Spontaneous and orientational phase transitions in quasiuniaxial magnetic films

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Results are presented of a theoretical and experimental study of inhomogeneous magnetic states in thin quasiuniaxial magnetic films in the vicinity of lines of phase transitions induced by changes of the temperature (near the Curie point) or of a magnetic field $H_\perp$ parallel to the developed film surface. The form of the phase diagram of such a film is determined for spontaneous and orientational phase transitions. The effect of magnetization fluctuations on the ordering of various types of domain structure is studied. It is shown that the decisive role in the amorphization of regular domain structures in phase transitions is played by magnetic dislocations and disclinations which can, by increasing the magnetic energy of the system, lower the free energy as a result of the increase of the entropy. Experiments on epitaxial films of mixed iron garnets yield results that are in good agreement with the conclusions of the theory.

This paper is devoted to a theoretical and experimental investigation of spontaneous (near the Curie point T_C) and orientational (induced by a magnetic field approximately perpendicular to the easy-magnetization axis (EMA) phase transitions (PT) in quasiuniaxial magnetic films with strong anisotropy. We report in Secs. 1–4 the results of the experiments and a PT theory developed without allowance for fluctuations; the influence of the fluctuations is considered in Secs. 5–7.

1. EXPERIMENTAL TECHNIQUE

The PT were experimentally investigated by a magneto-optical technique using iron garnet films grown by liquid-phase epitaxy on $Gd_3Ga_5O_{12}$ substrates with different crystallographic orientations. The block diagram of the setup for the investigation of orientational PT in the films at $T = 290$ K and in a magnetic field making a small angle with the developed surface of the samples is shown in Fig. 1. Light from source 1 (laser or incandescent lamp) passed through polarizer 3 and was focused by lens 4 on the investigated film 5 which was secured in an sleeve 6 that allowed the film to be rotated through an arbitrary angle φ_H around the optical axis. The domain structure (DS) in the film was viewed with a polarization microscope whose objective 7 was placed inside a sleeve 6 with the film 5, and whose analyzer 9 and ocular 10 were autonomously moved on an optical bench. A semitransparent mirror 11 was glued to the free-end surface of the sleeve 6 to permit reading of the film-rotation angle φ_H. The laser beam reflected from mirror 11 (with the lens 4 removed) was incident on a screen 2 provided with a circular scale on which the angle φ_H could be read accurate to $\pm 0.5°$. The magnetic field was produced by electromagnet 8 mounted on a rotating platform, so that the angle ϑ between the magnetic-field intensity vector **H** and the developed surface of the film could be varied in a range $\pm 25°$ (accurate to $\pm 0.1°$) to set the required ratio of the projections of **H** on the normal **n** ($H_\parallel = H \sin\vartheta$) and on the film surface ($H_\perp = H \cos\vartheta$). The maximum field intensity in the working gap was ≈ 10 kOe.

The spontaneous PT in the vicinity of T_C were investigated with a setup based on a polarization microscope with a heating stand that allowed the temperature T of the investigated object to be varied from 290 to 600 K. One junction of a copper-constantan thermocouple was glued to the film section in the field of view of the microscope. The second junction of the thermocouple was placed in a dewar with liquid nitrogen. The thermopower was measured with a digital voltmeter that could read the instantaneous temperature of the object accurate to 0.01 K. A constant magnetic field $H_\parallel \approx -150 - +150$ Oe was produced by a coil; the field $H_\perp$ (up to 500 Oe) was produced by a Permalloy-core electromagnet. The same setup could also be used to investigate orientational PT in a certain vicinity of the Curie point, for as $T \to T_C$ the effective uniaxial-anisotropy field H_u tends to zero (at $T = 290$ kOe the field H_u usually amounts to several kOe).[1)]

We determined in the experiments the phase diagrams of the films in the $(H_\parallel, H_\perp)$ plane (at constant T and φ_H) for orientational PT and in the $(H_\parallel, T)$ plane (at constant $H_\perp$ and φ_H) for spontaneous PT, i.e., the regions of existence of the

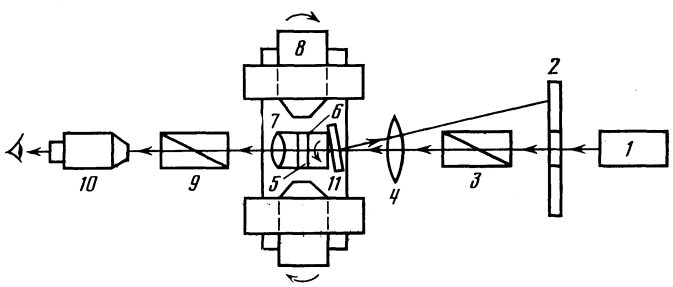

FIG. 1. Block diagram of setup for the investigation of orientational phase transitions.

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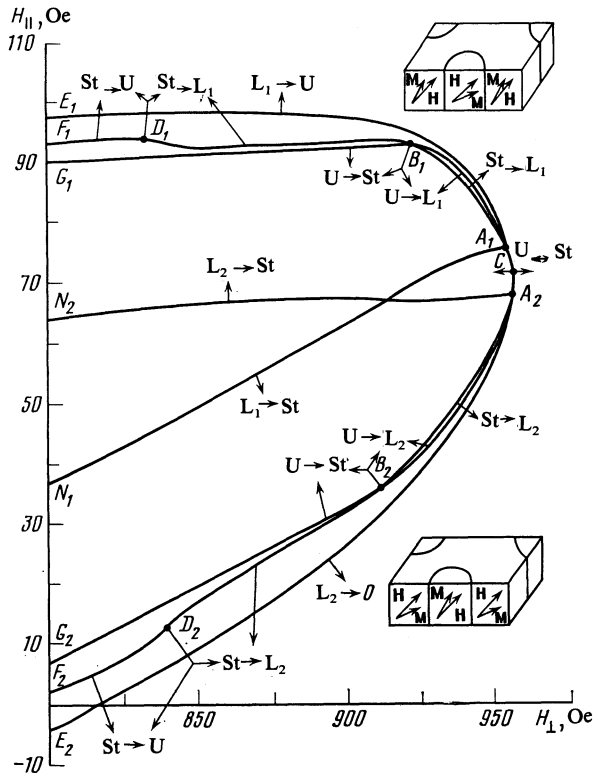

FIG. 2. Phase diagram for orientational PT in film No. 1 at $T = 290$ K.

homogeneous state, of a stripe (or labyrinth) DS, and of two "nonequipolar" lattices of magnetic-bubble domains (MBD), see the insets of Figs. 2 and 4.

2. ORIENTATIONAL PHASE TRANSITIONS

The phase diagram for orientational PT for $T = 290$ K and $\varphi_H = 234°$ in one of the investigated films, with composition $(YEu)_3(FeGa)_5O_{12}$ (henceforth designated film No. 1), having the (111) orientation), is shown in Fig. 2. The parameters of film No. 1 were: thickness $L = 6.1$ μm; magnetization $M = 10.5$ G; uniaxial, cubic, and rhombic anisotropic constants $K_u = 5.33 \cdot 10^3$, $K_c = 7.01 \cdot 10^2$, and $K_p = 2.92 \cdot 10^2$ erg·cm^{-3}, respectively, and angle of deflection of the EMA from the normal $\vartheta_u = 1.44°$.

The phase diagram was determined visually by observing the magnetic state of a small ($\sim 100 \times 100$ μm) section of the film. We determined first the stability loss line of the homogeneous state (line $G_1B_1A_1CA_2B_2G_2$ on Fig. 2 with arrows $U \rightarrow St$, $U \rightarrow L_1$, and $U \rightarrow L_2$), and the types of domain structures nucleated thereby. For most investigated films with (111) orientation (including film No. 1), at any fixed azimuth of the film ($\varphi_H = \text{const}$) there existed a certain critical value of the angle $\vartheta_c = \arctan (H_{\parallel}/H_{\perp})_c$, for which the transition from the homogeneous state to the inhomogeneous was of second order (without hysteresis or formation of a magnetization-reversal nucleus), and the transition field $H_{\perp c}$ was a maximum (point C on the diagram of Fig. 2). The DS created at $\vartheta = \vartheta_c$ and $H = H_c$ is amorphous and consists of isolated stripes (of finite length) of different "polarity," without a clearly pronounced preferred orientation. The length of the nonequipolar stripe domains varies in a wide range (and even individual MBD can exist), so that when the magnetic field is decreased to zero there is produced a so-called state with mixed polarity,[1] containing (in approximately equal proportions) isolated nonequipolar stripe domains and nonequipolar MBD (see photograph, Fig. 3a).

