Photon antibunching in a coherent light source and suppression of the photorecording noise

Yu. M. Golubev and I. V. Sokolov

Leningrad State University

(Submitted 10 February 1984)
Zh. Eksp. Teor. Fiz. **87**, 408–416 (August 1984)

The theoretical possibility of reducing the natural fluctuations in laser generation to lower the excitation noise of the working substance is discussed. Conditions can be cited under which the light from such a source is a quantum field and, in particular, in which there is considerable photon antibunching and the variance of the number of photons is smaller than that given by the Poisson distribution. The photorecording process has an interesting physical feature: the photocurrent noise at low frequencies turns out to be below the shot-noise level and can even be reduced to zero without additional light losses when detecting.

Much attention has been given recently in the literature to the ultimate possibilities of optical methods in a number of physically important problems. We may cite, for example, the problem of detecting gravitational waves,[1,2] optical manifestations of parity violation, and so on. It is well known that the principal limitations on the accuracy of many measurements are associated with the natural noise of coherent light. The principal sources of such noise are assumed to be the following:

1) the quantum statistical character of the elementary interaction of light with radiating particles (in the end, with atoms);

2) the random character of the processes that lead to field losses, which follow from the Callen-Welton fluctuation-dissipation[3,4]; and

3) the statistical nature of the elementary processes of atomic excitation by an active medium.

However, one can imagine a method of exciting matter in which the last of the noise sources listed above is eliminated (see Section 1). This leads to interesting physical consequences which, as far as we are aware, have not been discussed for the case of coherent radiation. The purpose of our work is to give a theoretical analysis of laser generation in the absence of noise due to excitation of matter.

Using the quantum theory of radiation, we show that the light from such a source is a quantum field. The variance of the number of photons associated with the noise turns out to be smaller than the variance of a Poisson distribution, which, under optimal conditions, describes the radiation field of a "ordinary" coherent source a well as the field in a coherent Glauber state. The form we obtained for the correlation function of the photocounts indicates antibunching of the photons, while the frequency spectrum of the photocurrent as observed by the light intensity fluctuations falls below the shot noise level at low frequencies.

It is known that the sensitivity of such methods as intensity-fluctuation spectrosocopy[5] (IFS) and intraresonator modulation spectroscopy[6] can be improved by reducing the noninformative part of the spectral power of the photocurrent. The usual way to extract the IFS signal as well as possible from the shot-noise background is to increase the quantum yield q of the photodetector—in the ideal case, to $q = 1$. An interesting and important feature of the light source considered in this work is that for ideal detection ($q = 1$) of the light from it, the photocurrent noise at low frequencies is not only below the shot-noise level, but, in principle, can be reduced to zero. This result is intemately associated with the antibunching of the photons and has a simple physical explanation, which is given in Section 4.

1. ELIMINATION OF THE NOISE DUE TO EXCITATION OF THE ACTIVE MEDIUM

Let us discuss the theoretical possibility of exciting the matter in such a manner that the number of atoms excited per unit time will not be random, i.e., that the excitation noise will be suppressed. The results of this discussion will indicate the range of applicability of our theory.

Let us consider an active medium in which the number of atoms that have the working transition remains constant, as would be the case, for example, if the medium were a doped crystal. This eliminates the randomness in the passage of the atoms into and out of the region of interaction with the radiation. The level scheme of an active atom is shown in Fig. 1. Let us assume that at first all the atoms are in the ground state 0. The perturbation v in the excitation channel $0 \to 3$ (e.g. an exciting light beam) is turned on at the time T_0. The intermediate level 3 decays rapidly at the rate γ_3 to the upper level 2 of the working transition $2 \to 1$. If the condition $\gamma_3 \gg v/\hbar$ for irreversible excitation is satisfied and the duration T_0 of the excitation is long enough to satisfy the condition $T_0 v^2/\hbar^2 \gamma_3 \gg 1$, then all the atoms will collect on the upper level of the radiative transition $2 \to 1$ during the time T_0. Then there will be no randomness in the initial inversion (to the extent that the above requirements are met).

