# Coherent bremsstrahlung, Pendellosung, and Raman emission of electrons in planar channeling

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The frequency and angular dependences of the intensity of radiation emitted by an electron moving in a planar channel in a crystal is calculated. The angle between the direction of motion and the atomic chains of the channeling plane is only slightly larger than the critical angle for axial channeling. In this case the inhomogeneity of the channeling plane is most pronounced. The conditions for resonant transformation of the electron wave function on account of the inhomogeneity is determined. It is shown that the changes in the frequency and angular dependences of the radiation from those in planar channeling in the one-dimensional interplanar-potential model are due to electron diffraction from the atomic chains of the channeling plane.

## 1. INTRODUCTION

This article is devoted to an investigation of the radiation of relativistic particles in a crystal in the case when the angle of incidence of the particle on the crystal is smaller than the planar-channeling angle relative to one transverse coordinate and somewhat larger than the axial-channeling critical angle relative to the other. This case can be regarded either as transitional between axial and planar channeling, or as planar channeling with account taken of the discrete character of the channeling plane. A particle moving along such directions feels strongly the inhomogeneity of the atomic plane and is scattered by the system of atomic chains of the channeling plane. Since the atomic chains are periodically arranged, the scattered waves produce diffraction maxima. This behavior is most typical of a particle moving near axes with large indices $(h, k, l)$. Indeed, the Bragg diffraction angle is $\theta_B \sim (h^2+k^2+l^2)^{1/2}$. On the other hand, the interatomic distance in the chain is $d = a_0(h^2+k^2+l^2)^{1/2}$, and the critical axial-channeling angle is $\psi_c \sim d^{-1/2}$. Therefore the ratio $\theta_B/\psi_c \sim (h^2+k^2+l^2)^{3/4}$, i.e., it increases with increasing reciprocal-lattice vector $\mathbf{H}_{nkl}$. However, at low energies of electrons with a relativistic factor $\gamma \approx 1$ to 10 the situation indicated can be realized also in motion near the principal axes of the crystal. Some features of the channeling of electrons under such conditions are noted in Refs. 1–3. In Refs. 4 and 5 was investigated the influence of the discrete character of the atomic chain on the axial channeling.

This paper deals in detail with the resonant case, when the condition

$$E_i(\mathbf{q}) = E_f(\mathbf{q}+\mathbf{g}) \tag{1}$$

is satisfied. Here $E_i(\mathbf{q})$ is the total energy of the electron in a planar channel at a level $i$ of transverse motion and with a quasimomentum projection $\hbar\mathbf{q}$ on the channeling plane, $g = (2\pi n_y/a_y, 2\pi n_z/a_z)$, $a_y$, and $a_z$ are the periods of the atomic lattice in the channeling plane, and $n_y$ and $n_z$ are nonnegative integers. The features of the electromagnetic radiation of the channeled electrons are in this case the following: the frequency-angle spectrum of the radiation changes; the intensity of the coherent bremsstrahlung and Pendellosung radiation increases compared with their values outside the channeling region; Raman scattering sets in. By changing the angle of incidence of the beam on the crystal near the condition (1) we can vary the radiation frequency. Investigation of electron channeling in the indicated situation extends both the possibilities of controlling the radiation parameters and the possibilities of investigating the properties of the crystal.

## 2. RESONANT RETUNING OF THE ELECTRON WAVE FUNCTION IN THE KINEMATIC APPROXIMATION

The motion of a relativistic electron in a crystal is described by the equation[6]

$$\left[\Delta - \frac{2EU(\mathbf{r})}{\hbar^2c^2}\right]\Psi(\mathbf{r}) = -k^2\Psi(\mathbf{r}), \tag{2}$$

where $E$ is the electron energy, $U(\mathbf{r})$ is the electrostatic potential of the atom lattice, and $k^2 = (E^2 - m_0^2c^4)/\hbar^2c^2$. In the planar-channeling regime it is possible to separate in the potential $U(\mathbf{r})$ the part due to the averaged potential $U_0(x)$ of the atomic plane; this part is the zeroth term of the expansion of the potential in a two-dimensional Fourier series:

$$U(\mathbf{r}) = U_0(x) + \sum_{\mathbf{g}\neq 0} U_{\mathbf{g}}(x)\exp(i\mathbf{g}\boldsymbol{\rho}), \tag{3}$$

where

$$U_{\mathbf{g}}(x) = \frac{1}{S}\int_S U(\mathbf{r})\exp(-i\mathbf{g}\boldsymbol{\rho})\,d\boldsymbol{\rho},$$

