Gyromagnetism of Cooper pairs in superfluid ³He-B

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The Einstein–de Haas gyromagnetic effect in ³He–B is analyzed. In a magnetic field, ³He-B acquires an orbital angular momentum comparable in magnitude to the spin but rotated from it by the matrix $R_{\alpha,i}$, which is an order parameter in ³He-B. Only a negligibly small part of the orbital angular momentum stems from the local rotational motion of Cooper pairs; most of it comes from a current flowing along the surface. This orbital angular momentum could be observed in an NMR experiment in a rotating apparatus, where it should make the orientation of the order parameter dependent on the rotation direction. The orientational effect would be negligibly small in large vessels, since it is due to a local gyromagnetism. It is suggested that the orbital angular momentum instead be measured in thin capillaries, where the orientational effect stems from a surface gyromagnetism, which is amenable to measurement.

1. INTRODUCTION

Gyromagnetic effects in superfluid ³He-B differ from the corresponding effects in ordinary media because of the unusual breaking of the invariance under rotations in the spin and orbital spaces and also because of the rigidity of the coherent state of the superfluid system. Because of the coherence in the motion of Cooper pairs, the quantum numbers of the state of the pair cannot be changed by a weak external perturbation; the system therefore opposes this perturbation, completely neutralizing it. Examples of this cancellation are well known in coherent systems; one is the response of a superconductor to a magnetic field which results in the complete displacement of the field, and another is the complete neutralization of friction during the flow of a superfluid liquid. A similar effect in superfluid ³He-B leads to a peculiar gyromagnetism, as was first pointed out by Leggett and Takagi.[1]

This effect can be seen in a simple model of ³He-B: a Bose condensate of molecules having the structure of a Cooper pair in ³He-B. A molecule consists of two ³He atoms and has a spin $S = 1$; the orbital angular momentum of the relative motion of the atoms in the molecule is $L = 1$. The state of the molecule is isotropic and has a total angular momentum $J = 0$. Since the spin-orbit interaction is negligibly weak, the states of the molecule are degenerate with respect to the choice of the relative orientation of the orbital and spin coordinate axes. A state in which the relative orientation of the axes is specified by the three-dimensional rotation matrix $R_{\alpha,i}$ is an eigenstate of the operator $\hat{J}_i = \hat{L}_i + R_{\alpha i}\hat{S}_\alpha$ with a zero eigenvalue, $J = 0$. Both the matrix $R_{\alpha i}$ and the quantum number $J = 0$ are identical for all molecules of the Bose condensate at equilibrium, with the consequence that the quantum state of the molecules is rigid on a macroscopic scale.

Let us assume that a magnetic field **H** is applied to this Bose liquid and induces a spin density of molecules $\mathbf{S} = \chi \mathbf{H}/\gamma$, where χ is the magnetic susceptibility of the liquid, and γ is the gyromagnetic ratio for the ³He nucleus. If the quantum number $J = 0$ is to be conserved, the system must react to an external field through the formation of an orbital angular momentum $L_i = (\chi/\gamma)H_\alpha R_{\alpha i}$, which cancels the change in J. The magnetic field thus causes a rotation of electrically neutral molecules. This analog of the Einstein–de Haas gyromagnetic effect is due exclusively to the rigidity of the coherent state.

A coherent analog of the Barnett effect arises in a similar way. When a liquid is rotating at an angular velocity $\mathbf{\Omega}$, a term $\delta\hat{H} = -\mathbf{\Omega}\cdot\mathbf{J}$ must be added to the Hamiltonian where the total angular momentum is

$$\hat{J}_i = \hat{S}_i + \hat{L}_i = \hat{S}_i - R_{\alpha i}\hat{S}_\alpha.$$

The magnetization which arises upon the rotation,[15]

$$M_\alpha = -\frac{\chi}{\gamma}\frac{\partial\langle H\rangle}{\partial\langle S_\alpha\rangle} = \frac{\chi\Omega_\alpha}{\gamma} - \frac{\chi}{\gamma}R_{\alpha i}\Omega_i,$$

contains two terms, the first corresponding to the standard Barnett effect and the second resulting from the rigidity of the coherent state with $J = 0$.

How is the coherent Einstein-de Haas effect modified in real ³He-B, i.e., in a Fermi liquid with Cooper pairing at the Fermi surface? We will show that in a Fermi liquid, where Cooper pairs overlap each other markedly, the coherent effect is conserved only in an integral sense, i.e., only when the orbital angular momentum is averaged over the entire volume of the liquid, including its boundaries. The local effect, in contrast, is exceedingly small, specifically, the local internal orbital angular momentum of the liquid, $\mathbf{L}^{\text{int}}$, contains an additional small factor on the order of $(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c)$, while the large integral angular momentum is produced by the macroscopic flow of ³He-B along the surface of the vessel.

The behavior exhibited by the orbital angular momentum of Cooper pairs in ³He-B in response to an external field is thus analogous to the behavior of the spontaneous orbital angular momentum of pairs in ³He-A (Ref. 2). In the latter medium each pair has an orbital angular momentum $\hbar\mathbf{l}$, where the unit vector **l** is common to all pairs. Although the integral orbital angular momentum of the liquid consisting of N ³He atoms is $\hbar N l/2$, the lion's share of this angular

momentum corresponds to the macroscopic current of the liquid along the surface. In contrast, the density of the local angular, momentum, corresponding to the internal rotation of pairs, is not $\hbar\rho l/2m_3$ as we would expect in the model of a Bose gas of molecules with a density $\rho/2m_3$ but instead contains the same additional small factor:

$$L \sim l\hbar \frac{\rho}{2m_3} \left(\frac{T_c}{\varepsilon_F}\right)^2 \ln \frac{\varepsilon_F}{T_c}.$$

Let us discuss the possibility of measuring the orbital angular momentum in ^{3}He-B in the rotating minilab at Otaniemi, Finland.[3] In our original paper,[4] where we had not yet distinguished between the integral and local angular momenta, we suggested that the orbital angular momentum might be detected by detecting a change in the orientation of the order parameter due to the gyromagnetic energy, which depends on this orientation,

$$F_{gm} = -\mathbf{\Omega L} \sim (\chi/\gamma) H_\alpha R_{\alpha i} \Omega_i.$$

A change in the orientation of the $R_{\alpha i}$ matrix upon a reversal of the rotation direction or the direction of the magnetic field should have been detectable by an NMR technique. A change was in fact detected,[5] but it turned out to be due to a coherent analog of the Barnett effect in ^{3}He-B. Specifically, the rotation of the vessel gives rise to quantized vortices which have a magnetic momentum concentrated in the cores of the vortices. It was this magnetic momentum which was detected and measured for the first time. The orbital angular momentum, on the other hand, which has a local orienting effect under these experimental conditions, is extremely small, on the order of the small factor $(T_c \varepsilon_F)^2 \ln(\varepsilon_F/T_c)$, and could not be detected.

