Effect of transverse inhomogeneities on the propagation of simultons (two-frequency light pulses) in three-level media

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As part of an investigation of the feasibility of experimentally observing simultons (multiparticle optical solitons in resonant multilevel media), the stability of their propagation in the presence of transverse inhomogeneities is investigated. The growth rates of various transverse inhomogeneities are obtained analytically. It is shown that under certain conditions a one-dimensional evolution of multiparticle pulses can be realized in experiment.

1. Problems involving the interaction between short multifrequency light pulses and resonant multilevel media have been attracting great interest of late.[1–6] Multifrequency operation enhances the possibility of laser separation of isotopes, of photostimulation of chemical reactions, as well as of spectroscopic investigations. The possibility of simultaneous stable propagation of two-frequency pulses with a common envelope

$$E \infty \operatorname{sech}\left(\frac{t-z/v}{\tau_p}\right)$$

(called simultons) in a three-level medium was investigated in Refs. 2–6. It was found that in media with a common lower or upper level (V or Λ configuration of the transitions) such a propagation is possible only if the oscillator strengths of the resonant transitions are equal, $\mu_1^2\omega_1 = \mu_2^2\omega_2$($\mu_{1,2}$ are the dipole moments and $\omega_{1,2}$ are the frequencies of the transitions). The application of the method of the inverse problem of scattering theory to a three-level system with common lower level (V configuration of the transitions) has shown that the "simulton" solution

$$E_{1,2} \infty \frac{c_{1,2}}{\tau_p} \operatorname{sech}\left(\frac{t-z/v}{\tau_p}\right), \qquad c_1^2+c_2^2=1$$

is the simplest soliton solution of the inverse problem.[5] Multisoliton solutions describe propagation and interaction of N simultons, such that their shape, velocity, and relative amplitudes remain unchanged. Thus, within the framework of the one-dimensional model, the simulton is a structurally stable soliton formation.

To our knowledge, no simultons have been observed in experiment so far. To observe simulton evolution one must choose a medium with resonant transitions that coincide with the frequencies of the known sources of coherent radiation. One possibility of such an experiment was recently indicated in Ref. 7. At the same time, simulton-propagation theory[2–5] is based on a one-dimensional model. An investigation of the propagation of monochromatic light pulses in two-level media[8–11] has shown that coherent propagation of pulses in resonantly absorbing media is unstable to the evolution of small-scale transverse perturbations. The transverse structure evolves as the pulse traverses in the medium a distance of the order of its length $L_p = v\tau_p$. The characteristic scale of the resultant transverse structure for resonant pulses coincides with the diffraction dimension $\sim(\lambda L_p)^{1/2}$, where λ is the wavelength of the light. In the case of nonresonant pulses, the instability growth rate depends on the magnitude and sign of the detuning of the light frequency from the transition frequency of the medium.[11] The decrease of the instability growth rate following the frequency detuning explained the experiments n which the propagation lengths of 2π pulses increased with increase of the detuning.[12]

Just as for a monochromatic 2π pulse, in the multifrequency case the pulse propagation length can be restricted by evolution of a transverse structure. The feasibility of realizing in experiment one-dimensional propagation of simultons depends on which of the processes, one-dimensional simulton evolution or development of transverse inhomogeneities, is the faster. The purpose of the present paper is an investigation of the stability of simultons to transverse perturbations.

2. We investigate the instability, using as an example a two-frequency pulse in a three-level medium (V configuration) (see Fig. 1). The system of equations for the envelopes $E_{1,2}$ of the resonant fields and the amplitudes $a_{1,2,3}$ of the levels that participate in the interaction with the field is given by

$$\left(\frac{\partial}{\partial z}+\frac{n}{c}\frac{\partial}{\partial t}+\frac{i}{2k_{1,2}}\Delta_\perp\right)E_{1,2}=-\frac{4\pi N\mu_{1,2}\omega_{1,2}}{cn}a_1\dot{a}_{2,3},$$
$$\frac{\partial a_1}{\partial t}=-\frac{1}{2\hbar}(\mu_1E_1a_2+\mu_2E_2a_3),\quad \frac{\partial a_{2,3}}{\partial t}=\frac{1}{2\hbar}\mu_{1,2}E_{1,2}a_1 \tag{1}$$