When ϑ deviates from ϑ_c (in either direction) the transition field decreases monotonically, the number of BMD with "convenient" orientation of the vector **M** increases in the produced DS, and the number of stripe domains and MBD of opposite polarity decreases smoothly, i.e., the mixed-polarity amorphous DS is transformed into one of the two amorphous BMD lattices (see Fig. 3b). Nonetheless, in a certain interval of the angle ϑ near ϑ_c the PT from the homogeneous to the inhomogeneous state continues to be of second order (line A_1CA_2 in Fig. 2). The average period d of the nucleating DS is a minimum at $\vartheta = \vartheta_c$ and increases insignificantly as $\vartheta \rightarrow \vartheta_{A1}$ and $\vartheta \rightarrow \vartheta_{A2}$. At $\vartheta = \text{const}$ the dependence of the average period on the magnetic field intensity is linear near the A_1CA_2 line, with d decreasing as H increases. Starting with certain characteristic values of ϑ_{A1} and ϑ_{A2} (points A_1 and A_2 in Fig. 2), a first-order PT (close to second-order) takes place and hexagonal MBD are created (L_1 at $\vartheta > \vartheta_{A1}$ and L_2 at $\vartheta < \vartheta_{A2}$).[2)] Formation of the lattices L_1 and L_2 is observed in the intervals $\vartheta_{B1} > \vartheta > \vartheta_{A1}$ and $\vartheta_{A2} > \vartheta > \vartheta_{B2}$ respectively (lines A_1B_1 and A_2B_2 of the diagram); at $\vartheta > \vartheta_{B1}$ and $\vartheta < \vartheta_{B2}$ the homogeneous state goes over, on loss of stability, into a labyrinth DS via a first-order PT (lines B_1G_1 and B_2G_2 in Fig. 2). The density of the BMD in the lattices L_1 and L_2 is a maximum at the points A_1 and A_2 of the diagram. The period of the lattice is then a minimum. On moving from points A_1 and A_2 to points B_1 and B_2 respectively as the stability of the homogeneous state is lost, the features of a first-order PT (nucleation of domains on defects, hysteresis phenomena, and others) are more and more strongly manifest; The diameter of the BMD and the periods of the lattices L_1 and L_2 increase rapidly.

After finding the position of the stability-loss line of the homogeneous state we determined the stability regions of the hexagonal MBD lattices L_1 and L_2. To this end the value of ϑ was set somewhat more than ϑ_{A1} for the lattice L_1 (or somewhat less than ϑ_{A2} for lattice L_2), and a transition from the homogeneous state into the lattice L_1 (or into L_2) was effected by lowering the field intensity from $H > H_c$ to any required value of H_0, with a simultaneous "jolting" by an alternating field $H \sim \parallel$ EMA of frequency 10–10^4 Hz. The amplitude of the alternating field and the duration of the jolting were chosen such as to ensure a maximum possible ordering of the lattice without a substantial change of the density of the BMD and without transforming them into elliptic domains. After obtaining a hexagonal MBD lattice at the chosen value H_0 of the constant magnetic field the angle ϑ was increased (for lattice L_1) or decreased (for lattice L_2) until the MBD collapsed (lines E_1A_1 and E_2A_2 in Fig. 2). The operation of producing a close-packed lattice of MBD was then repeated and the other stability limit was determined, corresponding to the transition from the hexagonal MBD lattice into a stripe (labyrinth) structure (lines N_1A_1 and N_2A_2 in Fig. 2).

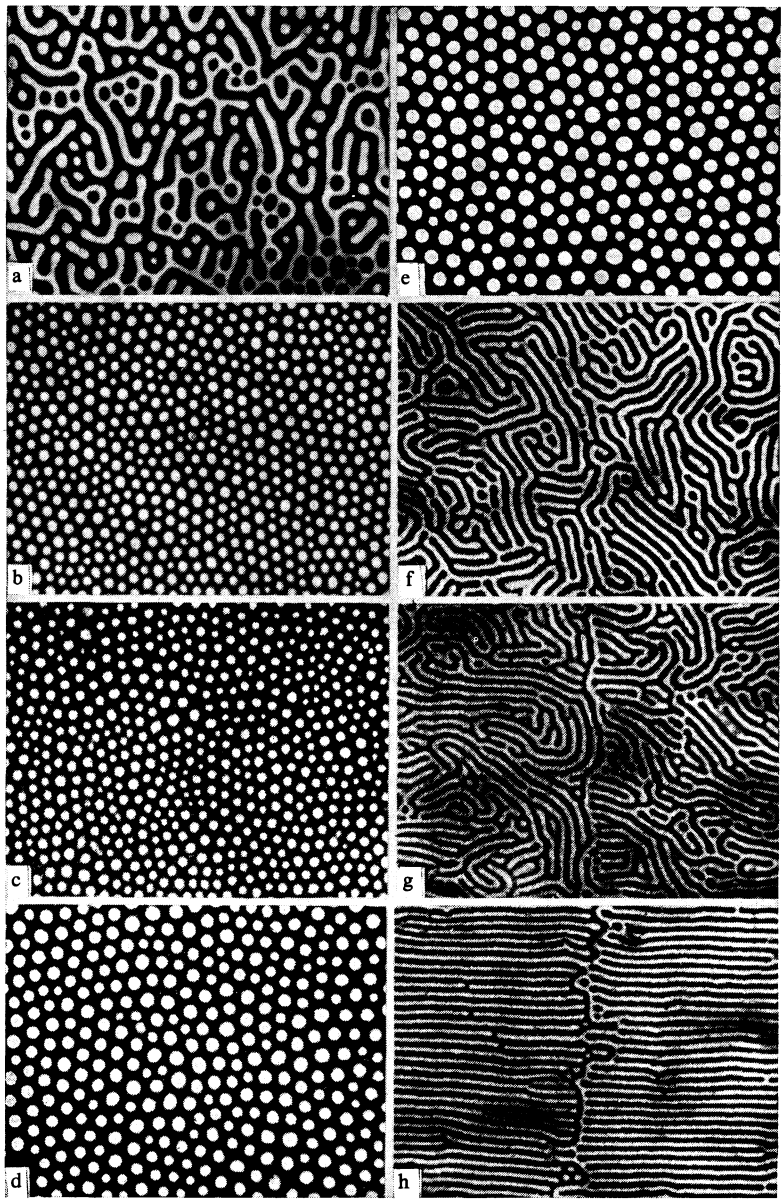

FIG. 3. Photographs of amorphous and quasiordered DS nucleated in iron garnet films in orientational (a, b) and spontaneous (c–h) PT.

The region of existence of a stripe (labyrinth) DS was determined in the following manner. At values $\vartheta \gtrsim \vartheta_{B1}$ (or $\vartheta \lesssim \vartheta_{B2}$) a DS was initiated from the homogeneous state, after which the required value of the magnetic field intensity H_0 was set and the sample was magnetically jolted to obtain a system of stripe domains with parameters close to the equilibrium values. The angle ϑ was then increased (decreased) either until the stripe DS vanished (first order transitions into a uniformly magnetized state; lines F_1D_1 and F_2D_2 of the diagram), or until the stripe DS was transferred into a hexagonal MBD lattice (first-order transitions; lines D_1A_1 and D_2A_2 with arrows $S \rightarrow L_1$ and $S \rightarrow L_2$). At larger differences between ϑ and ϑ_{A1} (or ϑ_{A2}), the BMD lattice produced from the stripe DS is characterized by a low domain density; as $\vartheta \rightarrow \vartheta_{A1}$ (or as $\vartheta \rightarrow \vartheta_{A2}$) the MBD density increases rapidly and a close-packed lattice is produced.

Variation of the angle φ_H (i.e., of the azimuthal orientation of the film) does not lead to a qualitative change in the forms of the phase diagrams. All the diagrams for the different values of the angle φ_H are topologically similar, and only the values of ϑ_c and H_c change. For all the diagrams, lines of like type converge (at $H_\perp = 0$) to identical points.

3. SPONTANEOUS PHASE TRANSITIONS IN THE VICINITY OF THE CURIE POINT

With respect to the behavior near the Curie point T_C, all the investigated films can be tentatively divided into four groups. A feature of the films of the first group is formation of an amorphous state with mixed polarity with a second-order PT from a homogeneous into an inhomogeneous state, as well as the presence of one nucleation region for each of the two nonequipolar BMD lattices. The phase diagrams of

such films near T_C, plotted in coordinates $H_\parallel$ and T, are qualitatively equivalent to the phase diagram of Fig. 2 (for orientational PT), apart from a change of the notation on the abscissa axis. The films of the second group differ from those of the first only in that for each of the lattices L_1 and L_2 there exist two regions of nucleation from the homogeneous state, separated by a line of a first-order PT from the homogeneous phase to a striped DS. In the films of the third group [usually with (110) orientation] there are no mixed-polarity states; each lattice has one nucleation region. The fourth group includes films with (110) orientation, in which only a system of oriented stripe domains is produced when the homogeneous state becomes unstable. The group to which a particular film belongs is determined exclusively by the character of the anisotropy. Anisotropy in the basal place hinders the nucleation of hexagonal MBD lattices and can even prevent it completely, as is attested by the existence of films of the fourth group.

The procedure for plotting the film phase diagrams near T_C is similar to that for orientational PT. One finds first the position of the stability-loss line of the homogeneous state. The film is heated to a temperature $T > T_C$ and is then cooled slowly in a fixed magnetic field $H_\parallel$. At the instant when the DS is produced, its type is determined by using the digital voltmeter that measures the thermopower, and the instantaneous transition temperature is determined. The regions where the stripe (labyrinth) DS and the hexagonal BMD lattices exist are determined by varying the field $H_\parallel$ as the film is slowly cooled (or heated). The equilibrium close-packed BMD lattices L_1 and L_2 and the stripe DS were produced by a method based on the use of orientational PT (see, e.g., the diagram in Fig. 2). At the chosen measurement temperature $T < T_C$, a current source was turned on and produced a magnetic field $H_\perp \approx 500$ Oe parallel to the developed surface of the film. The field $H_\perp$ was then smoothly decreased until the DS appeared and its type visually identified. If the DS type differed from the required one, a weak field $H_\parallel$ (of arbitrary polarity) was applied and the domain nucleation was repeated; when necessary, the polarity of the field $H_\parallel$ was reversed. Several "trial and error" operations yielded the DS of the required type. The field $H_\perp$ was then smoothly decreased to zero (and the sample was simultaneously jolted magnetically), and the field $H_\parallel$ was set at the required value. This procedure, in particular, nucleated hexagonal BMD lattices even in the films of the fourth group.

