$ is the energy of the electron in the undeformed metal, $\Lambda_{ik} = \lambda_{ik} - \bar{\lambda}_{ik}$ (the bar denotes averaging over the Fermi surface), $Q = en_0(1 - \operatorname{div}\mathbf{u})$ is the charge per unit volume of the strained lattice, n_0 is the equilibrium concentration of the electrons, $g(p,r,t)$ is the correction to the distribution function, which is sought in the form $f = f_0(\varepsilon_0 + \delta\varepsilon) + g(\mathbf{p},\mathbf{r},t)$. The electron concentration $n(\mathbf{r})$, which is determined by the locally equilibrium part of the distribution function $f_0(\varepsilon_0 + \delta\varepsilon)$, cancels the charge of the lattice Q. Therefore, the equation of electrical neutrality (2c) reduces to the condition

$$\int \frac{2d\mathbf{p}}{(2\pi\hbar)^3} g(\mathbf{p},\mathbf{r},t) = 0. \tag{2d}$$

The solution for g, obtained from the set of equations (2a) and (2b), does not satisfy the condition (2d)—the integral on the left side of (2d) is proportional to $(w/v_F)\Lambda_{xx}(\partial u/\partial x)$. This means that charge compensation does not occur and therefore a nonadiabatic electric field should appear, which assures the compensation of the charge as a result of the redistribution of the nonresonant particles. This field (as is shown in Sec. 4) has the order

$$e\Phi_1 \sim \frac{w}{v_F}\Lambda_{xx}\frac{\partial u}{\partial x} \ll \Lambda_{xx}\frac{\partial u}{\partial x}.$$

The distribution function g can be represented in the form $g = g_p + g_1$ with account taken of the field $e\Phi_1$. Here g is the distribution function of the resonant particles, g_p is the distribution function of the nonresonant particles, which is proportional to $e\Phi_1$ (see Sec. 4). In view of the smallness of the field $e\Phi_1$ in comparison with the deformation potential $\Lambda_{xx}(\partial u/\partial x)$, the term with g_i and $e\Phi_1$ can be omitted in the kinetic equation in determination of the distribution function of the resonant particles.

The equation for $g(p,r,t) \approx g_p$ follows from (2b):

$$\frac{\partial g}{\partial t} + v_x\frac{\partial g}{\partial x} - \frac{\partial}{\partial x}\left(\Lambda_{xx}\frac{\partial u}{\partial x}\right)\frac{\partial g}{\partial p_x} + \frac{g}{\tau_p} = -f_0'(\varepsilon_\perp)\frac{\partial}{\partial t}\left(\Lambda_{xx}\frac{\partial u}{\partial x}\right). \quad (3)$$

In the argument of f_0 we neglect the energy of longitudinal motion of the electrons in comparison with the transverse, and also the term $p_i v_k$ in the expression for $\delta\varepsilon$, which is possible in the case $w/v_F \ll 1$ and $\Lambda_{xx}(\partial u/\partial x) \ll \varepsilon_F$. The pulse propagates along an axis of symmetry of high order, parallel to the x axis. Transforming to dimensionless variables $\xi = (x - wt)/L$, $s = (v_x - w)/\tilde{v}$ and assuming that the dependence of u on x and t has the form $u = u(x - wt)$, we write down Eq. (3) in the form

$$s\frac{\partial g}{\partial \xi} - \frac{\partial V_1}{\partial \xi}\frac{\partial g}{\partial s} + ag = \frac{\partial V_1}{\partial \xi}\frac{V_0 w}{\tilde{v}}f_0'(\varepsilon_\perp), \quad (4)$$

where $V_0 = V(\xi_m, t)$ is the extremal value of the function

$$V(\xi, t) = \Lambda_{xx}(\partial u/\partial x),$$

ξ_m is the extremum point of the function $V_1 = V(\xi,t)/V_0$. We seek as a solution for (4) the function

$$g(E, \tau, \varepsilon_\perp) = \frac{V_0 w}{\tilde{v}}f_0'(\varepsilon_\perp)\int_{-\infty} d\tau' \frac{ds}{d\tau'} e^{-a(\tau-\tau')}, \quad (5)$$

where τ is the time of motion along the trajectory $d\tau = d\xi/s$, $E = s^2/2 + V_1(\xi)$ is an integral of the motion. With the help of (5), we can obtain expressions in different orders in a ($a \ll 1$ is the strong nonlinearity condition) for the distribution function of untrapped, trapped, and reflected particles in the region of resonant velocities.