(we recall that we are considering the case of equal transition oscillator strengths $\mu_1^2\omega_1 = \mu_2^2\omega_2$). Here N is the density of the resonant particles and n is the nonresonant refractive index. The one-dimensional simultaneous solution is of the form

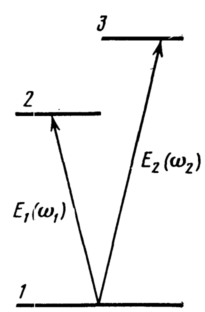

FIG. 1. Level diagram.

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$$E_{1,2}^{(0)}=c_{1,2}\frac{2\hbar}{\mu_{1,2}\tau_p}\operatorname{sech}\frac{t-z/v}{\tau_p}\quad (c_1^2+c_2^2=1),$$
$$a_1^{(0)}=-\operatorname{th}\frac{t-z/v}{\tau_p},\quad a_{2,3}^{(0)}=c_{1,2}\operatorname{sech}\frac{t-z/v}{\tau_p}. \tag{2}$$

The duration τ_p and the velocity v of the simulton are connected by the condition $cn/v-1=\Omega^2\tau_p^2$, where $\Omega^2=2\pi N\mu^2\omega/\hbar n^2$.

We transform to the dimensionless variables

$$\mathscr{E}_{1,2}=\mu_{1,2}E_{1,2}\tau_p/2\hbar,\ \tau=(t-z/v)/\tau_p,\ \xi=(1-nv/c)z/L_p,$$
$$\rho_\perp=[L_p/2k_1(1-nv/c)]^{-1/2}r_\perp$$

and linearize Eqs. (1) near the stationary solution (2) with respect to the small perturbations $\tilde{\mathscr{E}}_{1,2}$ and $\tilde{a}_{1,2,3}$, which we choose to be proportional to $\cos \varkappa r_\perp$,

$$\left(\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\tau}\right)\tilde{\mathscr{E}}_{1,2}-iq_{1,2}^2\tilde{\mathscr{E}}_{1,2}=-(a_1^{(0)}\tilde{a}_{2,3}^*+\tilde{a}_1a_{2,3}^{(0)}),$$
$$\frac{\partial\tilde{a}_1}{\partial\tau}=-(\mathscr{E}_1^{(0)}\tilde{a}_2+\mathscr{E}_2^{(0)}\tilde{a}_3+\tilde{\mathscr{E}}_1a_2^{(0)}+\tilde{\mathscr{E}}_2a_3^{(0)}), \tag{3}$$
$$\frac{\partial\tilde{a}_{2,3}}{\partial\tau}=\tilde{\mathscr{E}}_{1,2}^*a_1^{(0)}+\mathscr{E}_{1,2}^{(0)}\tilde{a}_1,$$

where

$$\mathscr{E}_{1,2}^{(0)}=c_{1,2}\operatorname{sech}\tau,\quad q_{1,2}^2=\frac{\varkappa^2}{2k_{1,2}}\frac{L_p}{1-nv/c}.$$

We note that all the coefficients in these equations are independent of ξ, so that the solutions of the linearized system are proportional to $\exp(\gamma\xi)$, where γ is the growth rate of the transverse perturbations. We consider two types of perturbation: "proportional"

$$(\tilde{\mathscr{E}}_{1,2})_{\rm pro}=c_{1,2}(\tilde{\varepsilon}+i\tilde{u}), \tag{4}$$

and "orthogonal"

$$(\tilde{\mathscr{E}}_{1,2})_{\rm ort}=\pm c_{2,1}(\tilde{f}+i\tilde{v}), \tag{5}$$

where $\tilde{\varepsilon}, \tilde{f}$ and $\tilde{u}, \tilde{v}$ are the real and imaginary parts of the perturbations. For perturbations consisting of proportional and orthogonal parts, the equations (3) can be reduced to the following system for the quantities

