# Measurement of the cross section for scattering of $p\mu$ atoms in gaseous hydrogen

V. M. Bystritskiĭ, V. P. Dzhelepov, V. I. Petrukhin, A. I. Rudenko, V. M. Suvorov, V. V. Fil'chenkov, N. N. Khovanskiĭ, and B. A. Khomenko

*Joint Institute for Nuclear Research*

(Submitted 19 January 1984)
Zh. Eksp. Teor. Fiz. **87**, 384–392 (August 1984)

The muon beam of the synchrocyclotron at the Joint Institute for Nuclear Research has been used in an experiment with gaseous hydrogen at a pressure of 41 atm to measure the cross section for scattering of $p\mu$ atoms by hydrogen molecules $\sigma(p\mu + H_2) = (42 \pm 8) \cdot 10^{-21}$ cm², which corresponds to a cross section for scattering by free protons $\sigma(p\mu + p) = (17.4 \pm 3.3) \cdot 10^{-21}$ cm².

Interest in the elastic scattering of $p\mu$ atoms in hydrogen

$$p\mu + H_2 \to p\mu + H_2 \quad (1)$$

has arisen in connection with the problem of determining the weak interaction constant by measurement of the rate of capture of the muon by the proton in a $p\mu$ atom. In interpretation of the results of such experiments it is necessary to take into account the factors which determine the population of the hyperfine-structure levels of the muonic atom, since the capture rates in them differ greatly.[1] To a significant degree these factors are related to the characteristics of the elastic scattering of $p\mu$ atoms in hydrogen (1). The first measurements[2,3] of the cross section for reaction (1), carried out by various methods, gave results which did not agree. Theoretical estimates[4–7] of the cross sections also did not agree with each other and revealed a great sensitivity to the models used.[7] Thus, the problem has acquired independent interest as a means of testing methods of calculation of processes in muonic atoms. To remove the uncertainties in the experimental data, it was necessary to have new measurements, which have been carried out by the Bologna-CERN group,[8,9] and also in the present work with use of a technique differing from that of Refs. 8 and 9.

## METHOD AND EXPERIMENTAL ARRANGEMENT

The experiment was performed in the muon beam of the synchrocyclotron at our institute. The method of measurement is based on use of the relation between the diffusion length of the $p\mu$ atom in gaseous hydrogen and the cross section for the scattering reaction (1). Let us consider the processes which determine this connection.

A negative muon which has stopped in gaseous hydrogen forms an excited $p\mu$ atom which in a time of the order $10^{-10}$ sec (at a hydrogen pressure of tens of atmospheres) drops to its ground state.[10] According to an estimate by Wightman,[11] initially the muonic atom has a kinetic energy of the order of 1 eV. During the time of de-excitation, various processes facilitate both the increase and the decrease of this energy,[12] and there are no definite data on the energy with which the $p\mu$ atom is formed in the ground state. Subsequently the muonic atom, diffusing in the gas, loses its energy $\mathscr{E}$ in collisions with hydrogen molecules until it is thermalized. Here for $\mathscr{E} > \Delta\mathscr{E}$ ($\Delta\mathscr{E} = 0.183$ eV is the energy of the hyperfine splitting of the ground state of the $p\mu$ atom) an important role is played by collisions with transitions $F = 0 \leftrightarrow F = 1$ ($F$ is the total spin of the $p\mu$ system), and at lower energies transitions to the upper state are excluded and the population of this level decreases. During the time of its existence, the muonic atom, moving along a broken trajectory, travels a total path $R$ and in so doing is removed from its place of formation by a distance $L$. The relation between these quantities is determined by the number of collisions of the muonic atom with hydrogen molecules and by the kinematic characteristics of the process (1). In the diffusion approximation (a large number of scattering events with a small energy loss in each) this relation has the form

$$L = R/(3ny)^{1/2}, \quad y = 1 - \overline{\cos\theta}, \quad (2)$$

where $n$ is the number of collisions (1) during the lifetime of the $p\mu$ atom; $\overline{\cos\theta}$ is the average value of the cosine of the lab scattering angle. In the processes considered, the conditions indicated above for applicability of the diffusion approximation cannot be satisfied (for example, the number of collisions turns out to be small), but the qualitative nature of the relation (2) is preserved. It follows from this relation that the quantity $L$ contains information on the scattering cross section $\sigma$, on which depend the number of collisions, the dynamics of slowing down of the $p\mu$ atom, and consequently the total range. Thus, the problem of measurement of the cross section can be solved by determination of the characteristics of the distribution of $p\mu$ atoms in range along a straight line, for example, the average length of these ranges $L$ (the diffusion length).

