Emission of photons by electrons and positrons passing through a thin single crystal

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We consider the radiation of particles (electrons and positrons) undergoing planar channeling in a single crystal of small thickness L. We show that for $L \sim \pi b/\theta_L$, where b is the lattice constant and θ_L is the Lindhard angle, in addition to the principal maxima of spontaneous radiation of channeled particles in the spectrum there are additional interference maxima, and the positions of all maxima of the radiation intensity depend on L. We discuss the dependence of the intensity of radiation at various frequencies on the crystal thickness.

1. INTRODUCTION

The motion of charged particles through single crystals along low-index crystallographic directions is determined to a significant degree by interaction with the continuous potential of the atomic strings or planes, averaged along the corresponding direction.[1] In the average potential the particles can be captured into a mode of finite transverse motion which is quasibound to the atomic strings or planes; this mode of motion has received the name channeling.[1] The spectrum of the transverse energies of particles in the periodic averaged potential has a band structure,[2] and the number of bands of quasibound (sub-barrier) states increases with the energy of the particles.

The motion of electrons and positrons during channeling can be likened to the motion of electrons in a one-dimensional atom (planar channeling) or in a two-dimensional atom (axial channeling). In analogy to the characteristic radiation which appears in atoms, channeled particles can emit photons in transitions from higher to lower bands of the spectrum of transverse energies, and the frequencies ω of the emitted photons observed at a definite angle θ_γ are related to the change of the transverse energy in the transition $\Delta E_\perp^{if} = E_\perp^i - E_\perp^f$ or, in a classical interpretation, they are related to the frequencies of the quasiperiodic motion of the particles in the channel.[3]

The radiation of channeled electrons and positrons has been intensively studied in recent years both theoretically and experimentally (see for example the reviews in Refs. 4–8). Nevertheless one cannot consider that this line of investigation is exhausted. This is due in particular to the fact that the properties of this radiation depend very strongly on the type of channeling (planar or axial), on the charge and the energy E of the particles, on the structure, orientation, and thickness of the crystals, and on other factors. For example, the characteristics of the motion of the particles in the crystals and, as a consequence, the properties of their radiation depend very substantially on the thickness of the crystals. The regime of stable quasiperiodic motion of a particle in a channel can be established only at depths $z \gg b/\theta_L$, where b is the channel width and θ_L is the critical angle for capture into the channeling regime, when the particle is able to complete at least several oscillations in the channel. Most of the existing theoretical studies are devoted specifically to the description of the radiation of particles executing stable and extended quasiperiodic motion. With increase of the crystal thickness it becomes necessary to take into account the influence of inelastic processes such as scattering and ionization loss on the characteristics of the motion of the particles and the spectrum of their radiation.[9] At depths of the order of the so-called dechanneling length L_d,[10] inelastic processes lead to dechanneling of particles and then also to destruction of the periodic nature of their transverse motion.

A special case is presented by the radiation of particles passing through very thin crystals ($L \lesssim b/\theta_L$ when the particles cannot complete even a single oscillation in the transverse direction. Apparently, insufficient attention has been devoted to this case in the literature. Among the existing studies we note Refs. 8 and 11, in which quantum-mechanical expressions have been introduced for the probabilities of radiation of particles undergoing planar channeling; these expressions are applicable also to description of the radiation in very thin crystals. We also note Refs. 12 and 6, in which the radiation of particles in interaction with short strings of atoms has been discussed in the classical approximation.[1)]

We note that the radiation of channeled particles in thin crystals, $L \sim b/\theta_L$, should have a number of interesting properties which we can suspect even on the basis of a classical interpretation of the problem. The radiation spectrum of a particle is made up as the result of interference of radiation from various portions of its trajectory. In a thick crystal the trajectory of the motion of a channeled particle is quasiperiodic, and interference makes the radiation at the principal frequencies significantly more intense than the radiation at all remaining frequencies. In thinner crystals, when the particle is not able to complete even a few oscillations in the channel, other interference peaks whose positions may depend on the thickness of the crystal should also remain appreciable in the radiation spectrum.

The present work has the purpose of considering in a quantum approach the problem of radiation of channeled particles in thin crystals when it is impossible to neglect radiation at other than the principal frequencies.