Let us assume that the lifetime τ_a of an atom on the working levels 2 and 1 is much shorter than the confinement

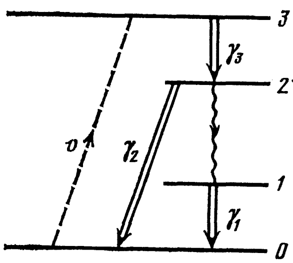

FIG. 1.

 0038-5646/84/080234-05$04.00

time C^{-1} of the radiation field in the optical resonator, i.e. $C^{-1} \gg \tau_a$, where C is the width of the resonator line. For the case in which $\gamma_1 \gg \gamma_2$, where γ_1 and γ_2 are the relaxation rates of the levels 1 and 2, and this is the case of importance for us, the rapid decrease of the number of atoms excited to the working levels will be assured provided the generation field E is not too weak, i.e. provided $\tau_a \sim \gamma_1 \hbar^2/(d_{21}E)^2 \ll C^{-1}$, where d_{21} is the transition dipole moment. In a saturating field, we have $\tau_a \sim \gamma_1^{-1}$. At a time of the order of τ_a after the arival of the exiting pulse, all the atoms will have taken part in the radiation at the working transition and will have returned to the ground state. We impose the condition $\tau_a \gg T_0$ so that no atom may be excited twice during a single pumping pulse.

If the exciting pulses follow one another periodically with the time interval $T \gg \tau_a$ between them, then the same number of atoms, i.e. all the atoms in the active region, will take part in each cycle. This also means that the components of the exitation noise that are slow on the time scale T_0 will be eliminated. The "cost" of suppressing the excitation noise is regular modulation of the light with the period T. However, the modulation depth will be small if the field is held in a high-Q resonator between the exciting pulses. We assume, further, that the confinement time C^{-1} of the light between the mirrors is long: $C^{-1} \gg T$. Thus, the characteristic times for the source we are describing stand in the following relations to one another: $C^{-1} \gg T \gg \tau_a \gg T_0$. The time trends of the principal processes are indicated in Fig. 2.

In what follows we shall consider only the statistical characteristics of the light and shall not be interested in its modulation, assuming that the regular modulation can either be eliminated or taken into account by electronic methods. That is precisely the situation when the photocurrent spectrum is observed with an instrument whose spectral resolution $\delta\omega$ is much smaller than C (as we shall see below, the width of the break in the power spectrum of the current is of the order of C). The time required for such a measurement will be longer than the time $\delta\omega^{-1}$, and in view of the fact that $\delta\omega^{-1} \gg C^{-1} \gg T$, it will be much longer than the period T of the excitation; the pulsations of the measured spectrum with the period T will therefore be vanishingly small. The fluctuations of the light intensity at the frequency $2\pi T^{-1}$ will of course contribute to the current spectrum, but in a frequency range of the order of $T^{-1} \gg C$ in which we are not interested.

2. DEVELOPMENT OF THE RADIATION FIELD IN THE ABSENCE OF EXCITATION NOISE

The condition $C^{-1} \gg \tau_a$ means that the atomic subsystem "trails" behind the field subsystem, and that makes it

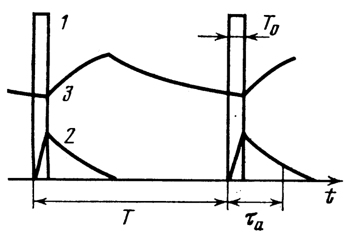

FIG. 2. Time dependences of the excitation intensity (1), the number of excited atoms (2), and the energy of the field in the resonator (3).

possible to obtain a closed kinetic equation for the density matrix of the generating field oscillator. For simplicity we shall consider the generation of a single traveling wave.