Both the present work and the previous measurements of the scattering cross section (1) are based on the method of analysis of the characteristics of distributions in the direct range, which is widely used in study of diffusion processes. The experiments vary in just which characteristics of a distribution are measured and by what means.

In the experiment of Dzhelepov *et al.*,[2] which was carried out with a high-pressure diffusion chamber, the ranges were measured in hydrogen containing impurities of alcohol vapor. Subsequently in the experiments of the Bologna-CERN group[3,8,9] data on the scattering cross section (1) were obtained from analysis of the yield and time distribution of the $\gamma$ rays arising as the result of diffusion of the $p\mu$ atoms to a thin metallic foil placed inside a gaseous hydrogen target. When the muonic atom reaches the foil, the muon is captured from it to an atom $Z$ of the foil material, forming an excited $Z\mu$ atom which, in dropping to its ground state, emits muonic x rays with energies characteristic of the given material. This idea made it possible to use electronic techniques for investigation of the diffusion of muonic atoms.

 0038-5646/84/080219-06$04.00  

The possibility of another arrangement follows from more recent work,[13] in which the rate of capture of negative muons was measured in hydrogen. In that work the authors considered as one of the background sources processes involving diffusion of $p\mu$ atoms to the walls of the working volume of the hydrogen target, which consisted of a CsI(Tl) scintillator. To determine the contribution of this background they used a normalization measurement with hydrogen containing a small admixture ($\sim 10^{-4}$) of xenon, which provides a significant number of captures of the muon from hydrogen to atoms of xenon. The closeness of the atomic numbers of the elements $^{53}$I, $^{54}$Xe, and $^{55}$Cs permits one to assume that the characteristics of the muonic-atom processes for CsI and Xe are the same. Thus, events which occur when a $p\mu$ atom reaches a wall of CsI in a hydrogen-xenon run are imitated over the entire volume, and the fraction of muons which experience capture can be determined and the result used for normalization.

In the present work we have used just such an arrangement. In a measurement with pure hydrogen "H" we determined what fraction of $p\mu$ atoms formed in the target working volume reached the target walls by diffusion. Obviously only muonic atoms produced in an adjacent layer of thickness $\sim L$ can reach the walls. The measurable yield of muonic x rays from Cs$\mu$ and I$\mu$ atoms is given by the expression

$$Y(\mathrm{H}) = \varepsilon \frac{N_0}{V} SLg = \varepsilon N_0 \alpha, \quad \alpha = \frac{SLg}{V}, \tag{3a}$$

where $\varepsilon$ is the probability of detection of the muonic x rays, $V$ is the volume of the working region of the hydrogen target, $S$ is the area of the surface which encloses it, $N_0$ is the number of stopped muons in the volume $V$ (it is assumed that the density of stoppings $N_0/V$ is the same ove the entire region), and $g$ is a factor which takes into account the details of the space-time characteristics of the diffusion process under real conditions. The product $\varepsilon N_0$ is determined from a hydrogen + xenon measurement ("H + Xe"), for which the yield of $\gamma$ rays from Xe$\mu$ atoms is

$$Y(\mathrm{H+Xe}) = \varepsilon N_0 \frac{\lambda_{\mathrm{Xe}}'}{\lambda_S}, \quad \lambda_S = \lambda_0 + \lambda_{pp}' + \lambda_{\mathrm{Xe}}'. \tag{3b}$$

Here $\lambda_0 = 0.455 \cdot 10^6\ \mathrm{sec}^{-1}$ is the rate of decay of the muon; $\lambda'_{pp}$ and $\lambda'_{\mathrm{Xe}}$ are the rate of formation of $pp\mu$ muonic molecules and the rate of capture of muons from $p\mu$ atoms into xenon, reduced to the density of hydrogen in the target. Combining Eqs. (3a) and (3b), we obtain

$$\alpha = \frac{Y(\mathrm{H})}{Y(\mathrm{H+Xe})} \frac{\lambda_{\mathrm{Xe}}'}{\lambda_S}. \tag{3c}$$

It can be seen from Eq. (3a) that the quantity $\alpha$, which is the fraction of $p\mu$ atoms which have reached the walls, is related to the parameter $L$ and consequently to the cross section for the scattering (1). On the other hand, Eq. (3c) indicates a means of determining $\alpha$ by means of the "H" and "H + Xe" measurements. We note that the relations between the measurable quantities and the diffusion parameters which have been given for explanation of the method reflect only the general relation between them, and for interpretation of the data of an actual experiment they can be used only at the qualitative level. The exact relations which take into account all aspects of the experimental arrangement are determined by numerical modeling in a computer.

