Dynamic damping of domain wall in a rhombic ferromagnet

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The dynamic damping of a domain wall in a rhombic ferromagnet having an energy that is exactly integrable in the one-dimensional case is investigated. The loss of exact integrability in the three-dimensional case is analyzed. It is shown that the wall in a three-dimensional magnet dissipates energy via two- and three-magnon processes. The contribution of the magnon scattering processes has a nonlinear dependence on the velocity v. The three-magnon damping force in the three-dimensional case is proportional to the velocity and contributes to the wall mobility. The general picture of wall relaxation at different temperatures is analyzed. The dependence of the velocity of the induced boundary motion on the external field is discussed. It is shown that this dependence is strongly nonlinear and has a quasi-saturation character at the velocity lower than the Walker limit. The experimental consequences of this fact are discussed.

INTRODUCTION

When it comes to describing the dynamics of real magnetically ordered crystals, the first question is that of the laws governing the motion of the domain walls (DW) and of solitary magnetic domains. Interest in this question has become particularly great recently. This is due, first, to the fact that in the theoretical description the DW constitutes a solitary wave of the magnetization field (magnetic soliton) and the study of DW dynamics is a basis for the development of soliton theory.[1] Second, this question is timely from the practical point of view in connection with the use of the rather high-velocity DW in various devices.[2,3]

In the most interesting case of high DW velocity in perfect magnets, the principal mechanism that determines the velocity of the stimulated motion of the DW is dynamic damping of DW by interaction with thermal quasiparticles, mainly magnons. The soliton character of the DW manifests itself in many models of magnets[1] by nonreflecting interaction of the magnons with the wall and leads to a distinct behavior of the DW relaxation properties. It was shown for a model of a uniaxial ferromagnet with simplest quadratic anisotropy ($w_a = \beta(M_x^2 + M_y^2)/2$, Ref. 4), the soliton character of the DW leads to a strongly nonlinear dependence of the magnon damping force on the wall velocity. Exact integrability manifests itself also in the kinetic properties of the model described by the sine-Gordon equation.[5]

In the present study we investigate dynamic damping of DW in a rhombic ferromagnet, with energy given by

$$W\{\mathbf{M}\}=\frac{1}{2}\int d\mathbf{r}\{\alpha(\nabla\mathbf{M})^2+\beta(M_x^2+M_y^2)+\rho M_x^2\}. \quad (1)$$

From the theoretical viewpoint this model is of interest because in the one-dimensional case it can be exactly integrated by the method of the inverse scattering theory.[6] It permits also a fairly complete investigation of the dynamics in the three-dimensional case. In particular, in the model (1) it is possible to describe consistently the changes of the wall structure as the wall moves, and also to analyze the magnons that are localized near the DW and describe the flexural oscillations of the DW. We shall show below that these magnons give a fairly large (and in some cases the decisive) contribution to the damping force.[1)] In particular, owing to the exact integrability of the system (1) in the one-dimensional case, the amplitude of magnon scattering by a DW in a real three-dimensional case is proportional to $\mathbf{k}_\perp^2$, where $\mathbf{k}_\perp$ is the magnon-momentum component in the DW plane, and differs from zero only for oblique incidence of the magnons. It is thus possible in this model to track the loss of total integrability on going to the three-dimensional case and to analyze the ensuing dissipative processes.

On the other hand, in view of the large maximum DW velocity and the absence of DW twisting, films with strong anisotropy of type (1) ($4\pi \ll \rho \ll \beta$) are vital for use in devices with magnetic bubble domains.[3] We investigate here the damping of DW by interaction with thermal magnons. We calculate the contribution of two- and three-magnon processes (with localized magnons taken into account) to the dynamic damping of DW. We discuss the overall picture of DW relaxation and the dependence of the stimulated velocity on the external field.

1. SMALL MAGNETIZATION OSCILLATIONS IN A MAGNET WITH A MOVING DOMAIN WALL

To describe the magnon damping of DW, we consider small magnetization oscillations against the background of a DW moving with velocity v. We represent for this purpose $\mathbf{M}(\mathbf{r},t)$ in the form $\mathbf{M}_0(\mathbf{r}-\mathbf{v}t)+\mathbf{m}(\mathbf{r},t)$. The first term describes the DW. It is convenient to consider the components of the vector $\mathbf{m}$ in a coordinate frame in which the magnetization quanitization axis $\mathbf{e}_3$ coincides with the direction of $\mathbf{M}_0(\mathbf{r}-\mathbf{v}t)$:

$$\begin{aligned} \mathbf{e}_1&=\mathbf{e}_x\cos\varphi+\mathbf{e}_y\sin\varphi,\\ \mathbf{e}_2&=\cos\theta(-\mathbf{e}_x\sin\varphi+\mathbf{e}_y\cos\varphi)-\mathbf{e}_z\sin\theta, \quad (2)\\ \mathbf{e}_3&=\sin\theta(-\mathbf{e}_x\sin\varphi+\mathbf{e}_y\cos\varphi)+\mathbf{e}_z\cos\theta. \end{aligned}$$