$$\psi=e^{-\gamma\xi}\int_{-\infty}^{\tau}\tilde{\varepsilon}\,d\tau,\quad u=e^{-\gamma\xi}\tilde{u},\quad f=e^{-\gamma\xi}\tilde{f},\quad v=e^{-\gamma\xi}\tilde{v},$$

namely

$$\left[\frac{d^2}{d\tau^2}-(1-2\operatorname{sech}^2\tau)\right]\begin{pmatrix}\psi\\u\end{pmatrix}=\gamma\frac{d}{d\tau}\begin{pmatrix}\psi\\u\end{pmatrix}+Q^2\begin{pmatrix}u\\-\dfrac{d^2\psi}{d\tau^2}\end{pmatrix}+R^2\begin{pmatrix}v\\-\dfrac{df}{d\tau}\end{pmatrix}, \tag{6}$$

$$\frac{d}{d\tau}\begin{pmatrix}f\\v\end{pmatrix}-\operatorname{th}\tau\int_{-\infty}^{\tau}\operatorname{th}\tau\begin{pmatrix}f\\v\end{pmatrix}d\tau$$
$$=\gamma\begin{pmatrix}f\\v\end{pmatrix}+P^2\begin{pmatrix}v\\-f\end{pmatrix}+R^2\begin{pmatrix}u\\-\dfrac{d\psi}{d\tau}\end{pmatrix}, \tag{7}$$

where

$$Q^2=c_1^2q_1^2+c_2^2q_2^2,\ P^2=c_1^2q_2^2+c_2^2q_1^2,\ R^2=c_1c_2(q_1^2-q_2^2).$$

Our problem, in analogy to Refs. 8 and 11, is to determine the spectrum of the eigenvalues $\gamma(q_1, q_2)$ and of the corresponding eigenfunctions that satisfy zero boundary conditions as $\tau\to\pm\infty$.

3. At $q_1=q_2=0$ the spectrum of the eigenvalues of the operator (6) was investigated quite fully (see, e.g., Ref. 13). It consists of a discrete level $\gamma=0$ and a continuum spectrum $\gamma=\pm i\lambda, \lambda\geqslant 2$. It can be shown that the eigenvalue spectrum of Eq. (7) at $q_1=q_2=0$, which corresponds to nonincreasing solutions at $\pm\infty$, coincides with the spectrum of the operator (6). We can therefore use standard perturbation theory and seek the corrections at small q_1 and q_2 for the fourfold degenerate eigenvalue $\gamma=0$. Corresponding to this eigenvalue are four linearly independent eigenvectors:

$$\psi_0^{(1)}=\operatorname{sech}\tau,\quad u_0^{(1)}=0,\quad f_0^{(1)}=0,\quad v_0^{(1)}=0, \tag{8a}$$
$$\psi_0^{(2)}=0,\quad u_0^{(2)}=\operatorname{sech}\tau,\quad f_0^{(2)}=0,\quad v_0^{(2)}=0, \tag{8b}$$
$$\psi_0^{(3)}=0,\quad u_0^{(3)}=0,\quad f_0^{(3)}=\operatorname{sech}\tau,\quad v_0^{(3)}=0, \tag{8c}$$
$$\psi_0^{(4)}=0,\quad u_0^{(4)}=0,\quad f_0^{(4)}=0,\quad v_0^{(4)}=\operatorname{sech}\tau, \tag{8d}$$

with (8a) corresponding to a small displacement of the initial position of the common envelope, (8c) to a small change of the relative amplitudes $c_1\to c_1+\alpha c_2$, $c_2\to c_2-\alpha c_1$ (α is the perturbation amplitude), and (8b) and (8d) to small shifts of the high-frequency content of the simulton.

