Spin nematics

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(Submitted 6 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 467–475 (August 1984)

We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid ^{3}He is discussed.

The onset of magnetic order in exchange magnets is accompanied by spontaneous symmetry breaking relative to the spin-space rotation group $O(3)$. In ordinary magnets the character of the ordering is such that the order parameter representative of the coordinate dependence of the average microscopic spin density is always transformed in accord with the vector representation of the spin group $O(3)$, i.e., in particular, it reverses sign under spin inversion whose role is played by the time-reversal transformation. It is precisely this last property which is usually regarded as the general criterion of magnetic ordering. Magnets differ in accordance with which representation of the pure space group of the crystal the order parameter is transformed. This is the unitary representation for ferromagnets, the one-dimensional non-unitary representation for collinear antiferromagnets, and so on (see Ref. 1).

We analyze in this paper the possible properties of exchange systems in which magnetic ordering leads to spontaneous breaking of the spin group $O(3)$, but invariance to time reversal is preserved. By virtue of this last property, the average microscopic spin density vanishes in the systems considered. The order parameter should be introduced as a characteristic of the transformation properties of the spin correlation functions and described by the tensor representation of the spin group $O(3)$. The symmetry of the considered systems is similar to the symmetry of ordinary nematics, the only difference being that we are dealing here with spin rather than coordinate space. We shall show below on the basis of general symmetry considerations that despite the invariance to time reversal, the magnetic properties of "spin nematics" are indistinguishable from the properties of antiferromagnets. Namely, all the phenomena typical of antiferromagnets, such as flopping and collapse of sublattices or zero longitudinal magnetic susceptibility at zero temperature, should be observed in spin nematics. The macroscopic equations that describe the low-frequency long-wave dynamic properties of spin nematics, particularly the spin-wave spectra, coincide with the corresponding equations for antiferromagnets and, just as in the latter, systems of the "easy axis" and "easy plane" type are possible here.

We consider also examples of systems in which states with spin-nematic symmetry are realized. The first is the quadrupole ordering, first observed by Blume and Hsieh, [2] of spins $s = 1$ localized at lattice sites. The interaction between which is characterized by the presence of a rather large biquadratic exchange. This system was considered in many papers.[2–5] Matveev[4] observed a spin-wave spectrum of antiferromagnetic type in the absence of relativistic interactions, as well as a phase transition corresponding to collapse of the sublattices of an antiferromagnet. We shall show how, in accord with the general properties noted above, there are produced in this system all the remaining properties typical of antiferromagnets—zero longitudinal susceptibility, a phase transition corresponding to sublattice flopping, etc. Quadrupole ordering is impossible for spin $s = 1/2$. We shall consider an example of a system of spins $s = 1/2$ localized in a lattice in which the spin-nematic state is produced via appearance of anisotropic spin correlations in the presence of a sufficiently large four-particle exchange interaction.

1. GENERAL PROPERTIES

Let $\mathbf{s}(\mathbf{r})$ be the microscopic spin density operator. In states with magnetic order invariant to time reversal, just as in the paramagnetic state, we have $\langle \mathbf{s}(\mathbf{r}) \rangle = 0$. We consider the spin-correlation function $\langle s_i(\mathbf{r}) s_k(\mathbf{r}') \rangle$. In the paramagnetic state, by virtue of the invariance to the spin group $O(3)$, this function reduces to the spin scalar

$$\langle s_i(\mathbf{r}) s_k(\mathbf{r}') \rangle = \delta_{ik} F(\mathbf{r}, \mathbf{r}').$$