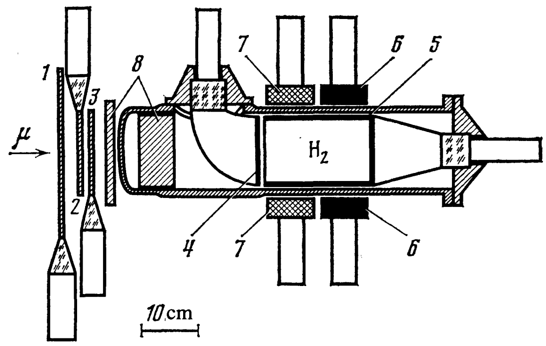


FIG. 1. Diagram of experimental apparatus: 1—guard detector; 2, 3, and 4—monitor counters; 5—cup counter enclosing the target working volume; 6 and 7—$\gamma$ and $e$ counters with respective scintillators of NaI(Tl) and plastic; 8—absorber to slow down muons.

The apparatus with the gaseous hydrogen target (a detailed description can be found in Refs. 14 and 15) is shown in Fig. 1. Muons passing through counters 2–4 are slowed down in flight by the absorbers 8 and stopped in the gas of the target. Stoppings are identified by operation of counters $234\bar{5}$. Products of the processes under study, which are emitted from the target, are detected by two $\gamma$ counters and five $e$ counters [scintillators of NaI(Tl) and plastic, respectively] in a time interval 0.38–10 $\mu$sec measured from the moment of stopping. The absence of other beam particles during this time and also for a period of 5 $\mu$sec before the stopping is monitored by guard counter 1. Discrimination of charged and neutral particles which hit the $\gamma$ and $e$ detectors is carried out by means of counter 5. Data on events, including time and pulse-height information from the detectors, was transferred, stored, and analyzed in an HP-2116C computer. Calibration and monitoring of the stability of the channels for measuring pulse heights were accomplished by periodic measurements of radiation from Po–Be and $^{60}$Co radioactive sources.

## MEASUREMENTS AND ANALYSIS

In the experiment we made the following runs (see also Table I):

1) The experiment with hydrogen—"H"—the principal measurement. Isotopically pure hydrogen (protium) with deuterium content no greater than $2 \cdot 10^{-6}$ was used, which excluded appreciable distortion of the result by diffusion of $d\mu$ atoms formed as a consequence of transfer of muons to deuterium. In detection of muonic x rays by the $\gamma$ counters, there is an additional contribution from the products of nuclear capture of muons into the muonic atoms Cs$\mu$, I$\mu$, and (in the normalization measurement) Xe$\mu$, but this only increases the probability of detection of cases of muon capture into these atoms (the delay of the capture with respect to the moment of formation of a heavy muonic atom and the cascade in it by $\sim 0.08\ \mu$sec can be neglected).

TABLE I.

| Run | Pressure, atmospheres | Monitor count in units of $10^6$ | Electron yield | $\gamma$-ray yield |
|---|---|---|---|---|
| H | 41.0 | 786.7 | 78 936 | 1085±73 |
| He | 47.0 | 640.7 | 63 173 | – |
| H+D | 43.9 | 1445.0 | 142 260 | – |
| H+D+Xe | 43.9 | 61.3 | – | 1360±60 |

2) The experiment with helium—"He"—the background for the main measurement. The amount of helium in the target provides conditions for stopping in it of muons which are equivalent to the "H" measurement, which is further monitored on the basis of the counting rate of electrons in the two runs. Actually in this measurement all events in hydrogen are imitated, with the exception of the diffusion of $p\mu$ atoms with their exit to the walls, and consequently the same background sources are acting as in "H".