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2. INTENSITY OF RADIATION OF PARTICLES IN PLANAR CHANNELING IN A CRYSTAL OF FINITE THICKNESS

Let us consider a lepton of mass m entering a crystal with momentum $\mathbf{p}_i$ and a relativistic energy $E_i = (\mathbf{p}_i^2 + m^2)^{1/2}$ at a small angle $\theta_x < \theta_L$ to close-packed atomic planes ZY. It emits in the crystal a photon ω at an angle θ to the direction of the momentum $\mathbf{p}_i$ and leaves the crystal with momentum $\mathbf{p}_f$ and energy $E_f = E_i - \hbar\omega$. The wave functions of the initial state Ψ_i and the final state Ψ_f of the lepton inside the crystal must be represented in the form of a superposition of Bloch functions of the transverse motion of the lepton in the periodic potential of the system of atomic planes[8,11] (here and below we use the system of units $\hbar = c = 1$):

$$\Psi_i=\sum_{s_i} a_{s_i}(p_{x_i})\Phi_{s_i k_i}(x) \times\exp\left\{ip_{y_i}y+i\left(z-\frac{L}{2}\right)(p_i^2-p_{y_i}^2-2E_iE_{x_i})^{1/2}\right\},$$
$$\Psi_f=\sum_{s_f} a_{s_f}(p_{x_f})\Phi_{s_f,k_f}(x)\exp\left\{ip_{y_f}y +i\left(z+\frac{L}{2}\right)(p_f^2-p_{y_f}^2-2E_fE_{x_f})^{1/2}\right\}. \tag{1}$$

Where the a_s are defined as the mixing coefficients of the wave functions (1) with plane waves at the front face (for Ψ_i) or the back fact (for Ψ_f) or the back fact (for Ψ_f) of the crystal:

$$a_s(p_x)=\int_0^b \frac{dx}{b}\Phi^*_{s,k}(x)\exp\{ip_xx\}; \tag{2}$$

here b is the period of the average potential of the atomic planes $U(x)$ and $\Phi_{s,k}$ are the eigenfunctions of the equation

$$\frac{d^2\Phi_{s,k}}{dx^2}=2E[U(x)-E_x(s,k)]\Phi_{s,k} \tag{3}$$

with boundary conditions of the Bloch type[2]:

$$\Phi_{s,k}(x)=e^{-ikb}\Phi_{s,k}(x+b),\quad \Phi'_{s,k}(x)=e^{-ikb}\Phi'_{s,k}(x+b). \tag{4}$$

To each allowed value of the transverse energy $E_x(s,k)$ there correspond two wave functions with quasimomenta k opposite in sign. The values of k in (1) and (2) are determined by the mixing conditions and can differ from the corresponding values of p_x only by amounts which are multiples of $2\pi/b$ (Refs. 2, 15); their difference is $k_i - k_f = K_x$, where $\mathbf{K}$ is the momentum of the emitted photon.

The functions $\Phi_{s_i k_i}$ and $\Phi_{s_f k_f}$ are solutions of Eq. (3) with various E values (i.e., E_i or $E_f \neq E_i$) and, generally speaking, belong to different orthonormalized sets,[7,8] within each of which the following normalization conditions are satisfied:

$$\int_0^{L_x}\frac{dx}{L_x}\Phi_{s,k}(x)\Phi^*_{s',k'}(x)=\begin{cases}\delta_{s,s'}, & k=k'\\ 0, & k\neq k'\end{cases}, \tag{5}$$

where $L_x \gg b$ is the width of the crystal along the channel X.

The differential intensity of the radiation of the leptons in the potential $U(x)$ can be written in the form[16]

$$dI=\frac{e^2(E_i+m)d^3\mathbf{K}d^3\mathbf{p}_f}{64\pi^4L_yL_xE_iE_f(E_f+m)}\delta(E_i-E_f-\omega)|M_{if}|^2, \tag{6}$$

$$M_{if}=\int d^3\mathbf{r}\Psi_f^*e^{-i\mathbf{K}\mathbf{r}}l_\mu^*\xi_f^+ \times\left[\sigma^\mu(\hat{\sigma},\hat{\mathbf{p}}_i-\mathbf{K})+\frac{E_f+m}{E_i+m}(\hat{\sigma},\hat{\mathbf{p}}_i)\sigma^\mu\right]\xi_i\Psi_i, \tag{7}$$

where $\hat{\mathbf{p}}_i = -i\hat{\nabla}$, $\hat{\sigma}$ are the Pauli matrices, [16] $\mathbf{l}$ is the photon polarization vector, and ξ are spinors which characterize the polarization of the lepton in the initial state and in the final state; $e^2 = 1/137$.