To find $\gamma(q_1, q_2)$ we must calculate the eigenfunctions at least up to first order in the parameter γ (since, as will be shown below, $\gamma\gg q_{1,2}^2$). At this accuracy we have

$$\begin{pmatrix}\psi\\u\\f\\v\end{pmatrix}=\begin{pmatrix}\beta_1(1+\gamma\tau/2)\\\beta_2(1+\gamma\tau/2)\\-\beta_2R^2/\gamma\\0\end{pmatrix}\operatorname{sech}\tau, \tag{9}$$

where β_1 and β_2 are constants as yet unknown.

The condition for the system (6), (7) to have a solution is orthogonality of the adjoined unperturbed equations. The adjoined equations for the operators (6) and (7) have the solution $\operatorname{sech}\tau$. We use this to find the connection between β_1 and β_2. Multiplying (6) by $\operatorname{sech}\tau$ and integrating with respect to τ we obtain a system of algebraic equations, and from the condition that it have nonzero solutions leads to the dispersion equation

$$\gamma^4=(16/3)Q^4. \tag{10}$$

One of the roots of (10), $\gamma=(2/3^{1/4})Q$, corresponds to an exponential growth of the perturbations

$$\tilde{\mathscr{E}}_{1,2}=\left[\alpha c_{1,2}\left(\frac{\operatorname{th}\tau}{\operatorname{ch}\tau}+\frac{i}{3^{1/2}}\frac{1}{\operatorname{ch}\tau}\right)+O(\alpha Q)\right]e^{\gamma\xi}\cos\varkappa r_\perp, \tag{11}$$

where α is the initial amplitude of the perturbations, and $O(\alpha Q)$ are terms of order αQ. This unstable mode coincides with the stable mode of a 2π pulse in a two-level medium and, of course, goes over into the latter when the simulton degenerates into a 2π pulse. In contrast to the two-level case, however, a perturbation of the other type ("orthogonal") is possible for the simulton. The corresponding branch of the dispersion equation yields a pure imaginary growth rate

$$\gamma=\pm i\gamma_0, \quad \gamma_0=\frac{P}{Q}q_1q_2, \tag{12}$$

and for the perturbation we obtain

$$\tilde{\mathscr{E}}_{1,2}=\Big[\pm c_{2,1}\frac{q_1q_2}{PQ}(\alpha_1\sin\gamma_0\xi+\alpha_2\cos\gamma_0\xi) +i\Big(\pm c_{2,1}+\frac{R^2}{Q^2}c_{1,2}\Big)(\alpha_1\cos\gamma_0\xi+\alpha_2\sin\gamma_0\xi)\Big]\cos\varkappa r_\perp, \tag{13}$$

where α_1 and α_2 are the initial amplitudes of the perturbation. The oscillatory character of the "orthogonal" perturbation can be easily understood. Indeed, for sufficiently small amplitude changes $c_1\rightarrow c_1+\alpha c_2$, $c_2\rightarrow c_2-\alpha c_1$ ($\alpha\ll 1$) the relation $c_1^2+c_2^2=1+\alpha^2\approx 1$, is valid, i.e., the simulton parameters are constant over the transverse cross section. This means that the simulton-component amplitude fronts that are perturbed in antiphase move at constant velocity over the cross section. The curving of either the amplitude or the phase front, however, leads to the appearance of diffracted oblique waves[8] that interfere with the fundamental wave and lead to weak (of the order of α) oscillations of its amplitude and phase. The character of the evolution of the perturbations remains oscillatory so long as the perturbations pertaining to the different frequency components of the simulton remain in "antiphase" (this will be dealt with below).

The principal role in the instability evolution is played thus by perturbations connected with the "proportional" curving of the amplitude and/or phase front of the simulton along the transverse coordinate. The point is that the oblique diffracted waves, which are due to the proportional curving of the amplitude (phase) fronts, are in phase. Interference of these waves with the fundamental wave leads to in-phase modulation of the simulton amplitudes. The pulse sections with the larger amplitude move more rapidly, so that the curvature of the wave front increases, leading to an increase of the modulation amplitude, and so on. In our case of a two-frequency pulse the perturbations of each of the fields evolve as a result of the interference of the oblique wave with a field of like frequency. The perturbations are therefore proportional to the relative amplitudes of the corresponding frequency components of the simulton and, in addition, increase jointly, since the different frequency components move jointly.