$S = a_ya_z$ is the volume of the two-dimensional unit cell. In the zeroth approximation, the motion of an electron in the planar-channeling regime is described by the equation

$$\left[\Delta - \frac{2EU_0(x)}{\hbar^2c^2}\right]\Psi_0(\mathbf{r}) = -k^2\Psi_0(\mathbf{r}). \tag{4}$$

The solution of (4) is

$$\Psi_0(\mathbf{r}) = \sum_n \int dq\, c_n(q)\, u_n(x)\exp(i\mathbf{q}_n\boldsymbol{\rho}), \tag{5}$$

where $q_n = \{q, (k^2+\varkappa_n^2-q^2)^{1/2}\}$, and $u_n(x)$ and $\varkappa_n^2$ are the eigenfunctions and eigenvalues of a Schrödinger equation with potential $U_0(x)$:

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$$\left[\frac{d^2}{dx^2}-\frac{2EU_0(x)}{\hbar^2c^2}\right]u_n(x)=\varkappa_n{}^2u_n(x). \tag{6}$$

The expansion coefficients $c_n(q)$ in (5) are determined by the continuity conditions on the interface ($z=0$):

$$c_n(q)=\frac{1}{2\pi}\int\Phi(x,y,0)\,u_n^*(x)\exp(-iqy)\,dx\,dy,$$

where $\Phi(\mathbf{r})$ is the wave function of the electron incident on the crystal. The first- approximation correction to the wave function in the perturbation-theory in terms of the potential $U_1(\mathbf{r})=U(\mathbf{r})-U_0(x)$ satisfies the following iteration equation:

$$\left[\Delta-\frac{2EU_0(x)}{\hbar^2c^2}+k^2\right]\Psi_1(\mathbf{r})=\frac{2EU_1(\mathbf{r})}{\hbar^2c^2}\Psi_0(\mathbf{r}). \tag{7}$$

Using the solution of Eq. (7), the wave function takes in first order the form

$$\Psi(\mathbf{r})=\Psi_0(\mathbf{r})+\Psi_1(\mathbf{r})=\sum_n\int dq c_n(q)$$

$$\times\left\{u_n(x)e^{i\mathbf{q}_n\boldsymbol{\rho}}+\frac{2E}{\hbar^2c^2}\sum_{m,\mathbf{g}\neq0}\frac{U_{gmn}u_m(x)\exp[i(\mathbf{q}_n+\mathbf{g})\boldsymbol{\rho}]}{q_m{}^2-(\mathbf{q}_n+\mathbf{g})^2}\right\}, \tag{8}$$

where

$$U_{gmn}=\int u_m^*(x)\,U_g(x)\,u_n(x)\,dx.$$

It is known[6] that the correlation of the different electron transverse-motion states vanishes at depths exceeding approximately one period of the transverse oscillations. We shall therefore describe the state of an electron in the crystal by the number $n$ of the transverse-motion level and by the two-dimensional quasimomentum $\hbar\mathbf{q}$. Taking into account the finite width of the transverse-motion levels, we can then represent the wave function of the electron in the crystal in the form

$$\Psi(\mathbf{r})=u_n(x)e^{i\mathbf{q}\boldsymbol{\rho}}+\sum_{m,\mathbf{g}\neq0}\frac{U_{gmn}u_m(x)\exp[i(\mathbf{q}+\mathbf{g})\boldsymbol{\rho}]}{E_{n\mathbf{q}}-E_{m\mathbf{q}+\mathbf{g}}+i\Gamma}, \tag{9}$$

where $\Gamma=\Gamma_n+\Gamma_m$ is the total width of the levels $n$ and $m$.

Thus, if the condition

$$|\mathbf{q}_n+\mathbf{g}|=q_m \tag{10}$$

is satisfied, retuning of the electron wave function takes place.