For an experimental observation of the orbital angular momentum we suggest carrying out the same NMR experiments but in a liquid in a system of thin tubes. In this case the orienting effect exerted on the order parameter by the orbital angular momentum will be an integral effect, by which we mean that it will include the surface gyromagnetism, which is amenable to measurement.

2. LOCAL AND INTEGRATED ORBITAL ANGULAR MOMENTA OF COOPER PAIRS

We wish to determine the difference between the local and integral properties of the angular momentum of Cooper pairs in an arbitrary superfluid state of ^{3}He. An arbitrary state is characterized by the order parameter $A_{\alpha i}$, which is a complex 3×3 matrix with a Latin letter used as an orbital index and a Greek letter used as a spin index. A Cooper pair may be represented qualitatively as a molecule with a spin $S = 1$ and an angular momentum $L = 1$. The order parameter $A_{\alpha i}$, which is a vector in both the spin and orbital spaces, specifies the wave function of this molecule.

The qualitative behavior of the orbital angular momentum can be seen most easily near T_c where we can use the Ginzburg-Landau functional, which consists of the condensation energy and a gradient energy. The condensation energy is[6]

$$F_{bulk} = \int dV \{-\alpha A_{\mu i}^* A_{\mu i} + \beta_1 A_{\mu i}^* A_{\mu i} A_{\nu j}^* A_{\nu j}$$

$$+ \beta_2 A_{\mu i}^* A_{\mu i} A_{\nu j}^* A_{\nu j} + \beta_3 A_{\mu i}^* A_{\nu i}^* A_{\nu j} A_{\mu j}$$

$$+ \beta_4 A_{\mu i}^* A_{\nu i} A_{\nu j}^* A_{\mu j} + \beta_5 A_{\mu i}^* A_{\nu i} A_{\nu j} A_{\mu j}^* \}. \quad (2.1)$$

Here $\alpha = \frac{1}{3} N(0)(1 - T/T_c)$, and the coefficients β are given in the weak-coupling approximation by

$$-2\beta_1 = \beta_2 = \beta_3 = \beta_4 = -\beta_5 = \frac{7\zeta(3)}{120\pi^2} \frac{N(0)}{T_c^2},$$

where $N(0) = m^* p_F/2\pi^2 \hbar^2$ is the state density on the Fermi surface for one spin projection.

Local angular momentum. To find the local orbital angular momentum in an arbitrary state $A_{\alpha i}$ it is sufficient to examine the gradient energy, which we write as follows, taking into account the motion of the normal component:

$$F_{grad} = \int dV \{K_1 D_i A_{\mu j}^* D_i A_{\mu j} + K_2 D_i A_{\mu i}^* D_j A_{\mu j} + K_3 D_i A_{\mu j}^* D_j A_{\mu i}\}. \quad (2.2)$$

Here $\mathbf{D}$ is the Galilean-invariant differentiation operator, which acts on the order parameter in the following way:

$$DA = \left(\nabla - \frac{2m_3}{\hbar} iv^n\right) A, \quad DA^* = ' \left(\nabla + \frac{2m_3}{\hbar} iv^n\right) A^*, \quad (2.3)$$

where v^n is the velocity of the normal motion. In the approximation of a symmetry of the particles and holes on the Fermi surface, the coefficients K_1, K_2, and K_3 are all equal:

$$K_1^0 = K_2^0 = K_3^0 = K^0 = \frac{7\zeta(3)}{240\pi^2} N(0) \left(\frac{\hbar v_F}{T_c}\right)^2.$$

When we take the asymmetry of the particles and holes near the Fermi surface into account, we find that these coefficients differ slightly, by a relative difference on the order of $(T_c/\varepsilon_F)^2 \ln(\varepsilon_F/T_c)$. This small difference gives us the magnitude of the local angular momentum of the Cooper pairs.[7,8] The local orbital angular momentum is determined from the reaction of the system to the angular rotation velocity $\mathbf{\Omega} = (1/2)\text{curl}v^n$. To find this reaction, we use an integration by parts to put (2.2) in the form

$$F_{grad} = \int dV \{K_1 D_i A_{\mu j}^* D_i A_{\mu j} + \frac{1}{2}(K_2 + K_3)$$

$$\times (D_i A_{\mu i}^* D_j A_{\mu j} + D_i A_{\mu j}^* D_j A_{\mu i})$$

$$+ \frac{1}{4}(K_2 - K_3)[A_{\mu j}(D_i D_j - D_j D_i)A_{\mu i}^* + A_{\mu j}^*(D_i D_j - D_j D_i)A_{\mu i}]\}. \quad (2.4)$$

Since we have

$$(D_i D_j - D_j D_i)A = -ie_{ijk}(\text{rot } v^n)_k(2m_3/\hbar)A,$$

the density of the internal angular momentum of the pairs, $\mathbf{L}^{int}$, is found by varying the third term in (2.4) with respect to $\mathbf{\Omega}$:

$$\mathbf{L}^{int} = -\frac{\delta F}{\delta \mathbf{\Omega}} = \frac{2m_3}{\hbar i}(K_2 - K_3)[\mathbf{A}_\mu^* \mathbf{A}_\mu]. \quad (2.5)$$

The expressions $\mathbf{\Omega} = (1/2)\text{curl}v^n$ do not appear in the first two terms in (2.4). In the equation for $A_{\alpha i}$ which is found by minimizing the Ginzburg-Landau functional, these terms lead to a dependence on v^n either in the trivial combination $v^s - v^n$ or in the symmetric form $\nabla_i v_j^n + \nabla_j v_i^n$, which van-

ishes in the case of a uniform rotation, $\mathbf{v}^n = [\mathbf{\Omega r}]$.