The nonscalar part of the correlation function

$$K_{ik}(\mathbf{r}, \mathbf{r}') = \langle s_i(\mathbf{r}) s_k(\mathbf{r}') - {}^1/_3 \delta_{ik} s_l(\mathbf{r}) s_l(\mathbf{r}') \rangle \quad (1)$$

can therefore be regarded as a characteristic of the symmetry of a magnetic order that is invariant to time reversal. Separating in K_{ik} the parts symmetric and antisymmetric in the indices i and k, we rewrite (i) in the form

$$K_{ik}(\mathbf{r}, \mathbf{r}') = e_{ikl} P_l(\mathbf{r}, \mathbf{r}') + Q_{ik}(\mathbf{r}, \mathbf{r}'), \quad (2)$$

where $P_i(\mathbf{r}, \mathbf{r}')$ are antisymmetric functions of the spatial arguments, and the Q_{ik} are symmetric in the indices i and k as well as in $\mathbf{r}$ and $\mathbf{r}'$, with $Q_{ii} = 0$.

We shall not engage here in classification in all the cases that are possible in principle. As is customarily done in the investigation of spontaneous breaking of the space group $O(3)$ in ordinary nematics,[6] we confine ourselves to structures having an axis of total axial spin symmetry.

Just as any exchange magnetic system, a spin nematic is

 0038-5646/84/080267-05$04.00

described by specifying the exchange symmetry group,[1] whose elements are products of certain spin rotations by elements of a certain space group G consisting of translations, rotations, and reflections. The group G itself determines the symmetry of the spin scalars in the system. Let the system considered be characterized by a nonzero spin pseudovector $P_i(\mathbf{r}, \mathbf{r}')$. We regard the components $P_i(G\mathbf{r}, G\mathbf{r}')$ at given initial $\mathbf{r}$ and $\mathbf{r}'$ as functions on a group G and represent them in the form of a sum of quantities that transform in accordance with irreducible representations of the group G. With the aid of the method described by Marchenko and one of us [1] it is easy to verify that for a structure with complete axial spin symmetry only one term in this sum differs from zero and corresponds to a certain one-dimensional (unitary or nonunitary) representation. This means that the structure in question can be characterized by an order parameter $\mathbf{p}$ ($|\mathbf{p}| = 1$) that transforms like a pseudovector under the action of the elements of the spin group $O(3)$ and in accord with one of the one-dimensional representations of the space group G. By virtue of the axial symmetry, the symmetric part Q_{ik} of the correlator (2) should take in this case the form

$$Q_{ik}(\mathbf{r}, \mathbf{r}') = Q(\mathbf{r}, \mathbf{r}')\,(p_i p_k - {}^1/_3\delta_{ik}). \tag{3}$$

The function $A(r, r')$ is a spin scalar and is therefore invariant to the group G. The pseudovector $\mathbf{p}$ thus determines completely the symmetry of the correlation function (1).

If $P_i(\mathbf{r}, \mathbf{r}') \equiv 0$, we have by virtue of the axial spin symmetry

$$Q_{ik}(\mathbf{r}, \mathbf{r}') = Q(\mathbf{r}, \mathbf{r}')\,(n_i n_k - {}^1/_3\delta_{ik}), \tag{4}$$

where $\mathbf{n}$ is a certain constant unit spin vector defined accurate to the sign, while $Q(\mathbf{r}, \mathbf{r}')$, as above, is a scalar with respect to all the transformations.

There exist thus two types of axisymmetric spin nematics, characterized by spin directors $\mathbf{p}$ or $\mathbf{n}$. In the first case $\mathbf{p}$ is a spin pseudovector that transforms in accord with a certain one-dimensional representation of the group G, with $\mathbf{p}$ and $-\mathbf{p}$ different. In the second case $\mathbf{n}$ can be regarded as a scalar relative to G, and the states $\mathbf{n}$ and $-\mathbf{n}$ are identical.