If we substitute Eq. (1) into Eq. (7), we can obtain general expressions for the probability of radiation by a channeled particle of a proton $\mathbf{K}$ in a crystal of finite thickness, which are given in Refs. 8 and 11. In the present work we restrict the discussion to the radiation of relativistic particles in a direction coinciding with the direction of the projection of $\mathbf{p}_i$ on the channeling plane, in the dipole approximation and without taking into account polarization effects. In this case the radiation intensity differential in the frequency ω and the angle θ for the case $\theta\to 0$ has the form

$$\frac{d^2I}{d\omega\theta d\theta}=e^2\omega^2\sum_{s_f}\left|\sum_{s_i}a_{s_i}(p_{x_i})\Delta_{if}d_{if}\Delta E_{x_{if}}\right|^2, \tag{8}$$

where

$$d_{if}=\frac{1}{E_i\Delta E_{x_{if}}}\int_0^b\frac{dx}{b}\Phi^*_{s_f,k_i}(x)\frac{d}{dx}\Phi_{s_i k_i}(x)$$

is the dipole matrix element of the radiative transition and

$$\Delta E_{x_{if}}=E_x(s_i,k_i)-E_x(s_f,k_i)$$

is the change of the transverse energy of the particle in the transition. The factor Δ_{if} in (8) arises from integration over the crystal thickness of the product of the z-components, which have a plane-wave form, of the wave functions of the leptons (1) and of the photon in the matrix element (7):

$$\Delta_{if}=\sqrt{\frac{2}{\pi}}\frac{1}{q_{if}}\sin\frac{q_{if}L}{2}\exp\left\{i\frac{L}{2}E_x(s_i,k_i)\right\}, \tag{9}$$

where $q_{if} = 1/2\omega\gamma^{-2} - \Delta E_{x_{if}}$ and $\gamma = E_i/m$ is the Lorentz factor.

In the triple sum (8) it is possible to separate subordinate sums which are diagonal and nondiagonal in the numbers of the initial states:

$$\left|\sum_{s_i}X_{if}\right|^2=\sum_{s_i}|X_{if}|^2+\sum_{s_i'}\sum_{s_i''\neq s_i'}X_{i'f}X^*_{i''f}.$$

One usually sets Δ_{if} proportional to $\delta(q_{if})$, which leads to the result that in the sum (8) only diagonal terms are kept (and not all of them, but only those for which $\Delta E_{x_{if}} > 0$), and in these terms it turns out to be possible to carry out the substitution

$$|\Delta_{if}|^2\to L\delta(q_{if}). \tag{10}$$

The substitution (10) signifies the appearance of a rigid connection between the amount of change of the transverse energy in the transition $\Delta E_{x_{if}}$ and the frequency ω of the photon radiated at a given angle θ. After the substitution (10), Eq. (8) coincides with the known expressions for the probability of radiation directly forward by channeled particles in thick crystals,[7,8] and it is found that at an angle $\theta\to 0$

photons are radiated only with frequencies $\tilde{\omega}_{if} = 2\gamma^2 \Delta E_{x_{if}}$

However, the substitution (10) is not always permissible, but only for sufficiently thick crystals $L \gg \pi(\Delta E_x^m)^{-1}$, where ΔE_x^m is the characteristic distance between the most closely spaced bands of the spectrum of transverse energies, which is achievable for electrons and for positrons, in the near-barrier region, and which amounts to $\sim \theta_L/b$.[8] Thus, the length L_0, which is critical for making the substitution (10), can be defined as

$$L_0 = \pi b/\theta_L = \pi b (E_i/2U_0)^{1/2}, \tag{11}$$

where U_0 is the depth of the periodic potential. This is the characteristic length of curvature of trajectories of the channeled particles, which at energies of the order of GeV reaches several microns (see the table). In crystals which do not correspond to the condition $L \gg L_0$, the quasiperiodic nature of the motion of the particle cannot be established, and instead of the well known formulas for the intensity of spontaneous radiation by a channeled particle from Refs. 3, 4, and 7 it is necessary to use the more general expressions from Refs. 8 and 11 or to use Eq. (8).