In the case of a potential barrier, the distribution function of the untrapped particles g_{untr} and reflected particles g_{ref} in zero order in a is determined by the simple expression

$$g = V_0\frac{w}{\tilde{v}}f_0'(\varepsilon_\perp)[s - s(-\infty)], \quad s(-\infty) = \pm[s^2 + 2V_1(\xi)]^{1/2}, \quad (6)$$

while the integral $|s| \geqslant [2(1 - V_1(\xi))]^{1/2}$ corresponds to the untrapped particles, and $|s| \leqslant [2(1 - V_1(\xi))]^{1/2}$ to the reflected; the plus sign corresponds to particles incident on the barrier from the left, while the minus sign applies to particles incident from the right.

The distribution function in the case of a potential well has the following form for trapped and untrapped particles:

$$g_{tr} = V_0\frac{w}{\tilde{v}}f_0'(\varepsilon_\perp)[-s + a\xi], \quad |s| \leqslant [2V_1(\xi)]^{1/2}, \quad (7)$$

$$g_{untr} = V_0\frac{w}{\tilde{v}}f_0'(\varepsilon_\perp)[-(s - \bar{s}) + a(\xi - \bar{\xi})], \quad |s| \geqslant [2V_1(\xi)]^{1/2}, \quad (8)$$

where

$$\bar{s} = \frac{\pi}{\varkappa K(\varkappa)}, \quad \frac{2}{\varkappa^2} - 1 = \frac{\bar{s}^2}{2} + V_1(\xi),$$

$$\bar{\xi} = \pi F(\xi/2, \varkappa)/K(\varkappa);$$

F and K are the incomplete and complete elliptic integrals of the first kind. By determining the complete distribution function with the help of Eqs. (7) and (8), and integrating it over the velocities of transverse motion, we can establish the fact that it has the form of a bell in the region of trapped particles, just as in the case of the monochromatic wave,[2] while in the region of untrapped particles it rapidly approaches the equilibrium function.

The concentration of resonant particles is found by integration of the distribution function g. Thus, for the potential barrier, and with the help of (6), we have

$$n_r(\xi, t) = \operatorname{sgn}(\xi - \xi_m)\frac{m^2 w}{\pi^2\hbar^3} \times \left[V\ln\frac{V_0^{1/2} + (V_0 - V)^{1/2}}{V^{1/2}} + (V_0(V_0 - V))^{1/2}\right]. \quad (9)$$

It must be noted that a contribution to (9) is also made by the reflected particles, the integral of the function g over the range of velocities corresponding to the untrapped particles is equal to zero. The concentration n_p, in accord with (9), vanishes at the point of the maximum of the function $V(\xi,t)$ and has the value $\pm m^2 w V_0/\pi^2\hbar^3$ at $\xi \to \pm\infty$. At the point where $V(\xi,t) = 0.305V_0(t)$, the concentration is an extremal one, and $n_{extr} = 1.1997 n_p(\pm\infty)$. With the help of (7) and (8), it is not difficult to establish the fact that in the case of a potential well, the concentration of resonant particles is equal to zero in zeroth order in a. It is not possible to carry out integration over the velocities in general form in the linear approximation in a, or to obtain an expression for the concentration in the field of a potential well of the type (9). For this reason, we limit ourselves in what follows to the calculation of the concentration in the principal (zeroth) order in a.

If the acoustic pulse consists of a region of compression and rarefaction, i.e., a well and a barrier located one after the other, then the calculations of the concentration are carried out according to the scheme given above. Here the formula (9) for the concentration of resonant particles is valid both in the region of the potential barrier and in the region of the potential well if we replace $V^{1/2}$ by $(-V)^{1/2}$ in the region where $V < 0$. In the region of the well there is an additional minimum concentration of the reflected particles n_r, which is connected, of course, with the increase in the velocity of the reflected particles incident on the well. A typical profile of the concentration of resonant electrons for potentials in the form of a well and a barrier is shown in Fig. 2.