The phase diagram of one of the investigated films of the first group (hereafter referred to as No. 2) near T_C is shown in Fig. 4. Film No. 2, 30 μm thick, had the composition $(YGdYbBi)_3(FeAl)_5O_{12}$ and was grown on a substrate with (111) orientation. The Curie temperature T_C (the abscissa of the point C in Fig. 4) for film No. 2 was 422.1 K. The phase diagram (Fig. 4) is symmetric about the abscissa axis, is topologically equivalent to the diagram of Fig. 2, and requires no additional clarification. The temperature dependence of the average period of the DS in the vicinity of the second-order PT lines (A_1CA_2 in Fig. 4) is linear.

Superposition of a magnetic field $H_\perp$ oriented parallel to the developed surface of the film leads to the following:

1. The diagram as a whole is shifted towards lower temperatures. Thus, for film No. 2 the decrease $\Delta T_C = f(H_\perp)$ of the transition temperature at $H_\perp = 360$ Oe is ≈ 4.5 K.

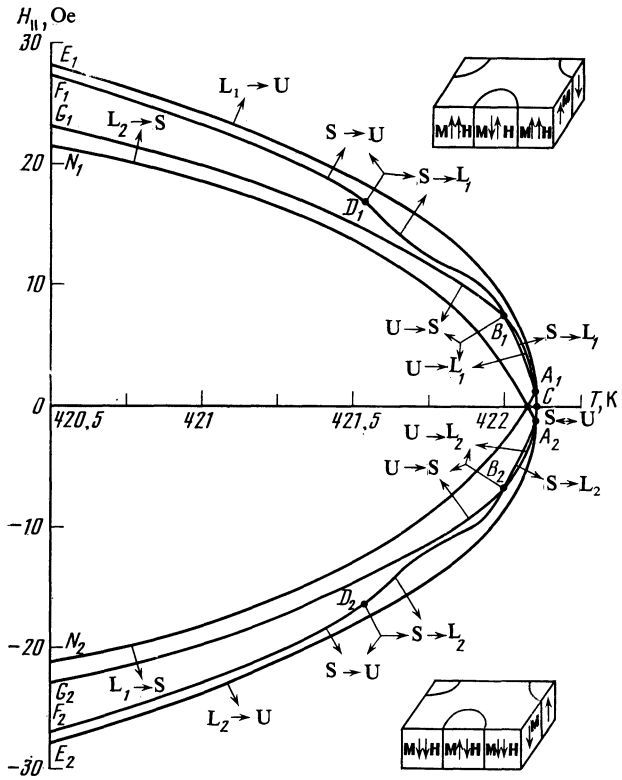

FIG. 4. Phase diagram for spontaneous PT in film No. 2 near $T_C = 422.1$ K.

2. The mirror symmetry of the diagram about the abscisso axis is lost. The point C is displaced away from this axis; the direction and magnitude of the displacement of $H_{\parallel c}$ are determined by the angle of deviation of the EMA from the normal, by the effective cubic-anisotropy field, as well as by the azimuthal orientation of the field $H_\perp$.

3. For films of the first group, in a definite interval of $H_\perp$, there exists a second region of nucleation of each of the MBD lattices L_1 and L_2 from the homogeneous state (via a first-order PT). This region is separated from the initial one (adjacent to the second-order PT line) by the region where the stripe DS is nucleated.[3)] With increasing $H_\perp$, the two lattice-nucleation regions overlap. The causes of the existence of the second region for nucleation of MBD lattices are not known at present. In the films of the fourth group there exists a critical value $|H_\perp|$ above which hexagonal lattices can be nucleated from the homogeneous state.

4. The field $H_\perp$ suppresses the amorphization of the DS. Thus, for example, in the films of the first and second groups one observes, when the field $H_\perp$ is increased, a transition from nucleation of an amorphous DS with mixed polarity to nucleation of an ordered stripe DS.

In conclusion, we note the following characteristic feature of the behavior of the DS in spontaneous and orientational PT of second order (or of first order close to second). If a regular (stripe or hexagonal lattice) DS is produced and then, by increasing the field or the temperature, a transition to the homogeneous state is effected at $\vartheta_{A1} \leqslant \vartheta \leqslant \vartheta_{A2}$, amor-

phization of the DS takes place in a narrow interval of variation of H or T directly ahead of the second-order PT. When the field or temperature is decreased there is produced on the line A_1CA_2 not a regular DS, but an amorphous state with mixed polarity, or else amorphous MBD lattices, and the concrete form of the DS is not duplicated when the experiments are repeated. This phenomenon is not due to defects of the investigated samples, and is observed in practically all films with (111) orientation, which have as a rule a weak anisotropy in the basal plane $K_p, K_c \ll K_u$ and $\vartheta_u \ll 1$, i.e., in these films the formation of amorphous DS is an inanielable attribute of a second-order PT and is due, as will be shown later, to the influence of the magnetization fluctuations. The anisotropy in the basal plane or (for spontaneous PT) the field $H_\perp$ suppress the fluctuations and contribute to nucleation of regular DS.

4. THEORY OF ORIENTATIONAL PHASE TRANSITIONS IN QUASIUNIAXIAL FERROMAGNETIC FILMS AT $T = 0$

The theory of DS nucleation in films of ferromagnets with uniaxial anisotropy and undergoing orientational PT was expounded in Refs. 2 and 3. The anisotropy of real objects, however, is frequently not uniaxial; thus, in epitaxial iron garnet films uniaxial anisotropy is always accompanied by a cubic anisotropy component and by anisotropy in the developed plane (rhombic component), and the EMA also deviates from the normal $\mathbf{n}$ to the surface.[1]

Consider an orientational PT in a quasiuniaxial iron garnet film of thickness L, grown on a substrate with (111), (110), or (100) orientation, using the coordinate system shown in Fig. 5 ($\mathbf{n}_u$ and $\mathbf{n}_p$ are respectively unit vectors along the EMA and the normal to the plane of the preferred orientation of the magnetization vector $\mathbf{M}$, and ϑ_u is the angle between the EMA and $\mathbf{n}$). We express the free energy at $T = 0$ in the form

$$F = 2\pi M_0^2 \int dv [D(\nabla_i m_k)^2 + f_A - 2(\mathbf{mh}) - (\mathbf{mh}_D)], \qquad (1)$$

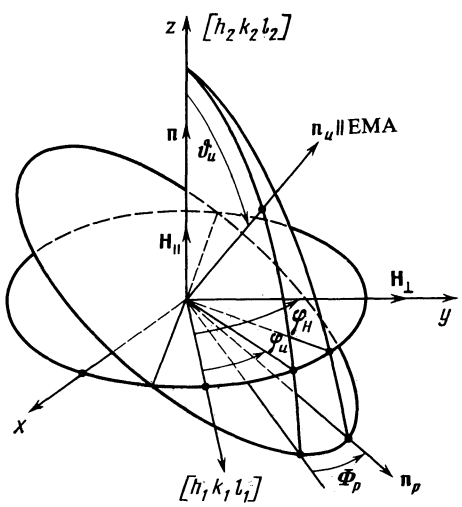

FIG. 5. Coordinate frame used in the theoretical analysis of orientational PT: $[h_1k_1l_1] = [11\bar{2}]$, [110] and [001]; $[h_2k_2l_2] = [111]$, [110] and [100] respectively for films with (111), (110), and (100) orientation.

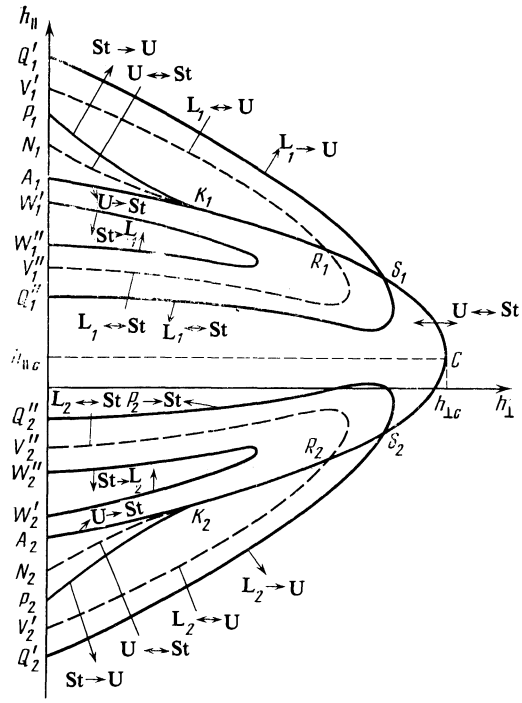

FIG. 6. Theoretical phase diagram for orientational PT.

where M_0 is the saturation magnetization, $D = A(2\pi M_0^2)^{-1}$ is the exchange constant, $m = \mathbf{M}M_0^{-1}$, $\mathbf{h} = \mathbf{H}(4\pi M_0)^{-1}$, $\mathbf{h}_D = \mathbf{H}_D(4\pi M_0)^{-1}$ is the dipole-interaction field, and f_A is the anisotropy energy density with uniaxial, rhombic, and cubic components f_u, f_p, and f_c, respectively:

$$f_A = f_u + f_p + f_c = -\beta_u(\mathbf{mn}_u)^2 + \beta_p(\mathbf{mn}_p)^2 - \beta_c[(\mathbf{mn}_{100})^2(\mathbf{mn}_{010})^2 + (\mathbf{mn}_{010})^2(\mathbf{mn}_{001})^2 + (\mathbf{mn}_{001})^2(\mathbf{mn}_{100})^2], \qquad (2)$$

where $\mathbf{n}_{hkl}$ is a unit vector along the axis $[hkl]$; $\beta_i = K_i(2\pi M_0^2)^{-1} > 0$ $(i = u, p, c)$.