### 3. INTENSITY AND SPECTRUM OF THE RADIATION

We shall examine the feature of the electromagnetic radiation from a channeled electron in the region of the resonance (10). The Hamiltonian of the interaction of the particle with the electromagnetic field is of the form

$$H_{int}=-\frac{1}{c}\int\hat{\mathbf{j}}\mathbf{A}\,d\mathbf{r}, \tag{11}$$

where $\hat{\mathbf{j}}$ is the operator of the electron-current density, and $\mathbf{A}$ is the vector-potential operator, which is expressed in the following manner in terms of the operators for photon creation ($\hat{a}^+_{\varkappa,\lambda}$) and annihilation ($\hat{a}_{\varkappa,\lambda}$):

$$\mathbf{A}=\sum_{\varkappa,\lambda}\left(\frac{2\pi\hbar c^2}{V\omega_{\varkappa,\lambda}}\right)^{1/2}\mathbf{e}^{(\lambda)}\{\hat{a}_{\varkappa,\lambda}\exp[i(\varkappa\mathbf{r}-\omega t)]+\text{H.c.}\};$$

here $V$ is the quantization volume, $\varkappa$ is the wave vector, $\omega$ is the frequency, and $\mathbf{e}^{(\lambda)}$ is the photon-polarization vector.

Using nonstationary perturbation theory, we obtain the following expression for the probability of the transition from the state $(n,\mathbf{q})$ into the state $(k,\mathbf{q}')$:

$$dw=\frac{2\pi}{\hbar}|W_\lambda(n\mathbf{q},k\mathbf{q}')|^2\delta(E_{n\mathbf{q}}-E_{k\mathbf{q}'}-\hbar\omega)\frac{V\,d^3\varkappa}{(2\pi)^3}. \tag{12}$$

Let the wave functions of the initial and final states be respectively

$$\Psi_{n\mathbf{q}}(\mathbf{r})=u_ne^{i\mathbf{q}\boldsymbol{\rho}}+U_{gmn}u_m(x)\exp[i(\mathbf{q}+\mathbf{g})\boldsymbol{\rho}], \tag{13a}$$

$$\Psi_{k\mathbf{q}'}(\mathbf{r})=u_k\exp(i\mathbf{q}'\boldsymbol{\rho}). \tag{13b}$$

The probability amplitude is then given by

$$W_\lambda(n\mathbf{q},k\mathbf{q}')=M_\lambda(n\mathbf{q},k\mathbf{q}')+\frac{U_{gmn}M_\lambda(m\mathbf{q}+\mathbf{g},k\mathbf{q}')}{E_{n\mathbf{q}}-E_{m\mathbf{q}+\mathbf{g}}+i\Gamma}. \tag{14}$$

The matrix elements of the interaction Hamiltonian can be represented in the form

$$M_\lambda(n\mathbf{q},k\mathbf{q}')=\langle k\mathbf{q}'|H_{int}^{(\varkappa,\lambda)}|n\mathbf{q}\rangle$$

$$=-\frac{ie}{m}\left(\frac{2\pi\hbar}{V\omega}\right)^{1/2}(n_{\varkappa,\lambda}+1)^{1/2}\delta(\mathbf{q}-\mathbf{q}'-\boldsymbol{\varkappa}_\parallel)$$

$$\times[m\Omega_{nk}e_x^{(\lambda)}x_{kn}+\hbar(\mathbf{e}^{(\lambda)}\mathbf{q})(\varkappa_xx_{kn}+i\delta_{kn})]; \tag{15}$$

here $\Omega_{nk}=-\hbar(\kappa_n^2-\kappa_k^2)/2m$, $n_{\varkappa,\lambda}$ is the number of photons with wave vector $\boldsymbol{\kappa}$ and polarization $\mathbf{e}^{(\lambda)}$, $\boldsymbol{\kappa}_\parallel$ is the projection of $\boldsymbol{\kappa}$ on the $(y,z)$ plane, and $(y,z),x_{kn}=\int u_k^*xu_n\,dx$ is the transition matrix element. The energy and momentum conservation laws lead to the following form of the frequency-angular dependence of the radiation, determined by the first term of (14)

