Spin current density waves and the weak ferromagnetism in itinerant magnets

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Some macroscopic physical properties of a new class of systems with the spin current density wave (SCDW) in the ground state are considered. Within the framework of the microscopic two-band model of a metal with congruent parts of the Fermi surface it is shown that in itinerant antiferromagnets a mutual reorientation of the magnetic moments of the sublattices may occur with the formation of the SCDW. This effect is analogous to the "weak ferromagnetism" predicted by Dzyaloshinskiĭ, but it does not involve any relativistically small quantity. The phase diagram for such systems is obtained. In the case of a special symmetry of the bands whose extrema coincide in momentum space, there may be a divergence in the nonlinear magnetic susceptibility of systems with SCDW (providing the angular-momentum interband transitions are allowed). If the dipole moment interband transitions are allowed, a system with SCDW can transform into the ferroelectric state. However, the effect is small to the extent that the spin-orbit coupling is.

1. INTRODUCTION

It was recently remarked by Gorbatsevich, Kopaev, and the author[1] that for phase transitions in crystals characterized by vector order parameters these order parameters can be subdivided with respect to time inversion and the spatial transformations of the crystal point group into four types: polar vectors which are either (a) even or (b) odd with respect to the operation of time inversion, or axial vectors which are either (c) even or (d) odd relative to this inversion. Types (a) and (d) are well known. In particular, the displacement vectors in structural phase transitions and the magnetic-moment vectors in magnetic phase transitions belong to the types (a) and (d) respectively.

An order parameter of the type (b) was considered in a series of papers.[1–5] The polar vector $\mathbf{T}$(π in the notation of Ref. 1) which is odd with respect to the operation of time inversion is dual to the antisymmetric part of the magnetoelectric tensor. Consequently, the state into which the system is transformed upon the formation of the vector $\mathbf{T}$ was said to be "magnetoelectric."

This work is devoted to a study of some types of states with the order parameter $\mathbf{G}$ of the type (c) (μ in the notation of Ref. 1). These states will be called "orientational" for reasons to be given below. Wherever it is not stated otherwise, it will be assumed that only commensurate structures are considered.

Some peculiarities of the transition into the orientational state of a magnetic system will be considered in Sec. 2 by means of a phenomenological Landau functional. The vector $\mathbf{G}$ is dual to the antisymmetric part of the weak ferromagnetism tensor which was introduced by Dzyaloshinskiĭ.[6] In the presence of the antiferromagnetic order characterized by a vector $\mathbf{L}$ of the type (d) a transition into a state with $\mathbf{G}\neq 0$ can cause a change in mutual orientation of the magnetic moments of the sublattices and the emergence of ferromagnetism. (The term "orientational state" is so named precisely because $\mathbf{G}$ has the property that its formation causes a change in the orientation of ordinary axial and polar vectors relative to their orientation in the state with $\mathbf{G}=0$.)

In Sec. 3 a correspondence between the orientational states of type (c) and the states with the imaginary interband triplet order parameter Δ^t_{Im} [the states with the spin current density waves (SCDW)] is established on the basis of a microscopic two-band model of metals with congruent segments of the electron and the hole Fermi surfaces. The explicit forms of the coefficients in the Landau free-energy functional are obtained and the region of the phase diagram in which the "weak" ferromagnetism can be realized is determined. This region is characterized by coexistence of the real interband triplet order parameters Δ^t_{Re}, which describes the states with the spin density waves (SDW), and the parameter Δ^t_{Im} (SCDW). The main result is that the tensor of the "weak" ferromagnetism is proportional to Δ^t_{Im} and that it does not contain any relativistically small quantity, in contrast to the known models (e.g., Refs. 7 and 8) where this tensor is due to the spin-orbit interaction. In the model with an interband electronic mechanism for the "weak" ferromagnetism an important role is played by the effects of the intraband exchange enhancement. These effects are taken into account within the microscopic analysis. This model can be generalized to the case of systems which besides the itinerant electrons also contain localized moments. The existence of an indirect exchange between the localized moments enhances the tendency towards the orientational state. This opens up a possibility of experimental observations of this state in a wide class of metallic magnets by analyzing related orientational effects.

We shall consider in Sec. 4 a two-band model of a semimetal with coincident extrema in the momentum space and with a nonzero interband matrix elements of the angular momentum operator. It will turn out that near the transition point into the orientational state the nonlinear magnetic susceptibility is divergent and that the orientational order parameter $\mathbf{G}$ is induced by the external magnetic field $\mathbf{H}$.

In Sec. 5 another class of orientational states is considered for uniaxial crystals. It is shown that when $\mathbf{G}\neq 0$ it is possible to realize in such systems unusual ferroelectric

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structures in which the polarization vector is orthogonal to the polarization axis. In this section a microscopic picture of such a ferroelectric state is formulated within the framework of a two-band model with hybridization.[1,5] Although this effect is small to the extent of the spin-orbit coupling is, the anomalies in the dielectric permeability can become significant near the transition temperature. When describing a transition into the orientational state one considers, in fact, the formation of an antisymmetric tensorial order parameter and it is understood that the author does not claim a priority in considering the general phenomenology of such systems. Since parallels between the phenomenology and the results of calculations within concrete microscopic models are very important for general physical understanding of the results, the elementary analysis within the Landau theory of second-order phase transitions, conducted in Secs. 3 and 5, is justifiable.

2. A PHENOMENOLOGICAL DESCRIPTION OF ORIENTATIONAL STATES

To clarify the physical meaning of the vector order parameter **G** of the type (c) it is first necessary to understand which physical quantities can be sources of **G** (i.e., conjugated to **G** in the free energy functional). Clearly, such sources can be, in principle, the vector products of any two vectors of the same type, either (a) or (b) or (d). In this work, we shall stay with two such cases of the greatest physical interest. In these cases the source of the parameter **G** is either the product $\mathbf{L}\times\mathbf{M}$ of two axial vectors of the type (d) or the product $\mathbf{U}\times\mathbf{V}$ of two polar vectors of the type (a). In the first case, which will be considered in this section, the relevant term in the free energy has the form

$$\delta F_1=\gamma\mathbf{G}[\mathbf{L}\times\mathbf{M}], \tag{1}$$

where γ is a proportionality constant. It is, of course, necessary that the expression (1) transforms according to the identity representation of the crystal space group (by definition of the quantities **G**, **L**, and **M**, δF_1 is invariant to the operation of time inversion). To be specific, we shall consider the case of a two-sublattice model of ferrimagnetism where **L** and **M** are the sublattice magnetic moment and the average magnetization of the crystal, respectively. The term (1) has in this case the structure which is similar to the structure of the Dzyaloshinskiĭ-Moriya term of the theory of weak ferromagnetism in which the vector **G** is dual to the tensor of the weak ferromagnetism (see, for example, Ref. 7, p. 246 of Russian original, or Ref. 9). A general expression for the three-parameter functional $F(\mathbf{G},\mathbf{L},\mathbf{M})$ can be written exactly to fourth (or even sixth) order in the small quantities **G**,**L**, and **M**. Nevertheless, we shall assume for simplicity that the system cannot transform into ferromagnetic state when $\mathbf{G}=\mathbf{L}=0$. That is, we shall assume that the coefficient of $\mathbf{M}^2$ in the free-energy functional is strictly positive, and that the coefficients of $\mathbf{G}^2$ and $\mathbf{L}^2$ can change their signs with, for example, a change in temperature. Then, among the fourth-degree terms, it is possible to drop those terms which contain **M** and the functional can be written in the simplified form