Following the procedure described in Refs. 2–4 and assuming that $\beta_u \geqslant \max\{1; \beta_p; \beta_c; h_\parallel\}$, $\vartheta_u \ll 1$ and $L \gg D^{1/2}$, we obtain the limits of the stability limits of the different magnetic states, as well as the PT lines between them on the $H_\perp H_\parallel$ plane, which are shown in Fig. 6. The line of the homogeneous-state loss of stability with respect to transition to a stripe DS (A_1CA_1 in Fig. 6) is described by the equation

$$h_\perp^{(U\to St)} = (1 - {}^3/_2\eta_c^2)\, h_{\perp c}, \qquad (3)$$

where $\eta_c^2 = (h_\parallel - h_{\parallel c})^2$, while $h_{\perp c}$ and $h_{\parallel c}$ are the coordinates of the point C of the phase diagram and are equal to[4)]

$$h_{\perp c} = \beta_u + {}^1/_2(\beta_p + \beta_{pc}) - \beta_{2c} - 2\pi L^{-1}D^{1/2}\mu^{-1/2}, \qquad (4a)$$

$$h_{\parallel c} = -\beta_{uc}(1 + \beta_u^{-1}) - \beta_{pc}^* - \beta_{1c}(1 + 3\beta_u^{-1}). \qquad (4b)$$

Here

$$\mu = 1 + (1 + 2m_{0x}^2)\beta_2^{-1}, \quad \beta_2 = (1 + {}^1/_2 m_{0z}^2)h_\perp - \beta_{pc} - 2\beta_{4c},$$
$$m_{0x} \approx (\beta_1 m_{0z} + {}^1/_2\beta_{ps} + \beta_{3c})(h_\perp - \beta_{pc} - \beta_{4c})^{-1},$$
$$m_{0z} \approx (h_\parallel + \beta_{uc} + \beta_{ps}^* + \beta_{1c})(1 + h_\perp - \beta_u - {}^1/_2\beta_p + \beta_{2c} - {}^1/_2\beta_{pc})^{-1},$$
$$\beta_1 = \beta_{us} + \beta_{ps}^* + 3\beta_{1c};$$

the constants β and β^* with first subscripts u and p, as well as the constants β_{ic} ($i = 1-4$), are proportional to β_u, β_p, and β_c, respectively and depend in general case on the angles ϑ_u, φ_u, Φ_p, φ_H (see Fig. 5) and the crystallographic orientation of the substrate. The critical period of the nucleating DS is

$$d^{(U\to St)}=2(\mu D\pi^2L^2)^{1/4},$$

and the wave vector of the soft mode is $\mathbf{k}_A=(2\pi/d^{(u\to St)})\mathbf{e}_x$.

The stripe DS loses stability to a transition into a homogeneous state (line P_1CP_2 in Fig. 6) at

$$h_\perp^{(St\to U)}=h_\perp^{(U\to St)}+\frac{1728}{4459}\varkappa_c^2(\eta_c^2\eta_k^{-2}-1)\theta(\eta_c^2-\eta_k^2), \tag{5}$$

where

$$\eta_k^2=(h_{\|k}-h_{\|c})^2={}^3/_4h_{\perp c}^{-1}\varkappa_c^2, \quad h_{\|k}=h_{\|c}\pm\sqrt{3}\varkappa_c(2h_{\perp c})^{-1/2},$$

$$\varkappa_c=(\pi^2L^{-2}D)^{1/4},$$

and $\theta(x)$ is the Heaviside function. At $\eta_c<\eta_k$ (inside the interval K_1CK_2 in Fig. 6) the field is $h_\perp^{(u\to 0)}=h_\perp^{(0\to u)}$. i.e., a second-order PT takes place. If, however, $\eta_c>\eta_k$ (outside the interval K_1CK_2) the PT is of first order. the points K_1 and K_2 of the diagram are tricritical with coordinates $h\|k$ and

$$h_{\perp k}=h_\perp^{(U\to St)}(h_{\|k}).$$

The line of the homogeneous state $\rightleftarrows$ stripe DS phase transitions (N_1CN_2 in Fig. 6) are described by the relation

$$h_\perp^{(U\rightleftarrows St)}=h_\perp^{(U\to St)}+\frac{1296}{4459}\varkappa_c^2(\eta_c^2\eta_k^{-2}-1)\theta(\eta_c^2-\eta_k^2). \tag{6}$$

The regions of the stability of the lattices R_1 and R_2 on the diagram of Fig. 6 are bounded by the lines $Q_1'S_1Q_2''$ and $Q_2'S_2Q_2''$, whose positions are determined by the equation

$$\Gamma^3-7{,}95\Gamma^2+(15{,}6\gamma-94{,}125b_c)\gamma\Gamma$$
$$+\gamma(0{,}8\gamma^2+36{,}45\gamma b_c+2736{,}75b_c^2)=0, \tag{7}$$

where

$$\Gamma=h_\perp-h_\perp^{(St\to U)}-3b_c, \quad b_c=\varkappa_c^2\beta_2^{-1}, \quad \gamma=512h_{\perp c}(\eta_cM_0/3\pi)^2.$$

On the sections $Q_1'S_1$, $Q_2'S_2$ and S_1Q_1'', S_2Q_2'' the lattices are unstable to transitions into an homogeneous state and a stripe DS. Investigation of the stripe DS with respect to transformation into a lattice yields the equations of the lines $W_1'W_1''$ and $W_2'W_2''$:

$$h_\perp^{(St\to L_{1,2})}=h_\perp^{(U\to St)}-\varkappa_c^2\{\tilde h_{\|w}^2-{}^3/_4\beta_2^{-1}\pm[\tilde h_{\|w}^2(\tilde h_{\|w}^2-{}^3/_2\beta_2^{-1})]^{1/2}\}, \tag{8}$$

where

$$\tilde h_{\|w}=16\beta_2^{3/4}(h_\|-h_{\|c})(3\pi\varkappa_c)^{-1}.$$

It is impossible to obtain the equations of the lines of the PT between the lattices and the homogeneous states ($V_1'R_1$ and $V_2'R_2$) and between the lattices and the stripe DS ($V_2''R_1$ and $V_2''R_2$)in explicit form; their positions can be determined, however, by numerical calculation. The points R_1 and R_2 of the phase diagram with coordinates

$$h_{\|R}\approx h_{\|c}\pm\beta_2^{-1}, \quad h_{\perp R}=h_\perp^{(U\to St)}(h_{\|R})$$

are triple points in which coexist homogeneous and a striped DS and one of the lattices. It was assumed in the calculations that the points R_1 and R_2 are located inside the interval K_1CK_2 (this is valid if $\beta_2\gg 1$, i.e., in the case of large uniaxial anisotropy).

The period of the hexagonal lattice of the domains in the vicinity of the PT lines depends little on the magnetic field, unlike the stripe DS, whose period on the PT line (N_1CN_2 in Fig. 6) satisfies the relation

$$d^{(U\rightleftarrows St)}=d^{(U\to St)}\left[1+\frac{108}{637}(\eta_c^2\eta_k^{-2}-1)\theta(\eta_c^2-\eta_k^2)\right].$$

The theoretical phase diagrams (dashed; the first-order PT lines are not shown) and experimental ones (solid lines) for film No. 1 are shown in Fig. 7. At $H_{\perp c}-H_\perp<5$ Oe the calculated stability-loss line of the homogeneous state practically coincides with that observed in experiment; at large deviations of $H_\perp$ from $H_{\perp c}$ the difference between the positions of diagram lines of like type does not exceed several

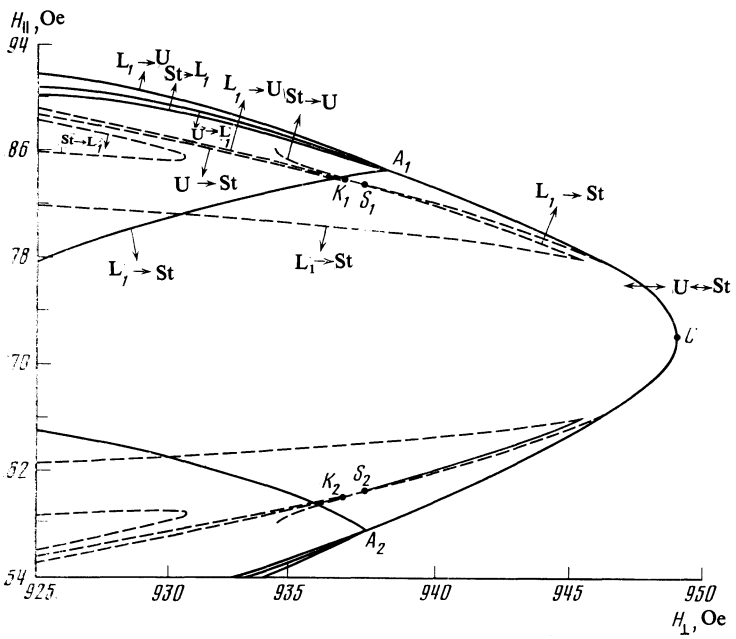

FIG. 7. Comparison of theoretical and experimental phase diagrams for film No. 1.

percent. The change of the character of the PT for nucleation of a DS in experiment (the points A_1 and A_2 takes place at H values close to those predicted by the theory (K_1 and K_2). For first-order PT, the difference between the theoretical and experimental diagrams is more noticeable. The reason is that the theory developed is valid in fields $H_\perp$ that satisfy the relation[2,3,5]

$$\Delta H_\perp = H_{\perp c} - H_\perp \ll 8\pi^2 M_0 D^{1/2} L^{-1} \mu^{-1/2}. \quad (9)$$

Estimates for film No. 1 ($D = 2.24\times 10^{-10}$ cm^2; $L = 6.1\,\mu$m; $\mu \approx 1$; $M_0 = 10.5$ G) yield $\Delta H_\perp \ll 20$ Oe, i.e., only qualitative agreement between theory and experiment can be expected in the entire range of variation of $H_\perp$ on Fig. 7.