Lamb and Scully[7,8] obtained a kinetic equation for a source of coherent light in which the lower working level is not the ground state; this equation can be written in the form

$$\dot{\rho} = r\hat{u}\rho. \tag{1}$$

Here ρ is the density matrix of the generating field, r is the average number of atoms excited to the upper working level per unit time, and $\hat{u}$ is an operator that arises in the solution of the problem of the interaction of a field with a single atom. This operator was in fact found in Refs. 7 and 8 (the explicit form of $\hat{u}$ will be given later). We shall temporarily ignore the decrement of the field due to the finite Q of the resonator and shall show that Eq. (1), as well as other quantum and semiclassical theories that yield similar results, lead to Poisson statistics for the excitation of the material and therefore contain an excitation-noise contribution. As far as can be judged from the literature, this circumstance has not been recognized before, since the original derivation of the Lamb-Scully equation (1) is not based explicitly on an average over the statistics of the excitation.

Let us assume that an atom was excited in the system at the time t_0, that it interacted with the field, and that it decayed to the ground state in the time τ_a. On the basis of the results of Refs. 7 and 8 we find that in this case the density matrix of the field will have undergone the transformation

$$\rho(t_0+\Delta t) = (1+\hat{u})\rho(t_0),$$

where $\Delta t \gg \tau_a$. If n atoms were excited during the time interval Δt, we would have

$$\rho(t_0+\Delta t) = (1+\hat{u})^n \rho(t_0), \tag{2}$$

where we have taken into account the fact that each atom finds the field in the state prepared by the preceeding atom. If the excitation process obeys Poisson statistics, we must average Eq. (2) with the weight $\exp(-r\Delta t)(r\Delta t)^n/n!$; this yields

$$\rho(t_0+\Delta t) = \exp(r\Delta t\hat{u})\rho(t_0).$$

On differentiating, we obtain Eq. (1). In the case of regular excitation, however, we set $n = r\Delta t$ in (2), and this leads to the equation

$$\dot{\rho} = r \ln(1+\hat{u})\rho, \tag{3}$$

which will be solved later on. The difference between Eqs. (3) and (1) is associated with the higher powers of the operator $\hat{u}$. Although this operator is generated by a single atom and is accordingly small, Eqs. (3) and (1) still do not reduce to one another, but describe different radiation statistics. This is associated with the following circumstance: the operator $\hat{u}$ can be expressed as a series

$$\hat{u} = \hat{u}_1\lambda + \hat{u}_2\lambda^2 + \ldots,$$

in which the small parameter λ is associated with the smooth dependence of the matrix elements $\rho_{n,n+m}$ on the subscript n. If the rms deviation of the number of photons from the mean value $\bar{n}$ is of the order of $\bar{n}^{1/2}$, then $\lambda \sim 1/\bar{n}^{1/2} \ll 1$. It turns out that if only the term proportional to λ is retained in the kinetic equation, the resulting equation will represent a semiclassical dynamics for the generation in the absence of radiation noise. At least the term of the order of λ^2 must be retained in order to describe the noise. In the expansion of $\ln(1+\hat{u})$ in powers of $\hat{u}$, therefore, at least the first two terms of the series must be retained.

It would seem that a description of regular excitation could be based on the Lamb-Scully equation (1), using a diagonal representation in coherent states. For the diagonal weight, Eq. (1) leads to the Fokker-Planck equation, in which the coefficients of the second derivatives with respect to the amplitude and phase variables determine the noise characteristics of the radiation. On proceeding to regular excitation one must subtract the contributions due to excitation noise of the medium from the coefficients (in the case of a laser with a uniform amplification line, the noise contributions to be subtracted are given in an appendix to the Russion translation of Ref. 8 by A. P. Kazantsev and G. I. Surdutovich, and for the case of a nonuniformly broadened line they are given in Ref. 9). It turns out, however, that this approach becomes meaningless for the most interesting range of the physical parameters (corresponding to a quantum field): the coefficient of the second derivative with respect to the field amplitude becomes negative. We therefore forgo the diagonal representation and solve Eq. (3) in the occupation number representation.