Here θ and φ are the angle variables that determine the magnetization distribution in the moving DW. This representation is convenient, for if θ and φ are solutions of the Landau-Lifshitz equations for $\mathbf{M}_0$, the dynamics of the variables m_i in terms of the Holstein-Primakoff operators a^+ and a is determined by a Hamiltonian that does not contain

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terms linear in a^+ and a (Ref. 4). This Hamiltonian can be written in the form

$$H=\int d\mathbf{r}\left\{\frac{\hbar}{2\mu_0}\left[(M_3\cos\theta-M_2\sin\theta)\frac{\partial\varphi}{\partial t}-M_1\frac{\partial\theta}{\partial t}\right]\right\}+W\{M_i\}.$$

The first term in this equation stems from the fact that by virtue of (2) the transformation from **M** to M_i depends explicitly on the time. $W\{M_i\}$ is the energy of the magnet (1), expressed in terms of the components M_i. The energy $W\{M_i\}$ is of the form

$$W\{M_i\}=\int d\mathbf{r}\{{}^1/_2\alpha(\nabla M_i)^2+{}^1/_2\alpha(\nabla\theta)^2(M_2^2+M_3^2)$$
$$+\alpha\nabla\theta(M_3\nabla M_2-M_2\nabla M_3)$$
$$+{}^1/_2\beta[M_0^2-(M_3\cos\theta-M_2\sin\theta)^2]$$
$$+{}^1/_2\rho[M_1\cos\varphi-(M_2\cos\theta+M_3\sin\theta)\sin\varphi]^2\}. \qquad (3)$$

In this equation we have taken it into account that in a moving DW we have $\varphi=\text{const}$ in accord with the Walker solution (see, e.g., Ref. 1), and have left out the terms with $\nabla\varphi$. The last term of this equation is due to the rhombic anisotropy and determines the difference between our problem and the model of Ref. 4.

In terms of the Holstein-Primakoff operators a^+ and a (see Ref. 7), the Hamiltonian (3) contains terms of second, third, etc. powers of these operators. Since $M_1, M_2 \propto a, a^+$, the contribution to the quadratic Hamiltonian is made by terms of the type M_1M_2, M_1^2, M_2^2, and M_0–M_3. An important circumstance that determines the specifics of a rhombic ferromagnet is the presence in $W\{M_i\}$ of a term of the form $\rho M_1M_2\cos\theta\sin\varphi\cos\varphi$. In a DW, the quantity $\cos\theta$ is not a localized function of the coordinates:

$$\cos\theta=\text{th}\frac{(x-vt)}{x_0(v)},\qquad x_0(v)=\left(\frac{\alpha}{\beta}\right)^{1/2}\left[1+\frac{\rho}{\beta}\sin^2\varphi\right]^{1/2}$$

[we assume that the DW moves along the x axis; $x_0(v)$ is the thickness of the DW, and $\varphi=\varphi(v)$]. Consequently, on going over to the spin-wave operators a_k^+ and a_k with the aid of the momentum representation [see Eq. (10) below] this term introduces into the amplitude of magnon scattering by a DW a singular part with a pole at $q\to 0$, where q is the momentum transfer. This means that even though this term contains the small factors ρ and $\sin\varphi$ (we recall that $\varphi\propto v/v_W$ at $v\ll v_W$, where $v_W=\rho\mu_0M_0x_0/\hbar$ is the Walker limiting DW velocity), its contribution cannot be analyzed by perturbation theory. Analysis below shows that this term makes a substantial contribution to the two-magnon damping force, and that this contribution does not contain the small factor ρ. The contribution of this term is therefore non-analytic in ρ and cannot be obtained by summing a finite number of terms of a perturbation-theory series.

For a correct analysis of this term we carry out a unitary transformation of the operators M_1 and M_2. We introduce the operators $\tilde{M}_1$ and $\tilde{M}_2$:

$$\tilde{M}_1+i\tilde{M}_2=(M_1+iM_2)e^{i\Phi} \qquad (4)$$

and choose Φ in such a way that the energy (3) does not contain dangerous terms of the type indicated. Corresponding to this condition is

$$\text{tg}\,\Phi=\text{tg}\,\varphi\cos\theta. \qquad (5)$$

With this choice, the last term of the energy (3) takes the form

$${}^1/_2\rho\{\tilde{M}_1(\cos^2\varphi+\sin^2\varphi\cos^2\theta)^{1/2}-M_3\sin\varphi\sin\theta\}^2. \qquad (6)$$

The transformation (4) corresponds to rotation of the system of unit vector (the reference frame) about the $\mathbf{e}_3$ axis. Thus, all three parameters, θ, φ, and Φ (which can be expressed in terms of the Euler angles that determines uniquely the position of the reference frame relative to the initial $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$) have been determined for a rhombic ferromagnet from physical considerations.