We note that if the radiation intensity were determined only by the diagonal part of (8), this would mean that inside the crystal, states corresponding to different s and k can be considered as independent, and instead of the functions $\Psi_{i(f)}$ of the type (1) it is possible to use wave functions of "pure" transverse states $\Phi_{s,k}(x)$. The coefficients $W_{s_i} = |a_{s_i}(p_x)|^2$, which are subject to the normalization condition

$$\sum_{s_i} W_{s_i} = 1, \tag{12}$$

have in this case the physical meaning of the probabilities of populating various initial states that depend on the angle of entry of the particle into the crystal. The nondiagonal part of (8) is an addition to the intensity of radiation of channeled particles, associated with quantum interference of radiative transitions from initial states with different transverse energy, and which is important in very thin crystals $L < L_0$.

3. RADIATION OF POSITRONS IN A THIN CRYSTAL

We shall consider in more detail the radiation of positrons which occurs in transitions between narrow sub-barrier bands (levels) in the dipole approximation when it is possible to neglect[15] in Eq. (8) the dependence on k and to consider that the wave functions of the initial and final states belong to the same orthonormalized set.

For positively charged particles a good approximation of the averaged potential of the atomic planes in many crystals is a harmonic potential of the type[4,7]

$$U(x) = 4U_0 (x-bn)^2/b^2, \quad |x-bn| \leqslant b/2, \quad n=0, \pm 1, \pm 2, \ldots, \tag{13}$$

for which the sub-barrier levels are equidistant and the dipole matrix elements of the radiative transitions are nonzero only for transitions between neighboring levels. In this case

$$E_x(s) - E_x(s-1) = \Delta E_x = 2\theta_L/b,$$
$$d_{s,s-1} = -d_{s-1,s} = {}^1/_4 b (s\Delta E_x/U_0)^{1/2}.$$

The radiation intensity (8) for sub-barrier transitions in the potential (13) has the form

$$\frac{d^2 I}{d\omega \theta d\theta} = \frac{e^2}{8\pi} b^2 \tilde{\omega}^2 \sum_{s=0}^{s_m-1} (s+1) \frac{\Delta E_x}{U_0} \left| a_{s+1}(p_{x_i}) \nu \frac{\sin \alpha(\nu-1)}{(\nu-1)} - e^{-2i\alpha} \left(\frac{s}{s+1}\right)^{1/2} a_{s-1}(p_{x_i}) \nu \frac{\sin \alpha(\nu+1)}{(\nu+1)} \right|^2, \tag{14}$$

where $S_m = [U_0/\Delta E_x + 1/2]$ is the number of sub-barrier states and

$$\alpha = \pi L/L_0, \quad \nu = \omega/\tilde{\omega}, \quad \tilde{\omega} = 2\gamma^2 \Delta E_x.$$

We shall assume that the positron beam enters the crystal strictly paralledl to the crystallographic planes ($p_{x_i} = 0$). We shall assume also that $s_m \gg 1$ and that the coefficients $a_s(0)$ for even s satisfy the relation (4.5) from Ref. 17:

$$\frac{a_s(0)}{a_{s+2}(0)} = \left(\frac{s+1}{s+{}^1/_2}\right)^{1/2},$$

while for odd s we have $a_s(0) =$. For $p_{x_i} \to 0$ the positrons are distributed practically only over the sub-barrier states.[4,7] Taking into account the normalization condition (12) and neglecting corrections of order $1/s_m$, the expression for the radiation intensity (14) can be reduced to the form

$$\frac{d^2 I}{d\omega \theta d\theta} = \frac{e^2}{24\pi} b^2 \tilde{\omega}^2 \lambda(\nu, \alpha), \tag{15}$$