3. PHOTON STATISTICS AND ANTIBUNCHING

Now let us consider the radiating system under such conditions that the effects of antibunching and of the narrowing of the distribution of the number of field quanta are maximal. We set $\gamma_2 = 0$ so that each active atom returns to the ground state only from the lower level of the working transition and necessarily gives up its excitation of the generating field. We shall assume that the generation takes place at the center of a homogeneous gain line. In this case the action of the operator $\hat{u}$ is given (see Ref. 8) by

$$(\hat{u}\rho)_{mn}=\frac{2(mn)^{1/2}}{m+n+\beta(m-n)^2}\rho_{m-1,n-1}-\rho_{mn}, \tag{4}$$

where β is the nonlinearity parameter, f is the constant coupling the field to the atomic transition,

$$\beta=\frac{|f|^2}{\gamma_1\gamma_{21}},\quad f=id_{21}\left(\frac{2\pi\hbar\omega_0}{V}\right)^{1/2}.$$

Here γ_{21} is the transverse relaxation constant, V is the volume of the resonator, and ω_0 is the frequency of the field. The kinetic equation (3) yields the following equation for the diagonal elements of the density matrix:

$$\begin{aligned}\dot{\rho}_{nn}=r\{(-\rho_{nn}+\rho_{n-1,n-1})-{}^1/_2(\rho_{nn}-\rho_{n-1,n-1})\\ +{}^1/_2(\rho_{n-1,n-1}-\rho_{n-2,n-2})\}\\ +C\{-n\rho_{nn}+(n+1)\rho_{n+1,n+1}\}.\end{aligned} \tag{5}$$

The field-damping contributions contain the factor C, the width of the resonator line. In writing Eq. (5) we have used the approximation $\ln(1+\hat{u})\approx\hat{u}-\hat{u}^2/2$. For nearly stationary states of the field, the contribution from $\hat{u}$ to the damping terms is of the same order of magnitude as the contribution from $-\hat{u}^2/2$; this is associated with the balance of gains and losses. On the one hand, therefore, the contribution from the term $-\hat{u}^2/2$ must be taken into account (it was noted above that this term must be taken into account when investigating fluctuations) and, on the other hand, we may assume that the higher powers of $\hat{u}$ will give only small corrections.

We introduce the generating function

$$G(z,t)=\sum_{n=0}^{\infty}\rho_{nn}(t)z^n.$$

Equation (5) yields an equation for $G(z,t)$ of the form

$$\frac{\partial}{\partial t}G=C(1-z)\left\{\bar{n}\frac{z-3}{2}G+\frac{\partial}{\partial z}G\right\},$$

whose general solution is not difficult to obtain:

$$G(z,t)=G(z_0,0)\exp\{{}^3/_2\bar{n}(z-z_0)-{}^1/_4\bar{n}(z^2-z_0^2)\}, \tag{6}$$

where $z_0=1+(z-1)\exp(-Ct)$ and $\bar{n}=r/C$. In what follows we shall denote a statistical average either by a bar or by angle brackets.

We shall show that the correlation function $g(t)$ for the photocounts can be expressed in terms of the generating function $G(z,t)$ as evaluated for a particular initial condition. As is well known,

$$g(t)=\langle a^+(0)a^+(t)a(t)a(0)\rangle. \tag{7}$$

We shall expand the average as the trace of the product of the Heisenberg field operators by the density operator. We goup the evolution operators of the system in such a manner that they describe the time development of a certain matrix, and turn to a closed description of the evolution of the field on the basis of the kinetic equation. We define the evolution operator $\hat{Q}(0,t)$ for the density matrix of the field so that $\rho(t)=\hat{Q}(0,t)\rho(0)$, and obtain