The macroscopic low-frequency properties of the spin nematics, particularly their behavior in magnetic fields that are weak compared with the exchange field, can be described on the basis of symmetry considerations alone. Reasoning similar to that expounded in Ref. 1 for the case of collinear antiferromagnets leads to the following Lagrange function:

$$L = \frac{1}{2\gamma^2}\chi_{ik}([\mathbf{n}\times\dot{\mathbf{n}}]_i + \gamma H_i)([\mathbf{n}\times\dot{\mathbf{n}}]_k + \gamma H_k) - \frac{1}{2}a_{\alpha\beta}\frac{\partial \mathbf{n}}{\partial x_\alpha}\frac{\partial \mathbf{n}}{\partial x_\beta} - U_{\text{an}}. \tag{5}$$

Here $\mathbf{H}$ is the external magnetic field, γ the gyromagnetic ratio, $a_{\alpha\beta}$ the inhomogeneous exchange tensor,

$$\chi_{ik} = \chi_\perp(\delta_{ik} - n_i n_k) \tag{6}$$

the magnetic-susceptibility tensor, $\chi_\perp$ the transverse susceptibility, and U_{an} the relativistic anisotropy energy. The magnetization of a spin nematic is equal to

$$\mathbf{M} = (\chi_\perp/\gamma)[\mathbf{n}\times\dot{\mathbf{n}}] + \chi_{ik}H_k. \tag{7}$$

The foregoing equations, especially formula (6) according to which a spin nematic has no longitudinal susceptibility (at zero temperature), do not differ in substance from the corresponding equations for collinear antiferromagnets. The only difference is that, by virtue of the invariance of the state to time reversal, weak ferromagnetism is impossible in spin nematics; furthermore, Eq. (7) cannot contain additional terms that are linear in the spatial derivatives of the order parameter. Equations (5)–(7) in which $\mathbf{n}$ is replaced by $\mathbf{p}$, hold also for spin nematics with director $\mathbf{p}$. Some difference between the two types of nematic is in the form of the anisotropy energy. In type-$\mathbf{n}$ nematics, just as in antiferromagnets, the expansion of U_{an} in powers of $\mathbf{n}$ has no linear terms. In the case of antiferromagnets this is due to symmetry with respect to time reversal, and in the case of $\mathbf{n}$ nematics to the identity of the states $\mathbf{n}$ and $-\mathbf{n}$. As a result, as usual, we have $U_{\text{an}} = (\alpha_{ik}/2)n_i n_k$, where α_{ik} is the anisotropy tensor ($\alpha_{ii} = 0$). A similar formula with $\mathbf{n}$ replaced by $\mathbf{p}$ holds formally for $\mathbf{p}$-type nematics only if the one-dimensional representation of the group G, according to which $\mathbf{p}$ is transformed, is not contained in the pseudovector representation. Otherwise there exists a relativistic invariant linear in the components of $\mathbf{p}$ and $U_{\text{an}} = \alpha_i p_i$, where α_i are constants. Let, for example, $\mathbf{p}$ transform in a uniaxial magnet like the z-component of a pseudovector. Then the anisotropy energy can be written in the form $U_{\text{an}} = \alpha_z p_z$. The energy minimum corresponds to $p_z = 1$ or $p_z = -1$, depending on the sign of α_z. At small deviations from equilibrium we have

$$U_{\text{an}} = -|\tilde{\alpha}_z|\{1 - {}^1/_2(p_x^2 + p_y^2)\},$$

which is analogous to antiferromagnets of the easy axis type.

2. EXAMPLES

Let exchange-interacting spins $s = 1$ be located at the sites of a crystal lattice. The most general pair interaction of the spins is described by the Hamiltonian

$$\mathcal{H} = -\sum_{a,b}\{J_{ab}\mathbf{s}_a\mathbf{s}_b + {}^1/_2 G_{ab} q_{ik}^{(a)} q_{ik}^{(b)}\}, \tag{8}$$

where $\mathbf{s}_a$ are matrices of unity spin,

$$q_{ik}^{(a)} = s_{ai}s_{ak} + s_{ak}s_{ai} - {}^4/_3\delta_{ik},$$

and the subscripts a and b number the lattice sites. Assuming that the exchange parameters J_{ab} and G_{ab} are positive and long-range, we shall use the self-consistent-field method, i.e., seek the wave function of the ground state in the form of the product

$$\Psi = \prod_a \psi(s_{az}), \tag{9}$$

where $s_{az} = +1, 0, -1$ are the values of the spin projections on the z axis, and $\psi(s_{az})$ is a single-node wave function defined by the condition that the energy be a minimum.