where the function

Crystal	Orientation	b, Å	U_0, eV, from Ref. 7	E_i	$\tilde{\omega}$ $(\theta\to 0)$ for $e+$	L_0, μm
Diamond (C)	(100)	0.90	12.4	40 MeV	42 keV	0.36
				1 GeV	5.3 MeV	1.8
				25 GeV	0.66 GeV	9.0
	(110)	1.28	22.8	40 MeV	40 keV	0.38
				1 GeV	5 MeV	1.9
				25 GeV	0.63 GeV	9.5
Silicon (Si)	(100)	1.36	13.1	40 MeV	30 keV	0.52
				1 GeV	3.7 MeV	2.6
				25 GeV	0.47 GeV	13
	(110)	1.92	22.9	40 MeV	27 keV	0.56
				1 GeV	3.4 MeV	2.8
				25 GeV	0.43 GeV	14

$$\lambda(\nu,\alpha)=\left(\frac{2\nu}{\nu+1}\right)^2\left[\frac{\sin^2 2\alpha}{4}+\frac{\sin^2\alpha(\nu-1)}{(\nu-1)^2}-\sin 2\alpha\frac{\sin\alpha(\nu-1)}{(\nu-1)}\cos\alpha(\nu+1)\right]$$

determines the shape of the positron radiation spectrum in a thin crystal.

For $L>L_0$ the function (15) has a principal maximum near the frequency $\tilde{\omega}$, and with increase of ω, the function oscillates, approaching a final limit $\sin^2 2\alpha$. The positions of the main and auxiliary maxima are determined by the condition

$$\partial\lambda/\partial\nu=0,\quad \partial^2\lambda/\partial\nu^2<0,$$

but for $L>L_0$ the auxiliary maxima of (15) are small.

The case $L\ll L_0$ (Fig. 1) is more interesting; here the main maximum is strongly shifted from the frequency $\tilde{\omega}$ to the energetic region, and the auxiliary maxima are comparable with it in order of magnitude.

If we integrate Eq. (15) over frequency, we obtain the integrated intensity of radiation of a collimated beam of positrons in the direction directly forward:

$$\frac{dI}{\theta d\theta}=\frac{e^2}{24\pi}b^2\tilde{\omega}^3\chi(\alpha),$$

$$\chi(\alpha)=\int_0^{\nu_m}d\nu\lambda(\nu,\alpha),\quad \nu_m=\frac{\omega_m}{\tilde{\omega}}. \tag{16}$$

The limit of integration over frequency in (16) ω_m is determined by the sensitivity threshold of the photon detector. From the conditions of applicability of the dipole approximation, this threshold must satisfy the condition

$$\omega_m<E_i/s_m\sim E_i\Delta E_x/U_0. \tag{17}$$

For $\omega>E_i/s_m$ the change in energy of the radiating particle becomes too great for us to assume that the wave functions of the initial and final states of the particle belong to the same orthonormalized set. For the same reason the changes of the transverse energy of the particle in transitions to the neighboring upper and lower levels in this case cannot be considered to be the same.

The form of the dependence (16) is shown in Fig. 2. The function $\chi(\alpha)$ breaks up into two components: a component linear in α which originates from integration of the second term in

$$\chi_L(\alpha)\approx\pi\alpha\quad(\text{for }\ \alpha>\pi),$$

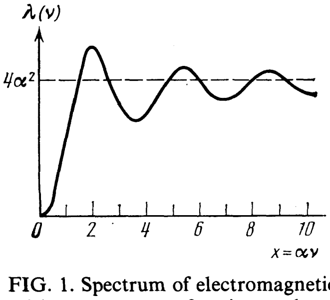

FIG. 1. Spectrum of electromagnetic radiation in the forward direction arising on passage of positrons through a monocrystalline film of thickness $L\ll L_0$ strictly along crystallographic planes ($\alpha\ll 1$).