$$F(\mathbf{G},\mathbf{L},\mathbf{M})=\alpha_1\mathbf{G}^2+\alpha_2\mathbf{L}^2+a\mathbf{M}^2 +\gamma\mathbf{G}[\mathbf{L}\times\mathbf{M}]+A_1\mathbf{G}^2\mathbf{L}^2+B_1(\mathbf{GL})^2+\beta_1\mathbf{G}^4+\beta_2\mathbf{L}^4. \tag{2}$$

We remark immediately that the coefficients in (2) are assumed such that all the transitions which will be considered below are second-order transitions. In the opposite case it would be necessary to keep the sixth-order terms in (2).

To be specific, we shall assume that the coefficient α_2 reverses sign before (i.e., at a higher temperature $T=T_L>T_G$) the coefficient α_1 does (we assume $\alpha_1\sim T-T_G$, $\alpha_2\sim T-T_L$, and $a>0$). However, generally speaking α_1 and α_2 are quantities of the same order of magnitude. We shall assume for simplicity that $\beta_1=\beta_2=\beta>0$. Then, it is not difficult to obtain from the minimization of the functional (2) a system of equations for the order parameters **G**, **L**, and **M**:

$$\begin{aligned}&\alpha_1\mathbf{G}+{}^1/_2\gamma[\mathbf{L}\times\mathbf{M}]+A_1\mathbf{G}\mathbf{L}^2+B_1\mathbf{L}(\mathbf{GL})+2\beta\mathbf{G}\mathbf{G}^2=0,\\ &\alpha_2\mathbf{L}-{}^1/_2\gamma[\mathbf{G}\times\mathbf{M}]+A_1\mathbf{L}\mathbf{G}^2+B_1\mathbf{G}(\mathbf{GL})+2\beta\mathbf{L}\mathbf{L}^2=0,\\ &a\mathbf{M}+{}^1/_2\gamma[\mathbf{G}\times\mathbf{L}]=0.\end{aligned} \tag{3}$$

We shall consider three possible types of solutions:

type I: $\mathbf{G}\perp\mathbf{L}$, $\mathbf{G}\neq0$, $\mathbf{L}\neq0$, $\mathbf{M}=-(\gamma/2a)\mathbf{G}\times\mathbf{L}$

$$\begin{aligned}&\alpha_1+(A_1-\gamma^2/4a)\mathbf{L}^2+2\beta\mathbf{G}^2=0,\\ &\alpha_2+(A_1-\gamma^2/4a)\mathbf{G}^2+2\beta\mathbf{L}^2=0;\end{aligned} \tag{4}$$

type II: $\mathbf{G}\|\mathbf{L}$, $\mathbf{G}\neq0$, $\mathbf{L}\neq0$, $\mathbf{M}=0$,

$$\begin{aligned}&\alpha_1+(A_1+B_1)\mathbf{L}^2+2\beta\mathbf{G}^2=0,\\ &\alpha_2+(A_1+B_1)\mathbf{G}^2+2\beta\mathbf{L}^2=0;\end{aligned} \tag{5}$$

type III: $\mathbf{L}\neq0$, $\mathbf{G}=0$, $\mathbf{M}=0$,

$$\alpha_2+2\beta\mathbf{L}^2=0. \tag{6}$$

The third type is clearly trivial: it simply describes a transition at the temperature $T=T_L$ into the state with $\mathbf{L}\neq0$.

It follows from equations (4) and (5) that the solutions of the types I and II can be realized under the condition that the determinants corresponding to the equations are nonzero,

$$\begin{aligned}&\mathrm{Det}_{\mathrm{I}}=4\beta^2\left[1-\frac{(A_1-\gamma^2/4a)^2}{4\beta^2}\right]\neq0,\\ &\mathrm{Det}_{\mathrm{II}}=4\beta^2\left[1-\frac{(A_1+B_1)^2}{4\beta^2}\right]\neq0.\end{aligned} \tag{7}$$

The solutions of the type II are interesting in themselves, although they do not lead to the ferromagnetic ordering. It is clear that regarding the orientation of **L** the system with $\mathbf{G}\|\mathbf{L}$ possesses the anisotropy of an "easy axis" type. However, we shall not consider these solutions, assuming that they are not realized since, for example, they are energetically less likely than the solutions of the type I (this can be obtained by an appropriate choice of B_1).

For the solutions of the type I, which realize "weak" ferromagnetism ($\mathbf{M}\neq0$), it is possible to obtain

$$\mathbf{G}^2=\frac{-\alpha_1+\delta_{\mathrm{I}}\alpha_2}{2\beta(1-\delta_{\mathrm{I}}^2)},\quad \mathbf{L}^2=\frac{-\alpha_2+\delta_{\mathrm{I}}\alpha_1}{2\beta(1-\delta_{\mathrm{I}}^2)}, \tag{8}$$

$$F_{\mathrm{I}}=-\frac{\alpha_1^2+\alpha_2^2-2\alpha_1\alpha_2\delta_{\mathrm{I}}}{4\beta(1-\delta_{\mathrm{I}}^2)},\qquad \delta_{\mathrm{I}}=\frac{A_1-\gamma^2/4a}{2\beta}, \tag{9}$$

while for the solutions of the type III we obviously have

$$\mathbf{L}^2=-\alpha_2/2\beta, \tag{10}$$

$$F_{\mathrm{III}}=-\alpha_2^2/4\beta. \tag{11}$$

For the solutions of the type II it is necessary to replace δ_I by $\delta_{II} = (A_1 + B_1)/2\beta$ in (8) and (9). It is easy to convince oneself by comparing the energies (9) and (11) for the interval in which (8) and (10) exist that $F_I - F_{III} = -(\alpha_1 - \delta_1\alpha_2)^2/4\beta(1-\delta_I^2)$. That is, for $1-\delta_I^2<0$ the solution of the type III is realized, while for $1-\delta_I^2>0$ the solution of the type I is realized. Therefore, we shall consider the case $1-\delta_I^2>0$ with $1-\delta_{II}^2<0$ so that a competition between the states I and II does not occur. It follows from equation (8) that for $-1<\delta_I<0$ the solution I exists in the interval $\alpha_2<0$, $\alpha_1<|\alpha_2\delta_I|$, while for $0<\delta_I<1$ it exists in the interval $\alpha_2<0, \alpha_1<\alpha_2\delta_1<0$. In this way, for $\delta_I<0$ the orientational state ($\mathbf{G}\neq 0$) appears already when $\alpha_1>0$, i.e., when $T>T_G$. The behavior of the system described by the functional (2) is the following. First, at the temperature T_L ($\alpha_2=0$), while $\mathbf{G}=0$, the antiferromagnetic order parameter $\mathbf{L}$ whose magnitude is determined by (10) emerges. Next, at the temperature T_c, given by the condition $-\alpha_1+\delta_I\alpha_2=0$, in addition to $\mathbf{L}$, $\mathbf{G}\neq 0$ (with $\mathbf{L}\perp\mathbf{G}$) emerges through a second order phase transition. At the same time, appears the parameter