$$\omega_1(\mathbf{n})=\frac{\Omega_{nk}}{1-(\hbar\mathbf{q}\mathbf{n}/mc)}=\frac{\Omega_{nk}}{1-\beta n\cos\theta_0} \tag{16}$$

and by the second

$$\omega_2(\mathbf{n})=\left[\Omega_{nk}-\hbar\left(\mathbf{q}+\frac{\mathbf{g}}{2}\right)\frac{\mathbf{g}}{m}\right]\Big/\left[1-\frac{\hbar(\mathbf{q}+\mathbf{g})\mathbf{n}}{mc}\right], \tag{17}$$

where

$$\mathbf{n}=\boldsymbol{\varkappa}c/\omega,\quad\cos\theta_0=\mathbf{q}\mathbf{n}/qn,\quad\beta=\hbar q/mc.$$

The resonance condition (10) for the coupling of the states $(n\mathbf{q})$ and $(m\mathbf{q}+\mathbf{g})$ can be rewritten in the form

$$\frac{\hbar}{m}\left(\mathbf{q}+\frac{\mathbf{g}}{2}\right)\mathbf{g}=\Omega_{nm}, \tag{18}$$

so that at exact resonance Eq. (17) takes the form

$$\omega_2(\mathbf{n})=\Omega_{mk}/(1-\beta n\cos\theta_g), \tag{19}$$

where $\cos\theta_g=(\mathbf{q}+g)\mathbf{n}/|\mathbf{q}+g|n$. Substituting (14) and (15) in (12) and integrating with respect to $\omega$ we obtain the following expression for the probability of emission into a unit solid angle in a transition between states with wave functions (13a) and (13b):

$$\frac{dw(n\mathbf{q}, k\mathbf{q}')}{do} = P_{nk}(\mathbf{q}, \varkappa) + \left| \frac{U_{gmn}}{\Gamma} \right|^2 P_{mk}(\mathbf{q}+\mathbf{g}, \varkappa), \quad (20)$$

where

$$P_{nk}(\mathbf{q}, \varkappa) = \frac{e^2 |x_{nk}|^2 \Omega_{nk}^3}{2\pi\hbar c^3} \frac{\cos^2\varphi(\cos\theta-\beta^2)+\sin^2\varphi(1-\beta\cos\theta)^2}{(1-\beta\cos\theta)^4}, \quad (21)$$

and the angles $\theta$ and $\varphi$ are respectively the azimuthal and polar angles of the wave vector $\varkappa$ in a coordinate frame with $z$ axis directed along the vector $\mathbf{q}$. The matrix element $U_{gmn}$ takes on the maximum value when $m = n$ and $n$ is an even number, since the maximum of $U_g(x)$ is reached at $x = 0$. For example, if the potential of an individual atom is approximated by the Coulomb screened potential $U_0(\mathbf{r}) = -Ze^2\exp(-\mu r)/r$, then $U_g(x)$ takes the form

$$U_g(x) = -\frac{Ze^2}{\pi S} \frac{1}{(\mu^2+g^2)^{1/2}} \exp(-(\mu^2+g^2)^{1/2}|x|). \quad (22)$$

At $(\mathbf{q} + \mathbf{g}/2)\cdot\mathbf{g} = 0$ the intensity of the lines corresponding to transitions from even levels will thus be higher in the direction of the diffracted beam than from odd levels. This circumstance is of interest for the identification of the radiation spectra of channeled particles.

The frequencies $\omega_1$ and $\omega_2$ [see (16) and (18)] coincide if the wave vector of the photon satisfies the condition

$$\frac{\Omega_{nm}}{\Omega_{nk}}\left(1 - \frac{\hbar\mathbf{q}\varkappa_{0\parallel}}{m\omega}\right) = \frac{\hbar\mathbf{g}\varkappa_{0\parallel}}{m\omega}. \quad (23)$$

The radiation intensity will increase by interference in the $\varkappa_0$ direction. The right-hand side of (20) will then acquire a term corresponding to the imaginary part of the matrix element $U_{gmn}$; this part differs from zero only in noncentrosymmetric crystals. In fact, the wave functions $u_n(x)$ can always be chosen real and $U^*_{gmn} = U_{gmn}$, if $U(\mathbf{r}) = U(-\mathbf{r})$. In centrosymmetric crystals the increase of the intensity in the $\varkappa_0$ direction at resonance will be determined by the ratio of the second term of (20) to the first at $\varkappa = \varkappa_0$.