Expression (2.5) can be written

$$\mathbf{L}^{int} = L_0 \langle \hat{\mathbf{L}} \rangle, \qquad (2.6)$$

where the angular-momentum operator $\hat{\mathbf{L}}$ acts on $A_{\alpha i}$ in accordance with

$$\hat{L}_i A_{\mu j} = (\hbar/i)\, e_{ijk} A_{\mu k}, \qquad (2.7)$$

and $\langle \vec{\mathbf{L}} \rangle$ is the expectation value of this operator in the state of the diatomic molecule having the same wave function $A_{\alpha i}$ as the Cooper pair:

$$\langle \hat{\mathbf{L}} \rangle = \frac{A_{\mu j}^* \hat{\mathbf{L}} A_{\mu j}}{A_{\nu i}^* A_{\nu i}} = \frac{\hbar}{i} \frac{[\mathbf{A}_\mu^* \mathbf{A}_\mu]}{A_{\nu i}^* A_{\nu i}}. \qquad (2.8)$$

In a Bose liquid of such molecules, the coefficient L_0 is equal to the density of molecules in the Bose condensate; i.e., $L_0 = \rho^s/2m_3$. In a real Fermi liquid, this quantity is much smaller, according to (2.5), on the order of the small particle-hole asymmetry:

$$L_0 = (2m_3/\hbar^2)(K_2 - K_3) A_{\mu i}^* A_{\mu i}. \qquad (2.9)$$

To show this, we note that since we have

$$K_2 - K_3 \sim K^0 \left(\frac{T_c}{\varepsilon_F}\right)^2 \ln \frac{\varepsilon_F}{T_c},$$

and since we have $K^0 (2m_3/\hbar^2)|A|^2 \sim \rho^s/m_3$ [as in ^{3}He-B, according to Ref. 6, we have $\rho^s \frac{10}{3} (2m_3/\hbar)^2 K^0 |A|^2)$, then

$$L_0 \sim \frac{\rho^s}{m_3} \left(\frac{T_c}{\varepsilon_F}\right)^2 \ln \frac{\varepsilon_F}{T_c}.$$

In other words, this coefficient is several orders of magnitude smaller than we would expect for the model of a Bose liquid of molecules.

The origin of such a large difference has been explained for the A phase of ^{3}He as resulting from the pronounced overlap of Cooper pairs in a superfluid Fermi liquid.[2] We recall that the state of a Cooper pair in the A phase is characterized by the quantum numbers $L_z = 1$, $S_z = 0$. The wave function of such a state is described by the order parameter $A_{\alpha i} \propto \hat{z}_\alpha (\hat{x}_i + i\hat{y}_i)$. The expectation value $\langle \hat{\mathbf{L}} \rangle$ for a diatomic molecule in such a state is $\hbar \mathbf{l} = \hbar \hat{\mathbf{z}}$, where the unit vector $\mathbf{l}$ runs along the quantization axis for the orbital angular momentum. As a result of the overlap of the pairs, their orbital motion transforms almost completely into a motion of the center of mass of the pairs along the surface of the vessel; only an insignificant part of the orbital motion corresponds to a local internal rotation of the pairs. The integral total orbital angular momentum of the A phase, on the other hand, which includes the angular momentum of the current along the surface, is the same as for a Bose liquid of molecules.[2]

We need to determine whether a similar assertion can be made for an arbitrary state of ^{3}He, with a nonzero $\langle \hat{\mathbf{L}} \rangle$.

Integral angular momentum. To calculate the integral angular momentum we consider the state of a liquid in a cylindrical vessel in which the order parameter is uniform everywhere except in a narrow layer of the liquid near the surface, where the order parameter may vary because of the boundary conditions. We assume as asymmetric distribution of $A_{\alpha i}$; then $\langle \hat{\mathbf{L}} \rangle$ is constant in the cylinder and directed along its axis (everywhere except in the boundary layer). Let us find the integral angular momentum in this state;

$$\mathscr{L} = \int dV [\mathbf{rj}].$$

An expression for the density of the superfluid current can be found by varying the gradient energy (2.2) or (2.4) with respect to $\mathbf{v}^n$. Since these expressions differ by a total derivative, the result for the current does not depend on whether we choose (2.2) or (2.4):

$$\mathbf{j}(\mathbf{r}) = -\frac{\delta F}{\delta \mathbf{v}^n(\mathbf{r})} = \frac{1}{2} \operatorname{rot} \mathbf{L}^{int} + \frac{2m_3}{\hbar i} K_1 [A_{\mu j}^* \nabla A_{\mu j} - \text{c.c.}]$$

$$+ \frac{m_3}{\hbar i} (K_2 + K_3)[\mathbf{A}_\mu^* (\nabla \mathbf{A}_\mu) + (\mathbf{A}_\mu^* \nabla) \mathbf{A}_\mu - \text{c.c.}]. \qquad (2.10)$$

For simplicity we consider the following distribution of the order parameter in the vessel. In the volume, the order parameter is constant, equal to $A_{\alpha i}^0$. It structure does not change in the boundary layer; it simply rotates in this layer in such a manner that the angular momentum of the pair $\langle \hat{\mathbf{L}} \rangle$ is perpendicular to the boundary at the surface. This circumstance eliminates a term which we have ignored in the surface current, which contains a gradient of the density[9] and which is proportional to $[\langle \hat{\mathbf{L}} \rangle, \nabla \rho]$. This term vanishes, since $\langle \hat{\mathbf{L}} \rangle$ and $\nabla \rho$ are parallel at the surface. Substituting the order parameter

$$A_{\mu i} = R_{ij}(\hat{\varphi}, \beta(r)) A_{\mu j}^0.$$

into (2.10), where R_{ij} is the rotation matrix for a rotation through an angle $\beta(r)$ around the $\hat{\varphi}$ axis [r, φ, z are cylindrical coordinates; $\beta(0) = 0$; $\beta(R) = \pi/2$; and R is the radius of the cylinder], we thus find the following expression for the azimuthal projection of the surface current:

$$j_\varphi^{surf} = \frac{m_3}{\hbar i}(K_2 + K_3)[\mathbf{A}_\mu^{*0} \mathbf{A}_\mu^0]_z \frac{d\beta}{dr} \sin \beta. \qquad (2.11)$$

The integral angular momentum is

$$\vec{\mathscr{L}} = \hat{\mathbf{z}} \int dV r j_\varphi^{surf} = \frac{2m_3}{\hbar}(K_2 + K_3)|A_\mu^0|^2 V \langle \hat{\mathbf{L}} \rangle, \qquad (2.12)$$

where V is the volume of the liquid. The same value of the angular momentum $\vec{\mathscr{L}}$ can be found for a uniform $\langle \hat{\mathbf{L}} \rangle$: The angular momentum $\mathscr{L}$ stems from the current associated with $\nabla \rho$.