Since the angle Φ depends on the time explicitly, the dynamics of the operators $\tilde{M}_i$ is described by a Hamiltonian that differs from the initial one by the term

$$\frac{\hbar}{2\mu_0}\int d\mathbf{r}(M_0-M_3)\frac{\partial\Phi}{\partial t}. \qquad (7)$$

The energy (3) expressed in terms of $\tilde{M}_i$ contains a number of terms with $\nabla\Phi$; we shall write out only those which contribute to the two-magnon Hamiltonian, e.g.,

$$\alpha\nabla\Phi(\tilde{M}_1\nabla\tilde{M}_2-\tilde{M}_2\nabla\tilde{M}_1)+{}^1/_2\alpha(\nabla\Phi)^2(\tilde{M}_1^2+\tilde{M}_2^2). \qquad (8)$$

This term is the result of a unitary transformation that removes the dangerous terms $M_1M_2\cos\theta$. Since

$$\nabla\Phi=-\nabla\theta\sin\theta\sin\varphi\cos\varphi(1-\sin^2\varphi\sin^2\theta)^{-1},$$

i.e., $\nabla\Phi$ decreases rapidly far from the DW, the term (8) can be investigated by standard perturbation theory for the interaction of a soliton with magnons.[4,5] We take the small parameter to be the ratio of the DW velocity v to the limiting Walker value v_W, in which case $\sin\varphi\approx\varphi\approx -v/2v_W$. We assume hereafter that $\rho\ll\beta$; this inequality holds in the case of greatest interest for practice, that of iron-garnet films in which $\rho<0.1\beta$ (Ref. 3).

2. MAGNON SCATTERING FROM A DW AND ITS CONTRIBUTION TO DYNAMIC DAMPING

Proceeding to the spin-wave description of the dynamics of a ferromagnet, we represent its Hamiltonian in the form

$$H=H_0+H_2+H_3+\ldots,$$

where H_n contains the product of n operators a and a^+. In the present section we study the contribution made to the damping force by two-magnon processes described by H_2.

Following the indicated transformations and simplifications that follow from the inequality $v\ll v_W$, we get for H_2

$$H_2=\varepsilon_0\int dr\left\{a^+\hat{L}a+\frac{\rho}{4\beta}(a^+a^++aa)+\frac{i\varphi x_0}{\text{ch}^2\xi}\left(a^+\frac{\partial a}{\partial x}-a\frac{\partial a^+}{\partial x}\right)\right.$$
$$\left.+\frac{\varphi^2}{\text{ch}^2\xi}\frac{2\rho}{\beta}\left[a^+a\left(-\frac{3}{2}+\xi\,\text{th}\,\xi+\frac{\beta}{2\rho}\frac{1}{\text{ch}^2\xi}\right)-\frac{1}{4}(a^+a^++aa)\right]\right\},$$
$$\xi=(x-vt)/x_0,\qquad \varepsilon_0=\hbar\omega_0=2\mu_0\beta M_0. \qquad (9)$$

The operator $\hat{L}$ has the form of a Schrödinger operator with nonreflecting potential

$$\hat{L}=-x_0^2\Delta+1+\rho/2\beta-2/\text{ch}^2\xi.$$

We expand the operators a and a^+ in terms of the eigenfunctions of the operator $\hat{L}$ (Ref. 8)

$$a(\mathbf{r})=\sum_k \psi_k a_k + \sum_{k_\perp} \psi_{k_\perp} a_{k_\perp}, \tag{10}$$

where

$$\hat{L}\psi_k = A_k\psi_k, \qquad \hat{L}\psi_{k_\perp}=A_{k_\perp}\psi_{k_\perp},$$
$$\psi_k=\frac{\operatorname{th}\xi - i\varkappa}{[\Omega(1+\varkappa^2)]^{1/2}}e^{i\mathbf{kr}}, \qquad \psi_{k_\perp}=\frac{1}{(2Sx_0)^{1/2}\operatorname{ch}\xi}e^{i\mathbf{k}_\perp \mathbf{r}_\perp}. \tag{11}$$

Here Ω is the volume of the ferromagnet, S the DW area, $\varkappa = x_0 k_x$,

$$A_k=1+\rho/2\beta+x_0^2\mathbf{k}^2, \qquad A_{k_\perp}=\rho/2\beta+x_0^2\mathbf{k}_\perp^2,$$

and $\mathbf{k}$ the magnon momentum. The wave functions ψ_k and $\psi_{k\perp}$ describe the volume and surface magnons, and $\mathbf{k}_\perp=(0,k_y,k_z)$.

Since the transformation (10) depends explicitly on the time, the Hamiltonian $\widetilde{H}_2$, which describes the dynamics of the operator a_k, contains, compared with (9), an additional term of the form[4]

$$\sum_{k,k'}\int d\mathbf{r}\frac{\partial\psi_{k'}^*}{\partial t}\psi_k a_{k'}^+ a_k.$$

Only this term determined the damping force in the uniaxial model of a ferromagnet.[4] In our problem, however, a contribution of the same order is made by the terms with φ and φ^2 in (9). A consistent allowance for all these terms leads in fact to the result indicated above, viz., to vanishing of the amplitude of the magnon scattering by a DW in the one-dimensional case.