$$\begin{aligned}g(t)&=\mathrm{Sp}\{a^+a\hat{Q}(0,t)(a\rho^{(c)}a^+)\}\\ &=\sum_{n,m=0}^{\infty}n\hat{Q}(0,t)_{nn}^{mm}(m+1)\rho^{(c)}_{m+1,m+1},\end{aligned} \tag{8}$$

where $\rho^{(c)}$ is the stationary density matrix of the field. At the same time, by the definition of the generating function we have

$$\frac{\partial}{\partial z}G|_{z=1}=\sum_{n=0}^{\infty}n\rho_{nn}(t)=\sum_{n,m=0}^{\infty}n\hat{Q}(0,t)_{nn}^{mm}\rho_{mm}(0). \tag{9}$$

It is evident that Eqs. (8) and (9) are the same provided the generating function is defined for the initial conditions

$$\rho_{mm}(0)=(m+1)\rho^{(c)}_{m+1,m+1},\quad G(z,0)=\frac{\partial}{\partial z}G^{(c)}(z),$$

where $G^{(c)}(z)$ is the limit as $t\to\infty$ of $G(z,t)$ as given by Eq. (6). With the aid of these relations we obtain the correlation function $g(t)$:

$$g(t)=\bar{n}^2-{}^1/_2\bar{n}e^{-ct}. \tag{10}$$

As is evident from (10), the probability for recording a photon in the time t following the initial photorecording event increases with the time, i.e. photon antibunching takes place in times of the order of the confinement time of the field in the resonator. The physical reason for antibunching in a laser without noisy excitation is the time correlation of the elementary atomic excitation processes (the suppression of excitation noise doubtless indicates strong correlation in the excitation).

Antibunching of photons in a laser whose lower working level is the gound state was predicted theoretically in a recent paper by Lugiato, Casagrande, and Pizzuto.[10] The suppression of pumping noise that we are considering is impossible in such a physical system; the antibunching there is due to other causes. The effect found in Ref. 10 unlike the effect in our system, is small and is attributed by the authors to collective phenomena in the interaction of the field with the active medium.

In discussing the result (10) it must be borne in mind that the initial description based on (3) is smoothed out over the characteristic time scale τ_a, and rapid variations of the field cannot be treated.

It is not difficult to find the moments of the photon-number distribution with the aid of the generating function (6). The mean square fluctuations of the number of photons in the case of steady-state generation is $\overline{\Delta n^2} = \bar{n}/2$, where $\Delta n = n - \bar{n}$, and turns out to be smaller by a factor of two than it would be for a Poisson distribution. It is natural to call such a state of the field a quantum state since the diagonal weight of the representation of the density matrix in coherent states is a generalized function, and the usual association of the diagonal weight with the amplitude distribution of a classical wave is not possible. We may also consider the moment $\overline{\Delta n^3}$ that describes the asymmetry of the distribution; for steady-state generation $\overline{\Delta n^3} = -\bar{n}/2$, and this differs from the value $\overline{\Delta n^3} = \bar{n}$ for the Poisson distribution.

4. REDUCTION OF THE PHOTORECORDING NOISE

Let us assume that the light leaves the resonator through a semitransparent mirror whose energy transmission coefficient is Λ and is converted into the photocurrent $i(t)$ at the collector with the quantum yield q. The measured quantity is the current correlation function $\langle \frac{1}{2}[i(0), i(t)]_+\rangle$ ($[\ldots]_+$ denotes the anticommutator), which can be expressed in terms of the Heisenberg field operators. If all the light flux of cross section S passes through the mirror into the photodetector, it follows from the general description of photodetection (see e.g., Refs. 11 and 12) that

$$\langle {}^1/_2[i(0), i(t)]_+\rangle = C_p\bar{n}\delta(t) + C_p^2\{g(t)\theta(t) + (t \leftrightarrow -t)\}, \quad (11)$$