$$\mathbf{M}=-(\gamma/2a)\,[\mathbf{G}\times\mathbf{L}]\neq 0.$$

If the magnetic field interaction term $-\mathbf{M}\cdot\mathbf{H}$ is added to the functional (2), it is easy to realize that the Curie-Weiss law for the temperature dependence of the magnetic susceptibility is valid in the temperature interval $T_c<T<T_L$ (for $\mathbf{H}$ noncollinear with $\mathbf{G}$). All our considerations can be followed in an analogous way when $T_G>T_L$. Then, a transition into the orientational state occurs first, followed, with already present $\mathbf{G}$ and $\mathbf{L}$, by the growth of the average magnetization. In the traditional analysis of the "weak" ferromagnetism[9] such sequence of transitions is assumed while the temperature T_G can be formally assumed larger than the melting temperature. That is, a real high-temperature phase already has a magnetic symmetry which allows for the existence of $\mathbf{G}$.

3. THE MICROSCOPIC MODEL

We shall consider a two-band model of a metal with congruent pieces of the electron and the hole Fermi surfaces. We shall include in the system Hamiltonian the intraband interactions of the density-density type as well as the interband interactions of two types: the density-density interactions and the interactions related to the interband transitions of pairs of particles. This gives

$$H=H_0+H_{int}, \tag{12}$$

$$H_0=\sum_{i,\mathbf{k},\alpha}\varepsilon_i(\mathbf{k})\,a^+_{i\mathbf{k}\alpha}a_{i\mathbf{k}\alpha},$$

$$H_{int}=\sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{q},\alpha,\beta\\ i,j,i',j'}} g_{iji'j'}\,a^+_{i\mathbf{k}\alpha}a^+_{j\mathbf{k}'\beta}a_{i'\mathbf{k}'+\mathbf{q}\beta}a_{j'\mathbf{k}'-\mathbf{q}\alpha}, \tag{13}$$

where i, j, i', and j' are band indices, α and β are spin indices, and $\varepsilon_i(\mathbf{k})$ are the dispersion laws. No special symmetry conditions are imposed on the wave functions of the bands 1 and 2. The interaction constants $g_{iji'j'}$ are chosen as

$$g_{1111}=g_{2222}=g_0, \quad g_{1221}=g_{2112}=g_1,$$
$$g_{1122}=g_{2211}=g_{1212}=g_{2121}=g_2. \tag{14}$$

In systems with the Hamiltonian (12) several types of phase transitions can take place. These phase transitions are described by the four interband order parameters $\Delta_{ij}^{\alpha\beta}$ ($i\neq j$) and also by the intraband parameters $\Sigma_{ii}^{\alpha\beta}$. In the present work we shall consider only the triplet order parameters which can be written in the spinor notation as

$$\hat\Delta_{12}=\hat\Delta_{Re}+i\hat\Delta_{Im}=\hat\Delta_{21}^+, \quad \hat\Sigma_{ii}=\hat\Sigma_{ii}^+,$$
$$\hat\Delta_{Re}=\Delta_{Re}{}^t\hat{\boldsymbol\sigma}, \quad \hat\Delta_{Im}=\Delta_{Im}{}^t\hat{\boldsymbol\sigma}, \quad \hat\Sigma_{ii}=\Sigma_{ii}\hat{\boldsymbol\sigma}, \tag{15}$$

where $\hat{\boldsymbol\sigma}$ is the vector whose components are the Pauli matrices.

The matrices $\hat\Delta_{ij}$ and $\hat\Sigma_{ii}$ are related to the Green's functions $G_{ij}^{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega_n)$,

$$\hat\Delta_{12}=T\sum_n\,[\hat G_{12}(\mathbf{r},\mathbf{r},\omega_n)g_1+\hat G_{21}(\mathbf{r},\mathbf{r},\omega_n)g_2],$$

$$\hat\Sigma_{11}=T\sum_n[\hat G_{11}(\mathbf{r},\mathbf{r},\omega_n)-\hat{\mathcal G}_{11}{}^0(\mathbf{r},\mathbf{r},\omega_n)]g_0$$
$$+[\hat G_{22}(\mathbf{r},\mathbf{r},\omega_n)-\hat{\mathcal G}_{22}{}^0(\mathbf{r},\mathbf{r},\omega_n)]g_2,$$
$$\hat\Sigma_{22}=T\sum_n[\hat G_{22}(\mathbf{r},\mathbf{r},\omega_n)-\hat{\mathcal G}_{22}{}^0(\mathbf{r},\mathbf{r},\omega_n)]g_0 \tag{16}$$
$$+[\hat G_{11}(\mathbf{r},\mathbf{r},\omega_n)-\hat{\mathcal G}_{11}{}^0(\mathbf{r},\mathbf{r},\omega_n)]g_2,$$
$$\omega_n=\pi T(2n+1), \quad n=0,\pm1,\ldots$$

It is known[2,10] that the order parameter Δ_{Re}^t describes the formation of the spin density waves (SDW), i.e., of the antiferromagnetic ordering, while the order parameter Δ_{Im}^t describes the formation of the spin current density waves (SCDW). The parameters Σ_{ii} describe the magnetization of the electrons in the band i (Stoner parameters). Let us consider the possibility of coexistence of Δ_{Im}^t, Δ_{Re}^t, and Σ_{ii}. The system of equations for the Green's functions in spinor notation is

$$\hat G_{11}=\hat{\mathcal G}_{11}+\hat{\mathcal G}_{11}\hat\Delta_{12}\hat G_{21}, \quad \hat G_{22}=\hat{\mathcal G}_{22}+\hat{\mathcal G}_{22}\hat\Delta_{21}\hat G_{12},$$
$$\hat G_{21}=\hat{\mathcal G}_{22}\hat\Delta_{21}\hat G_{11}, \quad \hat G_{12}=\hat G_{21}{}^+,$$
$$\hat{\mathcal G}_{jj}=\hat{\mathcal G}_{jj}{}^0+\hat{\mathcal G}_{jj}{}^0\hat\Sigma_{jj}\hat{\mathcal G}_{jj}, \quad \hat{\mathcal G}_{jj}{}^0=[i\omega_n-\varepsilon_j(\mathbf{k})]^{-1}\hat I, \tag{17}$$

where $\hat I$ is the unit matrix in the spin space.