3) For the normalization measurement (hydrogen + xenon) we used runs "H + D" and "H + D + Xe" with 7% admixture of deuterium, which was necessary for another experiment being carried out in parallel. From analysis of the time distributions of the $\gamma$ rays and electrons it was found that: $\lambda_S = (3.59 \pm 0.13) \cdot 10^6 \text{ sec}^{-1}$. Under the conditions of the measurement $\lambda'_{pp} = 0.29 \cdot 10^6 \text{ sec}^{-1}$, from which it follows that $\lambda'_{\text{Xe}} = (2.84 \pm 0.13) \cdot 10^6 \text{ sec}^{-1}$.

4) A set of events with an empty target "T" was used as background for all runs with the exception of "H".

5) Auxiliary measurements were made to check whether or not the average density of stoppings at the walls of the target working volume coincides with the density averaged over the entire volume. For this purpose we fastened to the inner surface of the scintillator of counter 5 by means of clamping rings a Mylar film, and the target was filled with helium. In another measurement (with the same amount of helium and with the rings) there was no film. The background measurement was a set with an empty target and rings. The difference in the electron count due to introduction of the material on the walls was assigned to the count from muons stopped in the gas, from which, taking into account the weight of the film and its relative stopping power, we drew a conclusion regarding the ratio of the densities of muon stoppings at the surface and in the volume. The measurements were made with films of thickness 60 and 25 $\mu$m. In the first case, for example, $4 \cdot 10^4$ electrons were recorded, and of them about 20% occurred at the Mylar. As a result it turned out that the density values coincide within 2–3%.

In analysis of the data we constructed time distributions of neutral particles recorded by the $\gamma$ detectors with pulse heights corresponding to $\gamma$-ray energies from 1 to 7 MeV. In Fig. 2 we show spectra obtained in the runs "H", "He", and "T", renormalized to the same conditions relative to the number of recorded electrons ("He" to "H") and to the monitor ("T" to "H"). The excess of the first histogram over the second is due to the effect under investigation—the departure of $p\mu$ atoms to the walls of the working volume and the detection of muonic x rays and other accompanying particles. Comparison of the "He" and "T" spectra permits us to judge the contributions of background from decay of muons stopped in the gas (the difference "He" or "T") and of the background from stoppings in the target structure. The time distribution of $\gamma$ rays from the effect was obtained by subtraction of the "He" spectrum from the "H" spectrum (Fig. 3). The yield $Y(\text{H})$ was determined in the time interval 0.38–7.6 $\mu$sec. This interval corresponds to the relative yield $\alpha_t$. To find $\alpha_t$, Eq. (3c) was modified to take into account the following factors:

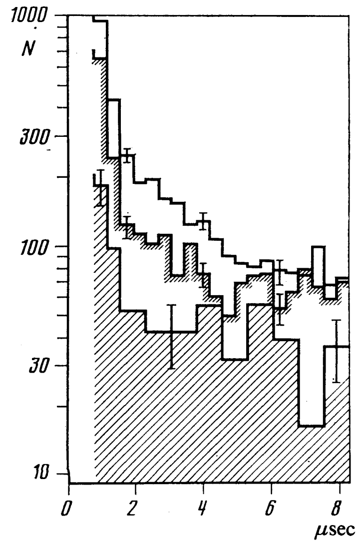


FIG. 2. Time spectra of $\gamma$ rays obtained in the runs "H", "He" (partially hatched), and "T" (completely hatched), renormalized to identical conditions.

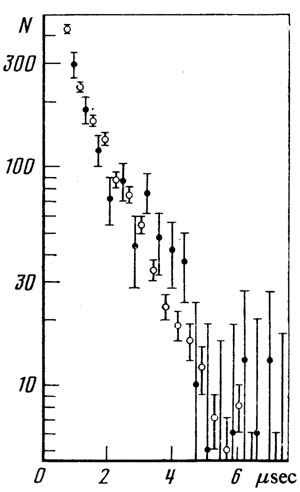


FIG. 3. Time spectra: ●—measured distribution of $\gamma$ rays from charge exchange of $p\mu$ atoms in the wall material; ○—similar distribution obtained by modeling with $\sigma = 42 \cdot 10^{-21}$ cm$^2$.