where $\theta(t) = 0$ when $t<0$ and $\theta(t) = 1$ when $t \geqslant 0$. The correlation function $g(t)$ found explicitely above occurs in (11). By analogy with the total field loss rate C, we call the quantity $C_p = cq\Lambda S/V$ the resultant field loss rate; it is the ratio of the number of photons converted into photoelectrons per unit time to the total number of photons in the resonator. We obtain the spectral power of the photocurrent noise,

$$(i^2)_\omega = \int_{-\infty}^{\infty} dt \langle {}^1/_2[i(0), i(t)]_+\rangle e^{i\omega t}$$

in the form

$$(i^2)_\omega = C_p\bar{n}\left(1 - \frac{C_pC}{C^2+\omega^2}\right), \quad (12)$$

from which the zero frequency contribution from the constant component of the current has been eliminated.

The first term corresponds to the photorecording shot noise. The ratio C_p/C of the resultant losses to the total losses is the probability that a photon which escapes from the resonator will produce a photoelectron. If the field losses due to absorption, diffraction, and the fact that the collector is not ideal are eliminated, then $C_p/C = 1$ and, as is evident from (12), the spectral power of the photocurrent noise vanishes at frequencies $\omega \ll C$. This means that the usual limitation of the sensitivity of the measurements associated with the photorecording shot noise is not present in the case of the light source described here.

In an actual case the spectral power of the current noise at low frequencies stands in the ratio $(C - C_p)/C$ to the shot noise level. Since the quantum yields of the best light collectors differ from unity by several dozen percent, a considerable reduction of the photodetection noise is possible in principle.

In order that the reasons for the inhibition of the low-frequency components of the current may be more clearly understood, we present some simple balance considerations that will show, in particular, how the action of such a source of fluctuations as the the randomness in the emission of photoelectrons at a given light intensity is compensated in the system.

As above, we assume that the atoms are excited uniformly (there is no excitation noise) and that the transition from the upper working level to the gound state is forbidden ($\gamma_2 = 0$). Since each atom is finally deexcited with the emission of a single photon, on a time scale that is slow compared with τ_a the rate of increase of the number of photons in the resonator due to the excitation is r and does not fluctuate. The number of photons in the resonator and the number of photoelectrons produced per second by the radiation are $n(t)$ and $i(t)$, respectively. Let us assume that $C_p/C = 1$, i.e. that each photon lost by the field is converted into a photoelectron. Then the balance of excitations in the system has the form

$$\dot{n} = r - Cn - f(t), \quad i = Cn + f(t). \quad (13)$$

The equation for the current contains not only the regular component, which is proportional to the light intensity, but also a random shot component $f(t)$, whose statistical properties, as is well known, are given by the correlation function $\overline{f(0)f(t)} = \bar{i}\delta(t)$. Since the light is completely recorded, the total number of excitations (the sum of the number of photons and electrons) does not fluctuate and is determined by the regular excitation $\dot{n} + i = r$. The random quantity $f(t)$ therefore occurs in the equation for the number of quanta, but with the opposite sign.

Let us introduce the small random deviations $\delta n = n - \bar{n}$ and $\delta i = i - \bar{i}$, where $\bar{n} = r/C$ and $\bar{i} = r$. Taking (13) into account, the power spectrum of the current can be written in the form

$$(\delta i^2)_\omega = (f^2)_\omega + 2\mathrm{Re}\, C(\delta nf)_\omega + C^2(\delta n^2)_\omega. \quad (14)$$

The spectrum (14) can be calculated with the aid of the solution of Eqs. (13). Retaining the order in which the contributions follow one another for clarity, we obtain

$$(\delta i^2)_\omega = C\bar{n}\left\{1 - \frac{2C^2}{C^2+\omega^2} + \frac{C^2}{C^2+\omega^2}\right\}. \quad (15)$$

It is evident from this that the suppression of the low-frequency components of the photocurrent is associated with the anticorrelation of the shot source of the current fluctuatons $f(t)$ and of the intensity fluctuations $\delta n(t)$. If, for example, the randomness in the times at which electrons are ejected should lead to a sudden increase in the current, the number of photons in the resonator would simultaneously decrease. In this case the regular component of the current [the component proportional to $\delta n(t)$] would decrease, and at low frequencies, where the time shift of these mutually connected processes is unimportant, there would be complete compensation.