The total angular momentum of a superfluid Fermi liquid is thus on the same order of magnitude, $\mathscr{L} \sim (\rho^s/m_3)V[\hat{\mathbf{L}}$, as that of a Bose liquid of molecules with the structure of Cooper pairs, but it is due nearly entirely to the surface current. Only an insignificant fraction of the orbital angular momentum, with a density

$$\mathbf{L}^{int} \sim \frac{\rho^s}{m_3}\left(\frac{T_c}{\varepsilon_F}\right)^2 \ln \frac{\varepsilon_F}{T_c} \langle \hat{\mathbf{L}} \rangle,$$

represents the local angular momentum of the internal rotational motion of the Cooper pairs.

3. ORBITAL AND SPIN ANGULAR MOMENTA OF COOPER PAIRS IN THE B PHASE

At equilibrium in ^{3}He-B the order parameter is proportional to the orthogonal matrix $R_{\alpha i}$:

$$A_{\mu i} = \left[\frac{\alpha}{2(3\beta_{12} + \beta_{345})}\right]^{1/2} R_{\mu i} e^{i\Phi}. \qquad (3.1)$$

Here $\beta_{12} = \beta_1 + \beta_2$, $\beta_{345} = \beta_3 + \beta_4 + \beta_5$, and $e^{i\Phi}$ is a phase factor. A diatomic molecule in a state with this wave function has the quantum number $J = 0$, where the operator $\hat{\mathbf{J}}$ is the total angular momentum in the coordinate system in which the spin axes are rotated from the orbital axes by the rotation matrix $R_{\alpha i}$:

$$\hat{J}_i = \hat{L}_i + R_{\alpha i}\hat{S}_\alpha. \tag{3.2}$$

It is easy to show that

$$\hat{J}A_{\mu i} = 0, \tag{3.3}$$

since the spin operator acts in the following way:

$$\hat{S}_\alpha A_{\mu i} = (\hbar/i)e_{\alpha\mu\nu}A_{\nu i}.$$

It can be seen from (3.2) and (3.3) that the expectation value of the orbital angular momentum in the state of the molecule with wave function (3.1) can be expressed in terms of the expectation value of the spin in this state:

$$\langle \hat{L}_i \rangle = -\langle \hat{S}_\alpha \rangle R_{\alpha i}. \tag{3.4}$$

This is of course a trivial relation for state (3.1), since both sides of (3.4) are zero: Cooper pairs have no spin or orbital angular momenta in an isotropic state in the B phase. This relation is nevertheless important, since it must also hold, by virtue of the rigidity of the wave function, for small perturbations which cause a deviation of $A_{\alpha i}$ from isotropic state (3.1). We would therefore expect that in a magnetic field in which an $\langle \hat{\mathbf{S}} \rangle \neq 0$ appears an orbital angular momentum of the Cooper pairs should also appear.

We turn now to the appearance of an orbital angular momentum in a magnetic field. In a field, the Ginzburg-Landau functional (2.1) is supplemented by two other terms. First, there is the energy $F_H^{(2)}$, which is quadratic in $\mathbf{H}$ and which describes the change in the paramagnetism of the Fermi liquid due to the formation of Cooper pairs:

$$F_H^{(2)} = g_H^{(2)} \int dV H_\mu H_\nu A_{\mu i}^* A_{\nu i},$$

$$g_H^{(2)} = \frac{7\zeta(3)}{24\pi^2} N(0) \left(\frac{\gamma\hbar}{T_c}\right)^2 \left(1 + \frac{Z_0}{4}\right)^{-2}. \tag{3.5}$$

Here $\gamma = -2.04\cdot10^4$ is the gyromagnetic ratio of the ^{3}He nucleus, and Z_0 is a Fermi-liquid parameter. Second, there is a term $F_H^{(1)}$ which is linear in $\mathbf{H}$ and which describes the interaction of the field with the spontaneous magnetic moment of the pairs[10]:

$$F_H^{(1)} = -\int dV\mathbf{M}^{(1)}\mathbf{H} = g_H^{(1)} ie_{\alpha\mu\nu}\int dV H_\alpha A_{\mu i}^* A_{\nu i}. \tag{3.6}$$

The coefficient $g_H^{(1)}$ is nonzero because of the slight asymmetry of the particles and holes near the Fermi surface:

$$g_H^{(1)} = \tfrac{1}{6}\gamma\hbar N'(0)\ln(\varepsilon_F/T_c), \tag{3.7}$$

where $N'(0)$ is the derivative of the state density with respect to the energy on the Fermi surface.

The change in the paramagnetic moment of the liquid upon the transition to the superfluid state is

$$\mathbf{M}^{(2)} = (\chi_B - \chi_n)\mathbf{H} = -\frac{\delta F_H^{(2)}}{\delta\mathbf{H}} = -g_H^{(2)}\frac{\alpha}{\beta_{12} + \tfrac{1}{3}\beta_{345}}\mathbf{H}. \tag{3.8}$$

Here χ_B and χ_n are the magnetic susceptibilities in the B phase and in the normal Fermi liquid, respectively. It might

seem quite natural to assume, as in Ref. 4, that if a pair spin density $\mathbf{M}^{(2)}/\gamma$ arises in the system then an orbital angular momentum density $L_i = -R_{\alpha i}M_\alpha^{(2)}/\gamma$. will arise simultaneously. However, the paramagnetic increment (3.5) to the condensation energy (2.1) does not give rise to nonunitary increments in state (3.1) with a nonvanishing expectation value of the spin,

$$\langle \mathbf{S} \rangle = A_{\mu i}^* \hat{\mathbf{S}}A_{\mu i}/|A_{\nu j}|^2. \tag{3.9}$$

The reason is, as can be seen from (3.5), that the paramagnetic moment differs from that of a normal Fermi liquid only because of the Cooper pairs in the state with a zero projection onto the field direction. The paramagnetic contribution to the free energy of a superfluid liquid can thus change only the weight of the state with a vanishing spin projection onto the field direction with respect to the weights of states with projections $S_z = \pm 1$; it cannot change the weights of the latter states with respect to each other. By virtue of (3.5), the state (3.1) becomes

$$A_{\mu i} = [\alpha/2(2\beta_{12} + \beta_{345})]^{1/2}e^{i\Phi}R_{\alpha i}(\delta_{\alpha\mu} + aH^2\delta_{\alpha\mu} + bH_\alpha H_\mu)$$

with the expectation value $\langle \mathbf{S} \rangle = 0$. Here $a \sim b \sim g_H^{(2)}/\alpha$ (see Ref. 6, for example). Although in a magnetic field there are more pairs with a spin projection $S_z = +1$ on the field direction than pairs with $S_z = -1$, because of the higher density of states the Fermi surface, the probability for pairing two particles with spins along the field direction in a state with $S_z = +1$ is still equal to the probability for pairing of two particles with spins counter to the field in a state with $S_z = -1$, as in the absence of a field.