In the representation (10), the two-magnon Hamiltonian takes the form

$$\widetilde{H}_2=H_0+H_2^{(1)}+H_2^{(2)}.$$

The Hamiltonian H_0 determines the magnon states adjusted to the specified position of the wall

$$H_0=\varepsilon_0\sum_k\left\{A_k a_k^+ a_k+\frac{B}{2}(a_k^+a_{-k}^+ + a_k a_{-k})\right\}$$
$$+\varepsilon_0\sum_{k_\perp}\left\{A_{k_\perp}a_{k_\perp}^+a_{k_\perp}+\frac{B}{2}(a_{k_\perp}^+a_{-k_\perp}^+ + a_{k_\perp}a_{-k_\perp})\right\}, \tag{12}$$

where A_k and $A_{k\perp}$ are given by (11) and $B=\rho/2\beta$.

This Hamiltonian can be diagonalized by the standard u–v transformation and reduced to the form

$$H_0=\sum_k \varepsilon_k b_k^+ b_k + \sum_{k_\perp}\varepsilon_{k_\perp} b_{k\perp}^{\pm} b_{k\perp}, \tag{13}$$

where ε_k and $\varepsilon_{k\perp}$ are respectively the energies of the volume and surface magnons, $\varepsilon_k=(A_k^2-B^2)^{1/2}$:

$$\varepsilon_k=\varepsilon_0[(1+x_0^2k^2)(1+\rho/\beta+x_0^2k^2)]^{1/2},$$
$$\varepsilon_{k_\perp}=\varepsilon_0 x_0|\mathbf{k}_\perp|[\rho/\beta+x_0^2k_\perp^2]^{1/2}. \tag{14}$$

The terms $H_2^{(1)}$ and $H_2^{(2)}$ determine the inelastic transitions between these states. We shall verify later that these terms make equal contributions to the damping force in term of the parameter $v/\omega_0 x_0$. For $H_2^{(1)}$ in terms of b_k and $b_{k\perp}$ it is easy to obtain

$$H_2^{(1)}=\sum_{1,2}\left\{U^{(1)}(1,2)b_1^+b_2+[\Psi^{(1)}(12)b_1^+b_2^+ + \text{H.c.}]\right\}$$
$$+\sum_{1,2_\perp}\{V^{(1)}(1,2_\perp)b_1^+b_{2\perp}+\Phi^{(1)}(1,2_\perp)b_1^+b_{2\perp}^+ + \text{H.c.}\}. \tag{15}$$

The amplitudes $U^{(1)}, V^{(1)}, \Psi^{(1)}$, and $\Phi^{(1)}$ are defined as

$$U^{(1)}(1,2)=R(1,2)(u_1u_2-v_1v_2),$$
$$V^{(1)}(1,2_\perp)=P(1,2_\perp)(u_1u_{2\perp}-v_1v_{2\perp}),$$
$$\Psi^{(1)}(12)=R(1,-2)(u_1v_2-u_2v_1),$$
$$\Phi^{(1)}(1,2_\perp)=P(1,-2_\perp)(u_1v_{2\perp}-v_1u_{2\perp}), \tag{16}$$

where $u_k, v_k; u_{k\perp} v_{k\perp}$ are the u–v transformation coefficients, $1\equiv k_1$,

$$R(1,2)=\Delta(\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})$$
$$\times\frac{\pi v}{2\rho\Omega_x}\frac{(\varkappa_1^2-\varkappa_2^2)[\beta(2+\varkappa_1^2+\varkappa_2^2)-\rho]}{[(1+\varkappa_1^2)(1+\varkappa_2^2)]^{1/2}\operatorname{sh}(\pi q x_0/2)}e^{-iqvt}$$
$$P(1,2_\perp)=\Delta(\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})\frac{i\pi v(1+\varkappa_1^2)^{1/2}}{2\rho(2x_0\Omega_x)^{1/2}}\frac{[\beta(1+\varkappa_1^2)-\rho]}{\operatorname{ch}(\pi\varkappa_1/2)}e^{-ik_{1x}vt}.$$