3. EVOLUTION OF THE PULSE

The expression found in the previous section for the concentration of electrons allows us to investigate the evolution of the pulse. As follows from Eq. (2a), the force exerted by the electrons on the lattice is determined only by the resonant electrons. Setting $\Lambda_{xx}(p)=\Lambda=$ const for all resonant particles, we rewrite (2a) in the form

$$\frac{\partial^2 u}{\partial t^2}-w^2\frac{\partial^2 u}{\partial x^2}=\frac{\Lambda}{\rho}\frac{\partial n_r}{\partial x}, \tag{10}$$

where n_r is determined by Eq. (9). Equation (10) describes the nonlinear collisionless damping of acoustic pulses. Thanks to the smallness of this damping in the parameter w/v_F, the solution of (10) can be carried out in the approximation of slowly changing profile.[3] We shall seek a solution in the form of a pulse traveling in the positive direction of the x axis and one slowly changing its shape, i.e.,

$$u(x-wt, t')=u(\zeta, t'),$$

where the function $u(\zeta,t')$ depends weakly on t, $\zeta=x-wt$ is the accompanying coordinate, t' is the "slowed" time. Transforming to the variables ζ, t and neglecting terms proportional to $\partial^2 u/\partial t'^2$, we reduce the order of Eq. (10):

$$\frac{\partial V}{\partial t}=-\frac{\Lambda^2}{2\rho w}\frac{\partial n_r}{\partial \zeta} \tag{11}$$

(here and below the prime on t is omitted). Introducing the additional notation

$$C(V(\zeta,t), V_0(t))=\frac{\Lambda^2}{2\rho w}\frac{\partial n_r}{\partial V}, \tag{12}$$

we can easily represent (11) n the form

$$\frac{\partial V}{\partial t}+C(V, V_0)\frac{\partial V}{\partial \zeta}=0. \tag{13}$$

Setting $V(\zeta,t)=V_0(t)V_1(\zeta,t)$ here, we rewrite (13) in the following form:

$$\frac{\partial V_1}{\partial t}+C(V_1(\zeta,t))\frac{\partial V_1}{\partial \zeta}+\frac{1}{V_0}\frac{\partial V_0}{\partial t}V_1=0, \tag{14}$$

where

$$C(V_1)=\operatorname{sgn}(\zeta-\zeta_m)C_0\left[\ln\frac{1+(1-V_1)^{1/2}}{V_1^{1/2}}-\frac{1}{(1-V_1)^{1/2}}\right], \tag{15}$$

$$C_0=\frac{3}{8}w\frac{\Lambda^2}{\varepsilon_F^2}\frac{mn_0}{\rho}v_F. \tag{15a}$$

A feature of this nonlinear equation is that it contains two unknown functions: $V_1(\zeta,t)$ and $V_0(t)$, which can be found from (14) with the initial conditions

$$V(\zeta_0,\ t=0)=V_0(0)\varphi(\zeta_0),\ \zeta=\zeta_0 \quad \text{at} \quad t=0, \tag{16a}$$

where $\varphi(\zeta)$ is the shape of the pulse at $t=0$, and the boundary condition

$$V_1(\zeta_m,\ t)=1. \tag{16b}$$

The change in the shape of the pulse is connected with the singularities of the function $C(V_1)$. As follows from (15), the function $C(V_1(\zeta,t))$ has a singularity at the maximum point, i.e., at $V_1(\zeta_m,t)=1$ and at $V_1(\zeta,t)\to 0$. Near the maximum $V_1(\zeta,t)$, we can set

$$C(V_1(\zeta,t))=-\operatorname{sgn}(\zeta-\zeta_m)C_0[1-V_1(\zeta,t)]^{-1/2}, \tag{17}$$

and near $V_1=0$, the singularity is a logarithmic one, $C(V_1)=-1/2C_0\ln V_1$. If the derivative $\partial V_1/\partial\zeta$ of the potential is finite at the point ζ, where $V_1(\zeta,t)=0$, then the force acting on the lattice $\partial n_p/\partial\zeta=C(V,V_0)(\partial V/\partial\zeta)$, tends to infinity. The origin of the mentioned singularities of the function $C(V_1)$ is discussed below. At $V_1(\zeta,t)=0.305$ we have $C(V_1)=0$.