The paramagnetic increment in the energy,

$$-\mathbf{M}^p\mathbf{H} = -(\chi_n\mathbf{H} + \mathbf{M}^{(2)})\mathbf{H}$$

is equivalent to an energy change $-\mathbf{L}\cdot\boldsymbol{\Omega}$ upon rotation at an angular velocity $\boldsymbol{\Omega} = \gamma\mathbf{H}$ with an angular momentum $\mathbf{L} = \mathbf{M}^p/\gamma$. The paramagnetic contribution to the B phase of ^{3}He describes the ordinary Einstein-de Haas effect, as in a normal Fermi liquid, and has no bearing on the specific effect which results from the rigidity of the coherent state of the pairs.

To describe the latter effect we return to expression (3.6), according to which the expectation value of the pair spin, $\langle S \rangle$ in (3.9), is related to the spontaneous magnetic momentum $\mathbf{M}^{(1)}$ of the pairs by

$$\mathbf{M}^{(1)} = (g_H^{(1)}/\hbar)|A_{\mu i}|^2\langle\hat{\mathbf{S}}\rangle. \tag{3.10}$$

If we ignore Fermi-liquid corrections, we can write this magnetic moment as

$$\mathbf{M}^{(1)} = \gamma L_0\langle\hat{\mathbf{S}}\rangle; \tag{3.11}$$

i.e., it contains the same small quantity L_0 as expression (2.6) for the internal orbital angular momentum of the pairs. This circumstance was first pointed out by Fomin.[11] The magnetic moment of Cooper pairs thus constitutes only an insignificant part of the magnetic moment of a Bose liquid consisting of molecules with the same wave function. The reason lies in the presence of two Fermi occupations with opposite spins, so that their magnetic moments cancel out almost perfectly. The internal orbital angular momentum of the Cooper pairs, L^{int}, and their spontaneous magnetization are thus weakened identically in a Fermi liquid in comparison with the

corresponding quantities for the corresponding Bose liquid of molecules.

The magnetic moment $\mathbf{M}^{(1)}$ and also the orbital angular momentum of the pairs arise from a magnetic-field-induced change in state (3.1), which can be found by minimizing the sum $F_{\text{bulk}} + F_H^{(1)}$:

$$A_{vi} = \left[\frac{\alpha}{2(3\beta_{12}+\beta_{345})}\right]^{1/2} R_{\mu i}e^{i\Phi}\left[\delta_{v\mu}+ie_{v\mu\alpha}H_\alpha\frac{g_H^{(1)}(3\beta_{12}+\beta_{345})}{2\alpha(3\beta_1-\beta_4+\beta_{35})}\right].$$

(3.12)

The expectation values $\langle \hat{L} \rangle$ and $\langle \hat{S} \rangle$ in a state of a diatomic molecule with a wave function of this type are

$$\langle \hat{L}_i \rangle = -R_{\mu i}\langle \hat{S}_\mu \rangle = -\frac{\hbar g_H^{(1)}(3\beta_{12}+\beta_{345})}{\alpha(3\beta_1-\beta_4+\beta_{35})}R_{\mu i}H_\mu.$$

(3.13)

Using (3.10) for the magnetic moment $\mathbf{M}^{(1)}$, expressions (2.6) and (2.9) for the local orbital angular momentum $\mathbf{L}^{\text{int}}$, and expression (2.12) for the integrated angular momentum $\overrightarrow{\mathscr{L}}$, we find the following expressions for these quantities:

$$\mathbf{M}^{(1)} \sim \chi_n \mathbf{H}\left(\frac{T_c}{\varepsilon_F}\right)^2 \ln^2\frac{\varepsilon_F}{T_c},$$

(3.14)

$$L_i^{\text{int}} = -\frac{1}{\gamma}M_\alpha^{(1)}R_{\alpha i},$$

(3.15)

$$\frac{\mathscr{L}_i}{V} \sim -H_\alpha R_{\alpha i}\frac{\chi_n}{\gamma}\ln\frac{\varepsilon_F}{T_c} \sim -\frac{1}{1-T/T_c}\frac{M_\alpha^{(2)}R_{\alpha i}}{\gamma}.$$

(3.16)

Near T_c, these quantities are independent of the temperature. The range of applicability of these expressions is of course restricted by the inequalities $g_H^{(1)}H < \alpha$ and $g_H^{(2)}H^2 < \alpha$, which are violated in the immediate vicinity of T_c. The first of these inequalities ensures that the B phase is stable with respect to the formation of the A_1 phase, while the second ensures stability with respect to the formation of the A phase.[10]

Expression (2.5) for the local angular momentum in a magnetic field is changed because of the appearance of a correction to the gradient energy which is linear in H:

$$F_{\text{grad}}^H = \frac{3}{2}\frac{\gamma\hbar}{\varepsilon_F}\frac{ie_{\alpha\mu v}}{1+Z_0/4}\int dV H_v\{\tilde{K}_1 D_i A_{\mu j}\cdot D_i A_{\alpha j}$$

$$+\tilde{K}_2 D_i A_{\mu i}\cdot D_j A_{\alpha j}+\tilde{K}_3 D_i A_{\mu j}\cdot D_j A_{\alpha i}\}.$$

(3.17)