Let now $k = n$. Then the first term of (20) vanishes and the second yields, according to (17) and (18), the radiation intensity at the frequency

$$\omega_2(\mathbf{n}) = \Omega_{mn}/[1-\hbar(\mathbf{q}+\mathbf{g})\mathbf{n}/mc].$$

A normal Doppler branch is excited if $\Omega_{mn} > 0$ and an anomalous one at $\Omega_{mn} < 0$. This radiation is due to collisions between the electron and the atomic chains of the channeling plane, and constitutes bremsstrahlung when channeled. Since the maxima of the wave functions $u_n(x)$ are reached near the atomic planes, the intensity of the channeled bremsstrahlung will exceed the unchanneled in an approximate ratio

$$|U_{gmn}|^2 \Big/ \left|\frac{1}{d}\int_0^d U_g(x)\,dx\right|.$$

The second circumstance that leads to an increase of the bremsstrahlung intensity on channeling is that $\Gamma$ in (20) is determined by the width of the transverse-motion levels, and not by the collision time $\Gamma_c$, as is the case outside the channel. The condition $\Gamma \ll \Gamma_t$ is always satisfied in the considered energy region.

Allowance for the fact that the final state of the electron can also be near a resonance of the type (18) does not alter substantially the radiation picture described above. An exception is the onset of the interference maximum (23), when $n = m$. In this case the final state will certainly be near a resonance point. This case will be investigated in detail in the last section.

## 4. TEMPERATURE DEPENDENCE OF THE RADIATION INTENSITY

The square of the matrix element $U_{gmn}$ in Eq. (20) for the radiation intensity must be averaged over the thermal vibrations of the atoms. Taking into account the deviations of the radius vectors $\mathbf{r}_i$ of the atoms from their equilibrium position $\mathbf{r}_{i0}$, the expression for the Fourier component of $U_g(x)$ can be written in the form

$$U_g(x) = \frac{1}{S}\int_S U(\mathbf{r})\exp(-i\mathbf{g}\rho)\,d\rho = \sum_{i=1}^{N} \bar{U}_g(x-x_i)\exp(-i\mathbf{g}\rho_i). \quad (24)$$

where

$$\bar{U}_g(x-x_i) = \frac{1}{\Sigma}\int_{\Sigma} U_0(x-x_i, \rho)\exp(-i\mathbf{g}\rho)\,d\rho,$$

$N$ is the number of unit cells in the channeling plane (we assume for simplicity a primitive unit cell), and $\Sigma = SN$. Taking (24) into account and assuming that the vibrations of the individual atoms are independent, we can represent $\langle|U_{gmn}|^2\rangle_{th}$ in the form[7,2]

$$\langle|U_{gmn}|^2\rangle_{th} = N^2 e^{-g^2u^2}\left|\int u_m^*(x)\,u_n(x)\langle\bar{U}_g(x-x_0)\rangle dx\right|^2 + N\left[\left\langle\left|\int u_m^*(x)\,u_n(x)\bar{U}_g(x-x_0)\,dx\right|^2\right\rangle - e^{-g^2u^2}\left|\int u_m^*(x)\,u_n(x)\langle\bar{U}_g(x-x_0)\rangle dx\right|^2\right], \quad (25)$$

where $u$ is the amplitude of the thermal vibrations. The first term in (25) reaches a maximum at zero temperature and falls of with rising temperature. The second term of (25), which is proportional to $N$, on the contrary, vanishes at zero temperature and increases with rise of temperature. The relation between the individual terms in (25) depends also on the crystal-lattice symmetry. Thus, in centrosymmetric crystals, where $U_0(\mathbf{r}) = U_0(-\mathbf{r})$, the first and last terms in (25) vanish if the levels $u_m(x)$ and $u_n(x)$ have different parity. On the other hand, the second term depends little on the crystal symmetry if the amplitude $u$ of the thermal vibrations is close to the average distance $\langle x^2\rangle^{1/2}$ between the electron and the plane. Thus, in centrosymmetric crystals the nonvanishing of the bremsstrahlung intensity is due to the second term in (25). In noncentrosymmetric crystals the terms in the square brackets cancel each other and the main contribution to the intensity of the bremsstrahlung is made by the first coherent term in (25).

