Mobility of ferromagnetic bodies in a resonant magnetic field

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The purpose of the paper is to draw attention to the existence of a mechanism whereby microwave fields exert a strong influence on the mechanical motions in a system of ferromagnetic particulates. It is shown that, upon the excitation of high-Q ferromagnetic resonances in the particles, the magnetic retardation of their relative motions is replaced by acceleration of appreciable magnitude. Because of this, a microwave field can be used to effect magnetic separation and control the mobility of magnetic bodies. The effect in question exhibits a nonlinear mechanism typical of physical systems of different natures, which makes it possible to effectively control the mobility of bodies, particles, impurities, etc. with the aid of high-frequency fields that excite internal resonances in the mobile objects or resonances in the surroundings. The distinctive features of the mechanism are investigated in the simplest formulation of the problem. The scales of the effect are estimated, and an experiment that can be performed on magnetic substances located in microwave fields is discussed.

I. INTRODUCTION

One of the aims of the present paper is to point out an interesting possibility of overcoming with the aid of high-frequency fields the magnetic cohesion of magnetic particulates that have come together. It is clear that a strong variable magnetic field that deflects the magnetic moments in the particles from their equilibrium positions through angles $\sim\pi/2$ can bring about a situation in which the attractive forces are replaced by repulsive ones. We shall discuss possibilities of another kind, which arise upon the excitation of magnetic resonances in the particles, and can manifest themselves even when the spin-precession angles in the particles are quite small. This force effect is due not to the replacement of attraction by repulsion but to the replacement of magnetic retardation by magnetic acceleration.

The effect in question is by itself interesting, and may be of interest in connection with its possible technological applications, e.g., for the control of the mobility of magnetic bodies and for effecting magnetic separation. It also exhibits a strong and universal nonlinear mechanism that allows the control of the mobility of bodies, particles, impurities, domain walls, etc., with the aid of high-frequency fields that excite internal resonances in the mobile objects or resonances in the surroundings.

Nonlinear interactions (with resonances) of a similar nature within the framework of the vibrational mechanism have been repeatedly considered to one extent or another in the literature before, but in connection with other physical problems. Here, as an example, we can cite the parametric Mandel'shtam-Papaleski motor,[1] whose operation is based on the nonlinear coupling between the rotational motions of a rotor and the resonant stator-excitation system.[1)] Similar vibrational mechanisms can also be perceived in the effects of excitation or suppression of low-frequency waves in dispersive media acted upon by high-frequency resonance fields, in the electromechanical-instability effects occurring in high-frequency cavity resonators, in the effects whereby the radiation damping of an atom changes when it is irradiated by monochromatic light that is in resonance with its internal vibrational modes. Various phenomena of this kind have been considered by many researchers, including the present author. In this connection, in Refs. 2 and 3 we developed an approach to their description which is basically suitable for the range of problems, in question here, concerning the mobility of magnetic bodies in a resonance field, and we shall develop the analysis, using this approach.

In this discussion we shall start by considering the simplest models, and then proceed to make some estimates suitable for the experimental situation of Ref. 4, in which it is noted that, when the ferromagnetic resonance (FMR) in unclamped ferromagnetic samples loosely contained in an ampoule is strongly excited, there occur a low-frequency "dance" of the samples and their separation. The explanation of these phenomena that is offered in Ref. 4, and supplemented in Ref. 5, turns out to be incorrect.[6] It is erroneously claimed in Refs. 4 and 5 that the forces and the moments of the magnetic forces acting on the ferromagnetic substances increase by many orders of magnitude upon the onset of the FMR, which reinforced the erroneous calculations. Here we only note that, in estimating the magnetic forces in the ferromagnetic substances on the basis of the well-known[7] expression for the force **F** acting on a particle with dipole moment **M** in a field **H**:

$$\mathbf{F}=(\mathbf{M}\nabla)\mathbf{H} \tag{1}$$

(or $\mathbf{F}=\nabla(\mathbf{M}\cdot\mathbf{H})$ in the case when $\nabla\times\mathbf{H}=0$), it is absolutely essential that we take the derivatives at fixed **M**. Since **F** in (1) can be regarded as a sum of forces acting on the elements of volume of the particle, **H** can be taken to be an effective internal field. Such a field includes the demagnetizing and anisotropy fields, which depend on the magnetization, which in turn depends on **H**(**x**). But these dependences do not affect **F**, since, in determining the magnetic forces, we should vary the magnetic energy as a function of all the generalized coordinated characterizing the state of the system

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with respect to the virtual displacements, i.e., with the remaining variables, including the spin variables, fixed. The disregard of the fixation condition can lead to appreciable errors, as happens in Refs. 4 and 5.