We now find the solution of (14) in the region $\zeta>\zeta_m(t)$. The characteristics of this equation are determined by integration of the set of ordinary differential equations

$$dt=\frac{d\zeta}{C(V_1)}=-\frac{V_0}{(\partial V_0/\partial t)}\frac{dV_1}{V_1}. \tag{18}$$

With account of the initial conditions (16a), we obtain the solution of (18) in the form

$$V_1V_0=V_0(0)\varphi(\zeta_0), \tag{19}$$

$$\zeta=\int_0^t dt'\,C\left(\frac{V_0(0)\varphi(\zeta_0)}{V_0(t')}\right)+\zeta_0, \tag{20}$$

or, by virtue of the relation $V(\zeta,t)=V_1V_0$,

$$V(\zeta,\ t)=V_0(0)\varphi(\zeta_0), \tag{21}$$

$$\zeta=\int_0^t dt'\,C\left(\frac{V(\zeta,t)}{V(\zeta_m(t'),t')}\right)+\zeta_0. \tag{22}$$

Further, eliminating ζ_0 from the set (21) and (22), we obtain, at $\zeta>\zeta_m(t)$:

$$V(\zeta,t)=V_0(0)\varphi\left[\zeta-\int_0^t dt'\,C\left(\frac{V(\zeta,t)}{V(\zeta_m(t'),t')}\right)\right]. \tag{23}$$

Similarly, in the region $\zeta<\zeta_m(t)$:

$$V(\zeta,t)=V_0(0)\varphi\left[\zeta+\int_0^t dt'\,C\left(\frac{V(\zeta,t)}{V(\zeta_m(t'),t')}\right)\right]. \tag{24}$$

Setting $\zeta=\zeta_m(t)$ in (23) and (24), and equating these expressions, we obtain

$$\varphi\left[\zeta_m(t)-\int_0^t dt'\,C\left(\frac{V(\zeta_m(t),t)}{V(\zeta_m(t'),t')}\right)\right]=\varphi\left[\zeta_m(t)+\int_0^t dt'C\left(\frac{V(\zeta_m(t),t)}{V(\zeta_m(t'),t')}\right)\right]. \tag{25}$$

With the help of (23) or (24) and (25), we can determine the dependence of the amplitude of the pulse on the time at small values of the time t, and also the law of motion of the maximum $\zeta_m(t)$. For this purpose, we assume that, near the maximum, the expansion of the function $\varphi(\zeta)$ in a series has the form

$$\varphi(\zeta)=1-\beta(\zeta/L)^2-\gamma(\zeta/L)^3. \tag{26}$$

Representing the function $\varphi(\zeta)$ in (25) in the form (26), we obtain

$$\zeta_m(t)=\gamma\frac{\Delta^2(t)}{2\beta},\quad \Delta(t)=\int_0^t dt'\,C\left(\frac{V(\zeta_m(t),t)}{V(\zeta_m(t'),t')}\right). \tag{27}$$

According to (27), the point of the maximum of symmetric pulse does not shift with the time. Substituting $\zeta_m(t)$ in Eq.

(23) or (23) at $\zeta = \zeta_m(t)$, we find, by taking (17) into account, the law of change of the amplitude in the principal approximation in t:

$$V(\zeta_m(t),\ t) = V_0(0)\,(1-2C_0\beta^{1/2}t/L). \tag{28}$$

It should be noted that (28) was obtained without account of the cubic term in (26). Account of the latter would have led the appearance of terms in (28) of higher order of smallness in t. If the pulse is symmetric and the function $\varphi(\zeta)$ close to the maximum has the form

$$\varphi(\xi) = 1-\beta(\xi/L)^{2n}, \tag{29}$$

where $n = 1,2,3,\ldots$, then, with the aid of (17) and (23), we can show that the amplitude of the pulse changes with time according to power law of the form

$$V(0,\ t) = V_0(0)\,(1-\alpha t^{\nu}). \tag{30}$$

Here

$$\alpha = \left[\beta^{1/2n}B\left(\frac{1}{\nu},\frac{1}{2}\right)\frac{1}{\nu}\frac{C_0}{L}\right]^{\nu},\quad \nu = \frac{2n}{n+1}, \tag{31}$$

where B is the beta function.