## 5. DYNAMIC SCATTERING

The intensity of the new radiation lines at resonance is inversely proportional to the squared widths of the transverse-motion levels. The level width is determined by several factors: the radiative lifetime of the particle at the level, the finite length of the crystal, and the band broadening. Assuming that the main cause is the finite size of the crystal (this assumption is natural for the energy range considered by us) we find that the intensity of the indicated radiation is proportional to the square of the crystal length. This dependence is the consequence of the kinematic approximation used by us. Allowance for dynamic effects in electron diffraction leads to a splitting of the energy levels of the transverse motion near the resonance. This eliminates the indicated dependence of the intensity on the crystal length $L$ if $L > L_0$, where $L_0$ is the extinction length determined by the dynamic interaction of the diffracted waves.

Let us determine the form of the wave functions of planarly channeled electrons with allowance for their diffraction by the atomic lattice of the channeling plane. We seek the solution of Eq. (2) with potential (3) in the form

$$\Psi(\mathbf{r}) = \sum_{\mathbf{g}} \varphi_{\mathbf{g}}(x) \exp[i(\mathbf{q}+\mathbf{g})\rho]. \tag{26}$$

Substituting (26) in (2) we obtain the following system of equations:

$$\frac{d^2}{dx^2}\varphi_{\mathbf{g}}(x) - \frac{2m}{\hbar^2}\sum_{\mathbf{g}'} U_{\mathbf{g}-\mathbf{g}'}(x)\varphi_{\mathbf{g}'}(x) = \left[\frac{2mE}{\hbar^2} - (\mathbf{q}+\mathbf{g})^2\right]\varphi_{\mathbf{g}}(x). \tag{27}$$

We confine ourselves for simplicity to the case of two-wave diffraction and to a resonance of the form $(n,\mathbf{q}) \to (n,\mathbf{q}+g)$. In this case the system (27) becomes

$$d^2\varphi_0/dx^2 + V_0\varphi_0 + V_1\varphi_{\mathbf{g}} = \varkappa^2\varphi_0, \quad d^2\varphi_{\mathbf{g}}/dx^2 + V_0\varphi_{\mathbf{g}} + V_1\varphi_0 = \varkappa_{\mathbf{g}}{}^2\varphi_{\mathbf{g}}, \tag{28}$$

where

$$V_0 = -2mU_0/\hbar^2, \quad V_1 = -2mU_{\mathbf{g}}/\hbar^2 = -2mU_{-\mathbf{g}}/\hbar^2,$$
$$\varkappa^2 = q^2 - k^2, \quad \varkappa_{\mathbf{g}}{}^2 = (\mathbf{q}+\mathbf{g})^2 - k^2.$$

Rewriting the system (28 in the form

$$\frac{d^2\varphi_+}{dx^2} + (V_0+V_1)\varphi_+ = \frac{\varkappa^2+\varkappa_{\mathbf{g}}{}^2}{2}\varphi_+ + \frac{\varkappa^2-\varkappa_{\mathbf{g}}{}^2}{2}\varphi_-,$$
$$\frac{d^2\varphi_-}{dx^2} + (V_0-V_1)\varphi_- = \frac{\varkappa^2+\varkappa_{\mathbf{g}}{}^2}{2}\varphi_- + \frac{\varkappa^2-\varkappa_{\mathbf{g}}{}^2}{2}\varphi_+, \tag{29}$$

where $\varphi_+(x) = \varphi_0(x) + \varphi_{\mathbf{g}}(x)$, $\varphi_-(x) = \varphi_0(x) - \varphi_{\mathbf{g}}(x)$, we see readily that each level splits into two in the potential $U_0(x)$. Thus, if $U_0(x) = \alpha x^2$ and $U_g(x) = \beta x^2$, at exact satisfaction of the Bragg conditions $(\mathbf{q} + \mathbf{g}/2)\mathbf{g} = 0$ the frequencies $\omega = (2\alpha/m)^{1/2}$ are replaced by $\omega_\pm = [2(\alpha \pm \beta)/m]^{1/2}$.