We recall that we are considering for both fields perturbations proportional to $\cos\varkappa r_\perp$, so that when the wave numbers are different, $k_1\neq k_2$ (i.e., $R\neq 0$), oblique waves pertaining to different components of a simulton propagate at different angles to the pulse propagation direction. As a result, purely "proportional" or purely "orthogonal" perturbations evolve into combinations of perturbations of both types, owing to the spatial mismatch. This can be easily seen from (9) and (13), as well as directly from the system (6), (7), where the coefficient R connects ψ and u with f and v. In addition, at $k_1\neq k_2$ one can expect the instability growth rate to decrease on account of the rescattering of the growing mode into a non-growing one. In fact, by calculating the eigenfunctions of the system (6), (7) up to terms of order Q^2 we obtain a more accurate expression for the growth rate

$$\gamma=(2/3^{1/4})Q-(3^{1/2}/4)R^4/Q^2. \tag{14}$$

It can be seen therefore that the larger the mismatch between the wavelengths of the simulton components, the slower the evolution of the transverse structure and the stronger the interaction between the two types of perturbation.

At $R=0$ (the simulton either degenerates into an ordinary 2 pulse and $c_1c_2=0$, or $k_1=k_2$) the next correction (of third order in Q) to the growth rate is equal to

$$\gamma^{(3)}=-(3^{1/4}/3)Q^3. \tag{15}$$

The maximum growth rate is reached at the limit $Q\sim 1$ of applicability of perturbation theory. This corresponds to a characteristic transverse dimension

$$\lambda_\perp\sim[L_p(c_1^2\lambda_1+c_2^2\lambda_2)]^{1/2}, \tag{16}$$

where λ_1 and λ_2 are the wavelengths.

4. We have thus shown that in the vicinity of the applicability of perturbation theory ($Q<1$) the most stable perturbation is of type (11) with a characteristic dimension (6). This, however, raises the question of perturbations with smaller transverse dimensions. It might seem that the increase of the number of degrees of freedom compared with the two-level case, the difference between the transition frequencies, and other factors can cause some perturbation with $Q>1$ to evolve more rapidly than (11). In addition, energy transfer between simulton components, or other nontrivial processes not accounted for in perturbations of the form (4) and (5), can occur. Interaction between the growing and oscillatory modes may lead to saturation of the instability at some finite level. To answer these questions, numerical experiments were performed. In the upshot, besides verifying the analytic results, the calculation should yield the maximum possible distance that a simulton can negotiate in a medium.

The numerical experiment consisted of the following: various sinusoidal small-amplitude perturbations with different wavelengths $\lambda_\perp$ were superimposed on a simulton that is homogeneous in the transverse direction and propagates along the z axis [see the system of equations (1)]. The calculation region was a rectangle with sides $\lambda_\perp$ and $15L_p$ along the

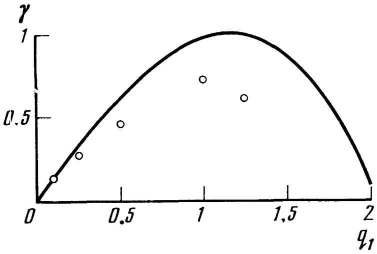

FIG. 2. Dependence of the growth rate of the perturbations on their transverse scale. Solid curve—calculated from Eqs. (14) and (15); points—results of numerical experiments. At $Q\gtrsim 2$ the perturbations no longer increase in the numerical calculations.