It is clear from estimates made with the aid of (1) that the orders of magnitude of the forces and the moments of the magnetic forces are not particularly changed under conditions of intense FMR excitation. This applies to the forces in a system of interacting magnetic bodies. As to the possibility of strong mechanical effects, such as the separation of two magnetic bodies, or their dance, it is due to the fact that the magnetic forces become essentially nonpotential under conditions of intense pumping of the FMR. As a result, the potential barriers can easily be surmounted, or there occurs an inverse phenomenon: enhanced damping of the mechanical motions. The pertinent analysis and estimates will be presented below.

ANALYSIS

The mechanism underlying the effect of the FMR on the mobility of magnetic substances is readily elucidated in the simplest case when the microwave field excites one spin mode of low intensity, so that the resonant-oscillation energy is adequately represented in terms of the normal-mode variables c and c^* by the approximation

$$\mathscr{H}=\omega_0 c^* c, \tag{2}$$

where $\omega_0=\omega_0(\mathbf{H})$ is the FMR frequency. Then the component F of the generalized forces that stems from the interaction with the resonance is equal to

$$F=-\frac{\partial\mathscr{H}}{\partial x}=-|c|^2\frac{\partial\omega_0}{\partial x}. \tag{3}$$

Here and below $F=\{F_\alpha\}$, $x=\{x_\alpha\}$ and, correspondingly, $\partial/\partial x\{\partial/\partial x_\alpha\}$. We can, for definiteness, conceive an interrelation between the motions (the translations as a whole and the rotation) of a single-domain ferromagnetic particle in which the ferromagnetic resonance has been excited.

Within the framework of the description of the spin-precession dynamics by the Hamiltonian (2), the quantity $|c|^2$ is an invariant, and does not change when $x(t)$ changes. Consequently, the function

$$U_N(x)=N\omega_0(x), \tag{4}$$

where $N=|c|^2=\text{const}$ plays the role of potential energy for the x motions. In other words, the entire effect of the force interaction with the FMR reduces to the elasticity expressed by the potential (4). Similarly, considering within the framework of the model (2) the deformation of the magnetic substances (substance), along with their motions, and including in the set $x=\{x_\alpha\}$ the amplitudes of the various deformation modes, we arrive at the expression (3) for the corresponding generalized forces, so that the effect of the resonance reduces to the potential (4) in this case as well.[2]

But the situation changes qualitatively if we taken into account the finiteness of the Q of the resonance. We shall assume that the losses are small, so that the magnitude of the Q of the FMR is given by the expression $Q=\omega_0/\gamma\gg 1$, where γ^{-1} is the magnetic-relaxation time, and take account of the pumping of energy into the c mode to compensate the losses through the application of an external variable field of frequency ω close to ω_0 in accordance with the model

$$\dot{c}+[\gamma+i\omega_0(x)]c=he^{-i\omega t}. \tag{5}$$

In the general case the $x(t)$ motions, besides bringing about the modulation $\omega_0(x(t))$, cause the pumping conditions to vary. We shall, for simplicity, neglect this variation here.

We have two typical situations: $\gamma\gtrsim\Omega_*$ and $\gamma\ll\Omega_*$, where Ω_* is a measure of the frequencies of the $\omega_0(t)$ modulation produced by the variation of $x(t)$. In the case when $\gamma/\Omega_*\to 0$ the dissipation and pumping of energy into the c mode over periods of time of the order of the times characterizing the motions of the magnetic substance are insignificant, and we essentially have an adiabatic situation, which is expressed by the result (4).