Let $u_{n\mathbf{g}}(x)$ be the orthonormalized eigenfunctions of the system (28). The general solution of Eq. (2) with potential (3) can then be written in the form

$$\Psi(\mathbf{r}) = \sum_n c_n \sum_{\mathbf{g}} u_{n\mathbf{g}}(x)\exp[i(\mathbf{q}_n+\mathbf{g})\rho], \tag{30}$$

where the coefficients $c_n$ are obtained from the boundary conditions and $q_n^2 = k^2 + \varkappa_n^2$. For example, if $U_0(x)$ and $U_g(x)$ are of the form

$$U_0(x) = \begin{cases} -U_0, & |x| \leqslant a, \\ 0, & |x| > a; \end{cases} \qquad U_{\mathbf{g}}(x) = \begin{cases} -U_1, & |x| \leqslant a, \\ 0, & |x| > a, \end{cases}$$

the functions even in $x$ can then be written, for zero deviation from the Bragg angle $[(\mathbf{q} + \mathbf{g}/2)\cdot\mathbf{g} = 0]$, in the form

$$\Psi_n^{(1)} = [\pi d(a+\varkappa_{n1}^{-1})]^{-1/2}\cos\mu_{n1}x\sin\frac{gy}{2}\exp(iq_{n1}z),$$
$$\Psi_n^{(2)} = [\pi d(a+\varkappa_{n2}^{-1})]^{-1/2}\cos\mu_{n2}x\cos\frac{gy}{2}\exp(iq_{n2}z), \tag{31}$$

where $\mu_{1,2} = (V_0 \mp V_1 - \varkappa^2)^{1/2}$, and the eigenvalues $\varkappa_{n1,2}$ are determined respectively from the conditions

$$\mu_{1,2}\,\mathrm{tg}\,(\mu_{1,2}a) = \varkappa. \tag{32}$$

The functions odd in $x$ are similar in form. When the condition $V_0 - \varkappa_0 \gg V_1$ is satisfied, the approximate solutions of (32) are

$$\varkappa_{n1,2}^2 = \varkappa_n{}^2 \mp V_1/(1+\varkappa_n a), \tag{33}$$

where $\varkappa_n$ are the solutions of (32) with $V_1 = 0$.

The radiation intensity is determined by the value of the matrix element $\langle ni|\exp(-i\varkappa\mathbf{r})\hat{\mathbf{j}}|mj\rangle$ [see (11)] The matrix element will depend on the parity of the initial and final wave functions with respect to the transverse coordinates $(x,y)$.

The energy of momentum conservation laws lead to the following expression for the angular dependence of the frequency

$$\omega = \frac{\Omega_{ni,mj}}{1-\beta n\cos\theta_B\cos\theta}, \tag{34}$$

where $\varkappa_z = \varkappa\cos\theta$, $\sin\theta_B = g/2q$.

The matrix elements of interest to us have, for a transition $\Psi_n^{(1)} \to \Psi_n^{(2)}$ between nearest states of like parity in $x$, the form

$$\langle n2|\exp(-i\varkappa\mathbf{r})\hat{j}_x|n1\rangle = \langle n2|\exp(-i\varkappa\mathbf{r})\hat{j}_z|n1\rangle = 0,$$
$$\langle n2|\exp(-i\varkappa\mathbf{r})\hat{j}_y|n1\rangle = \frac{\hbar g}{2}\left(-\frac{ie}{m}\right)\int u_{n2}(x)\,u_{n1}(x)\,dx, \tag{35}$$

where $u_{ni} = (\alpha + \varkappa_{ni}^{-1})^{-1/2}\cos\mu_{ni}x$.