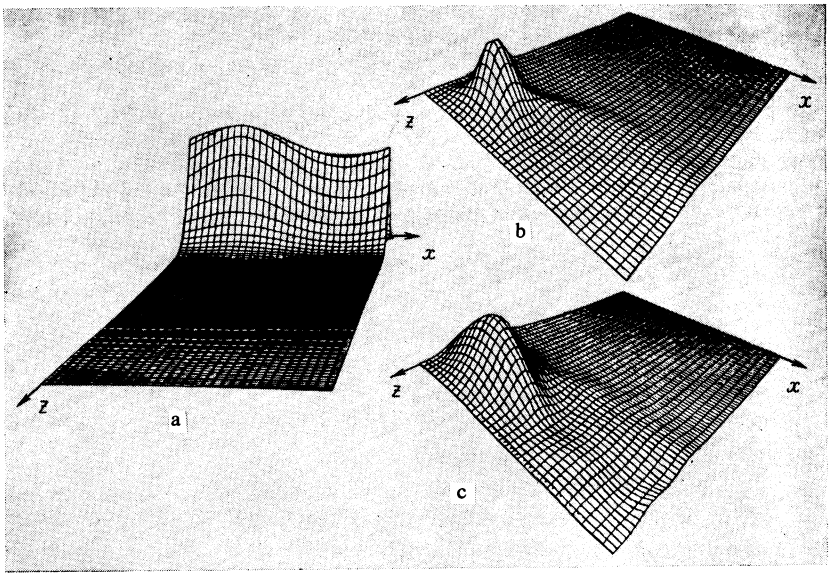

FIG. 3. Field intensities: a—common envelope $|\mathscr{E}|^2$ during the initial stage of the instability evolution ($|\mathscr{E}|_{1,2}|^2 = c_{1,2}^2 |\mathscr{E}|^2$; b, c—$|\mathscr{E}_1|^2$ and $|\mathscr{E}_2|^2$ respectively during the nonlinear stage.

x and z axes, respectively. Periodic boundary conditions were imposed at $x = 0$ and $x = \lambda_\perp$.

It was found that the analytic results agree well with the results of the numerical experiments. The calculations show that at sufficiently small Q all the initial perturbations first end up on the "growing" eigenfunction (11), and then evolve jointly with the growth rate (14), (15). Figure 2 shows a plot of the growth rate vs q_1 for a simulton with parameters $c_1 = 0.8$, $c_2 = 0.6$, $k_1/k_2 = 1/3$. In accord with the instability-evolution mechanism, the fastest to evolve are the perturbations with $Q \sim 1$. Perturbations with $Q \gg 1$ do not grow, as can also be demonstrated analytically in analogy with Ref. 8.

The evolution of the perturbations with the fastest growth was investigated numerically up to the nonlinear stage of the instability evolution, where the analytic results cited above no longer hold. Figures 3–5 show the distributions of the simulton intensity components and of the exitation energy of the medium during the nonlinear stage. It can be seen that the joint evolution of the perturbations leads to formation of radiation filaments and to a broadening of the maximum-excitation region. Through exchange of oblique waves, the excited regions contained in the medium go out of phase and cease to emit. Light is radiated only from the leading front of the excitation region and propagates in accord with linear diffraction theory.

The results can be easily generalized to include the case of an N-frequency simulton in an $(N+1)$-level medium. The fastest to grow in this case are perturbations with transverse dimension

$$\lambda_\perp^{(N)} \sim \left[L_p \sum_i^N c_i^2 \lambda_i \right]^{1/2}, \tag{17}$$

and the growth rate is, in first-order approximation,

$$\gamma^{(N)} = (2/3^{1/4}) \left(\sum_i^N c_i^2 q_i^2 \right)^{1/2}. \tag{18}$$

In all other respects the character of the instability development does not differ from the considered case of a two-frequency simulton.