In the approximation of a symmetry of the particles and holes on the Fermi surface, the coefficients $\tilde{K}_1$, $\tilde{K}_2$, and $\tilde{K}_3$ are all equal to each other and to the coefficients from (2.2): $\tilde{K}_1^0 = \tilde{K}_2^0 = \tilde{K}_3^0 = K^0$. The correction to $\mathbf{L}^{\text{int}}$ is found from the reaction of (3.17) to $(1/2)\text{curl}v^n$, as in the derivation of expression (2.5):

$$\delta\mathbf{L}^{\text{int}} = \frac{3\gamma m_3}{\varepsilon_F}\frac{e_{\alpha\mu v}H_v}{1+Z_0/4}(\tilde{K}_2-\tilde{K}_3)[\mathbf{A}_\alpha\cdot\mathbf{A}_\mu].$$

(3.18)

For the B phase, this quantity is

$$\delta L_i^{\text{int}} = \frac{2\gamma m_3}{\varepsilon_F}\frac{|A_{vj}|^2}{1+Z_0/4}(\tilde{K}_2-\tilde{K}_3)H_\alpha R_{\alpha i}.$$

(3.19)

Since the difference $\tilde{K}_2 - \tilde{K}_3$ differs from zero only because of an asymmetry in the particle and hole distributions,

$$(\tilde{K}_2-\tilde{K}_3) \sim K^0(T_c/\varepsilon_F)^2\ln(\varepsilon_F/T_c),$$

the result is only relatively small correction, on the order of

$(1 - T/T_c)$, to $\mathbf{L}^{\text{int}}$ in (3.15).

We see thus that the local internal orbital angular momentum of the pairs in the B phase is equal in order of magnitude to the spontaneous spin angular momentum of the pairs. In contrast, the integral orbital angular momentum of the pairs per unit volume is far greater than the spin angular momentum of a normal Fermi liquid in a magnetic field.

4. CAN THE ORBITAL ANGULAR MOMENTUM OF COOPER PAIRS IN ROTATING ^{3}He-B BE OBSERVED?

The orbital angular momentum of the Cooper pairs of the B phase in a magnetic field is very small. Even the integral angular momentum amounts to only a small fraction, on the order of $\hbar|\gamma|H/\varepsilon_F$, of the angular momentum produced by the current circulating around a single quantized vortex. This smal angular momentum is nevertheless amenable to observation by an NMR technique, which can measure extremely subtle effects in the B phase, including the magnetic moment of the cores of quantized vortices, which would amount under experimental conditions to something on the order of 10^{-11} of a nuclear Bohr magneton per atom.[5] The reason lies in the pronounced isotropy of the B phase, which makes it possible to detect even slight effects on the order parameter. In this section of the paper we examine the effect of the orbital angular momentum of Cooper pairs on the orientation of the order parameter.

The factor which primarily determines the properties of the NMR in the B phase is the weak spin-orbit ("dipole") interaction, which mixes the spin and orbital indices[6]:

$$F_D = g_D\int dV\left(A_{ii}^*A_{jj}+A_{ij}^*A_{ji}-\frac{2}{3}A_{ij}^*A_{ij}\right),$$

$$g_D = \frac{\pi}{10}\left(N(0)\gamma\hbar\ln\frac{\varepsilon_F}{T_c}\right)^2.$$

(4.1)

This interaction has the following important consequences: First, it partially lifts the degeneracy of states (3.1). Specifically, the minimum of dipole energy (4.1) corresponds to only those matrices $R_{\alpha i}$ which describe a rotation around an arbitrary axis $\mathbf{n}$, but through a fixed angle $\theta_0 = \arccos(-1/4)$:

$$R_{\alpha i}(\mathbf{n}, \theta_0) = \tfrac{1}{4}(-\delta_{\alpha i}+5n_\alpha n_i-\sqrt{15}e_{\alpha ik}n_k).$$

Second, the dipole interaction shifts the frequency of the transverse NMR with respect to the Larmor frequency $\omega_0 = |\gamma|H$, which depends on the relative orientation of $\mathbf{n}$ and $\mathbf{H}$. In sufficiently strong fields, $H > 25$ G, the frequency shift is given by[12]

$$\delta\omega = (\Omega_B^2/2\omega_0)\sin^2\beta,$$

where β is the angle between $\mathbf{n}$ and $\mathbf{H}$, and Ω_B is the Leggett frequency of the B phase, which is nonzero because of the dipole interaction. The NMR frequency is thus seen to be sensitive to the orientation of the vector $\mathbf{n}$.

The magnetic field and the boundaries have the primary orienting effects on the vector $\mathbf{n}$ in a vessel at rest. A magnetic field acts on $\mathbf{n}$ through the slight magnetic anisotropy which results from the dipole interaction; this interaction causes a slight perturbation of the order parameter (3.1), making it anisotropic. The magnetic-anisotropy energy is

$$F_{HD} = -a \int dV (\mathbf{nH})^2,$$

$$a = {}^{5}/_{4} g_H g_D / \beta_{345} \sim 10^{-5} \chi_n. \tag{4.3}$$

The interaction with the boundary results from the pronounced distortion of the B phase near the surface, which gives rise to a pronounced magnetic anisotropy in a surface layer with a thickness on the order of the coherence length

$$\xi = (K^0/\alpha)^{1/2} \sim 10^{-6} (1 - T/T_c)^{-1/2}.$$

The corresponding energy is[7]

$$F_s = -d \int dS (H_\alpha R_{\alpha i} \hat{s}_i)^2, \qquad d \sim \xi (\chi_n - \chi_B), \tag{4.4}$$

where $\hat{s}$ is the normal to the surface. The competition between the effects F_{HD} and F_s gives rise to a texture of the vector $\mathbf{n}$, which varies over a broad layer near the boundary, with a thickness on the order of the magnetic length $\xi_H^B \sim (K^0 \alpha / \beta a H^2)^{1/2}$. The thickness of this layer reaches a millimeter in the fields $H \approx 300$ G used in the experiments of Ref. 3 and 5.

The orienting effect of the orbital angular momentum on the vector $\mathbf{n}$ is seen when the vessel rotates. Let us assume that a vessel is rotating at an angular velocity Ω but that there are no vortices in it, so that the conditions $\mathbf{v}^s = 0$ and $\mathbf{v}^n = [\mathbf{\Omega r}]$ hold. We will discuss the conditions under which vortex formation is supressed below. A difference between the velocities of the superfluid and normal components in a rotating vessel, $\mathbf{v}^s - \mathbf{v}^n = -[\mathbf{\Omega r}]$, also perturbs the order parameter (3.1), giving rise to an additional magnetic anisotropy, whose energy can be written in terms of the parameter a for comparison with (4.3) (Ref. 7):

$$F_{flow} = -\frac{2}{5} \frac{a}{v_c^2} \int dV [H_\alpha R_{\alpha i} (v_i^s - v_i^n)]^2,$$

$$v_c = \frac{\hbar}{2 m_3 \xi_D}, \tag{4.5}$$

where $\xi_D = (K_0/g_D)^{1/2} \sim 10^{-3}$ cm is the dipole length.