By averaging the Hamiltonian (8) over the wave function (9) we obtain the energy per site:

$$E=-J\langle \mathbf{s}\rangle^2-\frac{G}{2}\langle q_{ik}\rangle^2. \tag{10}$$

Here

$$J=\sum_b J_{ab}, \quad G=\sum_b G_{ab},$$

the angle brackets denote averaging over the function $\psi(s_z)$, and the subscript z will hereafter be omitted. The wave functin $\psi(s_z)$ of unity spin is equivalent to the symmetric spinor ψ^{mn} $(m, n = 1, 2)$. We put

$$\psi^{mn}=\frac{1}{\sqrt{2}}(\sigma\sigma_y)_{mn}\psi,$$

where σ_{mn} are Pauli matrices and ψ is the unity-spin wave function in the vector representation and satisfies the normalization condition $\psi^*\psi=1$. The mean values in (10) are expressed in terms of the components of ψ as follows:

$$\langle s_i\rangle=-ie_{ikl}\psi_k^*\psi_l, \qquad \langle q_{ik}\rangle={}^2/_3\delta_{ik}-\psi_i^*\psi_k-\psi_k^*\psi_i. \tag{11}$$

The general phase factor of the wave function ψ can alway be chosen such that the vectors Re ψ and Im ψ become mutually perpendicular. We choose next a coordinate frame in which the x axis is directed along Re ψ and the z axis is perpendicular to the plane of Re ψ and Im ψ. Taking the normalization condition into account we obtain

$$\psi_x=\varphi, \quad \psi_y=i\lambda(1-\varphi^2)^{1/2}, \quad \psi_z=0, \tag{12}$$

where $0\leqslant\varphi\leqslant 1$ and $\lambda=\pm 1$.

Substitution of (12) in (11) and (10) yields

$$\langle s_x\rangle=O(\varphi^2), \quad \langle s_y\rangle=-2\ \mathrm{Im}\ \varphi_z, \quad \langle s_z\rangle=2\ \mathrm{Im}\ \varphi_y. \tag{13}$$

If $G<J$ the energy is a minimum at $\varphi=2/\sqrt{2}$. Then $\langle s_z\rangle=1$, i.e. the ground state corresponds to an ordinary ferromagnet. If, however, $G>J$, the minimum is reached at two points: $\varphi=0$ and $\varphi=1$. In both cases we have $\langle \mathbf{s}\rangle=0$. This is the so-called quadrupole ordering.[2-5] In the considered simplified model the function (1) differs from zero only at $\mathbf{r}=\mathbf{r}'$. Therefore $P_i(\mathbf{r}, \mathbf{r}')=0$ and the order parameter is $Q_{ik}=\langle q_{ik}\rangle$ or the spin director $\mathbf{n}$ determined by the formula $Q_{ik}=-2(n_in_k-(1/3)\delta_{ik})$. At $\varphi=0$ and $\varphi=1$ we have respectively $n_y=1$ and $n_x=1$. In the general case at $G>J$ the ground state of the Hamiltonian (8) is characterized by the equality $\langle \mathbf{s}\rangle=0$ and is infinitely degenerate in the directions of the vector $\mathbf{n}$, with the states $\mathbf{n}$ and $-\mathbf{n}$ identical.