From the system of equations (17) and (16), using the notation (15), it is possible to establish the explicit form of the free energy functional F for the order parameters $|\Delta_{Re}^t|, |\Delta_{Im}^t|, |\Sigma_{ii}| \ll \max(T,\mu)$, where μ is the noncongruency parameter of the electron and the hole Fermi surfaces ("chemcial potential"). We shall assume for simplicity that $\mu=$ const (the system is in contact with a reservoir of infinite capacity). Then, to lowest orders in the order parameters

$$F/2N(0)=\alpha_1(\Delta_{Im}{}^t)^2+\alpha_2(\Delta_{Re}{}^t)^2+\gamma_1\Delta_{Im}{}^t[\Delta_{Re}{}^t\times\Sigma_+]$$
$$+\gamma_2\Delta_{Im}{}^t[\Delta_{Re}{}^t\Sigma_-]+a_+\Sigma_+{}^2+a_-\Sigma_-{}^2+A_1(\Delta_{Im}{}^t)^2(\Delta_{Re}{}^t)^2$$
$$+B_1(\Delta_{Im}{}^t\Delta_{Re}{}^t)^2+\beta[(\Delta_{Im}{}^t)^4+(\Delta_{Re}{}^t)^4], \tag{18}$$
$$\Sigma_\pm={}^1/_2(\Sigma_{11}\pm\Sigma_{22}).$$

The coefficients in (18) have the form

$$\alpha_1=\tau_{\rm Im}\left(1-\frac{4\mu}{\pi T}\varphi_1\right),\quad \tau_{\rm Im}=\frac{T-T_{\rm Im}}{T_{\rm Im}},$$
$$\alpha_2=\tau_{\rm Re}\left(1-\frac{4\mu}{\pi T}\varphi_1\right),\quad \tau_{\rm Re}=\frac{T-T_{\rm Re}}{T_{\rm Re}},$$
$$a_\pm=(1/\bar g_\pm-1)>0,\quad \bar g_\pm=(g_0\pm g_2)N(0),$$
$$\gamma_1=\frac{2}{\varepsilon_F}\varphi_0,\quad \gamma_2=\frac{4}{\pi T}\varphi_1,\quad \beta=\frac{1}{2}\frac{\varphi_2}{(\pi T)^2},$$
$$A_1=3\frac{\varphi_2}{(\pi T)^2},\quad B_1=-2\frac{\varphi_2}{(\pi T)^2}. \tag{19}$$

The temperatures $T_{\rm Re}$ and $T_{\rm Im}$ are defined through the effective coupling constants $g_{\rm Re(Im)}=g_1\pm g_2$ by the equation

$$2\left[g_{\rm Re\,(Im)}N(0)\right]^{-1}=\int_0^{\varepsilon_F}\left[\operatorname{th}\frac{\zeta+\mu}{T_{\rm Re\,(Im)}}+\operatorname{th}\frac{\zeta-\mu}{T_{\rm Re\,(Im)}}\right]\frac{d\zeta}{\zeta}. \tag{20}$$

Furthermore, $N(0)$ is the density of states at the Fermi level while

$$\varphi_0=\sum_{n\geqslant 0}^{N_{max}}(2n+1)^{-1}\left(1+\frac{\mu^2}{\omega_n^2}\right)^{-1},\quad N_{max}=\left[\frac{\varepsilon_F}{\pi T}\right],$$
$$\varphi_1=\sum_{n\geqslant 0}^{\infty}\frac{\mu}{\omega_n}(2n+1)^{-2}\left(1+\frac{\mu^2}{\omega_n^2}\right)^{-2}, \tag{21}$$
$$\varphi_2=\sum_{n\geqslant 0}^{\infty}\left(1-3\frac{\mu^2}{\omega_n^2}\right)(2n+1)^3\left(1+\frac{\mu^2}{\omega_n^2}\right)^{-3}.$$

Let us consider in detail the origin of the third order invariants proportional to $\gamma_{1,2}$. The quantities $\gamma_{1,2}$ can be expressed in terms of the Green's functions $\mathcal{G}_{ii}^0$ as

$$\gamma_{1,2}=T\sum_{n,\mathbf{k}}\left[\mathcal{G}_{11}^0(\mathbf{k},\omega_n)\left(\mathcal{G}_{22}^0(\mathbf{k}+\mathbf{Q},\omega_n)\right)^2\right.$$
$$\left.\mp\left(\mathcal{G}_{11}^0(\mathbf{k},\omega_n)\right)^2\mathcal{G}_{22}^0(\mathbf{k}+\mathbf{Q},\omega_n)\right], \tag{22}$$

where $\mathbf{Q}$ is the wave vector which connect the congruent parts of the electron and the hole Fermi surfaces (in the commensurate case which is only considered here the vector $\mathbf{Q}$ is at the same time the wave vector of the SDW and SCDW). When integrating over the momentum in (22), in order to calculate γ_1 it is necessary to take into account that the density of states $N(\zeta)$ should not be replaced by the constant $N(0)$ because in such cases $\gamma_1=0$. In evaluating (22) one takes

$$\varepsilon_1(\mathbf{k})=-\varepsilon_2(\mathbf{k}+\mathbf{Q})=\zeta=\frac{k^2}{2m^*}-\varepsilon_F,$$

that is,

$$N(\zeta)=N(0)\left[1+\frac{\zeta}{\varepsilon_F}\right]^{1/2}\approx N(0)\left(1+\frac{\zeta}{2\varepsilon_F}\right)$$

in the interval of the momenta $k\approx k_F$. Here $\varepsilon_F=k_F^2/2m^*$, where m^* is the effective mass of the carriers. The integration in (22) is cut off at $|\zeta|\lesssim\varepsilon_F$. We note that the coefficient γ_1 equals zero in a one-band model. This can be easily proved by simply changing the variables $\mathbf{k}\to\mathbf{k}+\mathbf{Q}$ and by noting that the spectrum is even, $\varepsilon_i(\mathbf{k})=\varepsilon_i(-\mathbf{k})$. In principle, when calculating the coefficients A_1, B_1, β, and γ_2 of the functional (18) it is also necessary to include the corrections

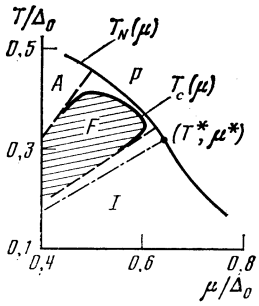

FIG. 1. P denotes the paramagnetic phase, A denotes the antiferromagnetic phase, F is the "weak"-ferromagnetism phase, and I is the incommensurate phase.

related to the energy dependence of the density of states $N(\zeta)$. One can convince oneself that the contribution of these corrections is a small quantity of order $[\ln(\varepsilon_F/\pi T)]^{-1}\ll 1$ relative to the accounted terms proportional to γ_1.