In the quasistatic limit, i.e., for $\Omega_*/\gamma\to 0, |c|^2\neq\text{const}$ when $x(t)$ varies, and the quantity $\overline{|c|^2}$ follows the course of the resonance curve:

$$\overline{|c|^2}=N_0\left[1+\left(\frac{\omega_0(x)-\omega}{\gamma}\right)^2\right]^{-1}, \tag{6}$$

where N_0 is the value of $\overline{|c|^2}$ at $\omega=\omega_0$; the bar denotes averaging over the period $2\pi/\omega$. We neglect the difference between $\bar{x}$ and x, which is justified when the frequency ω is so high that the amplitudes of the high-frequency jitters $x_\sim=x-\bar{x}$ are small compared to the scales of the characteristic changes in $\omega_0(x)$. For the averaged forces we obtain from (3) and (6) the expression

$$\bar{F}=-\overline{|c|^2}\frac{\partial\omega_0}{\partial x}=-\frac{\partial U_0}{\partial x},$$

where the potential $U_0(x)$ differs significantly from the potential (4), and is equal to

$$U_0(x)=N_0\gamma\,\text{arctg}\,\frac{\omega_0(x)-\omega}{\gamma}. \tag{7}$$

Thus, in this limit the force effect of the resonance reduces to a potential effect. The profiles of the forces characterized by the potentials (4) and (7) in the case when the dependence $\omega_0(x)$ is monotonic in the resonance region are shown in Fig. 1.

Nonpotential corrections to $\bar{F}$ appear when the ratio Ω_*/γ is finite. Thus, we obtain in the case of small Ω_*/γ the approximate expression

$$\bar{F}=-\partial U_0/\partial x-\Gamma\dot{x}, \tag{8}$$

where the matrix $\Gamma=\Gamma(x)$ has the elements

$$\Gamma_{\alpha\beta}=\frac{\partial\omega_0}{\partial x_\alpha}\frac{\partial\omega_0}{\partial x_\beta}\frac{2\xi}{(1+\xi^2)^2}\frac{P}{\omega_0\gamma^3}. \tag{9}$$

The parameters ξ and P are equal to

$$\xi=[\omega_0(x)-\omega]/\gamma,\quad P=2\gamma\omega_0\overline{|c|^2}$$

and are respectively the dimensionless detuning of the resonance and the power scattered by the magnetic system in the steady-state FMR regimes for a given $x=\text{const}$; the quantity $\overline{|c|^2}$ is given by (6). The approximation is in fact equivalent

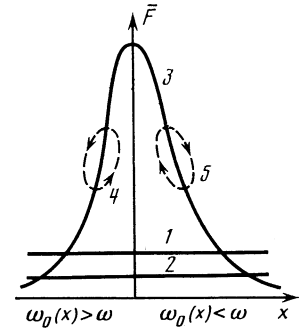

FIG. 1. The forces $\bar{F}$ for different natures of the x motion. The curves 1 and 2 depict the forces in the adiabatic limit for two different initial conditions; 3) the forces in the quasistatic limit; 4) and 5) the forces that obtain in periodic x motions of frequency $\Omega \ll \gamma$ in regimes that occur on the $\omega_0(x) > \omega$ and $\omega_0(x) < \omega$ sides of the resonance.

to the computation of the forces $\bar{F}(t)$, (3), using the $\overline{|c|^2}$ values given by (6), with x taken at the instant $t - t_0$, where the delay time $t_0 = 2(1 + \xi^2)^{-1}\gamma^{-1}$.

These results are easily arrived at if account is taken of the fact that the solution (5) can, in the case when $x(t)$ varies smoothly, be represented in the form.

$$c(t)=e^{-i\omega t}\left[\frac{d}{dt}+D(t)\right]^{-1}h$$
$$=e^{-i\omega t}\int_{-\infty}^{t}\exp\left[-\int_{\tau}^{t}D(\tau')d\tau'\right]h\,d\tau$$
$$=e^{-i\omega t}\left(1-D^{-1}\frac{d}{dt}+D^{-1}\frac{d}{dt}D^{-1}\frac{d}{dt}-\ldots\right)D^{-1}h, \qquad (10)$$

where

$$D=D(t)=\gamma[1+i\xi(x(t))].$$

The expression on the right hand side of (10) is a power series in the parameter $\Omega_*/|D|$, and, up to terms $\sim\Omega_*^2/|D|^2$, we obtain (8).