The orbital angular momentum contributes the orientation of $\mathbf{n}$ because of the $\mathbf{n}$ dependence of the gyromagnetic energy. This energy arises, first, because of the internal orbital angular momentum of the pairs,

$$F_{gm}^{int} = -\int dV \mathbf{\Omega L}^{int} = \frac{4}{5} a \varkappa^{int} \int dV (H_\alpha R_{\alpha i} \hat{\Omega}_i). \tag{4.6}$$

Here $\hat{\Omega}$ is a unit vector along $\mathbf{\Omega}$, and the new parameter $\varkappa^{int}$ is given in order of magnitude by the following expression, according to (3.15):

$$\varkappa^{int} \sim \frac{\chi_n \Omega}{\gamma a} \left(\frac{T_c}{\varepsilon_F} \ln \frac{\varepsilon_F}{T_c} \right)^2. \tag{4.7}$$

Second, there is a component of the gyromagnetic energy which stems from the surface current (2.11) created by the orbital motion of the Cooper pairs:

$$F_{gm}^{surf} = \int dV (\mathbf{v}^s - \mathbf{v}^n, \mathbf{j}^{surf})$$

$$= -\mathbf{\Omega} \vec{\mathscr{L}} = \frac{2}{5} a \frac{\varkappa^{surf}}{\Omega} e_{ijk} \int dS_i H_\alpha R_{\alpha j} (v_k^s - v_k^n). \tag{4.8}$$

Here, according to expression (3.16) for the integral angular momentum $\vec{\mathscr{L}}$, we have

$$\varkappa^{surf} \sim \frac{\chi_n \Omega}{\gamma a} \ln \frac{\varepsilon_F}{T_c}. \tag{4.9}$$

The surface gyromagnetism (4.8) can also be interpreted in terms of a surface magnetic moment

$$M_\alpha^{surf} \sim \frac{\chi_n}{\gamma} e_{ijk} \hat{s}_i R_{\alpha j} (v_k^s - v_k^n),$$

$$F_{gm}^{surf} = -\int dS \mathbf{M}^{surf} \mathbf{H}, \tag{4.10}$$

produced by a countercurrent $\mathbf{v}^s - \mathbf{v}^n$ near the boundary of the liquid.

Since the gyromagnetic energy is linear in $\mathbf{\Omega}$, its orienting effect depends on the direction of the rotation. It is this dependence which led to the detection of the magnetic moment $\mathbf{M}^{core}$ of the cores of vortices in the B phase; this moment also makes a gyromagnetic contribution,[5]

$$F_{gm}^{core} = -\int dV \mathbf{M}^{core} \mathbf{H} n = \frac{4}{5} a \varkappa^{core} \int dV (H_\alpha R_{\alpha i} \hat{\Omega}_i). \tag{4.11}$$

Here $n = (4 m_3 / 2\pi\hbar)\Omega$ is the density of vortices. The value $\varkappa^{core}$ found experimentally[5] agrees in order of magnitude with the theoretical estimate of Ref. 5, which can also be derived from (4.8) by treating the vortex as a cylindrical surface with a radius on the order of ξ in a flow $|\mathbf{v}^s - \mathbf{v}^n| \sim \hbar / 2 m_3 \xi$. The magnetic moment per unit length of the cortex is then given in order of magnitude by the following expression, according to (4.10):

$$\frac{M_\alpha^{core}}{L} \sim \frac{\chi_n}{\gamma} \int \frac{dS_i}{L} e_{ijk} R_{\alpha j} (v_k^s - v_k^n) \sim \frac{\hbar \chi_n}{m_3 \gamma} R_{\alpha i} \hat{\Omega}_i. \tag{4.12}$$

Multiplying this expression by the density of vortices, we find the following estimate of the gyromagnetic parameter of a system of vortices:

$$\varkappa^{core} \sim \frac{\chi_n \Omega}{\gamma a} \ln \frac{\varepsilon_F}{T_c} \sim \varkappa^{surf}. \tag{4.13}$$

The effect of the vortex cores, $\varkappa^{core}$, is considerably greater than the local orienting effect of the internal orbital angular momentum, $\varkappa^{int}$, but it is of the same order of magnitude as $\varkappa^{surf}$. The component $\varkappa^{surf}$, however, could not be observed under the experimental conditions of Ref. 5. The orbital angular momentum in (4.8) has an orienting effect only near the surface; the effect falls off exponentially into the interior of the liquid over a distance ξ_H^B. Experimentally the radius of the vessel is much greater than ξ_H^B, so that the NMR signal, which comes primarily from the central part of the vessel, could not detect the surface effect of the orbital angular momentum. Furthermore, the vortices cancel the countercurrent $\mathbf{v}^s - \mathbf{v}^n$ at the surface almost completely.

For observation of the surface gyromagnetic effect of the orbital angular momentum it will be necessary to carry our NMR experiments in a liquid in a system of thin tubes with a radius $R \lesssim \xi_H^B$. In this case the effect of the boundary is not attenuated. To simplify the analysis of the experimental data, we impose an even stricter condition on the capillary radius:

$$R \lesssim (a/d) (\xi_H^B)^2, \tag{4.14}$$

where d is the coefficients of the surface energy F_s in (4.4) $(d / a \sim 3$ mm; Ref. 12). In this case the field of the vector $\mathbf{n}$ is

uniform over the entire capillary, since the gradient energy prevents a deformation of this field. As a result, all the orienting effects have an integrated influence on the vector $\mathbf{n}$.