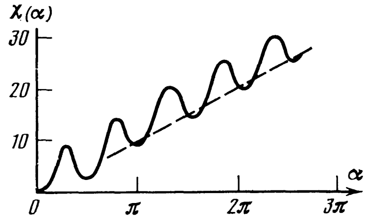

FIG. 2. Radiation intensity directly forward, integrated over frequency, as a function of crystal thickness (positrons, $\theta_x=0$, $\nu_m=10$).

and an oscillatory term $\chi_{osc}(\alpha)$. The main contribution to $\chi_{osc}(\alpha)$ for $\alpha>\pi$ is from the integral of the first term in (15). The scale of the oscillations is determined by the value of ν_m. In the background of the linearly rising component of the radiation, these oscillations should be quite distinct up to a thickness $\sim L_0\nu_m$, which for a limiting threshold frequency ω_m (17) amounts to

$$L_{osc}<L_0(m/\gamma U_0). \tag{18}$$

Near the limit of applicability of the dipole approximation in the energies of the particles ($\gamma\sim m/U_0$ according to Refs. 5–7) the condition (18) is already violated at $L\sim L_0$.

4. RADIATION FROM A DIVERGING BEAM OF PARTICLES IN A THIN CRYSTAL

The oscillations in the differential spectrum of the radiation for an ideally collimated beam (15) die out rapidly with departure from the main frequency. This is a consequence of quantum interference of radiative transitions to a given level from the neighboring upper and lower states, which compensates the intrinsic oscillations of the probabilities of these transitions in the high-frequency region. For particles entering the crystal at angles $\theta_x\sim\theta_L$ to the atomic planes, an inverse dependence of the coefficients $a_s(p_x)$ on s is realized, in which the values of $|a_s|$ for the lower levels turn out to be smaller than for higher levels of the same parity.[15,17,18] In this case in at least one of the terms of the sum (14) the second term will turn out to be substantially less than the first term and for these terms there will be no compensation of the oscillations for $\nu\gg 1$. Since the radiation of different particles entering the crystal at different angles is incoherent, we can expect that in bombardment of a crystal by diverging beams of particles, oscillations in the spectrum of electromagnetic radiation which arises will be observed up to higher frequencies than for $\theta_x=0$.

Let us consider radiation which occurs in passage through a thin crystal of a beam of leptons with angular divergence $\Delta\theta$. In this case the probability (8) must be averaged over the initial momenta of the particles,

$$\int dp_xQ(p_x)\frac{d^2I}{d\omega\theta d\theta}=e^2\omega^2\sum_{s_f,s_i,s_i'}d_{if}d^*_{i'f}\Delta_{if}\Delta^*_{i'f}\Delta E_x^{if}\Delta E_x^{i'f}$$
$$\times\iint_0^b\frac{dx\,dx'}{b^2}\Phi^*_{s_i}(x)\Phi_{s_i'}(x')\int dp_xQ(p_x)e^{ip_x(x-x')}, \tag{19}$$

where $Q(p_x)$ describes the distribution of the particles in transverse momentum in the beam. Here as before we restrict the discussion to transitions only between sub-barrier states, for which the wave functions Φ_s do not depend on the quasimomentum and therefore can be taken outside the integral over dp_x. We shall assume that the particles in the beam are uniformly distributed in momentum in the interval from $-E_i\Delta\theta/2$ to $+E_i\Delta\theta/2$, where $\Delta\theta \gg \theta_L$. Since the wave functions of the sub-barrier states Φ_s change substantially in distances of the order $b/s > (E_i\theta_L)^{-1}$, the integral over dp_x in (19) can be considered proportional to a δ function and the expression (19) is simplified:

$$\left\langle \frac{d^2I}{d\omega d\theta} \right\rangle \sim e^2\omega^2 \frac{2\pi}{bE_i\Delta\theta} \sum_{s_f} \sum_{s_i} |d_{if}|^2 |\Delta_{if}|^2 |\Delta E_x^{if}|^2. \qquad (20)$$

The result (20) essentially means that on averaging over $\Delta\theta > \theta_L$ the quantum interference of transitions between different states is suppressed, and the average probabilities of population of sub-barrier bands $\overline{W}_s = \langle |a_s|^2 \rangle$ turn out to be identical for all s and[18] equal to

$$\overline{W}_s = 2\pi(bE_i\Delta\theta)^{-1}.$$

For ultrarelativistic positrons in a potential (13) the expression (20) becomes

$$\left\langle \frac{d^2I}{d\omega d\theta} \right\rangle \sim e^2\tilde{\omega}^2 \left(\frac{b}{4}\right)^2 \frac{\theta_\pi}{\Delta\theta} \bar{\lambda}(\nu, \alpha), \qquad (21)$$

where

$$\bar{\lambda}(\nu, \alpha) = \frac{\nu^2}{2} \left[\frac{\sin^2\alpha(\nu-1)}{(\nu-1)^2} + \frac{\sin^2\alpha(\nu+1)}{(\nu+1)^2} \right].$$