The approximations made in the above treatment are justified by the agreement of the result (15) with conclusions derived from a rigorous quantum theory.

5. PHYSICAL CONDITIONS FOR THE REALIZATION OF A QUANTUM FIELD

Let us discuss the requirements on the physical parameters of the active medium for the quantum character of the radiation field, the antibunching of the photons, and the suppression of the photodetection noise to be maintained, even if only partly. Up to now we have assumed that the relaxation channel of the upper working level is closed ($\gamma_2 = 0$) and that there is no frequency mismatch ($\Delta = \omega_{21} - \omega_0 = 0$, where ω_{21} is the transition frequency and ω_0 is the field frequency). In the general case, when $\gamma_2 \neq 0$, the excited atoms may lose their excitations in a nonradiative manner. Since the elementary quenching processes are random, the light noise will increase in the end. The detuning of the frequency of the radiation from the center of the amplification line acts similarly. As Δ increases, the effectiveness of the interaction of the atom with the field decreases and the 2→0 decay plays a larger part.

In what follows we shall assume the relaxation constants γ_1, γ_2, and γ_{21} and the frequency mismatch Δ to be arbitrary. We note that as Δ increases, the lifetime on the working levels may increase, and the requirement $C^{-1} \gg \tau_a$ must be borne in mind. We define the dimensionless intensity I and the unsaturated amplification factor A for a uniform amplification line as follows:

$$I = \frac{2|f|^2 \bar{n}(\gamma_1+\gamma_2)\gamma_{21}}{\gamma_1\gamma_2(\gamma_{21}^2+\Delta^2)} \qquad A = \frac{2r|f|^2\gamma_{21}}{\gamma_2(\gamma_{21}^2+\Delta^2)}$$

where the average number of photons, $\bar{n}$, is to be found from the condition $A = (1 + I)G$ for steady-state generation. A calculation similar to the one given above yields the following expression for the variance of the number of photons:

$$\overline{\Delta n^2} = \bar{n}(1+\xi), \qquad \xi = I^{-1} - \frac{1}{2}\frac{\gamma_1}{\gamma_1+\gamma_2}. \qquad (16)$$

The negative contribution to ξ arose as a result of the suppression of the excitation noise. It is evident from (16) that the variance is smaller than that for a Poisson distribution, i.e. the radiation field is a quantum field provided $I > 2(\gamma_1 + \gamma_2)/\gamma_1$. For a given excitation rate and fixed relaxation constants, an increase in the detuning Δ worsens the conditions for the manifestation of the quantum character of the field, since the dimensionless intensity decreases.

We find the correlation function of the photocounts in the form

$$g(t) = \bar{n}^2 + \bar{n}\xi e^{-\Gamma t}, \qquad (17)$$

where $\Gamma = CI/(1 + I)$. As is evident, the quantum character of the field also implies antibunching of the photons.

The spectral power of the photocurrent noise is

$$(i^2)_\omega = C_p\bar{n}\left\{1+2\xi\frac{C_p\Gamma}{\Gamma^2+\omega^2}\right\}.$$

This result enables us to estimate, under realistic conditions, the photocurrent noise at low frequencies, which is due both to incomplete recording ($C_p/C < 1$) and to the randomness in the transfer of energy from the material to the field when $\gamma_2 \neq 0$.