The frequency-angular dependence of the radiation is given according to (33) and (34) by the expression

$$\omega = \frac{2U_1/\hbar(1+\varkappa_n a)}{1-\beta n\cos\theta_B\cos\theta}.$$

Thus, only the $y$ projection of the transition current density differs from zero, and consequently the radiation is due to diffraction of the electron in the channeling plane and has the same properties as Pendellosung radiation of a diffracted electron.[8]

The matrix element of the operator $[\exp(-i\varkappa\cdot\hat{\mathbf{r}})]\mathbf{j}$ on going from the state $\Psi$ [Eq. (31)] into a state with wave function

$$\Psi_m^{(1)} = [\pi d(a+\varkappa_{m1}^{-1})]^{-1/2}\sin\mu_{m1}x\sin\frac{gy}{2}\exp(iq_{m1}z)$$

is of the form

$$\langle m1|\exp(-i\varkappa\mathbf{r})\hat{j}_y|n1\rangle=0,$$
$$\langle m1|\exp(-i\varkappa\mathbf{r})\hat{j}_x|n1\rangle=ie\Omega_{n_1,m_1}x_{n_1,m_1}\delta(q_{n_1}-q_{m_1}'-\varkappa_z),$$
$$\langle m1|\exp(-i\varkappa\mathbf{r})\hat{j}_z|n1\rangle=-iev_{1z}\varkappa_x x_{n_1,m_1}\delta(q_{n_1}-q_{m_1}'-\varkappa_z).$$

Consequently, the transition between levels of like parity in $y$ and of unlike parity in $x$ corresponds to interference of transitions of the type (23) between discrete levels of transverse motion of channeled particles. The presence of diffraction manifests itself only in an insignificant change of the radiation frequency. In fact, according to (33) and (34) we have

$$\omega=\left[\Omega_{nm}+\frac{U_1}{\hbar}\left(\frac{1}{1+\varkappa_n a}-\frac{1}{1+\varkappa_m a}\right)\right]\Big/\ (1-\beta n\cos\theta_B\cos\theta),$$

where $\Omega_{nm}=-\hbar(\varkappa_n^2-\varkappa_m^2)/2m$.

The last type of possible transitions corresponds to radiative coupling of levels with unlike parity both in $x$ and in $y$. Consider, for example, the transition $\Psi_n^{(2)}\to\Psi_m^{(1)}$. The corresponding matrix elements are

$$\langle m1|\exp(-i\varkappa\mathbf{r})\hat{j}_x|n2\rangle=\langle m1|\exp(-i\varkappa\mathbf{r})\hat{j}_z|n2\rangle=0,$$
$$\langle m1|\exp(-i\varkappa\mathbf{r})\hat{j}_y|n2\rangle=\frac{e\varkappa_x}{m}x_{n2,m1}\frac{\hbar g}{2}\delta(q_{n2}-q_{m1}'-\varkappa_z).$$

From the very form of the matrix elements it follows that Raman emission will be observed in this case. In fact, the only projection of the transition current at $\varkappa\cdot\mathbf{g}=0$ is a product of matrix elements of the transition with respect to the $x$ and $y$ coordinates. The expression for the frequency $\Omega_{n2,m1}$ is

$$\Omega_{n2,m1}=-\frac{\hbar(\varkappa_n{}^2-\varkappa_m{}^2)}{2m}-\frac{U_1}{\hbar}\left(\frac{1}{1+\varkappa_n a}+\frac{1}{1+\varkappa_m a}\right).$$

The change of frequency compared with the kinematic approximation is thus determined by the diffractive splitting.

## 6. CONCLUSION

Our calculations of the angular and frequency dependences of the radiation intensity under the conditions indicated in the Introduction have shown that the differences from the case of planar channeling in the model of a one-dimensional interplanar potential are due to diffraction of the electron from the atomic chains of the channeling plane, which results in a Doppler branch of radiation with a frequency maximum along the propagation direction of the diffracted electron wave. In the directions defined by the condition (23), this branch interferes with the primary one. Localization of the electron wave function near the atomic planes increases the intensity of the coherent bremsstrahlung and Pendellosung radiation. The increase of the bremmstrahlung intensity is due also to the fact that the number of coherently scattering atoms is now determined not by the collision time (i.e., by the longitudinal momentum transfer), but by the reciprocal width of the transverse-motion levels. The distinguishing feature of the Pendellosung radiation produced as a result of dynamic diffraction of the electrons is that dynamic splitting takes place now of each of the levels in the channel. Therefore, in electron transitions between dynamic sublevels of one level there can occur in the channel, besides the purely Pendellosung radiation, also Raman radiation in transitions of electrons between dynamic sublevels of unlike parity and belonging to different levels in the channel.


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