The radiation spectra (21) for various crystal thicknesses are shown in Fig. 3. For $\alpha > \pi/2$ the principal maxima of the radiation are somewhat shifted from the frequency $\tilde{\omega}$, but the magnitude of the shift does not exceed the half-width of the principal maximum

$$\Delta\omega_0 \sim \tilde{\omega}(L_0/2L).$$

Additional energetic maxima are located approximately equidistant and the distances between them are

$$\Delta\omega \approx \tilde{\omega}(L_0/L). \qquad (22)$$

At high frequencies, $\bar{\lambda}(\nu)$ oscillates between values $\bar{\lambda} = \sin^2\alpha$ and $\bar{\lambda} = \cos^2\alpha$.

It is worthwhile to discuss in more detail the case of radiation from a diverging particle beam in a very thin crystal $L \ll L_0$ (Fig. 3b). In this case the frequency of the principal maximum differs substantially from $\tilde{\omega}$, and the positions of all maxima are determined by the expression

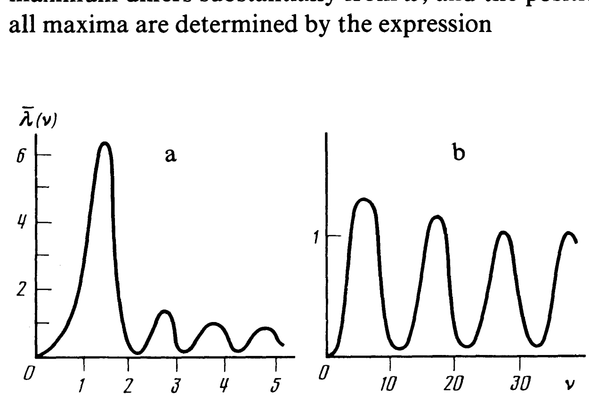

FIG. 3. Radiation spectra for sub-barrier transitions from a diverging beam of positrons in crystals of various thicknesses for $\alpha = 3$ (a) and $\alpha = 0.3$ (b).

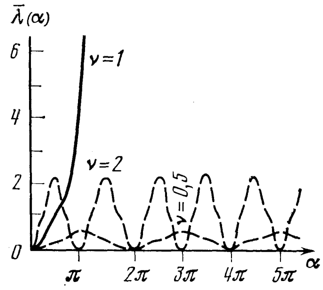

FIG. 4. Radiation intensity of positrons as a function of crystal thickness at various frequencies ($\Delta\theta > \theta_L$, the sub-barrier component).

$$\omega_n = (2\pi\gamma^2/L)(2n+1), \quad n = 0, 1, 2, 3\ldots, \qquad (23)$$

which does not depend on the parameters of the averaged potential. The point is that for crystals with $L \ll L_0$ the factor $|\Delta_{if}|^2$ in (20), which determines the shape of the radiattion spectrum, is approximately

$$(8\gamma^4/\pi\omega^2)\sin^2(L\omega/4\gamma^2)$$

and can be taken outside the summation over the states. In this way for $L \ll L_0$ the shape of the radiation spectrum turns out not to depend on the form of the averaged potential $U(x)$, and this means that it is similar for electrons and positrons. The positions of the maxima in the spectrum (21) for $L \ll L_0$ depend only on L and γ, and their magnitudes depend only weakly on L. It is as if the spectrum (21) is formed as the result of interference of two radiation waves excited when the leptons cross the crystal boundaries, which have approximately identical amplitude and which differ in phase by $\pi + \omega(L/2\gamma^2)$ (here $L/2\gamma^2$ is the time of traversal of the crystal in the rest system of the lepton).