It follows from the theory that in an ideal crystal at $T = 0$ direct nucleation of lattices form the homogeneous state is impossible, although the first-order PT line R_1V_1' (R_2V_2') lies above (below) the stability-loss line of the homogeneous state (see Fig. 6). In the experiments, however, nulceation of amorphous and hexagonal MBD lattices is observed in practically all films (except for spontaneous transitions in films of the fourth group). The apparent reason is that the experiments are performed at $T \neq 0$. Inasmuch as below (above) the line R_1V_1' (R_2V_2') the lattice has lower energy than the homogeneous state, the fluctuation energy at finite temperature may turn out to be sufficient to surmount the barrier between the homogeneous and inhomogeneous states. Another cause of the nucleation and amorphous and hexagonal lattices by fluctuations may be the formation of magnetic defects such as dislocations and disclinations (see below). At high density of such defects the nucleating DS constitute aggregates of isolated segments of stripe domains, which can be regarded as a ready-made nucleus of an amorphous lattice.

The theory developed in the present section can be generalized to include the case of spontaneous PT near T_c. To this end we must add to the density of the free energy (1) of the system the term

$$\Delta f = -\tilde{\xi} m^2 + {}^1/_2 \tilde{\delta} m^4,$$

where $\tilde{\xi} = \delta \tilde{M}^2(T) M_0^{-2}$, $\tilde{M}(T)$ is the equilibrium value of the magnetization at temperature T in an unbounded medium, $\tilde{\delta}$ is the homogeneous exchange-interaction constant, and $M_0 = \tilde{M}(0)$. It is necessary next to use the Landau-Khalatnikov equation. Leaving out the intermediate steps, we present the final results, confining ourselves for simplicity to the case when $f_c = 0$ and $\mathbf{n}_u$ and $\mathbf{n}_p$ (see Fig. 5) lie in the xz plane. For example, the equation for the homogeneous-state stability-loss line and the expressions for the wave vector of the critical mode and for the critical period of the DS are

$$T^{(U\to St)} = T_{0c} - (\xi_0')^{-1}(3m_{0z}^2 + m_{0x}^2 + 2\pi D^{1/2} L^{-1} \mu_T^{-1/2}),$$
$$|\varkappa_A| = [\pi^2 D (\mu_T L^2)^{-1}]^{1/4},\ d = 2\pi D^{1/2} \varkappa_A^{-1}.$$

Here $\xi_0' = (\partial\tilde{\xi}/\partial T)_{T=T_{0c}}$, $\mu_T = 1 + \beta_{2T}^{-1}$;

$$\varkappa_A \| \mathbf{e}_y \perp \mathbf{h}_\perp \text{ and } \beta_{2T} = \beta_u + \tilde{\delta} m_0^2, \text{ if } 2m_{0x}^2 - \beta_p > 0;$$
$$\varkappa_A \| \mathbf{e}_x \| \mathbf{h}_\perp \text{ and } \beta_{2T} = \beta_u - \beta_p + \tilde{\delta}(m_0^2 + 2m_{0x}^2), \text{ if } 2m_{0x}^2 - \beta_p < 0;$$

T_{0c} is the Curie point of an unbounded sample. The remaining results can be obtained from the corresponding equations for the orientational PT, by using the substitutions

$$h_{\perp c} \to 2\tilde{\delta}, \quad h_\perp^{(i\to k)} \to T^{(i\to k)} \xi_0', \quad (i, k = \mathrm{U,St,L}),$$
$$\beta_2^{-1} \to |2m_{0x}^2 - \beta_p| \beta_u^{-2};$$

in the case $2m_{0x}^2 - \beta_p > 0$ we must interchange x and y.

5. DISLOCATION-DISCLINATION MECHANISM OF AMORPHIZATION OF DOMAIN STRUCTURES IN SECOND-ORDER PHASE TRANSITIONS

In the analysis of the behavior of DS in the vicinity of PT lines either the fluctuation corrections to the expression for the free energy $F = U - TS$ of the magnet are completely disregarded or only the nonsingular fluctuations of small amplitude are taken into account. This leaves out of consideration the possible formation, in a regular DS of singular defects (dislocations and disclinations) and of irregular large-scale distortions such as curvature of the domain walls, which, while increasing the magnetic energy U, can nontheless lower the free energy system as a result of the increase of the entropy S.

If we confine ourselves to thin films that receive a small contribution from the optical modes that are due to thickness quantization, we can take the fluctuations into account by the approach developed in Refs. 6–11 (see also Ref. 12) for two-dimensional systems.[5)] The onset of stripe DS in quasiuniaxial films following orientational PT is accompanied by breaking of the symmetry of the homogeneous state with respect to translations and reflections in the plane, and also with respect to rotations about the normal to the surface. The system is therefore characterized at $T = 0$ by three types of order parameter, which can be called respectively translational, modulational, and orientational.[6)] Since the translation group is continuous (a Goldstone mode exists), and the reflection and rotation groups are discrete, only modulational and rotational long-range order can be preserved in the domain phases for a two-dimensional magnet at $T \neq 0$. The spectrum of the fluctuations in a film with a DS is characterized by the presence of soft optical modes that restore the symmetry with respect to reflection in the plane, and of acoustic modes that restore the symmetry with respect to translations. The system symmetry "in the mean" is restored at $T \neq 0$ also on account of formation of dislocations and disclinations in the DS and on account of large-scale nonperiodic violation of the orientational order.

At $T = 0$ the DS of a magnet is fully ordered (in the region where it exists), and this phase can be likened to a "ferrimagnetic" crystal (FC).[7)] An arbitrarily small temperature rise from $T = 0$ transforms the magnet DS into a phase characterized by the presence of static fluctuations of the "spin-wave" type, and also by the presence of dislocations of opposite sign that are bound into pairs—dislocation dipoles. The binding energy of the dislocations in the pairs is determined by the effective rigidity constants c_{ij} of the DS, which are functions of the magnetic field $H_\perp$ for orientational PT and of the magnet temperature T for spontaneous PT. Since dislocation dipoles merely renormalize the correlation function without changing the law that governs their

decrease at large distances, the correlation function has a power-law variation in the far zone. This means that, besides the modulational and orientational order, there is preserved in the system also a definite translational order that can be named, in analogy with the case of an elastic medium with dislocations,[13] a topological translational order. We refer to such an inhomogeneous state of a magnet as a Berezinskiĭ-Kosterlitz-Thouless (BKT) ferrimagnetic phase.[6-8]

In orientational PT, the effective DS rigidity constants decrease with increasing field $H_\perp$ (because of the approach to the lines of a second-order PT or a first-order PT close to second order), which brings about a lowering of the binding energy of the dislocation pairs. If $T \neq 0$, the average energy of the thermal fluctuations becomes equal, at a certain characteristic field value $H_\perp = H_{\perp 1}$ to the binding energy, and the dislocation dipoles dissociate. This means that at $H_\perp > H_{\perp 1}$ a transition takes place into a ferrimagnetic liquid crystal (FLC) phase with free dislocations (i.e., with bound disclinations), a phase having modulational and topologically orientational order. The FLC phase is stable in the field interval $H_{\perp 1} < H_\perp < H_{\perp 2}$, where $H_{\perp 2}$ is the dissociation field of the disclination dipoles. At $H_\perp > H_{\perp 2}$ the inhomogeneous state of the magnet is similar to a ferrimagnetic liquid (FL), in which only modulational order exists. With further increase of the field, at $H_\perp = H_{\perp 3} > H_{\perp 2}$, the modulation order is upset, and the magnet goes over into the paramagnetic-liquid (PL) phase.

In the case of spontaneous PT, one must speak not of the dissociation fields of the dislocation and disclination dipoles, but of their dissociation temperature T_1 and T_2. If the spontaneous PT are observed in an ideally uniaxial film in the absence of the field $H_\perp$, the rotation group for the homogeneous state becomes continuous and the initial inhomogeneous state of the magnet at $T \neq 0$ (far from the PT lines) is not the KBT phase, but the FLC phase. A similar situation obtains also for orientational PT in films with strong uniaxial anisotropy ($\beta_u \gg 1$).