The phase diagram $T(\mu)$ for the considered model can be easily constructed using the results of the phenomenological analysis of Sec. 2 (see Fig. 1). It is easy to see that the coexistence condition for $\Delta_{\rm Re}^t$ and $\Delta_{\rm Im}^t$ is

$$1-\frac{1}{4\beta^2}\left(A_1-\frac{\gamma_1^2}{4a_+}-\frac{\gamma_2^2}{4a_-}\right)^2>0. \tag{23}$$

By considering the explicit dependences of the coefficients A_1, γ_1, γ_2, and β on T and μ it is possible to determine exactly the phase-transition line for the state with coexisting $\Delta_{\rm Re}^t$ and $\Delta_{\rm Im}^t$ (shaded region in Fig. 1). As the noncongruency parameter μ is increased, the coefficients A_1 and β are decreases while γ_2 is increased so that the conditions (23) is fulfilled in the region of (T,μ) approximately given by

$$\varphi_1^2<a_-\varphi_2<2\varphi_1^2\quad(\gamma_1\ll\gamma_2). \tag{24}$$

The coexistence of $\Delta_{\rm Re}^t$ and $\Delta_{\rm Im}^t$ leads to the formation of the average magnetization $\mathbf{M}$. A direct calculation shows that

$$\mathbf{M}=\frac{2}{1-\bar g_+}\mu_B\gamma_1[\Delta_{\rm Im}^t\times\Delta_{\rm Re}^t], \tag{25}$$

where μ_B is the Bohr magneton. We emphasize that we consider everywhere the case $\bar g_\pm<1$, i.e., the Stoner criterion for the itinerant ferromagnetism is not fulfilled.

Therefore, at the transition line $T_c(\mu)$ that bounds the shaded region in the figure, a transition occurs from the antiferromagnetic phase (SDW state, $\Delta_{\rm Re}^t\neq 0$) into the ferromagnetic phase in which the average magnetization $\mathbf{M}$ is oriented perpendicular to the SDW ($\mathbf{M}\perp\Delta_{\rm Re}^t$). This phase can be conditionally called the "weak"-ferromagnetism phase (in accordance with the formal analogy with the traditional "weak" ferromagnets), although there exists no relativistic "weakness" in our case. With a change of sign of φ_2 (and, accordingly, of β, A_1, and B_1) it would seem that the situation becomes even more favorable for the coexistence of SDW and SCDW. However, the situation is more complicated because within our model the coefficient of the gradient term [which is not written in (18)] changes its sign at the same time as the signs of β, A_1, and B_1 change. Consequently, to the

right of the Lifshitz point (the point T^*, μ^* in the figure) the system goes over into a phase with incommensurate structure. A detailed calculation of the coefficients with the gradient terms of different degrees was completed in Ref. 13. The present analysis regarding the existence of Δ^t_{Re} and Δ^t_{Im} turns out to be analogous to the analysis of the coexistence between the SDW and the CDW (charge density waves) which was carried out in Ref. 13. In particular, the favorable configuration in the incommensurate phase is $\Delta^t_{\mathrm{Re}} \perp \Delta^t_{\mathrm{Im}}$, in which the spatial modulations of Δ^t_{Re} and Δ^t_{Im} are displaced for $\pi/2$. That is

$$\Delta_{\mathrm{Re}}{}^t(\mathbf{r}) = \Delta_{\mathrm{Re}}{}^t \cos \mathbf{q}_0\mathbf{r}, \quad \Delta_{\mathrm{Im}}{}^t(\mathbf{r}) = \Delta_{\mathrm{Im}}{}^t \sin \mathbf{q}_0\mathbf{r}, \tag{26}$$

where $\mathbf{q}_0$ is the wave vector of the incommensurate structure,

$$q_0{}^2 = -a_1/2a_2; \quad a_1 = {}^1/_6 v_F{}^2 (\pi T)^{-2} \varphi_2,$$
$$a_2 = {}^1/_{40} v_F{}^4 (\pi T)^{-4} \varphi_4,$$
$$\varphi_1 = \sum_{n \geqslant 0}^{\infty} \left(10 \frac{\mu^2}{\omega_n{}^2} - 5 \frac{\mu^4}{\omega_n{}^4} - 1 \right) (2n+1)^{-5} \left(1 + \frac{\mu^2}{\omega_n{}^2} \right)^{-5}, \tag{27}$$

and v_F is the Fermi velocity. In the structure described by formula (26) the magnetic-moment density $\mathbf{m}(\mathbf{r})$, which is proportional to $\Delta^t_{\mathrm{Re}} \times \Delta^t_{\mathrm{Im}}$, is modulated with the period doubled relative to the SDW and the SCDW,

$$\mathbf{m}(\mathbf{r}) = \mathbf{m} \sin 2\mathbf{q}_0\mathbf{r}.$$

In this way, the magnetization averaged over the whole crystal is zero and there is a distinctive domain structure of typical dimension $L \sim (2q_0)^{-1}$.

It is easy to see that in the systems with the Hamiltonian (18) the question about coexistence of $\Delta^t_{\mathrm{Re}} \| \Delta^t_{\mathrm{Im}}$ is analogous to the question about coexistence of Δ^s_{Re} and Δ^s_{Im}. The latter was considered in detail in Ref. 5. It was shown that within the interval of commensurate structure, where the Landau expansion is valid, the coexistence is not possible. Analogous result about the impossibility of coexistence of commensurate $\Delta^t_{\mathrm{Re}} \| \Delta^t_{\mathrm{Im}}$ is also obtained in our case as long as the expression (18) is valid. The analysis shows that in the region of incommensurate structure the solution $\Delta^t_{\mathrm{Re}} \perp \Delta^t_{\mathrm{Im}}$ is the preferred one sufficiently near the Lifshitz point.

It is possible to suggest another mechanism for the formation of the intraband order parameter $\mathbf{\Sigma}_+$ which is not related to the Stoner exchange intergral g_+. Namely, we shall let the system have the localized magnetic moments $\mathbf{S}$ which interact with the itinerant electrons,

$$H_{s-d} = \sum_{j,i,\mathbf{k},\mathbf{q}} J_{ji}(\mathbf{S}_\mathbf{q}, \sigma)_{\alpha\beta} a^+_{i\mathbf{k}\alpha} a_{j\mathbf{k}+\mathbf{q}\beta}. \tag{28}$$

Here, J_{ij} are corresponding coupling constants and $\mathbf{S}_\mathbf{q}$ are the Fourier components of the density of the localized moments. For simplicity, we shall neglect the iterband constants J_{12} and J_{21}. Then, we have an ordinary system with the RKKY exchange and the order parameter which describes the emergence of the ferromagnetism is $\mathbf{\Sigma}_+ = J \langle \mathbf{S} \rangle$, where $J = J_{11} = J_{22}$ and $\langle \mathbf{S} \rangle$ is the average density of the localized moments.