Here $\Delta(\mathbf{k})$ is the Kronecker delta and $q=k_{1x}-k_{2x}$. The first term in $H_2^{(1)}$ describes the scattering of volume magnons by the wall, the second the transformation of a volume magnon into a surface one, and so forth. It can be verified that the Hamiltonian $H_2^{(1)}$ makes no contribution to the damping force in the leading order of perturbation theory. For terms of the type $b_k^+ b_{k'\perp}$ this is obvious. As for the scattering processes, we note that their contribution in first-order perturbation theory is determined by a formula such as (19) below. From an analysis of the conservation law in this equation it follows that $\varkappa_1^2-\varkappa_2^2 \propto v(\varkappa_1-\varkappa_2)$. Consequently the contribution of $H_2^{(1)}$ to the damping force contains the same power of the velocity as $H_2^{(2)}$ (the same situation obtains in the problems of Refs. 4 and 5). It is therefore necessary in the calculation of the damping force to take $H_2^{(2)}$ into account in first-order perturbation theory, and with it the two next orders of perturbation theory in $H_2^{(1)}$ (Ref. 4). The Hamiltonian $H_2^{(2)}$ contains the terms of the same type as $H_2^{(1)}$, but in contrast to $H_2^{(1)}$ the corresponding amplitudes $U^{(2)}, V^{(2)}, \Psi^{(2)}$, and $\Phi^{(2)}$ are proportional to the square of the DW velocity and do not contain the factor $\varkappa_1^2-\varkappa_2^2$.

To avoid lengthy calculations in the next orders of perturbation theory, we use the method proposed in Ref. 5. We carry our a unitary transformation of the operators b_k to new operators c_k in accord with the formula $b_k=S^+c_kS$, $S=\exp(iK)$, $K^+=K$. We choose the operator S to satisfy the condition that the Hamiltonian, written in terms of c and c^+, contain no terms linear in v. It suffices for this purpose to choose

$$H_2^{(1)}+i[H_0,K]=0.$$

For the two-magneton Hamiltonian we obtain ultimately (see Ref. 5 for details)

$$H_{int}=H_2^{(2)}-\frac{i}{2}[K,H_2^{(1)}]+\frac{\partial K}{\partial t}.$$

The Hamiltonian H_{int} is quadratic in v, but contains terms of like order in the parameter ρ/β. It suffices to take H_{int} into account in first-order perturbation theory; contribution to the damping force are then given only by the terms of the type $c_k^+ c_{k'}$, (Refs. 4,5). The amplitudes for this term are easily written. It follows from an analysis of (19) that we need only the value of the amplitude at $k_{1x} = -k_{2x}$, i.e., at $\mathbf{k}_2 = \tilde{\mathbf{k}}_1, \tilde{\mathbf{k}}_\perp = \mathbf{k}_\perp \; \tilde{k}_x = -k_x$. Then

$$U_{eff}(\mathbf{k}, \tilde{\mathbf{k}}) = U^{(2)}(\mathbf{k}, \tilde{\mathbf{k}}) + \lim_{k_1 \to \tilde{k}} \left[\frac{i}{\varepsilon_{\tilde{k}} - \varepsilon_1} \frac{d}{dt} U^{(1)}(\mathbf{k}, \mathbf{k}_1) \right]$$
$$+ \sum_p \left\{ \frac{U^{(1)}(\mathbf{k}, \mathbf{p}) U^{(1)}(\mathbf{p}, \tilde{\mathbf{k}})}{\varepsilon_k - \varepsilon_p} - 4 \frac{\Psi^{(1)}(\mathbf{k}, \mathbf{p}) \overset{*}{\Psi}{}^{(1)}(\mathbf{p}, \tilde{\mathbf{k}})}{\varepsilon_k + \varepsilon_p} \right\}$$
$$+ \sum_{p_\perp} \left\{ \frac{V^{(1)}(\mathbf{k}, \mathbf{p}_\perp) \overset{*}{V}{}^{(1)}(\mathbf{p}_\perp, \tilde{\mathbf{k}})}{\varepsilon_k - \varepsilon_{p\perp}} - \frac{\Phi^{(1)}(\mathbf{k}, \mathbf{p}_\perp) \overset{*}{\Phi}{}^{(1)}(\mathbf{p}_\perp, \tilde{\mathbf{k}})}{\varepsilon_k + \varepsilon_{p\perp}} \right\}. \quad (17)$$

The first term in this equation is due to $H_2^{(2)}$. The second and third describe the effective allowance for the next order of perturbation theory in $H_2^{(1)}$. Here $U^{(1)}, V^{(1)}, \Psi^{(1)}$, and $\Phi^{(1)}$ are the amplitudes contained in $H_2^{(1)}$ and defined by Eqs. (16).

Changing in (17) from summation over $\mathbf{p}$ to integration and calculating the corresponding integrals with respect to p_x, we obtain the sought expression for U_{eff}. This calculation turns out to be quite complicated and can be effectively carried out only after expansion in powers of the small parameter ρ/β. It turns out then that the terms proportional to $(\beta/\rho)^2$ and β/ρ vanish identically. The first nonvanishing term of U_{eff} does not contain ρ and can be represented in the form

$$U_{eff}(\mathbf{k}, \tilde{\mathbf{k}}) = \frac{\pi \hbar v^2 k_\perp{}^2 x_0}{8\omega_0 \Omega_x (1 + k^2 x_0{}^2)} \left\{ \frac{\pi (1+\varkappa^2)}{(1 + x_0{}^2 k^2 + x_0{}^2 k_\perp{}^2) \operatorname{ch}^2(\pi\varkappa/2)} \right.$$
$$\left. - \int_{-\infty}^{+\infty} \frac{dp}{(1+p^2)} \frac{(p^2 - \varkappa^2)(2 + p^2 + \varkappa^2)}{(1+\varkappa^2)(2 + \varkappa^2 + p^2 + 2k_\perp{}^2 x_0{}^2) \operatorname{sh}[\pi(p-\varkappa)/2] \operatorname{sh}[\pi(p+\varkappa)/2]} \right\} \cdot e^{-i2k_x v t}. \quad (18)$$