In the scheme described above for the transformation of the inhomogeneous states of quasiuniaxial films as the PT lines are approached, we took into account only nonsingular small-amplitude periodic static fluctuations, as well as singular DS perturbations such as magnetic dislocations and disclinations. In the case considered here, however, it appears that large-scale aperiodic nonsingular disturbances of the orientational ordering in DS are also possible. The possibility of such an amorphization of a regular stripe DS in an ideally uniaxial film follows from the simple considerations presented below. For slight bending of the domain walls ($R_c \gg d$, where R_c is the average curvature radius and d is the period of the DS), the energy of a DS with an effective rigidity constant c increases, in analogy with the case of a bent elastic plate,[14] by an amount $U \approx cd^4 R_c^{-2}$, whereas the entropy contribution to the energy in the case of random disposition of the bends is equal to

$$TS = T \ln \{N! [n! (N-n)!]^{-1}\},$$

where $N = \sigma_p R_c^{-2}$ is the maximum possible number of loops that fill completely the surface of a plate of area σ_p (for a given d); n is the number of loops. Variation of the free energy $F = U - TS$ with respect to n and N yields an equilibrium loop density $n/N \approx 0.22$ and an equilibrium curvature radius $\tilde{R}_c = d^2 c^{1/2} T^{-1}$. It can be seen that even such rough estimates yield qualitatively correct results, viz., the existence of bends of domain walls at $T \neq 0$ and the decrease of the average curvature radius with increasing temperature (for spontaneous PT at constant T—with increase of the field $H_\perp$). We shall not analyze here more accurately the influence of the large-scale DS distortions on PT in films, and assume that the bending of the domain walls is suppressed, say by a sufficiently strong anisotropy in the basal plane.[8)]

6. INHOMOGENEOUS MAGNETIC STATES AT $T \neq 0$

To study the behavior of the system at finite temperatures we determined the spin-wave spectrum in a ferromagnetic plate having a regular DS that produces a periodic potential in the xy plane. The solution of the equation of motion of the magnetization with corresponding boundary conditions was represented in the form of a superposition of Bloch functions:

$$\tilde{\mathbf{m}}(\mathbf{r}, t) = \sum_{n,m=-\infty}^{+\infty} \left(\mathbf{A}_{nm} \cos \frac{\pi z}{L} + \mathbf{B}_{nm} \sin \frac{\pi z}{L} \right) \times \exp[i(\mathbf{k}_{nm} \mathbf{r}_\perp - \omega t)] + \text{c.c.}, \tag{10}$$

where $\mathbf{k}_{nm} = \mathbf{Q} + n\mathbf{k}_1 + m\mathbf{k}_2$, and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the basis vectors of the reciprocal lattice. Calculation of the spectrum (by the procedure developed in Refs. 3 and 15) has shown that the existence of DS with dimensionality 1 and 2 leads to the appearance, in the spin-wave spectrum, of respectively one (longitudinal) and two (longitudinal and transverse) acoustic modes (displacement oscillations of the domain walls) as well as of soft optical modes (oscillations of the magnetization inside the domains).

At $T = 0$ it is necessary to add to expression (1) for the free energy a fluctuation part ΔF that can be calculated by expressing the static fluctuations of the magnetization in the form of series (10) at $\omega = 0$. Confining ourselves to the harmonic approximation, we reduce ΔF to the canonical form

$$\Delta F = \frac{V}{2} \sum_{i=1}^{\infty} \sum_{Q} \Omega_{iQ}^2 |X_{iQ}|^2, \tag{11}$$

where

$$\Omega_{iQ}^2 = 2\pi (\omega_i / \omega_M)^2, \quad \omega_M^2 = \mu \beta_2 (4\pi g M)^2;$$

X_{iQ} are the normal coordinates expressed as linear combinations of the expansion coefficients (10), i is the number of the mode, and g is the magnetomechanical ratio. Singular contributions to ΔF are made, first, by soft optical modes whose correlation radius tends to infinity when the phase stability-loss lines are approached, and second, by acoustic modes that destroy the translational order in the DS.[9)]

We consider first the fluctuations of the first type. According to the thermodynamic theory of fluctuations[16]

$$\langle |X_{iQ}| \rangle = T (V \Omega_{iQ}^2)^{-1},$$

whence

$$\langle X_i(r)X_i(0)\rangle=\sum_Q e^{iQr}\langle|X_{iQ}|^2\rangle$$
$$=\frac{T}{\pi L(\alpha_x^{(i)}\alpha_y^{(i)})^{1/2}}K_0\left[\left(\frac{x^2}{x_i^2}+\frac{y^2}{y_i^2}\right)^{1/2}\right], \quad (12)$$

where the correlation radius is

$$\mathbf{r}_i=\Omega_{i0}^{-1}\{\sqrt{\alpha_x^{(i)}};\ \sqrt{\alpha_y^{(i)}}\}.$$

For a stripe DS ($i=1$) and for a hexagonal lattice ($i=1, 2, 3$) we have respectively

$$\alpha^{(1)}=4\pi D\{4;\ \beta_2^{-1}\};\quad {}^1/_2\alpha^{(1)}=\alpha^{(2)}={}^1/_3\alpha^{(3)}=4\pi D\{1;\ 1\},$$

where $K_0(x)$ is a Macdonald function. As $h_\perp\to h_\perp^{(i\to k)}$ the correlation radius changes like

$$r_i\sim|h_\perp-h_\perp^{(i\to k)}|^{-1/2}.$$

The Ginzburg criterion for the applicability of the Landau theory, say for a stripe DS $\rightleftarrows$ homogeneous phase transition, takes in this case the form

$$(h_\perp^{(U\to St)}-h_\perp)h_{\perp c}^{-1}\gg 0.1(\alpha_x^{(1)}\alpha_y^{(1)})^{-1/2}L^{-1}M^{-2}T. \quad (13)$$

Substituting in (13) typical parameters of the films at $T=300$ K, we find that the Landau theory is valid if $H_\perp^{(U\to St)}-H_\perp\gg 0.01-0.1$ Oe. This estimate holds also for other fluctuations of the first type.

We proceed now to analyze the fluctuations of the second type. Since the translation group is continuous and corresponds to Goldstone branches of the spectrum, there is no long-range translational order in the DS at $T\neq 0$. Films in an inhomogeneous state at $T\neq 0$ are equivalent to quasi-two-dimensional crystals without a long-range order, but with nonzero rigidity modulation. This makes it possible to use the approach developed for two-dimensional crystals in Refs. 8–11, and modify this approach to fit the problem considered here. Following Refs. 8–11, we shall show that the phase diagram of the film, in coordinates $(H_\perp, H_\parallel, T)$, breaks up into four regions corresponding to the following states: 1) ferrimagnetic BKT phase—quasiregular DS with bound magnetic dislocations of opposite sign; 2) FLC phase, in which the dislocation pairs dissociate; 3) FL phase, in which the disclination pairs dissociate; 4) PL phase, characterized to total absence of order.

We use next in place of the normal coordinates $\{X_i\}$ a system of variables $\{\mathbf{u}_i(\mathbf{r})\}$ that describe the displacement of a point with a given magnetization value from its position in the regular DS, i.e., we represent the magnetization distribution $\mathbf{m}(\mathbf{r})$ in the form

$$\mathbf{m}(\mathbf{r})=\sum_{i=1,2,3}\mathbf{m}_i\exp\{i\mathbf{k}_i[\mathbf{r}_\perp+\mathbf{u}_i(\mathbf{r}_\perp)]\}\cos\frac{\pi z}{L}+\text{c.c.}, \quad (14)$$

where $\mathbf{m}_i$ is the amplitude of the magnetization in the regular DS, and $\mathbf{k}_3=-\mathbf{k}_1-\mathbf{k}_2$. The fluctuating part of the free energy for a stripe DS and for a lattice can then be respectively expressed as follows:

$$\Delta F_{St}={}^1/_2\int dr_\perp[c_x(\nabla_x u)^2+c_y(\nabla_y u)^2+c_0(\mu k_A^2)^{-1}(\nabla_y u)^4], \quad (15a)$$

$$\Delta F_L={}^1/_2\int dr_\perp c_{iklm}u_{ik}u_{lm}, \quad (15b)$$

where $k_A^2=\pi(\mu DL^2)^{-1/2}$. For a stripe DS we have $u\equiv u_x$, and the effective rigidity constants, connected by the relations $c_x=4c_0$ and $c_y=(1-\mu^{-1})c_0$, decrease on approaching the line of the second-order transition (or of the first order close to the second) in the following manner

$$c_0=16\pi Lk_A^2(h_\perp^{(St\to U)}-h_\perp)[9h_{\perp c}(1-\eta_c^2\eta_k^{-2})]^{-1}$$

far from the critical points, i.e., at

$$(h_\perp^{(St\to U)}-h_\perp)\ll\varkappa_c^2(1-\eta_c^2\eta_k^{-2});$$

in the vicinity of the critical points, when

$$1\gg(h_\perp^{(St\to U)}-h_\perp)\gg\varkappa_c^2(1-\eta_c^2\eta_k^{-2}),$$

the variation of c_0 is of the form

$$c_0=2.216\varkappa_c(h_\perp^{(St\to U)}-h_\perp)^{1/2}h_{\perp c}^{-1}.$$

For a two-dimensional lattice we have

$$u_{ik}={}^1/_2(\nabla_i u_k+\nabla_k u_i),\quad c_{11}=3\mu^*[1-(36\beta_2)^{-1}],$$
$$c_{22}=3\mu^*[1+5(12\beta_2)^{-1}],\quad c_{12}=c_{44}=\mu^*[1+(12\beta_2)^{-1}],$$

where in the case of a hexagonal lattice ($\beta_2\to\infty$)

$$\mu^*=24\pi\varkappa_c^2LM^2[(8h_\parallel/45\pi)+(3\sqrt{5}\,h_{\perp c})^{-1}(h_\perp^{(L\to U)}-h_\perp)^{1/2}]^2.$$

The condition for the transition from the BKT phase to the FLC phase can be deduced approximately in the following manner.[11] On formation of a solitary magnetic dislocation with the minimum possible Burgers vector $\mathbf{B}_1\approx\pm\mathbf{d}_1$, where

$$\mathbf{d}_1=4\pi\varkappa_c^{-1}(D/3)^{1/2}(a_x\mathbf{e}_x+a_y\mathbf{e}_y)$$