The functional (18) is also valid in this case (when $\mathbf{\Sigma}_- = 0$). However, now the coefficients a_+ depends on temperature. If it is assumed that the Curie temperature $\tilde{T}_c$, which is determined only by the RKKY exchange interaction, is of the same order of magnitude as T_{Re} and T_{Im}, then in the mean-field approximation

$$a_+ = 1 - \tilde{T}_c/T, \quad \tilde{T}_c = {}^1/_3 J^2 N(0) S(S+1). \tag{29}$$

Clearly, for $\tilde{T}_c < T_{\mathrm{Re,Im}}$ the ferromagnetic order does not appear in the absence of the SDW and the SCDW. However, the smallness of a_+ can lead to the expansion in the phase diagram of the region of the "weak" ferromagnetism.

We need to emphasize that the considered model (in the absence of localized spins) has peculiarities related to the simultaneous vanishing of the coefficients A_1, B_1, β and of the coefficient a_1 of the gradient term. This is due to the extreme idealization of the model (in particular, other bands are ignored, scattering is neglected, the density of states is idealized, etc.). However, even with these conditions the state of "weak" ferromagnetism is possible. It is only important that the mechanism for the formation of SCDW has a purely electronic origin related to the characteristic topology of the Fermi surface, while the formation of the antiferromagnetic and the ferromagnetic order parameters can come from completely different sources. This substantially broadens the class of systems in which there is, possibly, an electronic mechanism for the "weak" ferromagnetism.

4. THE FORMATION OF "ORIENTATIONAL" STATES IN AN EXTERNAL MAGNETIC FIELD AND THE ANOMALY IN THE NONLINEAR MAGNETIC SUSCEPTIBILITY

We shall consider a special case of a two-band model of a semimetal with the extrema of the bands 1 and 2 overlapping at the point $\mathbf{k}_0$ in momentum space. We shall suppose that the symmetry of the Bloch wave functions at the extrema of the bands allows for a nonzero interband matrix element of the orbital momentum operator

$$\mathbf{L}_{12} = \int U^*_{1\mathbf{k}_0} [\mathbf{r} \times -i\nabla] U_{2\mathbf{k}_0} d\mathbf{r} = \mathbf{L}_{21}{}^*. \tag{30}$$

Such model was considered in Ref. 14. If the wave functions $U_{1\mathbf{k}_0}$ and $U_{2\mathbf{k}_0}$ can be chosen purely real, then the vector $\mathbf{L}_{12}$ will be purely imaginary, i.e., $\mathbf{L}_{12} = \mathbf{L}_{21} = i\mathbf{L}$. In the models with such band symmetry the formation of the orbital ferromagnetism is possible when the imaginary singlet order parameter Δ^s_{Im} is nonzero (note that the magnetic moment $\mathbf{M} \sim \mathbf{L}\Delta^s_{\mathrm{Im}}$).[14] The magnetic susceptibility of the system diverges as the transition point is approached according to the Curie-Weiss law $\chi \sim (T - T^s_{\mathrm{Im}})^{-1}$, where T^s_{Im} is the transition temperature.

In contrast to Ref. 14, we shall assume that the state with $\Delta^t_{\mathrm{Im}} \neq 0$ is realized earlier (i.e., at the higher temperature T^t_{Im}) than the state with $\Delta^s_{\mathrm{Im}} \neq 0$ (i.e., $T^s_{\mathrm{Im}} < T^t_{\mathrm{Im}}$). Next, let us assume that the states with the real order parameters are not at all realized in the considered temperature interval (this can be obtained by introducing the scattering on charged impurities). Then, in the lowest approximation in Δ^t_{Im} and Δ^s_{Im} and in the external field $\mathbf{H}$ it is possible to write

$$F/2N(0) = \alpha_{\mathrm{Im}}{}^t (\Delta_{\mathrm{Im}}{}^t)^2 + \alpha_{\mathrm{Im}}{}^s (\Delta_{\mathrm{Im}}{}^s)^2 + \eta_t (\Delta_{\mathrm{Im}}{}^t \mathbf{H})(\mathbf{LH}) + \eta_s \Delta_{\mathrm{Im}}{}^s (\mathbf{LH}) + C \Delta_{\mathrm{Im}}{}^s (\Delta_{\mathrm{Im}}{}^t \mathbf{H}). \tag{31}$$

Thus, we notice that the invariants of the type $(\Delta^t_{\rm Im}\cdot\mathbf{L})\mathbf{H}^2$ are formally absent from (31). Such invariants appear in our model only as a result of the spin-orbit interaction. In addition, the invariants linear in $\mathbf{H}$ and which contain $\Delta^t_{\rm Re}$ or $\Delta^s_{\rm Re}$ [of the form $\Delta^t_{\rm Im}\cdot(\Delta^t_{\rm Re}\times\mathbf{H})$, $\Delta^s_{\rm Re}(\Delta^t_{\rm Re}\cdot\mathbf{H})$, etc.] are dropped from (31) because their contribution is assumed small as a result of the large difference between the transition temperatures $T^s_{\rm Re,Im}$ and $T^t_{\rm Re,Im}$ ($T^s_{\rm Re,Im}\ll T^t_{\rm Re,Im}$). It is necessary to keep the terms containg $\Delta^s_{\rm Im}$ already for $T^s_{\rm Im}\ll T^t_{\rm Re}$ because in the weak magnetic fields precisely these terms determine the linear magnetic susceptibility.