Since, according to (9), $\Gamma_{\alpha\beta} = \Gamma_{\beta\alpha}$, the forces $-\Gamma\dot{x}$ are frictional forces that are linear in the velocity. All the eigenvalues of the matrix Γ change sign simultaneously when the sign of the detuning ξ is changed, since, for arbitrary $\omega_0(x)$ functions and arbitrary real $y = \{y_\alpha\}$, the quadratic form

$$\sum_{\alpha,\beta}\frac{\partial\omega_0}{\partial x_\alpha}\frac{\partial\omega_0}{\partial x_\beta}y_\alpha y_\beta=\left(\sum_{\alpha}\frac{\partial\omega_0}{\partial x_\alpha}y_\alpha\right)^2\geqslant 0. \qquad (11)$$

When $\xi > 0$ we have retardation; when $\xi < 0$, acceleration: a buildup of the x motions. We can elucidate the physics of such behavior by referring to Fig. 1. As a result of the retardation of the c-oscillation mode, in response to the variation of x, the contour 5 has a clockwise direction of circulation, while the contour 4 has an anticlockwise direction. The work done by the forces $\bar{F}$ over a period of the $x(t)$ variations, which is equal to the area under the contour, is negative when $\omega_0(x) > \omega$, and leads to the quenching of the x oscillations, but positive when $\omega_0(x) < \omega$, and leads to the buildup of these oscillations. This result does not depend on the sign of $\partial\omega_0/\partial x$ (in Fig. 1 the reversal of the sign of $\partial\omega_0/\partial x$ is equivalent to the reversal of the signs of both x and $\bar{F}$).

The magnitude of Γ (the eigenvalues) on the steep sections of the resonance curve increases, as compared to the nonresonance case, like Q^3 for a given P level (which, for $Q = 10^3$, comes up to 10^9!).

As Ω_* increases, the magnetic-friction coefficient Γ becomes frequency dependent. We can form an idea about this by analyzing the response of the resonance to weak perturbations $\delta x = x - x^0$ of the $x = x^0 = \text{const}$ state. From (5) we obtain for the response in the case when $\delta x \to 0$ the expression

$$\delta c(t)=-i\sum_{\alpha}\frac{\partial\omega_0}{\partial x_\alpha}\int_{-\infty}^{t}\exp\{-(\gamma+i\omega_0)(t-\tau)\}c(\tau)\delta x_\alpha(\tau)d\tau, \qquad (12)$$

where in the integrand $\omega_0 = \omega_0(x^0)$ and $c(\tau) = e^{i\omega\tau}D^{-1}h$ is the steady-state c-oscillation mode for $\delta x = 0$. From (3) and (12) we obtain for the averaged response of the magnetic forces to the perturbations δx, which can be represented in the spectral form

$$\delta x(t)=\int_{-\infty}^{\infty}e^{-i\Omega t}\delta x(\Omega)d\Omega,$$

the expression

$$-\delta\bar{F}(t)=\bar{F}|_{\delta x=0}-\bar{F}$$
$$=K\delta x(t)+\int_{-\infty}^{\infty}e^{-i\Omega t}[\Lambda(\Omega)-i\Omega\Gamma(\Omega)]\delta x(\Omega)d\Omega, \qquad (13)$$

where the first term on the right-hand side is the elasticity of the magnetic forces in the adiabatic approximation, which quantity can be expressed in terms of a potential:

$$K_{\alpha\beta}=\frac{\partial^2}{\partial x_\alpha\partial x_\beta}U_{\bar{N}}, \qquad U_{\bar{N}}=\bar{N}\omega_0(x);$$

here $\bar{N}$ is given by the expression (6) with $x = x^0$. The integral in (13) is due to the response of the FMR, i.e., it represents recoil forces. The matrices $\Lambda(\Omega)$ and $\Gamma(\Omega)$ are given by the expressions