($a_x=\sqrt{3}/2$ and $a_y=0$ for a stripe DS; $a_x=0$ and $a_y=1$ or $[1-(6\beta_2)^{-1}]$ for an hexagonal and anisotropic lattice, respectively) the system energy increases by

$$\delta U=(g_0B_1^2/4\pi)\ln(R/r_{\perp 0}),$$

where $\mathbf{R}$ is the radius vector in the film plane, $r_{\perp 0}\sim d_1$ is the dimension of the dislocation core, and g_0 is the effective shear modulus of the DS and is expressed in terms of the rigidity constant; for example, $g_0=(c_xc_y)^{1/2}$ for a stripe DS. Stipulating that this increase of the energy be offset by the entropy contribution

$$\delta U\leqslant TS=T\ln(R/r_{\perp 0})^2,$$

we obtain a criterion for the dissociation of dislocation pairs, in the form

$$g_0\leqslant 8\pi T. \quad (16)$$

Substituting in (16) the values of the system parameters, we determine the positions of the lines of the transition from the BKT phase to the FLC phase: $h_{\perp 1}=f(h_\parallel)$. For a stripe DS far from the critical points and in their vicinity we have respectively

$$\frac{\Delta h_{\perp L}}{h_\perp^{(St\to U)}}=\frac{h_\perp^{(St\to U)}-h_{\perp 1}}{h_\perp^{(St\to U)}}=\begin{cases}\dfrac{9\beta_2}{16\pi^2}\left(\dfrac{T}{T^*}\right)(1-\eta_c^2\eta_k^{-2}); & (17a)\\[2ex] 2.09\cdot 10^{-3}\dfrac{h_{\perp c}\beta_2}{Dk_A^2}\left(\dfrac{T}{T^*}\right)^2; & (17b)\end{cases}$$

and for a hexagonal lattice

$$\frac{\Delta h_{\perp L}}{h_\perp^{(L\to U)}}=\frac{h_\perp^{(L\to U)}-h_{\perp 1}}{h_\perp^{(L\to U)}}=\frac{135}{128\pi^2}\left(\frac{T}{T^*}\right)\left[1-\frac{64\sqrt{6}}{135}\left(\frac{T^*}{T}\right)^{1/2}h_\parallel\right]^2, \quad (17c)$$

where $T^* = LDM^2$; the equations for an anisotropic lattice are too long to cite here.[10)]

Even though there is no long-range order in a two-dimensional solid, in the BKT phase the magnetization correlation function $G(\mathbf{r}_\perp - \mathbf{r}_\perp')$ falls off exponentially only in the near zone, namely (for a stripe DS)

$$G(\mathbf{r}_\perp - \mathbf{r}_\perp') \infty \begin{cases} \exp[-2q(v|x-x'|)^{1/2}], \\ \text{if} \quad |y-y'| \ll (d_1|x-x'|)^{1/2} \ll d_1\beta_2^{1/2}; \\ \exp(-qv|y-y'|), \\ \text{if} \quad (d_1|x-x'|)^{1/2} \ll |y-y'| \ll d_1\beta_2^{1/2}, \end{cases}$$

where

$$q = k_A^2 T(16\pi^2\beta_2 c_x c_y)^{-1/2}; \quad v = \pi k_A \mu^{1/2}.$$

In the far zone, when

$$\min\{(d_1|x-x'|)^{1/2}; \quad |y-y'|\} \gg d_1\beta_2^{1/2},$$

the fall-off follows a power law:

$$G(\mathbf{r}_\perp - \mathbf{r}_\perp') \, |\tilde{\mathbf{r}}_\perp - \tilde{\mathbf{r}}_\perp'|^{-2q\sqrt{\beta_2}},$$

where $\tilde{\mathbf{r}}_\perp = r_\perp\{1/2; \beta_2^{-1/2}\}$. The dislocation dipoles only renormalize the rigidity constants of the system, and leave the variation of $G(\mathbf{r}_\perp - \mathbf{r}_\perp')$ unchanged. Similar relations hold also for two-dimensional lattices.

With increasing β_2, the region of the exponential change of the correlation function broadens, and in the limit as $\beta_2 \to \infty$ existence of a BKT phase is not possible at all. The latter holds also for spontaneous transitions in ideally uniaxial films.

Screening of the interaction between dislocations bound into pairs and separated by distance $R_\perp = |\mathbf{r}_{\perp 1} - \mathbf{r}_{\perp 2}|$ by other dislocation dipoles leads to renormalization of the DS effective rigidity constants, which become functions of $R_\perp$. Using the method of renormalized Wilson groups,[8-11] we obtain the following equations for the renormalized magnetic field $h_\perp(R_\perp) = h_{\perp R}$ that determines the $g_0(R_\perp)$ dependence.

a) Stripe DS

$$\frac{dh_{\perp R}}{dl} = \frac{8\pi^2 y^2}{\Delta h_{St}}(h_\perp^{(St\to U)} - h_{\perp R})^2, \quad \frac{dy}{dl} = \frac{2y}{\Delta h_{St}}(h_{\perp R} - h_{\perp 1}) \qquad (18a)$$

far from the critical points and

$$\frac{dh_{\perp R}}{dl} = \frac{16\pi^2 y^2}{(\Delta h_{St})^{1/2}}(h_\perp^{(St\to U)} - h_{\perp R})^{1/2},$$

$$\frac{dy}{dl} = \frac{2y(h_{\perp R} - h_{\perp 1})}{\Delta h_{St} + [\Delta h_{St}(h_\perp^{(St\to U)} - h_{\perp R})]^{1/2}} \qquad (18b)$$

in the vicinity of the critical points.

b) Hexagonal lattice

$$\frac{dh_{\perp R}}{dl} = \frac{9\sqrt{5}\pi^2 y^2 A m_{1z}^3(h_{\perp R})}{m_{1z}^2(h_{\perp 1})(h_\perp^{(L\to U)} - h_{\perp R})}$$

$$\frac{dy}{dl} = \frac{8y[m_{1z}(h_{\perp 1}) + m_{1z}(h_{\perp R})](h_{\perp 1} - h_{\perp R})}{3\sqrt{5}m_{1z}^2(h_{\perp 1})[(h_\perp^{(L\to U)} - h_{\perp R})^{1/2} - (\Delta h_L)^{1/2}]^2} + 2\pi B y^2. \qquad (18c)$$

Here $y(R_\perp) \ll 1$ is an auxiliary variable proportional to the probability of thermal excitation of the dislocation dipole and satisfying the boundary condition.

$$y(r_{\perp 0}) = y_0 = \exp(-E_c/T),$$

where E_c is the energy of the core of a single dislocation:

$$l = \ln(R_\perp/r_{\perp 0}), \quad A = e^2[2I_0(2) - I_1(2)], \quad B = eI_0(2);$$

I_0 and I_1 are modified Bessel functions; the boundary condition for $h_{\perp R}$ is

$$h_{\perp R}(r_{\perp 0}) = h_\perp.$$

Account is taken in (18c) of the contribution from the bound triplets of dislocations with zero resultant Burgers vector.

Analysis of Eqs. (18) by the method developed in Refs. 8–11 shows that the external magnetic field $h_{\perp 1}^{(e)}$ corresponding to a transition from the BKT phase to the FLC phase is connected with the internal field $h_{\perp 1}$ defined by Eqs. (17) by the relation

$$h_{\perp 1}^{(e)} = h_{\perp 1}(1 - py_0),$$

where we have for a stripe DS

$$p = 2\pi\Delta h_{St} h_{\perp 1}^{-1} f$$

($f = 2$ or 1 respectively close to and far from the critical points), while for a hexagonal lattice we have

$$n = \frac{3\pi}{4}\sqrt{\frac{5}{2}}\, h_{\perp 1}^{-1}(\Delta h_L)^{-1/2}[B - (B^2 + 4A)^{1/2}].$$

The corrections to the rigidity constants to account for screening are

$$[(h_{\perp 1}^{(e)} - h_\perp)(ph_{\perp 1})^{-1}]^\nu,$$

where $\nu = 1/2$ for a stripe DS and an anisotropic two-dimensional lattice and $\nu = 0.369$ for a hexagonal lattice. The rigidity constants vanish on the PT lines.

Estimates show that at typical values of the parameters used in the experimental films the shift of the field of the transition from the BKT phase to the FLC phase on account of the screening amounts to $\sim 10^{-1}$ of the value of Δh_{St} or Δh_L.

In the FLC phase with free dislocations (i.e., with bound disclinations), the correlation function falls off exponentially:

$$G(\mathbf{r}_\perp - \mathbf{r}_\perp') \infty \exp[-|\mathbf{r}_\perp - \mathbf{r}_\perp'| R_+^{-1}],$$

where the characteristic length R_+, equal to

$$R_+ \sim r_{0\perp} \exp\{[(h_\perp - h_{\perp 1}^{(e)})(ph_{\perp 1})^{-1}]^{-\nu}\},$$

determines the average distance between the free dislocations. The domain structure can be regarded as a BKT phase in the length range $R < R_+$ and as an FLC phase at $R > R_+$.