a) In the "H + D + Xe" measurement the $\gamma$-ray yield $Y'(\mathrm{H+D+Xe})$ was determined in the interval 0.38–7.0 $\mu$sec, in which occurs a fraction $f = 0.256$ of all cases of capture of muons to xenon, and therefore the total yield (without time limitations) is

$$Y(\mathrm{H+D+Xe}) = Y'(\mathrm{H+D+Xe})/f;$$

b) The value of $Y(\mathrm{H+D+Xe})$ must be reduced to the number of muonic atoms produced in the "H" run with a different amount of gas in the target; the ratio of the stoppings in the measurements "H + D" and "H + D + Xe" is equal to the ratio of the monitor counting rates M(H + D)/M(H + D + Xe), and for "H" and "H + D" it can be determined in terms of the ratio of the numbers of electrons recorded $E(\mathrm{H})/E(\mathrm{H+D})$. As a result the substitution has the form

$$Y(\mathrm{H+Xe}) \rightarrow \frac{Y'(\mathrm{H+D+Xe})}{f}\,\frac{E(\mathrm{H})\,\mathrm{M}(\mathrm{H+D})}{E(\mathrm{H+D})\,\mathrm{M}(\mathrm{H+D+Xe})}.$$

Finally we obtain

$$\alpha = \frac{Y(\mathrm{H})f}{Y'(\mathrm{H+D+Xe})}\,\frac{\lambda_{\mathrm{Xe}}'}{\lambda_{\varepsilon}}\,\frac{E(\mathrm{H+D})\,\mathrm{M}(\mathrm{H+D+Xe})}{E(\mathrm{H})\,\mathrm{M}(\mathrm{H+D})}$$

$$= 0.0125 \pm 0.0010.$$

## MODELING OF THE DIFFUSION PROCESS

To determine what scattering cross section corresponds to the $\alpha_t$ value found, and also to find the sensitivity of the result to various assumptions regarding the characteristics of the process (1), we carried out a numerical modeling by the Monte Carlo method of the events occurring in the target after formation of the $p\mu$ atom. Here we assumed the following:

1. Muon stoppings are uniformly distributed in the target working volume, which has a cylindrical shape 120 mm in diameter and 195 mm long.

2. Although the initial energy of the $p\mu$ atom is greater than $\Delta\mathscr{E} = 0.18$ eV, the cross section for inelastic interactions of the muonic atom in hydrogen at $\mathscr{E} > \Delta\mathscr{E}$ is so great that the atom rapidly loses its energy without being able to travel any significant distance. Modeling was begun at $\mathscr{E}_0 = 0.18$ eV.

3. In the region $\mathscr{E} \leqslant 0.18$ eV the scattering cross section is assumed to be constant, although actually it depends on the energy. In this way the modeling of the processes and the interpretation of the experimental results were carried out in the approximation of an "effective cross section," the value of which is the weighted mean of the actual dependence $\sigma(\mathscr{E})$ in the region $\mathscr{E} \leqslant 0.18$ eV.

4. In a collision with a hydrogen molecule the muonic atom, wich has a wavelength smaller than the size of the molecule, actually interacts with one of its atoms, but senses that it is bound. On the other hand, energies 0.04–0.18 eV are sufficient to excite rotational levels of the molecule (with spacing 0.01 eV), and for this reason we cannot speak of purely elastic collisions with a molecule as a whole. Therefore it is assumed that the process (1) can be described as isotropic scattering, in the center-of-mass system, of the $p\mu$ atom by a particle with some effective mass $M$, where $m_p < M < 2m_p$ ($m_p$ is the proton mass).

5. To the molecule with effective mass $M$ we assigned a thermal motion in accordance with a Maxwellian distribution with a mean square energy 0.038 eV.

In the program which accomplished the modeling we specified as initial conditions the following: the initial energy $\mathscr{E}_0$ of the $p\mu$ atom, the cross section $\sigma$ for scattering by the molecule, the effective mass $M$ of the molecule, and also the parameters describing the conditions of the experiment. The main result of the operation of the program was the time distribution of collisions of $p\mu$ atoms with the walls, from which we obtained a model estimate of the relative yield $\alpha_m(\mathscr{E}_0, \sigma, M)$. In addition, we constructed distributions of the other quantities describing the scattering and diffusion of the muonic atoms. We made a total of about forty Monte Carlo runs with various values of the cross section and effective mass in the ranges $20 \leqslant \sigma/(10^{-21}\,\mathrm{cm}^2) \leqslant 70$, $1 \leqslant M/m_p \leqslant 2$. It turned out that for $\mathscr{E}_0 = 0.18$ eV the values obtained for $\alpha_m$ are satisfactorily described by the following empirical dependence ($\sigma$ is in units of $10^{-21}$ cm$^2$):

$$\alpha_m(\sigma, y) = Ae^{-B\sigma y},\quad A = 1{,}82\cdot 10^{-2},\quad B = 1.96\cdot 10^{-2}. \qquad (4)$$