Integrating all the orienting energies over the volume of the capillary, we find the following effective orienting effect:

$$F = F_{HD} + F_{flow} + F_s + F_{gm}^{surf} = aV\{-(\mathbf{nH})^2 + (H_\alpha R_{\alpha i}\hat{\Omega}_i)^2$$

$$\times [{}^1/_{10}(\Omega R/v_c)^2 + d/Ra] + {}^4/_5 \varkappa^{surf} H_\alpha R_{\alpha i}\hat{\Omega}_i\}. \quad (4.15)$$

By minimizing this effect we can find the equilibrium orientation of $\mathbf{n}$ in a rotating vessel. In particular, for the quantity $\sin^2\beta$ which appears in the NMR shift in (4.2), we find the following equation, which expresses this shift in terms of the orientational parameters and the angle (μ) between $\mathbf{H}$ and $\mathbf{\Omega}$:

$$\lambda_{eff}\left[u\cos 2\mu \pm \frac{u^2 - {}^1/_2}{(1-u^2)^{1/2}}\sin 2\mu\right]$$

$$+ \frac{\varkappa^{surf}}{H}\left[\cos\mu \pm \frac{u}{(1-u^2)^{1/2}}\sin\mu\right] = 1,$$

$$\lambda_{eff} = {}^1/_4(\Omega R/v_c)^2 + {}^5/_2(d/Ra),$$

$$u = 1 - {}^5/_4\sin^2\beta. \quad (4.16)$$

This equation is of the same form as the equation which determines the orienting effect of quantized vortices.[5] The only differences are that the vortex parameter λ_{vort}, which results from the pronounced anisotropy in the vortex core, is replaced by the parameter λ_{eff}, which depends on the radius of the vessel, and $\varkappa^{core}$ is replaced by $\varkappa^{surf}$, which is on the same order of magnitude. Since it has been found possible to measure $\varkappa^{core}$, which is a measure of the gyromagnetism of vortices, we can hope that it will also be possible to detect $\varkappa^{surf}$, which characterizes the surface gyromagnetism.

The capillary dimension R must be chosen at the optimum value, such that the parameter λ_{eff} is not too large. We must therefore take the maximum possible radius allowed by condition (4.14), i.e., $R \sim (a/d)\xi_H^2$, which amounts to 0.2–0.3 mm at low pressures. Here λ_{eff} would be about an order of magnitude greater than λ_{vort}.

At this capillary radius, a vortex-free state in a rotating vessel at the angular velocities $\Omega \sim 1$ rad/s which are used would be metastable, but it follows from experimental data for a vessel with $R = 2.5$ mm that the vortex formation time in the B phase is quite long, on the order of 1 min (Ref. 13). In narrow capillaries the lifetime of the vortex-free state should be much greater if only because the linear velocity of the superfluid flow at the surface of a capillary would be an order of magnitude lower. Consequently, vortex formation can be ignored during the experiments in such capillaries.

5. CONCLUSION

The gyromagnetism in superfluid ³He-B is a consequence of the rigidity of the wave function of a Cooper pair with the quantum number $J = 0$. Since this quantum number is conserved, a magnetic field which produces an average spin gives rise to an orbital angular momentum of the Cooper pairs. The behavior of this orbital angular momentum is extremely specific and sharply different from the behavior which would be expected in a Bose liquid of molecules having the same wave function as the Cooper pairs in ³He-B.

Essentially the entire orbital angular momentum of the pairs is concentrated in a current which flows over the surface of the vessel; only an insignificant part of this angular momentum, $\sim(T_c/\varepsilon_F)^2$, which results from the slight asymmetry of the particles and holes near the Fermi surface, is due to the local rotational motion of the liquid. This behavior of the orbital angular momentum of the B phase, which is characteristic of any superfluid Fermi liquid, including the A phase, shows that all earlier calculations on the gyromagnetism in the B phase[14–16,4] were correct only in an integral sense, i.e., only in an integration over the entire volume, including the surface layer of the liquid. The local gyromagnetic effect, in contrast, is very small.

The gyromagnetism was found in Refs. 14–16 through a solution of the Gor'kov equations in a zero magnetic field. The solution was carried out by means of a gauge transformation which eliminated the dependence of the order parameter on the coordinates and the time. This method is valid for calculating the terms in the current which are linear in the spatial and time derivatives of the order parameter. The term in the current due to the orbital angular momentum of the B phase is $- 1/2e_{ikl}\nabla_k(R_{\alpha l}S_\alpha)$, according to Ref. 16, where S_α is the spin angular momentum of the liquid, proportional to the time derivative of the order parameter. It was concluded from the form of this term that in the presence of a magnetization γS the B phase would have a local orbital angular momentum $L_i = - R_{\alpha i}S_\alpha$. We reached exactly the same conclusion in a previous paper[4] by calculating the reaction of the current to a magnetic field at a constant value of the order parameter $R_{\alpha i}$.

A calculation of the reaction of the current to the field incorporating the coordinate dependence of the order parameter $R_{\alpha i}$, however, shows that the corresponding term in the current is of the form $- 1/2e_{ikl}R_{\alpha i}\nabla_k S_\alpha$; i.e., the order parameter has been taken through the derivative sign. This result means that the angular momentum $L_i \sim - R_{\alpha i}S_\alpha$ exists not locally but in an integral sense, in a manner precisely analogous to the situation in the A phase, where the corresponding term in the current is $1/2e_{ikj}l_j\nabla_k(\rho/2m_3)$. The difference in the expressions for the current, $- 1/2e_{ikl}S_\alpha\nabla_k R_{\alpha i}$, is of second order in the spatial and time derivatives of the order parameter and thus goes beyond the range of applicability of the gauge-transformation method used in Ref. 16. That method therefore cannot be used to distinguish a local gyromagnetism from a surface gyromagnetism.

The orbital angular momentum which arises in the B phase in a magnetic field could be detected in experiments with a rotating vessel, in which the gyromagnetic energy should give rise to a change in the orientation of the order parameter and thus a characteristic shift of an NMR signal, which depends on the rotation direction. In wide vessels this orientational effect would be small, since the signal comes from the central part of the vessel, where the local gyromagnetic energy is negligbly low. It would be necessary to seek a shift of the NMR frequency in a liquid in a system of narrow capillaries, where the surface gyromagnetic orientational effect would be dominant. We estimate the optimum radius of the capillaries to be on the order of 0.2–0.3 mm.

G. E. Volovik and V. P. Mineev   978

Since the surface gyromagnetism can be interpreted as the appearance of a surface magnetic-moment density

$$M_\alpha^{surf} \sim \frac{\chi_n}{\gamma} e_{ijk} \hat{s}_i R_{\alpha j}(v_k{}^s - v_k{}^n)$$

in the presence of a countercurrent $\mathbf{v}^s - \mathbf{v}^n$, this effect could also be measured in experiments with a superfluid flow in a channel.

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