The explicit expression for η_t, η_s, and C are the following:

$$C=\eta_t=(4\mu_B/\pi T)\varphi_1, \qquad \eta_s=\varphi_0. \tag{32}$$

It is easy to obtain from (31) the magnetic susceptibility $\chi(\mathbf{H})$ in which it is necessary to keep the field dependence to order $\mathbf{H}^2$ (we consider the magnetic field to be the only small quantity in the problem). For the case $\mathbf{H}\|\mathbf{L}$ we have

$$\chi(\mathbf{H})=\frac{\mathbf{L}^2\eta_s^{\,2}}{2\alpha_{\rm Im}{}^s}+\mathbf{L}^2\mathbf{H}^2\frac{\eta_t^{\,2}(\alpha_{\rm Im}{}^s)^2+{}^1/_8\eta_s^{\,2}C^2-\alpha_{\rm Im}{}^s\eta_s\eta_t C}{\alpha_{\rm Im}{}^t(\alpha_{\rm Im}{}^s)^2}. \tag{33}$$

It follows from (33) that near the transition point into the state with the imaginary triplet order parameter ($\alpha^t_{\rm Im}\to 0$) the $\mathbf{H}^2$ term in the nonlinear magnetic susceptibility diverges while the linear susceptibility

$$\chi(0)=\mathbf{L}^2\eta_s^{\,2}/2\alpha_{\rm Im}{}^s$$

does not experience any anomaly near the phase transition point ($\alpha^s_{\rm Im}>0$ as $\alpha^t_{\rm Im}\to 0$.) This effect is related to the induction of the order parameter $\Delta^t_{\rm Im}$ in the external fields,

$$\Delta_{\rm Im}{}^t=-\mathbf{H}(\mathbf{LH})\frac{2\alpha_{\rm Im}{}^s\eta_t-\eta_s C}{4\alpha_{\rm Im}{}^t\alpha_{\rm Im}{}^s}. \tag{34}$$

Therefore, a transition into the orientational state can, in principle, be observed through the Curie-Weiss anomaly of the nonlinear magnetic susceptibility $\chi(\mathbf{H})$. It is understood that the formula (33) is not valid for sufficiently strong fields $\mathbf{H}$ and in the immediate vicinity of the transition point where it is essential to renormalize $T^t_{\rm Im}$ relative to $\mathbf{H}^2$. It is easy to show that the criterion for the validity of (33) is $4\alpha^t_{\rm Im}\chi^s_{\rm Im}\gg C^2\mathbf{H}^2$.

5. FORMATION OF SPONTANEOUS MAGNETIZATION WITHIN THE "ORIENTATIONAL" STATE

The invariants in the free energy which are linear in the parameter $\mathbf{G}$ can be formed by having two polar vectors $\mathbf{U}$ and $\mathbf{V}$:

$$\delta F=\lambda\mathbf{G}[\mathbf{U}\times\mathbf{V}], \tag{35}$$

where λ is a proportionality constant.

We shall assume that $\mathbf{U}=\mathbf{E}$ is the constant electric field strength and $\mathbf{V}$ is some polar vector characterizing the crystal anisotropy of the system (for example, it can be the unit vector $\mathbf{n}$ along the direction of the polar axis). The free energy functional F of such a system depends only on one parameter and has the form

$$F=\alpha\mathbf{G}^2+\alpha'(\mathbf{Gn})^2+\lambda\mathbf{G}[\mathbf{E}\times\mathbf{n}]+\beta\mathbf{G}^4+\beta'(\mathbf{G}n)^4+\beta''\mathbf{G}^2(\mathbf{Gn})^2. \tag{36}$$

We shall let the coefficients in (36) be such that the transition into the configuration $\mathbf{G}\perp\mathbf{n}$ is the most likely (the type of systems having an "easy plane"). Then, below the transition point which is determined by $\alpha=0$ (where $\alpha\sim T-T_{\rm G}$) the spontaneous polarization

$$\mathbf{P}=\lambda[\mathbf{G}_0\times\mathbf{n}], \qquad \mathbf{G}_0{}^2=-\alpha/2\beta \tag{37}$$

is formed perpendicular to the polarization axis $\mathbf{n}$. As the transition point is approached, $T\to T_{\rm G}+0$, there is a Curie-Weiss anomaly in the perpendicular component of the dielectric permeability relative to the direction $\mathbf{n}$:

$$\varepsilon_{jk}=\frac{\lambda^2}{2\alpha}(1-\delta_{jz})\,\delta_{jk}. \tag{38}$$

Here, $k,j=x,y,z$ and the z-axis coincides with the vector $\mathbf{n}$.

Therefore, the orientational state can be accompanied with the emergence of the ferromagnetic order in a direction perpendicular to the polar axis. It is, of course, assumed here that there is no ordering along this axis. If, however, a "normal" ferroelectric transition with the polarization direction along $\mathbf{n}$ occurred earlier, then the formation of the parameter $\mathbf{G}$ will be accompanied by a rotation of the polarization vector for some angle ψ relative to the polar axis.

We move to a microscopic model which illustrates these phenomenological conclusions. We shall consider a two-band model of a semimetal with the extrema of bands 1 and 2 coinciding at the point $\mathbf{k}_0$ in momentum space, and with a nonzero interband momentum matrix element $\mathbf{P}_{12}$. This model was explored in detail in Refs. 1–5. The effective Hamiltonian of this system in an external field of strength $\mathbf{E}$ has the form

$$\hat H=\hat H_0+\hat H_{s0} \tag{39}$$

$$\hat H_0=\sum_{\mathbf{k}}\begin{bmatrix}\left(\varepsilon_1(\mathbf{k})-ie\mathbf{E}\dfrac{d}{d\mathbf{k}}\right)\hat I & \dfrac{1}{m}\mathbf{P}_{12}\mathbf{k}\hat I-\hat\Delta_{21}\\ \dfrac{1}{m}\mathbf{P}_{21}\mathbf{k}\hat I-\hat\Delta_{21} & \left(\varepsilon_2(\mathbf{k})-ie\mathbf{E}\dfrac{d}{d\mathbf{k}}\right)\hat I\end{bmatrix}\hat\sigma, \tag{40}$$

where m is the electron mass and

$$\hat H_{s0}=-\frac{e\hbar^1}{(2mc)^2}\sum_{\mathbf{k}}\begin{bmatrix}[\mathbf{E}\times\mathbf{k}] & [\lambda\times\mathbf{k}]+[\mathbf{E}\times\mathbf{P}_{12}]\\ [\lambda\times\mathbf{k}]+[\mathbf{E}\times\mathbf{P}_{21}] & [\mathbf{E}\times\mathbf{k}]\end{bmatrix} \tag{41}$$

is the spin-orbit interaction Hamiltonian for the assumed symmetry of the bands 1 and 2. In the formula (41)

$$\lambda=\int U^*_{1\mathbf{k}_0}\frac{\partial V}{\partial\mathbf{r}}U_{2\mathbf{k}_0}d\mathbf{r},$$
$$\mathbf{P}_{12}=-\mathbf{P}_{21}=\int U^*_{1\mathbf{k}_0}(-iV_r)\,U_{2\mathbf{k}_0}\,d\mathbf{r}, \tag{42}$$

where $V(\mathbf{r})$ is the crystal potential. The vector λ is collinear with $\mathbf{P}_{12}$ and the matrices $\hat\Delta_{12}$ and $\hat\Delta_{21}$ have the structure considered in Sec. 3. In the absence of the electric field $\mathbf{E}$ one of the four types of order parameter considered above can appear in systems with the Hamiltonian (39). It was shown in Ref. 3 that when a singlet order parameter $\Delta^s_{\rm Im}$ which describes the CCDW (charge current density wave) is formed, a uniform current is absent from the system, i.e., $\langle\mathbf{j}(\mathbf{r})\rangle=0$. In analogy with Ref. 3, it can be shown that a uniform spin current density is absent ($\langle\hat{\mathbf{j}}_s(\mathbf{r})\rangle=\langle\hat\sigma\times\mathbf{j}(\mathbf{r})\rangle=0$) when a

uniform triplet order parameter Δ_{Im}^{t} (which describes SCDW) is formed. This follows simply from the fact that the proof for the absence of the uniform current in the system goes through independently of the spin direction (see Ref. 3).