In Fig. 4 we have shown the intensity of radiation directly forward from a diverging beam of positrons at various frequencies, as a function of the crystal thickness. The radiation at the principal frequency $\omega = \tilde{\omega}$ rises in proportion to L^2 as long as the photon detector resolution $\delta\omega$ remains less than the half-width of the principal radiation line $\Delta\omega_0$ For $\delta\omega > \Delta\omega_0$ the detector records the integrated intensity of radiation at all frequencies within the principal line, which rises in proportion to L in accordance with the theory.[3-5,7] The intensity of radiation at frequencies other than $\tilde{\omega}$ oscillates appreciably with change of the crystal thickness. The intensity of electromagnetic radiation directly forward, integrated over frequency, for a divergent positron beam with $\Delta\theta > \theta_L$ rises linearly with the thickness and does not experience significant oscillations comparable in magnitude with the linearly rising component as long as $\nu_m \gg 1$.

It is necessary to mention again that the radiation described in the present section, which arises in passage of divergent beams of relativistic leptons through thin single crystals, is due to transitions only between sub-barrier states. Therefore it can be observed only in the background of the possibly more intense radiation of superbarrier particles, which for $\Delta\theta > \theta_L$ should be greater than that of sub-barrier particles.[6,7,18] Some information on the properties of the radiation of superbarrier particles in thin crystals can be found in Refs. 6 and 12.

5. CONCLUSIONS

It is necessary to estimate the limits of applicability of our results. The approach based on use of the wave functions (1), with the mixing coefficients (2) and integration of the matrix element (7) over the crystal volume, is inapplicable in the case in which the effective length of formation of the radiation l (Refs. 5 and 6) exceeds the crystal thickness. For $l > L$ one cannot neglect the components of the wave functions of the radiating particles which are scattered outside the crystal, and it is necessary to integrate (7) over all space.[5] According to Refs. 5 and 6, for sufficiently high frequencies of radiation $l \sim \gamma^2 \omega^{-1}$. Substituting $\omega = \nu \tilde{\omega}$, we find that the condition $L \gg l$ will be satisfied if

$$L \gg L_0 / 2\pi\nu. \tag{24}$$

For $L \sim L_0$, the condition (24) is satisfied for the energetic ($\omega > \tilde{\omega}$) auxiliary maxima. It is satisfied also for $L \ll L_0$ in the region of the radiation maxima, which occur in this case at frequencies $\omega \gg (2\pi/L)\gamma^2$ given by Eq. (23).

The averaged-potential model was introduced by Lindhard[1] for description of the passage of particles through crystals at angles $\theta_x < \theta_L$ to the atomic planes or axes, although it has been used also for $\theta_x > \theta_L$ (so-called quasi-channeling—see Ref. 10). The averaging (20) over entry angles $\Delta\theta_x \gg \theta_L$ simplifies the expressions for the radiation intensity in the averaged potential of the atomic planes. It is possible, however, that the behavior at high frequencies of the spectra (21) will change in a model which is improved for $\theta_x \gg \theta_L$, which takes into account superbarrier transitions.

In this work it has been assumed that the thicknesses of the crystals investigated $L \sim L_0$ are less than the lengths characteristic of inelastic processes leading to a change of state of the channeled particles. The situation when this is not the case requires special discussion.

The quantitative calculations in this work were carried out in the dipole approximation. In principle the auxiliary maxima in the radiation spectra of leptons in thin crystals should be preserved also in the nondipole case, although the formulas which determine their locations and heights will change quantitatively. It should be noted that the use of the dipole approximation requires special caution in discussion of radiation at sufficiently large angles θ to the forward direction.

A distinctive feature of the spectra of electromagnetic radiation of fast charged particles channeled in thin crystals turns out to be the presence of auxiliary, approximately equidistant interference maxima at frequencies appreciably different from the principal maxima. An important fact is that the positions of these maxima depend substantially on the crystal thickness. In particular, the distances between neighboring auxiliary maxima are inversely proportional to the thickness L, as shown by Eqs. (23) and (22). An attractive idea would be to use this cirmumstance for measurement of the thicknesses of thin single-crystal films, including films which are not transparent for optical radiation.

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[1)]The model problem of radiation accompanying transition of an electron from a free state to a channeled state near a crystal boundary has been discussed in Ref. 13. In application to real processes, however, it is unquestionably necessary to take into account the difference of the initial wave function of the particle from a plane wave, which has correctly been noted in Ref. 14.

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