4. NUMERICAL SOLUTION. DISCUSSION OF THE RESULTS

The evolution of a pulse of complex shape has been studied by the method of numerical integration of Eq. (11). Graphs are shown in Fig. 3 that reflect the evolution of a pulse of the form $V(x) = -A\,|x|x\exp(-|x|)$. As is seen from the drawing, in the region of a potential barrier the evolution takes place in the same way as in the case of a solitary pulse: the point of the maximum shifts in the direction of gentler slope, the deformation in the region of the maximum decreases and at $x > x_1$, where x_1 is determined by the condition $V(x_1,t) = 0.305V_0(t)$, and an increase occurs in the deformation. The character of the evolution in the region in which the transition from a potential well to a potential barrier takes place is determined by the singularity of the right side of (11) at $V = 0$. If the derivative $\partial V/\partial x$ is finite at this point, then $\partial n_r/\partial x$ has a logarithmic singularity [see (15)]. The latter, as has already been noted, is connected with the discontinuity of the electron distribution function on the boundary of the region of velocities of the reflected and trapped particles. Because of the singularity of the function $\partial n_p/\partial v$ at $V = 0$, a rapid increase in the deformation, i.e., $V(x,t)$ takes place in the transitional region from well to barrier. Because of this, as is seen from Fig. 3, an additional maximum appears near the point $x = 0$. In the solution of Eq. (11), the formation of a group of trapped particles in the region between the principal maximum $V^{(1)}_{\max}$ and the additional maximum $V^{(2)}_{\max}$ has been taken into account. It can be shown that the concentration of resonant particles to the left of the additional maximum, where $V \leqslant V^{(2)}_{\max}$ has the form

$$n_r(V) = \frac{m^2w}{\pi^2\hbar^3}\left\{\left[V^{(1)}_{max}(V^{(1)}_{max}-V)\right]^{1/2} - \left[V^{(2)}_{max}(V^{(2)}_{max}-V)\right]^{1/2} + V\ln\left|\frac{(V^{(1)}_{max})^{1/2}+(V^{(1)}_{max}-V)^{1/2}}{(V^{(2)}_{max})^{1/2}+(V^{(2)}_{max}-V)^{1/2}}\right|\right\}. \tag{32}$$

As follows from Eq. (32), the function

$$C(V, V^{(1)}_{max}, V^{(2)}_{max}) \propto \partial n_r/\partial V$$

$V^{(2)}_{\max}$ acquires a root singularity with the appearance of the additional maximum at $V = V^{(2)}_{\max}$. This can in turn lead to the formation of a new maximum. Thus, within the damping time of the pulse, several maxima can be formed on the curve $V(\zeta,t)$. As is seen from the drawing, $V(\zeta,t)$ changes comparatively slowly in the region of the potential well, while the point of the minimum remains practically unchanged for small time intervals.

The electron collisions eliminate the singularity of the right side of Eq. (11). Actually, close to the separatrix, the times of motion to the point of return of the trapped and untrapped particles tend to infinity according to the same law. Since the character of the motion of the trapped and untrapped particles is the same, the distribution function, which is determined by Eq. (5), is continuous in velocity space. The width of the transition region is determined from the condition of equality of the time of motion of the electrons $T(E)$ and the collision time τ_r. Therefore, the value of the singularity at the points $V = 0$ and $V = V^{(2)}_{\max}$ and, correspondingly, the number of maxima that can be formed in the damping time are determined by the shape of the potential and the relaxation time τ_r.

It should be noted that the singularities of the evolution of a pulse of longitudinal sound that are discussed here and also in Sec. 3 are characteristic only for the nonlinear regime. In the linear regime, when the parameter a is much greater than unity, the wave equations in the collisionless ($v_F\tau_r \gg L$) and the collision ($v_F\tau_r \ll L$) regimes takes the form

$$\frac{\partial^2 u}{\partial t^2} - w^2\frac{\partial^2 u}{\partial x^2} = C_0 w\int\frac{dx'}{x-x'}\frac{\partial u}{\partial x'}, \tag{33}$$