The parameter $y$ defined in Eq. (2) was uniquely related to the effective mass of the molecule:

| | | | | |
|---|---|---|---|---|
| $M/m_p$: | 1,00 | 1,25 | 1,40 | 2,00 |
| $1-y=\cos\theta$: | 0.52 | 0,46 | 0,43 | 0,35 |

## EXPERIMENTAL RESULTS

Curves 1 and 2 in Fig. 4 show the relation $\alpha_m(\sigma)$ for extreme values of the effective mass of the molecule respectively $M = m_p$ and $M = 2m_p$, and on the ordinate axes we have marked the experimental value of $\alpha_t$ with its statistical error. From these data we can draw a conclusion as to the limits between which the desired cross section lies. To reduce the uncertainty it is necessary to indicate more precisely the effective mass of the hydrogen molecule. If we take it into account that in collisions with a muonic atom the rotational levels of the molecule are easily excited while the vibrational levels are not excited, we can consider a classical

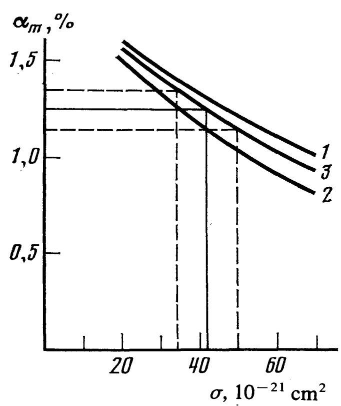


FIG. 4. The relation $\alpha_m(\sigma)$ for the following effective masses of the molecules: curve 1—$M = m_p$, curve 2—$M = 2m_p$, curve 3—$M = 1.25m_p$. On the ordinate axis we have shown the measured value of $\alpha_t$ and on the abscissa we have shown the resulting value of $\sigma(p\mu + \mathrm{H}_2)$.

analog of the molecule in the form of a dumbbell—two atoms located on the axis with a fixed distance between them. The muonic atom elastically collides with one of the atoms, and here the molecule, depending on its orientation with respect to the line of the collision, appears to be a particle with various masses: from $M = m_p$ (when the impact line is perpendicular to the dumbbell axis) to $M = 2m_p$ (impact along the axis). Modeling of collisions with such molecules participating in thermal motion (having five degrees of freedom) showed that the average characteristics of the scattering process are equivalent to cases of collision of the muonic atom with particles having a mass $M = 1.25m_p$. Curve 3 in Fig. 4 corresponds to the relation $\alpha_m(\sigma)$ for this effective mass. Then from the experimental value of $\alpha_t$ we obtain the result—the effective cross section for scattering of $p\mu$ atoms by hydrogen molecules,

$$\sigma(p\mu+H_2)=(42\pm8)10^{-21}\ \mathrm{cm}^2. \tag{5}$$

In Fig. 3 together with the measured time spectrum of $\gamma$ rays produced by capture of muons into atoms of the wall material, we have given the corresponding distribution which is the result of the modeling with $\sigma = 42\cdot10^{-21}$ cm$^2$. As one can see, the two spectra are in good agreement. We shall give some characteristics of the diffusion of $p\mu$ atoms under these conditions:

$R$=3,6 мм; $L$=1,4 мм; $n$=11.

At an initial energy $\mathscr{E}_0 = 0.18$ eV a muonic atom is thermalized essentially in 2–3 collisions, and from this it follows that the main contribution to the effective cross section is from the scattering cross section at thermal energies. The modeling also showed that a change of the initial energy of the $p\mu$ atom in the range 0.04–0.5 eV changes only very slightly the relation $\alpha_m(\sigma)$, and this means also only a slight change in the result (5).