$$\Lambda_{\alpha\beta}(\Omega)=-\frac{\overline{|c|^2}}{\gamma}\frac{\partial\omega_0}{\partial x_\alpha}\frac{\partial\omega_0}{\partial x_\beta}\left[\frac{\xi_+}{1+\xi_+^2}+\frac{\xi_-}{1+\xi_-^2}\right],$$
$$\Gamma_{\alpha\beta}(\Omega)=\frac{\overline{|c|^2}}{\gamma^2}\frac{\partial\omega_0}{\partial x_\alpha}\frac{\partial\omega_0}{\partial x_\beta}\left[\frac{\gamma/\Omega}{1+\xi_+^2}-\frac{\gamma/\Omega}{1+\xi_-^2}\right], \qquad (14)$$

where we have introduced the notation $\xi_\pm = [\omega_0 - (\omega \pm \Omega)]/\gamma$. The matrices $\Lambda(\Omega)$ and $\Gamma(\Omega)$, being real, symmetric and even in Ω, have respectively the meaning of coefficients of elasticity and friction for the magnetic forces produced by the reaction of the FMR regime in response to weak harmonic x perturbations of frequency Ω. It is characteristic that in (14), the expressions in the square brackets change sign when the sign of ξ is changed and the factors attached to them from matrices with nonnegative eigenvalues (on account of the fact that the quadratic form (11) is of fixed sign). Therefore, the eigenvalues of $\Lambda(\Omega)$ and $\Gamma(\Omega)$ are all of one, or the other, sign, depending on the tuning of the FMR.

The positiveness of the eigenvalues of Λ implies that the forces that are proportional to Λ in (13) strive to return the system to the initial state x-x^0. The friction matrix $\Gamma(\Omega)$, like its limiting values Γ, (9), for $\Omega \to 0$, represents, when $\omega_0 \to \omega$,

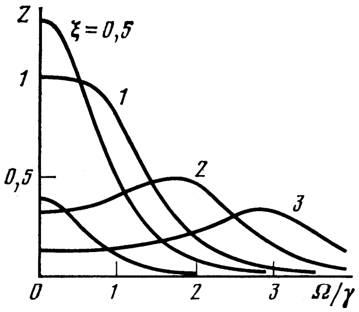

FIG. 2. Behavior of the friction coefficient F as a function of Ω and ξ in the case when $P = \text{const}$ (lowest curve: $\xi = 0.1$).

forces that damp the perturbations and, when $\omega_0 < \omega$, forces that lead to their buildup. The Ω dependence of the eigenvalues of Γ is duplicated by the behavior of the quantity.

$$Z(\xi, \Omega/\gamma) = \frac{\gamma/\Omega}{1+\xi_+^2} - \frac{\gamma/\Omega}{1+\xi_-^2} = \frac{4\xi}{(1+\xi_+^2)(1+\xi_-^2)}, \tag{15}$$

which is depicted in Fig. 2. The expression (15) attains its maximum at $\Omega/\gamma \to 0$ and $\xi = 1/\sqrt{3}$, where $Z = Z_{\max} = 3\sqrt{3/4}$.

Notice that the model (2), which forms the basis of the description of the interaction with the resonance, does not take account of all the possible modes of coupling between c and x, even without the framework of the single-mode, linear—in the c oscillations—magnetic system (with $x = \text{const}$): the variables c and c^*, having been chosen as the normal-mode variables for some geometry $x = x^0$, cease in the general case to be such variables for $x \neq x^0$, so that instead of (2) we can have

$$\mathscr{H} = \omega_0 c^* c + (vc + v^* c^*),$$

where v is a function of x, that vanishes at $x = x^0$. We are, however, investigating the conditions under which the subsystem x is sufficiently inertial, so that the high-frequency jitters $x_\sim = x - \bar{x}$ are negligibly small in comparison with the other characteristics—for the problem—scales. In this case the effect of the additional terms in $\mathscr{H}$ on the renormalization of the elasticity and friction at resonance is insignificant. This, for example, can easily be followed by approximating v by a linear—in x—function. Then, by carrying out a linear transformation of the variables c, c^*, and x, we can get rid of these terms in $\mathscr{H}$. As for the renormalization of the parameters that results from the transformation, it is insignificant when the frequencies of the x and c motions are widely spaced, and the oscillations $x_\sim$ are relatively small: we essentially have the previous picture of force interactions with the resonance.