One can also calculate the field-strength spectrum of the generating field in the framework of our treatment; it has the form

$$(a^2)_\omega = \bar{n}\frac{D}{D^2+\omega^2} + \frac{1}{4}\xi\frac{\Gamma}{\Gamma^2+\omega^2}, \qquad (18)$$

where

$$D = \frac{C}{4\bar{n}}\left\{1 + I\frac{\gamma_2}{\gamma_{21}^2}\frac{(\gamma_{21}^2+\Delta^2)}{(\gamma_1+\gamma_2)} + \frac{I}{(1+I)}\frac{\Delta^2}{\gamma_{21}^2}\frac{(\gamma_1-\gamma_2)}{(\gamma_1+\gamma_2)} - \frac{I}{2(1+I)}\frac{\Delta^2}{\gamma_{21}^2}\frac{\gamma_1}{(\gamma_1+\gamma_2)}\right\}.$$

Frequency pulling has been omitted from Eq. (18), the spectrum being shifted to zero frequency. The suppression of excitation noise leads to a certain decrease in the half width D of the line; this is responsible for the negative sign of the last term in the above expression for D. There is no narrowing of the generating line at the center of the amplification contour. If the frequency mismatch is large ($\Delta \gtrsim \gamma_{21}$) and the conditions $\gamma_1 \gg \gamma_2$ and $I \gg 1$ characteristic of a quantum field are satisfied, then the greatest narrowing (by about a factor of two) is reached when $\gamma_1 \gg \gamma_2$.

The spectrum (18) also has an enegertically negligible contribution arising from the amplitude fluctuations. This contributions, which plays a part only in the remote wings, enters with a negative weight in the case of a quantum field.

In concluding, we note that the scheme of regular noise-free excitation of matter used in this work was called to our attention by E. B. Aleksandrov (in connection with the problem of suppressing the photorecording noise of spontaneous emission). We also thank E. B. Aleksandrov for discussing the results of the work.

[1]I. M. Belousova, L. F. Vitushkin, I. P. Ivanov, *et al.*, Usp. Fiz. Nauk **134**, 170 (1981) [Sov. Phys. Usp. **24**, 441 (1981)].
[2]L. P. Grishchuk and M. V. Sazhin, Zh. Eksp. Teor. Fiz. **84**, 1937 (1983) [Sov. Phys. JETP **57**, 1128 (1983)].
[3]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical physics), Moscow, Nauka, 1964 [Engl. Transl. Pergamon Press, London, 1969.
[4]Volnovye i fluktuatsionnye protsessy v lazerakh (Wave and fluctuation processes in lasers), Yu. L. Klimontovich, Editor, Nauka, Moscow, 1974.
[5]E. B. Aleksandrov, Yu. M. Golubev, A. V. Lomakin, and V. A. Noskin, Usp. Fiz. Nauk **140**, 547 (1983) [Sov. Phys. Usp. **26**, 643 (1983)].
[6]A. A. Mak, O. A. Orlov, and V. I. Ustyugov, Kvantovaya elektronika **9**, 2412 (1982) [Sov. J. Quantum Electron. **12**, 1574 (1982)].
[7]Marion O. Scully and Willis E. Lamb Jr., Phys. Rev. **159**, 208 (1967).
[8]F. Arecchi, M. Scully, G. Haken, and W. Weidlich, Quantum fluctuations of laser radiation (Russ. transl.) Mir, Moscow, 1974.
[9]Yu. M. Golubev, I. V. Sokolov, and V. P. Gryaznevich, Opt. Spektrosk. **51**, 875 (1981).
[10]L. A. Lugiato, F. Casagrande, and L. Pizzuto, Phys. Rev. **A26**, 3438 (1982).
[11]D. F. Smirnov and I. V. Sokolov, Zh. Eksp. Teor. Fiz. **70**, 2098 (1976) [Sov. Phys. JETP **43**, 1095 (1976)].
[12]D. F. Smirnov, I. V. Sokolov, and A. S. Troshin, Vestnik LGU, seriya fiz.-khim. (Leningrad State University Bulletin, Physical-chemical series) **10**, 36 (1977).

Translated by E. Brunner