Let us estimate the possible influence of an admixture of deuterium in the hydrogen (a concentration $c_d < 2\cdot10^{-6}$) on the measurement. A fraction of the muons will be captured from $p\mu$ atoms into deuterons with a rate $\lambda'_d = c_d\varphi\lambda_d$ [$\lambda_d \approx 10^{10}$ sec$^{-1}$ (Refs. 16–18) is the capture rate reduced to the density of liquid hydrogen], and here $p\mu$ atoms are formed with an initial energy 45 eV. As a result of the difference of the straight-line ranges of $p\mu$ and $d\mu$ atoms (respectively $L$ and $L'$) the probabilities of their reaching the walls $\alpha$ and $\alpha'$ are also different. Actually some average yield will be measured:

$$\bar{\alpha}=(1-\delta)\alpha+\delta\alpha',$$

where $\delta = \lambda'_d/(\lambda_0 + \lambda'_d + \lambda_{pp}')$, and the relative error in the measurement of $\alpha$ will be:

$$\frac{\Delta\alpha}{\alpha}=\frac{\bar{\alpha}-\alpha}{\alpha}=\delta\frac{\alpha'-\alpha}{\alpha}.$$

If we take it into account that $\alpha \sim L \sim \sigma^{-1/2}$, we obtain

$$\frac{\Delta\sigma}{\sigma}=-2\frac{\Delta\alpha}{\alpha}=2\delta\frac{\alpha-\alpha'}{\alpha}. \tag{6}$$

Under the conditions of the present experiment $\delta < 2\cdot10^{-3}$, $\alpha \approx 2\cdot10^{-2}$.

In order to determine $\alpha'$, let us see what the straight-line ranges of $d\mu$ atoms are. Since the cross section for the scattering

$$d\mu+d\rightarrow d\mu+d \tag{7a}$$

is of the order of $10^{-19}$ cm$^2$,[9,10] it is easy to see that in hydrogen at a pressure of 40 atm and with $c_d \ll 10^{-3}$ this process will not appear, and consequently it is necessary to take into account only collisions with protons:

$$d\mu+p\rightarrow d\mu+p. \tag{7b}$$

According to calculations[4,6,20] the cross section for the process (7b) depends strongly on the energy of the $d\mu$ atom: amounting to (5–20) $\cdot 10^{-21}$ cm$^2$ at $\mathscr{E}_{d\mu} \geqslant 5$, it falls off by an order of magnitude or more in the region 1–3 eV (the Ramsauer effect). For the conditions considered this means that the $d\mu$ muonic atom slows down rapidly in collisions with protons to $\mathscr{E}_{d\mu} \approx 2$ eV, and in so doing travels a distance of 2–4 mm, and then moves along a straight line without scattering for about 3 cm. Under the conditions of our geometry a range $L' \approx 3$ cm corresponds to a yield $\alpha' \approx 0.25$. Thus, we find from Eq. (6) that the admixture of deuterium in the hydrogen leads to a reduction of the measured cross section by no more than 5%.

We emphasize that in the experiment we are investigating the scattering of muonic atoms by molecular hydrogen, and depending on the means of normalization (to the number of molecules or atoms of hydrogen) the result can be presented in the form of the cross section for scattering of a $p\mu$ atom by a hydrogen molecule $\sigma(p\mu + H_2)$ or the cross section for scattering by a bound proton $\sigma(p\mu + H)$. Obviously $\sigma(p\mu + H_2) \equiv 2\sigma(p\mu + H)$. At the same time in theoretical studies one usually calculates the cross section for scattering by free protons $\sigma(p\mu + p)$. For comparison of theory and experiment one can use the relation found by the method of Men'shikov[21]:

$$\sigma(p\mu+H_2)/\sigma(p\mu+p)=2{,}42, \tag{8}$$

and then from our measurements it follows that:

$$\sigma(p\mu+p)=(17.4\pm3.3)10^{-21}\ \mathrm{cm}^2.$$

In Table II we have given the latest experimental and theoretical values for the cross section of reaction (1). In

TABLE II.

| | Hydrogen pressure, atm | $\sigma(p\mu+H_2)$ *, $10^{-21}$ cm$^2$ | $\sigma(p\mu+p)$, $10^{-21}$ cm$^2$ |
|---|---|---|---|
| Bologna-CERN[8,9] | 26 | 29,8±2,8 | 12,3±1,2 ** |
| Present work | 41 | 42±8 | 17,4±3,3 ** |
| Theory[20] | – | – | 19 |

*The values have been normalized to the density of molecules in the conditions of the experiment. **Found from Eq. (8).