We shall assume that the critical temperature T_{Im}^{t} corresponding to the formation of the parameter Δ_{Im}^{t} is very high. Within the two-band model with coincident extrema, the parameter Δ_{Re}^{s} is induced in the presence of an external field,[15] and a component of the polarization along $\mathbf{P}_{12}$ emerges,

$$\mathbf{P}_{\parallel}=\frac{1}{\alpha_{\mathrm{Re}^s}}|\mathbf{P}_{12}|\,(|\mathbf{P}_{12}|\mathbf{E})A,$$
$$A=\left[\frac{1}{3}\frac{v_F^2}{(\pi T)^2}\frac{m^*}{m}\right]^2\varphi_2^2,\quad \alpha_{\mathrm{Re}^s}=\frac{T-T_{\mathrm{Re}^s}}{T_{\mathrm{Re}^s}}\left(1-\frac{4\mu}{\pi T}\varphi_1\right), \tag{43}$$

where T_{Re}^{s} is the critical transition temperature into the usual ferromagnetic state (the state with $\Delta_{\mathrm{Re}}^{s}\neq 0$). With the account of the spin-orbit interaction $\hat{H}_{s0}$ another component of the polarization perpendicular to $\mathbf{P}_{12}$ emerges. It is related to the induction by the electric field of the imaginary triplet order parameter Δ_{Im}^{t} with

$$\mathbf{P}_{\perp}=\frac{1}{\alpha_{\mathrm{Im}^t}}[|\mathbf{P}_{12}|,[\mathbf{E},|\mathbf{P}_{12}|]]B,$$
$$B=\frac{e^2\hbar^2}{(2mc)^4}\varphi_0^2,\quad \alpha_{\mathrm{Im}^t}=\frac{T-T_{\mathrm{Im}^t}}{T_{\mathrm{Im}^t}}\left(1-\frac{4\mu}{\pi T}\varphi_1\right). \tag{44}$$

In calculating B only the dominant contribution, related to the term in $\hat{H}_{s0}$ proportional to $\hat{\boldsymbol{\sigma}}\cdot(\mathbf{E}\times\mathbf{P}_{12})$, was kept. The contributions related to the remaining terms, such as the interference terms of second order in $\hat{H}_0$ and $\hat{H}_{s0}$, contain a logarithmically small quantity $[\ln(\varepsilon_F/\pi T)]^{-1}$ compared with the dominant term.

Although the coefficient B is nonzero only because of the spin-orbit interaction, its contribution to the polarization $\mathbf{P}_{\perp}$ is the most important for $\mathbf{E}\perp\mathbf{P}_{12}$ and $T\approx T_{\mathrm{Im}}^{t}$. Then, the component of the dielectric permeability parallel to $\mathbf{P}_{12}$ does not experience any anomaly while the perpendicular component diverges according to the Curie-Weiss law as $T\to T_{\mathrm{Im}}^{t}$.

Clearly, the question arises as to how favorable is the formation of a state with Δ_{Im}^{t} within the considered model and in the absence of an external field. Realistically, the electron-phonon interaction in the electronic ferroelectrics is, evidently, much too strong for the state with $\Delta_{\mathrm{Im}}^{t}\neq 0$ to be favorable, i.e., $T_{\mathrm{Re}}^{s}>T_{\mathrm{Im}}^{t}$. However, even in this case one can hope that with the formation of Δ_{Im}^{t} in the background of Δ_{Re}^{t} and $\mathbf{E}\perp\mathbf{P}_{12}$ it would be possible to observe a growth of the perpendicular component of the polarization $\mathbf{P}_{\perp}$ as well as the characteristic break in the temperature dependence of $\mathbf{P}\perp(T)$ at the ferroelectric transition point T_{Re}^{s}. We remark that in the systems with the Hamiltonian (39) the electronic spectrum becomes spin-polarized at the transition into the orientational state. Let $\mathbf{E}=0$ and let us neglect the spin-orbit interaction. By directing the quantization axis along Δ_{Im}^{t} it is easy to see that in the transformed phase

$$\tilde{\varepsilon}_{1,2}^{\uparrow}(\mathbf{k})=\pm\left[\zeta_{\mathbf{k}}^2+\left(\frac{|\mathbf{P}_{12}|\mathbf{k}}{m}-\Delta_{\mathrm{Im}^t}\right)^2\right]^{1/2},$$
$$\tilde{\varepsilon}_{1,2}^{\downarrow}(\mathbf{k})=\pm\left[\zeta_{\mathbf{k}}^2+\left(\frac{|\mathbf{P}_{12}|\mathbf{k}}{m}+\Delta_{\mathrm{Im}^t}\right)^2\right]^{1/2}. \tag{45}$$

Clearly, such rearrangement of the spectrum leads to a change in the selection rules for the optical interband transitions and this may manifest itself in the studies of the polarized light absorption

6. CONCLUSION

In connection with the results obtained in this work for the systems with SCDW (and also in previous works[1-5] for the CCDW) some clarifications of the terminology "charge current density wave" (CCDW) and "spin current density wave" (SCDW) appear necessary. The concepts CCDW and SCDW were used in many papers to denote the excited states of the systems with the CDW and the SDW ground states, respectively (see, for example, the review in Ref. 16). The excited CCDW and SCDW states are accompanied, according to Ref. 16, with the formation of "macroscopic nondissipative" charge or spin currents.

The terms CCDW and SCDW were used in the present work in their original meaning (see Ref. 10 and also Ref. 2), i.e., they refer to the ground states of a crystal. At the transitions into the CCDW or the SCDW states a rearrangement of the charge or spin currents occur on the scale of the crystal's unit cell. This is analogous to rearrangements in the charge or spin densities occurring at the transitions into the CDW or the SDW states. However, there is no formation of any uniform macroscopic charge or spin currents in the ground states with CCDW or SCDW (a periodic, stationary distribution of charge or spin currents appears in the incommensurate structures). The possibilities of realizing equilibrium states with the CCDW were presented earlier[1-5] while those with the SCDW were presented now. They have nothing in common with the methods of generating nonequilibrium "nondissipative currents" considered in Ref. 16. The systems with the CCDW or the SCDW can be completely classified within the framework of the known magnetic symmetry groups and their existence does not contradict any of the general physical principles. The interesting properties which are predicted, perhaps offer a stimulus for a broad experimental investigation or for a new interpretation of already available results.

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