When considering DS defects such as magnetic disclinations, the orientational order can be characterized by the quantity

$$\tilde{m}_z(\mathbf{r}_\perp) = 2m_{1z} \exp[in\Phi(\mathbf{r}_\perp)] + \text{c.c.},$$

where $\Phi(\mathbf{r}_\perp)$ is the angle between $\mathbf{e}_x$ and the vector that joins the centers of the neighboring domains, and n is the order of the system symmetry axis. For a stripe DS we have $n = 2$ (180-degree disclinations) and $\Phi = -\partial u/\partial x$; for a hexagonal lattice $n = 6$ (60-degree disclinations) and

$$\Phi = {}^1/_2 e_{ij}(\partial u_j/\partial x_i),$$

where e_{ij} is a two-dimensional completely antisymmetric tensor.

A long-range orientational order is present in the BKT phase; on going to the FLC phase, the correlation begins to decrease in accord with the power law

$$G(\mathbf{r}_\perp - \mathbf{r}_\perp') = 4m_{1z}^2 \exp\{in[\Phi(\mathbf{r}_\perp) - \Phi(\mathbf{r}_\perp')]\} \sim |\mathbf{r}_\perp - \mathbf{r}_\perp'|^{-n^2T/2\pi\Omega(h_\perp)},$$

where $\Omega(h_\perp)$ is the Frank constant, which has a singularity as $h_\perp \to h_{11}^{(e)}$. The dissociation of the disclination dipoles, i.e., the transition to the FL phase, takes place in a field $h_\perp = h_{\perp 2}$ determined from the condition

$$\Omega(h_{\perp 2}) = 2n^2 T\pi^{-1}.$$

In the FL phase, the correlation function falls off exponentially:

$$G(\mathbf{r}_\perp - \mathbf{r}_\perp') \sim \exp[-|\mathbf{r}_\perp - \mathbf{r}_\perp'|/\bar{R}_+^{-1}],$$

where

$$\bar{R}_+ \sim \exp[\text{const}\,(h_\perp - h_{\perp 2})^{-1/2}]$$

is the distance between the free disclinations. The renormalization of the field $h_{\perp 2}$ on account of the screening is carried out by the method developed in Refs. 8–11.

7. DISCUSSION OF RESULTS

The experimentally observed behavior of the DS in PT in the investigated films confirms fully the main conclusions of the theory. Figures 3c–e show photographs of the DS in film No. 2 for a spontaneous PT ($H_\perp = 0$ Oe, $H_\parallel = 4$ Oe); they illustrate the change of the form of the two-dimensional lattice on moving smoothly away from the stability-loss line of the homogeneous state as the temperature is lowered. It can be seen that near the transition line (Fig. 3c; $T = 422.08$ K) the lattice becomes completely amorphous; when T is lowered, the DS becomes gradually ordered (Figs. 3d, e; $T = 418$ and 414 K, respectively). The width of the region where the amorphous DS exists correlates well with the predictions of the theory (see Sec. 6). One can trace on the photographs also the increase, with decreasing temperature, of the average curvature radii of the lines joining the centers of neighboring domains. This effect confirms the qualitative theoretical arguments (see Sec. 5) concerning large-scale destruction of the order in the DS.

The presence of magnetic defects "mixes up" the various types of DS. Thus a stripe DS with high magnetic-dislocation density is essentially indistinguishable from an amorphous two-dimensional lattice. Analysis of the singularities of the experimentally observed transition from the nucleation of a state with mixed polarity to nucleation of amorphous two-dimensional lattices (see Figs. 3a, b) suggests that there exists a certain critical dislocation-dipole density above which the dipole dissociation becomes avalanche-like. This situation can probably be described within the framework of percolation theory.

In the analysis of the results of experiments on nucleation of DS in real samples, it must be borne in mind that the identification of a DS as belonging to one type or another (amorphous or ordered) depends on dimensions of the observation field (or on the transverse dimensions of the sample). In addition, the external parameters ($H_\perp$ or T) always vary in experiment at a nonzero rate, so that allowance must be made for the relation between the time τ_a during which the system goes through the amorphization interval and the time τ_f in which the DS is formed. The time τ_a can be decreased by increasing the rate of change of the external parameters as well as by increasing the temperature (or field) of the dissociation of the bound pairs of magnetic defects; this temperature depends on the anisotropy in the basal plane, and for spontaneous transitions also on the field $H_\perp$. For example, if the field $H_\perp$ of the spontaneous transitions is gradually increased, a change takes place from nucleation of a fully amorphized DS ($\tau_a > \tau_f$) to nucleation of a quasiregular stripe DS ($\tau_a < \tau_f$), containing single dislocations (Figs. 3f–h); the values of T, $H_\perp$ and $H_\parallel$ in cases f–h are respectively: $T = 422$ K $= 0.1$; 0.08; 0.01 K; $H_\perp = 0$;4.5; 18 Oe; $H_\parallel = 0.55$; 2.2 Oe).

The processes of establishment and destruction of the order in DS of real films are generally speaking independent, owing to the presence of crystal-structure defects (dislocations, etch pits, melt inclusions, and others), which contribute to the appearance of magnetic dislocations and disclinations in regular DS when $H_\perp$ or T is increased, but prevent restoration of the order in amorphous DS when $H_\perp$ or T is decreased.

A magnetic dislocation wall oriented perpendicular to the field $H_\perp$ is sometimes observed in DS nucleated in spontaneous second-order PT in films having a large uniaxial anisotropy ($\beta_u \gg \max\{1; \beta_p; B_c\}$ (Figs. 3g, h). When the temperature is lowered the dislocation wall is rapidly dispersed by crowding out the dislocations from the sample.

Various types of magnetic defects observed in a stripe DS and in two-domain lattice domains (single or coupled dislocations, 180- and 60-degree disclinations), nucleated in orientational and spontaneous PT, can be easily discerned on Fig. 3. A distinguishing feature of magnetic defects is that they can exist in two "dual" modifications (replacement of the "black" domains by "white" ones).

The foregoing theoretical and experimental analysis of the behavior of DS in quasiuniaxial magnetic films in PT shows that they constitute a very convenient object for the study of the influence of fluctuations on the ordering of two-dimensional systems. These effects were heretofore investigated experimentally only in superconductors and in liquid crystals. In contrast to these objects, the DS of epitaxial iron garnet films permit a direct utilization of visual observation methods, so that complete and reliable information can be obtained by relatively simple means on the behavior of the system in critical points.

The described regularities of the amorphization of DS remain apparently valid also for PT of first-order close to second. Similar phenomena can occur also in nonferromagnetic systems, e.g., in ferroelectrics and in solid solutions.

We note that nucleation of amorphous DS in second-order PT was observed not only in epitaxial iron-garnet films, but also in other magnets (magnetoplumbite, barium

ferrite, cobalt; see, e.g., Refs. 1 and 20–22), but this fact was reported there without explanation.

[1)]At $H_\perp \neq 0$ the spontaneous PT can be observed, owing to the temperature dependence of the field H_u, at any temperature $0 \lesssim T \lesssim T_C$. The transition temperature T^* can be approximately determined from the condition $H_u(T^*) \approx H_\perp$.

[2)]The locations of the points A_1 and A_2 on the diagram are approximate, since replacement of one type of DS by another (i.e., a transition from an amorphous to a hexagonal lattice) is smooth in a certain interval of ϑ. The same pertains to points B_1, B_2, D_1, and D_2 of the diagram in Fig. 2, but the position of the point C (the angle ϑ_c) is determined with very high accuracy.

[3)]Two regions of nucleation of the lattices L_1 and L_2 are observed in films of the second group also at $H_\perp = 0$.

[4)]The $h_{\perp c}(\varphi_H)$ and $h_{\parallel c}(\varphi_H)$ dependences are used to determine the parameters that characterize the anisotropy of the films.

[5)]The two-dimensionality criterion for the magnetic films considered by us coincides with the condition for the validity of the theory developed in Sec. 4 [see Eq. (9)].

[6)]By orientational order is meant for a stripe DS ordering of the domain walls, and for a hexagonal lattice the ordering of the "bonds" joining the centers of neighboring MBD.

[7)]The concept of a "ferrimagnetic" crystal was used to emphasize the existence of a periodic DS with unequal (in the general case) amplitudes of the magnetization M_z in neighboring domains.

[8)]The theory expounded below is valid for films with weak anisotropy in the basal plane in a space region $R \ll \tilde{R}_c$.

[9)]The regular fluctuation correction to F can be taken into account if

$$r_i^{(s)} \gg r_i^{(r)} \sim D^{1/2} \varkappa_c^{-1} (n-1)^{-1/2},$$

where $n > 2$ and $r_i^{(s)}$ and $r_i^{(r)}$ are respectively the correlation radii of the "singular" and "regular" fluctuations. These relations, which are valid at

$$\varkappa_c^2 \gg |h_\perp - h_\perp^{(i \to k)}|,$$

are satisfied in the region of validity of the theory developed in Sec. 4.

[10)]Estimates of the intervals $\Delta h_{St}/h_{\perp c}$ and $\Delta h_L/h_{\perp c}$ for typical parameters of iron garnet films at $T = 300$ K yield values 10^{-3}–10^{-4}, i.e., the amorphization of quasiregular DS in the region of transitions of second order (and of first order close to second), takes place in an exceedingly narrow vicinity of the PT lines. On departure from the critical points into the region of a first-order transition, the rigidity constants of the DS increase, therefore the "melting" of the DS may also become unobservable. Thus, for example, a two-dimensional amorphous lattice exists only at

$$h_\parallel < h_\parallel^* = (45\sqrt{6}/128)\,(T/T^*)^{1/2}.$$

Estimating $h_\parallel^*$ for the investigated films, we obtain the value 10^{-1}–10^{-2}, in good agreement with the experimentally observed width of the region of existence of amorphous lattices in orientational PT (see Sec. 2).

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