Refs. 8 and 9 the result was presented in the form of the cross section $\sigma(p\mu + \mathrm{H})$, and in the table we have given twice this value. As one can see, the experimental and theoretical values of the cross section for scattering of $p\mu$ atoms in hydrogen agree satisfactorily in terms of the errors. We note, however, that the two experimental results were obtained on assumptions which differ in several respects (for example, in Refs. 8 and 9 the value of the effective mass of the hydrogen molecule was set equal to $2m_p$). Therefore it would be premature to combine these results by statistical summation.

In conclusion the authors express their gratitude to S. S. Gershteĭn, L. I. Ponomarev, A. I. Men'shikov, and V. S. Melezhik for helpful discussions and valuable remarks.

[1] S. S. Gershteĭn, Zh. Eksp. Teor. Fiz. **36**, 1309 (1959) [Sov. Phys. JETP **9**, 927 (1959)]. H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959).
[2] V. P. Dzhelepov, P. F. Ermolov, and V. V. Fil'chenkov, Zh. Eksp. Teor. Fiz. **49**, 393 (1965) [Sov. Phys. JETP **22**, 275 (1966)].
[3] A. Alberigi Quaranta, A. Bertin, G. Matone, *et al.*, Nuovo Cimento **47B**, 72 (1967).
[4] S. Cohen, D. L. Judd, and R. J. Riddell, Phys. Rev. **119**, 384 (1960).
[5] Ya. B. Zel'dovich and S. S. Gershteĭn, Usp. Fiz. Nauk **71**, 581 (1960) [Sov. Phys. Uspekhi **3**, 593 (1961)].
[6] A. V. Matveenko and L. I. Ponomarev, Zh. Eksp. Teor. Fiz. **59**, 1593 (1970) [Sov. Phys. JETP **32**, 871 (1971)].
[7] A. V. Matveenko, L. I. Ponomarev, and M. P. Faĭfman, Zh. Eksp. Teor. Fiz. **68**, 437 (1975) [Sov. Phys. JETP **41**, 212 (1975)].
[8] A. Bertin, I. Massa, M. Piccinini, *et al.*, Phys. Lett. **78B**, 355 (1978).
[9] A. Bertin, M. Capponi, I. Massa, *et al.*, Nuovo Cimento **72A**, 225 (1982).
[10] V. E. Markushin, Zh. Eksp. Teor. Fiz. **80**, 35 (1981) [Sov. Phys. JETP **53**, 16 (1981)].
[11] A. S. Wightman, Phys. Rev. **77**, 521 (1950).
[12] M. Leon and H. A. Bethe, Phys. Rev. **127**, 636 (1962).
[13] V. M. Bystritskiĭ, V. P. Dzhelepov, P. F. Ermolov, *et al.*, Zh. Eksp. Teor. Fiz. **66**, 43 (1974) [Sov. Phys. JETP **39**, 19 (1974)]. V. V. Fil'chenkov, Dissertation, Dubna, Joint Institute for Nuclear Research, 1-7900, 1974.
[14] V. M. Bystritskiĭ, V. P. Dzhelepov, V. I. Petrukhin, *et al.*, Zh. Eksp. Teor. Fiz. **71**, 1680 (1976) [Sov. Phys. JETP **44**, 881 (1976)].
[15] V. M. Bystritskiĭ, V. P. Dzhelepov, P. F. Ermolov, *et al.*, Dubna, JINR, 13-7246, 1973.
[16] V. P. Dzhelepov, P. F. Ermolov, E. A. Kushnirenko, *et al.*, Zh. Eksp. Teor. Fiz. **42**, 439 (1962) [Sov. Phys. JETP **15**, 306 (1962)].
[17] E. J. Bleser, E. W. Anderson, L. M. Lederman, *et al.*, Phys. Rev. **132**, 2679 (1963).
[18] A. Bertin, M. Bruno, and A. Vitale, Lett. Nuovo Cimento **4**, 449 (1972).
[19] V. P. Dzhelepov, P. F. Ermolov, V. I. Moskalev, *et al.*, Zh. Eksp. Teor. Fiz. **47**, 1243 (1964) [Sov. Phys. JETP **20**, 841 (1965)].
[20] V. S. Melezhik, L. I. Ponomarev, and M. P. Faĭfman, Dubna, JINR, R4-83-89, 1983.
[21] A. I. Men'shikov, Preprint *IAÉ* No. 3811, Institute of Atomic Energy, Moscow, 1983.

Translated by Clark S. Robinson