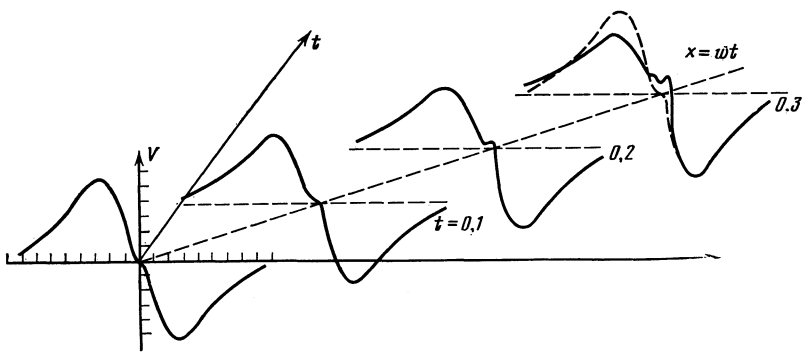

FIG. 3. Evolution of the deformation $\partial u/\partial x = V/\Lambda_{xx}$ for a pulse given at the initial instant of time by the function $-x|x|e^{-|x|}$, where x is measured in units of L and t in units of L/C.

$$\frac{\partial^2 u}{\partial t^2} - w^2 \frac{\partial^2 u}{\partial x^2} = 2C_0 v_F \tau_r \frac{\partial^3 u}{\partial t\, \partial x^2}. \tag{34}$$

Using the Fourier representation, we can show that the asymptotes of the solutions of the linear equations (33) and (34) at small t are,

$$u(x,t) = u(x-wt,0) - \frac{C_0 t}{2} \int \frac{dx'}{x'-x+wt} \frac{\partial u(x',0)}{\partial x'}, \tag{35}$$

$$u(x,t) = u(x-wt,0) + C_0 t \frac{v_F \tau_r}{4} \frac{\partial^2 u(x-wt,0)}{\partial x^2}. \tag{36}$$

respectively. A comparison of Eqs. (35) and (36) with formulas (28), (29), and (30), and with the results of computer calculations, shows a substantial difference in the character of the damping of the pulse in the nonlinear and linear regimes.

Up to now, we have assumed that the electrons (with the exception of the group of electrons with energies $E \to 0$ and $E \to V_{\max}$) do not undergo collisions in momentum space, i.e., over scales $\sim L$. The collisions become important at distances of the order of $L/a = \tilde{v}\tau_r$ from the center of the pulse. Since the electrons, which react reasonably with the sound, move almost perpendicularly to the direction of propagation of the pulse, this distance in metals is much smaller than the free path length $v_F \tau_r$. With the help of Eq. (3), it is not difficult to show that, over scales of the order of L/a, where $V = 0$, the electron distribution function has the form

$$g(s,\xi) = -2 \frac{V_0 w}{\tilde{v}} f_0'(\varepsilon_\perp)\, \theta(s\xi) \exp\left[-\frac{a}{s} (\xi - \operatorname{sgn}\xi) \right] \tag{37}$$

($\theta(x)$ is the theta function). The concentration of resonant particles, obtained by integration of the function (37) over the momenta, is

$$n_r(\xi) = \frac{m^2 w V_0}{\pi^3 \hbar^3} F(\xi),$$
$$F(\xi) = \operatorname{sgn}\xi \int_0^{\sqrt{2}} ds\, s \exp\left[-\frac{a}{s} (\xi - \operatorname{sgn}\xi) \operatorname{sgn}\xi \right]. \tag{38}$$

The time dependence of V_0 and $a = (V_0/m)^{1/2}$ is determined by Eqs. (28) and (30). Substituting (38) in (11) and using Eq. (30), which determines the amplitude of the symmetric pulse $V_0(t)$, we find that at large distances from the center of the pulse $\xi \sim L/a$,

$$V(\zeta,t) = 2\sqrt{2} \frac{C_0 t}{\tilde{v}(0)\tau_r} V_0(0) \operatorname{sgn}\zeta \frac{e^{-\varkappa/\sqrt{2}}}{\varkappa} \left[1 - \frac{\alpha}{4} \frac{\varkappa t^\nu}{(\nu+1)} \right],$$
$$\varkappa = a(\zeta=0) \left(\frac{\zeta - L \operatorname{sgn}\zeta}{L} \right). \tag{39}$$

Thus, according to (39), at small times and large distances, an increase in the deformation takes place. As has already been noted in the Introduction, this deformation is connected with the transfer of momentum from the electrons to the lattice in collisions.