ESTIMATES

Let us briefly sketch a picture of the motions of a magnetic particle in a potential well under the action of a microwave FMR pump. Keeping ourselves within the framework of the above-described single-mode model for the magnetic resonance, we have, for $\Omega_* \ll |D|$, the following equations describing the mechanical motions:

$$m\ddot{x} + \partial U/\partial x = -\Gamma\dot{x}. \tag{16}$$

Here $U = U_0(x) + U_1(x)$, where the potential $U_1(x)$ represents the forces in the absence of the pump field; $U_0(x)$ and $\Gamma(x)$, the same quantities figuring in (8), are proportional to the power P scattered by the magnetic substance; and m is the inertia matrix.

When $\omega_0(x) > \omega$, the forces $-\Gamma\dot{x}$ retard the motions, and the particle will settle at the minimum of the potential U. When $\omega_0(x) < \omega$, the friction coefficient Γ is negative, and the particle will try to get out of the potential wells no matter what shape they have. The eigenvalues of the matrix $T = m\Gamma^{-1}$ determine the times scales involved in the development of the instability and, consequently, the time it takes the particle to get out of the potential wells.

Notice that the particle under consideration can be a "composite" one, i.e., a system of magnetic bodies whose geometry is described by the coordinates $x = \{x_\alpha\}$, and whose dynamics is sufficiently inertial, so that $\Omega_* \ll |D|$. As the inertia coefficients m decrease and the steepness of the potential relief $U_1(x)$ increases, the frequencies Ω_* increase, and the approximation can lose its validity. For frequencies Ω_* that are high compared to $|D|$, we approach the adiabatic situation, and the probability of overcoming the corresponding potential barriers falls. Let us note further that, within the framework of the equations (16), the effect is described in its pure form, no account being taken of the mechanical friction. The latter leads to the appearance of a threshold for the power P, above which the forces $-\Gamma\dot{x}$ are, under conditions of favorable FMR tunings, not compensated by the mechanical friction, and instability develops.

Let us estimate the time scale T characterizing the separation of two magnetic spheres under the conditions, indicated in Ref. 4, of FMR excitation. The spheres, which were YIG pellets with diameter $d \lesssim 1$, magnetization parameters $4\pi m_0 = 1750$ G, and density $\rho = 5.17$ g/cm^3, were loosely enclosed in a glass ampoule located in the antinode region of a magnetic field of frequency $\omega = 9.5$ GHz. A constant magnetic field of intensity $H_0 = 3.3 \times 10^3$ Oe was applied in the direction perpendicular to the microwave field, and the FMR region for the samples was traversed by slowly varying this constant field. As the FMR was approached from the region of low H_0 values, the spheres were observed to separate and dance with a frequency of several hertz. The magnetic resonance excited in the samples was nearly homogeneous, and we shall not take the other spin modes into consideration in the estimates.

Choosing as x the distance R between the centers of the spheres, we find that the parameter $(\partial\omega_0/\partial x)/\omega_0$ figuring in (9) is $\sim 3/R$, since the dominant contribution to it is made by the dependence of ω_0 on the field induced by one sphere in the other, a field which varies like R^{-3} (in the dipole approximation). As a result, if we use the above-indicated numerical data, and take $R = d = 1$ *mm and* $m = (\pi/3)d^3\rho$, then we obtain in the case when the power of $P = 10^{-3}$ W and we have the optimal FMR tuning $\xi = 1/\sqrt{3}$ the order-of-magnitude estimate

$$T \sim \frac{(1+\xi^2)^2}{2\xi} \frac{R^2}{9} \frac{\gamma^3 m}{\omega_0 P} \sim 2\,\text{sec}.$$

For powers in the region of tenths of a watt, the time T lies in the region of hundredths of a second. No values are given in Ref. 4 for the power P and the time T, but the estimates obtained are apparently characteristic of experiment, judging by the figures given for the total microwave power (it was largely absorbed by the employed microwave cacity resonator, whose Q value was $\sim 10^3$, which was more than an order of magntiude higher than the Q value for the FMR) and the resonance-traversal speed of the H_0 field.