The effective amplitude (18) turned out to be proportional to $\mathbf{k}_\perp^2$, i.e., U_{eff} vanishes in the one-dimensional case. Let us examine the meaning of this result in greater detail. The vanishing of the amplitude U_{eff} and the absence of dissipation should not cause surprise, for in the one-dimensional case the model (1) can be integrated exactly by the method of inverse problem of perturbation theory.[6] It follows from this fact that the scattering amplitude of a spin wave (including a nonlinear one) vanishes exactly on a domain wall that moves with arbitrary velocity $v < v_W$ and at any ratio of ρ and β. Our calculations, adapted for the analysis of the three-dimensional case, reflect this fact.

In the three-dimensional case the model (1) is no longer integrable. As a result there appear at $\mathbf{k}_\perp \neq 0$ a nonzero amplitude of magnon scattering by a DW (18) and a finite contribution to the damping force. A memory of the exact integrability magnifests itself in the dependence of the effective amplitude on the vanishing and in its vanishing at $v = 0$ or $k_\perp = 0$, i.e., $U_{\text{eff}} \propto v^2 k_\perp^2$.

We proceed now to the actual calculation of the two-magnon damping force F_2. For F_2 at $v \ll \omega_0 x_0$ we easily obtain

$$F_2 = \pi v \sum_{1,2} q^2 \left(-\frac{\partial n}{\partial \varepsilon} \right) |U_{eff}(\mathbf{k}_1, \tilde{\mathbf{k}})|^2 \delta(\varepsilon_1 - \varepsilon_2),$$
$$n = n(\varepsilon) = (e^{\varepsilon/T} - 1)^{-1}, \quad (19)$$

where n is the equilibrium Bose distribution function.

Analytic calculation of the integrals in (19), with allowance for the fact that the effective amplitude itself contains integration, is possible only in the limiting cases of high ($T \gg \varepsilon_0$) and low ($T \ll \varepsilon_0$) temperatures:

$$F_2 = \left(\frac{v}{\omega_0 x_0} \right)^5 \frac{T}{x_0^5} \begin{cases} (T/2\pi\varepsilon_0)^2 \exp(-\varepsilon_0/T), & T \ll \varepsilon_0 \\ \eta_1, & \varepsilon_0 \ll T \end{cases}, \quad (20)$$

where η_1 is an elaborate integral whose numerical value is $5.23 \cdot 10^{-5}$.

The additional small factor $(T/\varepsilon_0)^2$ appeared at low temperatures because the amplitude is proportional to $\mathbf{k}_\perp^2$.

3. CONTRIBUTION OF THREE-MAGNON PROCESSES TO THE DAMPING FORCE

Proceeding to the study of the contribution of three-magnon processes to the damping force, we note the following. The Hamiltonian H_3 does not contain the singular terms discussed above. The damping force due to H_3 is in the three-dimensional case a regular function of ρ/β, and we confine ourselves to the first nonvanishing term, which does not depends on ρ. In the three-dimensional case H_3 leads to a linear dependence of the damping force on the velocity. To calculate it we can disregard in the present section the changes produced in the DW structure by its motion. With allowance for the foregoing remarks, we obtain the expression

$$H_3 = i\varepsilon_0 x_0 a_0^{3/2} \left(\frac{2}{s} \right)^{1/2} \int d\mathbf{r}\, a^+ a \frac{d}{dx} \left\{ \frac{a^+ - a}{\operatorname{ch} \xi} \right\}, \quad (21)$$

where a_0 is the interatomic distance and s is the spin of the atom.

In the representation (10), the Hamiltonian (21) takes the form of a sum:

$$H_3 = H^{(3v)} + H^{(2v+s)} + H^{(v+2s)}, \quad (22)$$

where the superscripts indicate the magnon types that participate in the process, while v and s denote the volume and surface magnon, respectively. We obtain for $H^{(3v)}$

$$H^{(3v)} = \sum_{1,2,3} \{ \Phi_1(2,13) c_2^+ c_1 c_3 + \text{H.a.} \}, \quad (23)$$

where

$$\Phi(2,13) = -\frac{i\pi\varepsilon_0 x_0 a_0^{3/2} (1+\varkappa_2^2)^{1/2} (1 + \varkappa_1^2 + \varkappa_3^2 - \varkappa_2^2) \Delta(\mathbf{k}_{1\perp} + \mathbf{k}_{3\perp} - \mathbf{k}_{2\perp})}{2\Omega_x [2s\Omega(1+\varkappa_1^2)(1+\varkappa_3^2)]^{1/2} \operatorname{ch}[\pi(\varkappa_1 + \varkappa_3 - \varkappa_2)/2]}$$
$$\times \exp[ivt(k_{1x} + k_{3x} - k_{2x})].$$

This operator describes the conversion of one exchange magnon into two others and the inverse processes: $v_1 + v_3 \leftrightarrow v_2$.