In conclusion, we estimate the value of the nonadiabatic part of the electric field. We represent the potential of the longitudinal electric field that accompanies the sound pulse in the form

$$\Phi = \Phi_0 + \Phi_1, \tag{40}$$

where Φ_0 is the adiabatic electric field, which is found from the condition of compensation of the lattice charge by the nonresonant electrons, i.e.,

$$e\Phi_0 = -\lambda_{xx} (\partial u/\partial x);$$

Φ_1 is the nonadiabatic part of the field, which determines the charge compensation of the resonant particles. We represent the increment to the locally-equilibrium distribution function in the form

$$f = f_0(\varepsilon_0 + \delta\varepsilon) + g, \quad g = g_r + g_1. \tag{41}$$

Here g_r is the distribution function of the resonant particles, satisfying Eq. (3). The equation for the distribution function of the nonresonant particles, linearized with respect to g_1 and $e\Phi_1$ [which follows from (2b)] in the variables $v_x' = v_x = w$ and $\zeta = x - wt$ has the form

$$v_x' \frac{\partial g_1}{\partial \zeta} - \frac{1}{m} \frac{\partial(\delta\varepsilon)}{\partial \zeta} \frac{\partial g_1}{\partial v_x'}$$
$$= \frac{\partial}{\partial \zeta} (e\Phi_1) \left[(v_x' + w) \frac{\partial f_0}{\partial \varepsilon} + \frac{1}{m} \frac{\partial g_r}{\partial v_x'} \right]. \tag{42}$$

In the case of a strong nonlinearity ($a \to 0$) we have for g_r, in accord with (6),

$$g_r = -mw[v_x' - v_x'(-\infty)] \frac{\partial f_0}{\partial \varepsilon},$$
$$v_x'(-\infty) = \pm \left(v_x'^2 + \frac{2\delta\varepsilon}{m} \right)^{1/2}. \tag{43}$$

The distribution function of the resonant particles g_r differs from zero in a narrow range of velocities $\tilde{v}$ near $v_x' = 0$. Therefore, in accord with (42), the distribution function of the nonresonant particles will have an increment connected with it only in this region. In the nonresonant region of velocities, where $v_x' \sim v_F$, we can keep only the first term on the right side of Eq. (42). On the left side of the equation, we can neglect the second term. Actually, the ratio of the second part to the first part has the order

$$\frac{\partial(\delta\varepsilon)}{\partial\zeta} \frac{\partial g_1}{\partial v_x'} \Big/ v_x' \frac{\partial(e\Phi_1)}{\partial\zeta} f_0' \sim \frac{w}{v_F} \frac{V_0}{\varepsilon_F} \ll 1,$$

while the ratio of the second term of the left side to the first is

$$\frac{\partial(\delta\varepsilon)}{\partial\zeta} \frac{\partial g_1}{\partial v_x} \Big/ v_x' \frac{\partial g_1}{\partial\zeta} \sim \frac{w}{v_F} \frac{V_0}{\varepsilon_F} \ll 1.$$

Therefore, we obtain the following simple expression for g_1:

$$g_1 = e\Phi_1 (\partial f_0/\partial\varepsilon). \tag{44}$$

Further, substituting (43) and (44) in the equation of electrical neutrality (2d), we obtain the expression for the nonadiabatic part of the field:

$$e\Phi_1 = -n_r/N(\varepsilon_F), \tag{45}$$

where n_r is the concentration of resonant particles, determined in (9), $N(\varepsilon_F)$ is the density of electron states on the Fermi surface. As is seen from (45), the nonadiabatic part of the field is small in comparison with the adiabatic part in the ration w/v_F and is not taken into account in Eq. (3).

The effects discussed in this paper can be observed at input intensities of the order of 100 W/cm^2 at $L = 10^{-3}$ cm for sufficiently pure samples with $\tau_r = 5\times 10^{-9}$ s. Here the length $L/a = 5\times 10^{-3}$ cm, while the characteristic velocity of motion of the points of the pulse profile (in a system moving with the pulse) is $C_0 \sim 10^2$ cm·s^{-1}.

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[1)]A theory of nonlinear damping of a monochromatic sound wave was developed in Ref. 1.

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