CONCLUSION

We have considered in the present paper the characteristics of the effect of ferromagnetic resonance on the mobility of magnetic bodies, using a small number of parameters and simple mathematical tools. The main idealization involved here lies in the assumption that the resonance subsystem c is single-mode linear (for $x = \text{const}$) system. It is significant that we obtain in the case of a multimode system, when, instead of (2), we take as the basis of the description the model

$$\mathscr{H} = \sum_{i,k} \omega_{ik}(x) c_i^* c_k,$$

matrix analogs—in the indices of the c variables—of the above-obtained expressions, and are able to establish a number of relatively simple general laws governing the effect of the resonances on the mobility. These analogs and the role of the nonlinearity of the resonances will be considered elsewhere. Here we only note that the above-considered single-mode model gives an ideal about the scales of the changes that occur in the magnetic friction in the case of slightly nonlinear magnetic resonances. But the limits of variation of the magnetic friction can change significantly under conditions of intense excitation of the resonances, when their anharmonicity appears (in the $x = \text{const} =$ case). By our estimates, the anharmonicity, which can be modeled by including terms of fourth order in c in the Hamiltonian $\mathscr{H}$, can, in the case when the anharmonicity terms have the right sign, lead to a mobility that is many orders of magnitude higher than the estimates given in the text for the same level of P in a certain, albeit narrow, region of resonance detunings. Also effective for the selective control of the mobility of magnetic substances are the regimes in which we have paramagnetic pumping of the magentic resonances, when these resonances have low thresholds.

Notice that the effect of the resonant fields on the mobility of small objects moving in viscous media e.g., the control by a high-frequency field of the structure of a ferroliquid, may, on the face of it, seem ineffective, since the magnetic forces, like body forces, decrease with decreasing body dimensions d much faster than the Stoke viscous forces, which decrease linearly with d. But the magnetic friction coefficient Γ can decrease relatively slowly. According to (9), the dependence on d enters into the expression for Γ largely through the factor

$$\left(\frac{1}{\omega_0}\frac{\partial\omega_0}{\partial x}\right)^2 P \sim \frac{1}{l^2} P,$$

where the inhomogeneity scale l can be of the order of d (as in the above-considered example of two magnetic spheres). The admissible values of the power P dissipated in the particle do not decrease faster than d^3: in the case of normal heat transfer through the surface, it decreases like d^2. Therefore, the quantity Γ can decrease even more slowly than the Stokes forces.

Concerning the other objects, let us note that the approach described in the present paper can easily be used to estimate the effect of resonant magnetic fields in polydomain samples on the mobility of the domain walls. In the case of high-grade samples possessing a high-Q spin system, we can expect the effect of the resonances in the domains or walls on the mobility of the latter to be appreciable. The experimental observation of an increase in the mobility of domain walls under resonance conditions is reported in Ref. 8, but the mechanisms underlying this increase are not considered in detail, and there may be other factors here.[3)] The mechanism considered in the present paper is, possibly, also of interest in connection with the establishment of ordered motions of impurities and dislocations in substances.

[1)] In the light of the effects under discussion, the operation of such a motor in the case when $\omega \gg \Omega_\bullet$ (where ω is the frequency of the power supply and $\Omega_\bullet$ characterizes the smoothness of the variations of the angle $x(t)$ of rotation of the rotor) can be described as the continuous overcoming by the coordinate x of a periodic potential relief with a pole period, that occurs as a result of the presence of appreciable negative friction for the rotational motions, which friction is produced by the nonlinear interaction with the resonance.

[2)] Strictly speaking, the expression (2)–(4) and the subsequent ones do not reflect the characteristics of the interactions that expressly involve the magnetic resonance, and we arrive at the same conclusions when we analyze interactions with resonances of a different nature.

[3)] One of these factors is the vibrotranslation mechanism, similar to the well-studied mechanism in mechanics,[9] in which the "dry" friction is, as it were, converted into viscous friction. In such a mechanism, in contrast to the one under discussion, the overcoming of the potential barriers occurs as the result of the increase of the level of the high-frequency vibrations $x_\sim = x - \bar{x}$, and not as the result of the buildup of the slow $\bar{x}$ motions. The role of high-frequency resonance then amounts simply to this: It brings about an increase in the $x_\sim$ level. The sign of the effect does not depend on the sign of the resonance detuning, and its effectiveness rapidly decreases with increasing frequency ω of the influences.

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