The Hamiltonian $H^{(2v+s)}$ describes processes of the type $v \leftrightarrow v' + s$:

$$H^{(2v+s)} = \sum_{1,2,3_\perp} \{\Phi_2(1,2,3_\perp) c_1^+ c_2 c_{3\perp}^{\pm} + \text{H.a.}\}, \tag{24}$$

where

$$\Phi_2(1,2,3_\perp) = \frac{\pi\varepsilon_0 x_0 a_0^{3/2}(1+\varkappa_2^2)^{1/2}(\varkappa_1^2-\varkappa_2^2)\Delta(\mathbf{k}_{1\perp}+\mathbf{k}_{3\perp}-\mathbf{k}_{2\perp})}{2\Omega_x[sSx_0(1+\varkappa_1^2)]^{1/2}\,\mathrm{sh}[\pi(\varkappa_1-\varkappa_2)/2]} \times \exp[-ivt(k_{1x}-k_{2x})].$$

Finally, the operator $H^{(v+2s)}$ describes processes of the type $s + s' \leftrightarrow v$:

$$H^{(v+2s)} = \sum_{1_\perp,2,3_\perp} \{\Phi_3(1_\perp,2,3_\perp) c_{1\perp}^{+} c_2 c_{3\perp}^{+} + \text{H.a.}\}, \tag{25}$$

where

$$\Phi_3(1_\perp,2,3_\perp) = -\frac{i\pi\varepsilon_0 a_0^{3/2}(1+\varkappa_2^2)^{1/2}\Delta(\mathbf{k}_{1\perp}+\mathbf{k}_{3\perp}-\mathbf{k}_{2\perp})}{4(2s\Omega)^{1/2}\,\mathrm{ch}[\pi\varkappa_2/2]}\exp(ik_{2x}vt).$$

After standard calculations we obtain for the damping force

$$\begin{aligned} F_3 = {} & \frac{4\pi}{S}\sum_{1,2,3}[n_2(n_1+1)(n_3+1)-n_1n_3(n_2+1)]|\Phi_1|^2 \\ & \times(k_{1x}+k_{3x}-k_{2x})\delta(\varepsilon_1+\varepsilon_3-\varepsilon_2-(k_{1x}+k_{3x}-k_{2x})v) \\ & +\frac{2\pi}{S}\sum_{1,2,3_\perp}[n_2(n_1+1)(n_{3\perp}+1)-n_1n_{3\perp}(n_2+1)]|\Phi_2|^2 \\ & \times(k_{1x}-k_{2x})\delta(\varepsilon_1+\varepsilon_{3\perp}-\varepsilon_2-v(k_{1x}-k_{2x})) \\ & +\frac{4\pi}{S}\sum_{1_\perp,2,3_\perp}[n_2(n_{1\perp}+1)(n_{3\perp}+1)-n_{1\perp}n_{3\perp}(n_2+1)]|\Phi_3|^2 \\ & \times k_{2x}\delta(\varepsilon_{1\perp}+\varepsilon_{3\perp}-\varepsilon_2+vk_{2x}). \end{aligned} \tag{26}$$

We shall write out the final expressions for two limiting cases.

At low temperatures $T \ll \varepsilon_0$ we obtain

$$F_3 = \frac{v\hbar}{2^9 s x_0^4}\left(\frac{T}{sJ_0}\right)^{3/2} \times e^{-\varepsilon_0/T}\left\{\frac{3e^{-\varepsilon_0/T}}{\pi\sqrt{2}\,\mathrm{ch}^2(\pi/2)}\left(\frac{T}{\varepsilon_0}\right)^{5/2} + \frac{2\sqrt{2}}{\pi^3}\left(\frac{T}{\varepsilon_0}\right)^{3/2} + \frac{\sqrt{\pi}}{2}\right\}, \tag{27}$$

where $sJ_0 = \varepsilon_0(x_0/a)^3$ is an energy of the order of the Curie energy. The first, second, and third terms in the curly brackets describe respectively the contributions $H^{(3v)}$, $H^{(2v+s)}$, $H^{(v+2s)}$. It follows from (27) that the main contribution to the damping force F_3 is made at low temperatures by processes in which one volume and two surface magnons participate, $v \leftrightarrow s + s'$. This contribution is larger than the contribution of processes of the type $v_1 + v_3 \leftrightarrow v_2$ considered in Ref. 4.