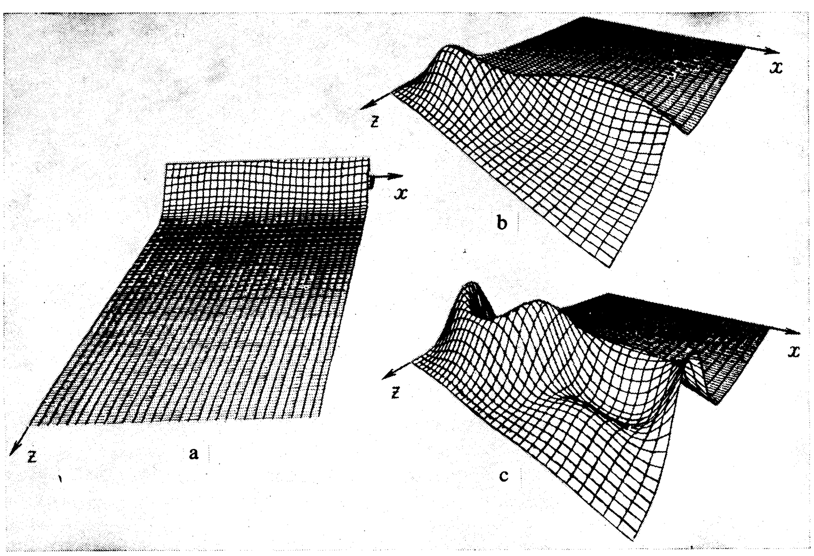

FIG. 4. Excitation of medium: a—common envelope $|a|^2$ during the initial stage ($|a_{2,3}|^2 = c_{1,2}^2 |a|^2$); b, c—$|a_2|^2$ and $|a_3|^2$ respectively in the nonlinear stage.

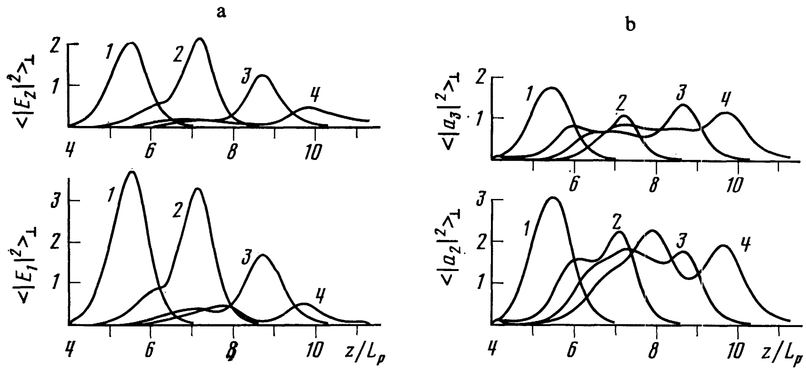

FIG. 5. Field intensities (a) and level populations (b) averaged over the transverse coordinates. Curves 1–4 pertain to four successive instants of time.

In conclusion, we recall once more that we have dealt in the present paper with a purely "soliton" situation (transitions with equal oscillator strengths). Joint propagation of two-frequency pulses is possible also in media with arbitrary ratios of $\mu_1^2\omega_1$ and $\mu_2^2\omega_2$ (Ref. 4). Recent investigations,[6] however, have shown that such pulses are not solitons in the sense of structural stability. The numerical analysis in Ref. 6 has shown that these pulses decay not only as a result of development of a transverse structure, but also via one-dimensional collisions with other similar pulses. The interaction is accompanied by a redistribution of photons having different frequency components, by creation of new two-frequency pulses, and by other "non-soliton" effects. The transverse structure propagates in similar fashion. The maximum growth rates occur in perturbations having transverse dimensions intermediate between those of the individual frequency components.

The formation of one-dimensional simultons and their interaction take place, as shown by the analytic and numerical investigations,[5,6] over resonant-region dimensions of the order of several pulse lengths. The evolution of the transverse structure is determined by the level of the initial perturbations. The length over which total disintegration of the simulton takes place increases logarithmically with decreasing perturbation amplitude. At a perturbation level of several percent ($\tilde{\mathscr{E}}/\mathscr{E}_0 \sim 0.05$) this length amounts to 15–20 pulse lengths. A similar transverse instability limits the distribution length of single-frequency 2π pulses in two-level media.[14–16] Coherent propagation of single-frequency 2π pulses was investigated in media with dimensions up to 20 pulse lengths. To increase the total simulton-interaction length in the experiments one can use intermediate spatial filters. It can thus be concluded from the results of the present paper that experimental realization of multifrequency solitons in resonant media is feasible.

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