At high temperatures $T \gg \varepsilon_0$ the force is given by

$$F_3 = \frac{\hbar v T^2}{8\pi s x_0^4 (sJ_0)^{3/2}\varepsilon_0^{1/2}}\left\{\frac{1}{9\pi^2}\ln^2\frac{T}{\varepsilon_0} + \eta_2 + \frac{1}{5}\right\}, \tag{28}$$

where the sequence of the terms is the same as in (27). Here η_2 stands for an elaborate mutliple integral whose numerical value is 0.14. If the slowly varying function $\ln(T/\varepsilon_0)$ is disregarded, the contributions of all three terms have like temperature dependences.

Since $\varepsilon_0 \approx 0.3$ K at $H_A = \beta M_0 \simeq 2$ kOe, the characteristic value at $T \approx 300$ K is $\ln(T/\varepsilon_0) \approx 6$–7. Consequently the contributions of all types are comparable at room temperature.

It follows from the presented analysis that the flexural vibrations of the DW (surface magnons) make the decisive contribution to the damping force at low temperatures, but they must also be taken into account at room temperatures.

4. CONCLUSION

Let us discuss the general picture of the DW relaxation in a rhombic ferromagnet of energy (1). This model is exactly integrable in the one-dimensional case. Analysis has shown that the soliton relaxation in this model is due to violation of the exact integrability on going to the three-dimensional case.

The damping force consists of two- and three-magnon parts F_2 and F_3. The effective amplitude of magnon scattering by a DW, which determines F_2, is equal to zero in the one-dimensional case. In the three-dimensional case the memory of the exact integrability of the system manifests itself in the absence, from the damping force F_2, of a term linear in the velocity. The relation $F \propto v$ is the result of only three-magnon processes, because their amplitude is more strongly modified in the three-dimensional case than in the one-dimensional.[5] Indeed, the amplitude (23), which describes processes involving three volume magnons, contains a factor $1 + x_0^2(k_{1x}^2 + k_{3x}^2 - k_{2x}^2)$. In the one-dimensional case this is rewritten as $\varepsilon_1 + \varepsilon_3 - \varepsilon_2$, and the amplitude vanishes on the mass shell. At the same time, in the three dimensional case this factor, with account taken of the conservation law in (26), is $x_0^2(k_{1\perp}^2 + k_{3\perp}^2 - k_{2\perp}^2)$, and the amplitude makes a finite contribution to F_3. The contribution of the three-magnon processes, however, contains additional factors that are small compared with F_2, of the type T/T_C and ε_0/T_C, where $T_C \sim sJ_0$ is the Curie temperature of the magnet. This means that the contribution F_2 of the two-magnon processes which is small in the parameter $v/\omega_0 x_0$, can become comparable with the contribution of the three-magnon processes at a sufficiently low velocity $v = v_*(T)$. From (27) and (28) we can obtain

$$\begin{aligned} v_*(T) &= \omega_0 x_0 \frac{\pi^{5/8}}{4s^{1/4}}\left(\frac{\varepsilon_0^2}{TT_C}\right)^{3/8}, \quad T \ll \varepsilon_0, \\ v_*(T) &= 10\omega_0 x_0\left(\frac{\varepsilon_0}{T_C}\right)^{1/8}\left(\frac{1{,}75T}{8\pi s T_C}\right)^{1/4}, \quad T \gg \varepsilon_0. \end{aligned} \tag{29}$$

This value of the velocity determines the transition from a linear velocity dependence of the damping force to an essentially nonlinear one. The transition from the regime $F \propto v$ and $F \propto v^5$ leads to a change of the experimentally determined dependence of the velocity of the stimulated motion of the DW on the external field: $v \propto H^{1/5}$ at $v > v_*(T)$ (Refs. 4 and 5). The transition to the $v \propto H^{1/5}$ regime is similar to velocity saturation in a strong field. Such a saturation is frequently observed already at $v < v_W$ in experiments on iron garnet films.[3]

Let us estimate $v_*(T)$. It can be seen from (29) that $v_*(T)$, unlike in a uniaxial ferromagnet, has a nonmonotonic temperature dependence. Using the values $H_A \approx 2$ kOe, i.e., $\varepsilon_0 \approx 0.3$ K and $T_C \sim 10^3$ K, which are typical of epitaxial iron-garnet films, we find that $v_* \sim v_W$ at room temperature. The conditions for realizing the effect at low temperatures are more favorable, since v_* has a minimum at $T \sim \varepsilon_0$. Using (29) at $T \sim \varepsilon_0$, we obtain $v_* \sim 10^{-2}\omega_0 x_0 \ll v_W$. It can also be seen from (29) that the nonlinearity of $v(H)$ manifests itself most strongly in weakly anisotropic magnets with high Curie temperature.

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[1)] The model of a uniaxial magnet with dipole interaction, considered in Ref. 4, is equivalent in the one-dimensional case to the model (1), but cannot make allowance for the contribution of the DW flexural oscillations to the damping force because the interaction has a nonlocal character in the three-dimensional case.

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