Let the system be in an external magnetic field H directed along the z axis. For states described by Eqs. (12) this the case of a field transverse to $\mathbf{n}$. An extra term $-\gamma(H)\langle s_z\rangle$ is added to expression (10) for the energy and leads, when (11) and (12) are taken into account to an additional term $-2\lambda\gamma H\varphi(1-\varphi^2)^{1/2}$ in (13). If $H<H_c$, where $H_c=2(G-J)/\gamma$, the minimum of the energy corresponds to $\lambda>0$ and $\varphi(1-\varphi^2)^{1/2}=H/2H_c$. In the presence of a field the energy E is decreased by $\gamma H^2/2H_c$, from which it follow that the nematic state is characterized by a field-independent transverse susceptibility $\chi_\perp=\gamma/H_c$ per atom.[4] At $H>H_c$ the energy minimum corresponds to $\varphi=1/\sqrt{2}$ and $\langle s_z\rangle=1$. The field H_c is thus[4] the critical magnetic field of the phase transition from the nematic to the usual completely polarized state. This transition is analogous to collapse of antiferromagnet lattices.

It is also easy to see that in the nematic state, just as in collinear antiferromagnets, the longitudinal susceptibility is zero. Indeed, let $\psi_x=1$ and $\psi_y=\psi_z=0$ in the absence of a field; this is equivalent to $n_x=1$. We consider closely lying states, putting $\psi_x=1+\varphi_x$ and $\psi_{y,z}=\varphi_{y,z}$, where $|\varphi|\ll 1$. In the approximation linear in φ the first of formulas (11) yields

$$E=-{}^4/_3G+4(G-J)\varphi^2(1-\varphi^2).$$

Thus, the contribution $-\gamma H_x\langle s_x\rangle$ to the energy from the longitudinal magnetic field does not contain terms linear in the deviations of the state of the system from the initial minimum. In a sufficiently weak field the system remains therefore in exactly the same state as in the absence of a field.

Weak relativistic interactions can be accounted for by introducing in the Hamiltonian the one-ion-anisotropy energy. This leads to the appearance in the energy (10) of a relativistic term $(\alpha/4)Q_{xx}=-(\alpha/2)n_x+\mathrm{const}$. The system behaves like an easy-axis antiferromagnet at $\alpha>0$ and like an easy-plane one at $\alpha<0$. In a magnetic field longitudinal with respect to $\mathbf{n}$, a phase transition should be observed accompanied by flopping of $\mathbf{n}$ in a plane perpendicular to the field. The corresponding critical field $H_0=(\alpha/\chi_\perp)^{1/2}$ $(\alpha>0)$ is perfectly analogous to the case of antiferromagnets.

We consider now another example, which is interesting because both nematic-type states with directors $\mathbf{n}$ and $\mathbf{p}$ are realized in it; it involves besides spins 1/2, for which quadrupole ordering is impossible. Let the unit cell of the crystal contain two atoms with $s=1/2$, and let the exchange Hamiltonian be

$$\begin{aligned}\mathcal{H}=-\sum_{a,b}\{&J_{ab}^{(1)}\boldsymbol{\sigma}_a^{(1)}\boldsymbol{\sigma}_b^{(1)}+J_{ab}^{(2)}\boldsymbol{\sigma}_a^{(2)}\boldsymbol{\sigma}_b^{(2)}+J_{ab}^{(3)}\boldsymbol{\sigma}_a^{(1)}\boldsymbol{\sigma}_b^{(2)}\\&+G_{ab}^{(1)}(\boldsymbol{\sigma}_a^{(1)}\boldsymbol{\sigma}_b^{(1)})(\boldsymbol{\sigma}_a^{(2)}\boldsymbol{\sigma}_b^{(2)})+G_{ab}^{(2)}(\boldsymbol{\sigma}_a^{(1)}\boldsymbol{\sigma}_b^{(2)})(\boldsymbol{\sigma}_a^{(2)}\boldsymbol{\sigma}_b^{(1)})\\&+G_{ab}^{(3)}(\boldsymbol{\sigma}_a^{(1)}\boldsymbol{\sigma}_a^{(2)})(\boldsymbol{\sigma}_b^{(1)}\boldsymbol{\sigma}_b^{(2)})\}.\end{aligned} \tag{14}$$

The subscripts a and b number here different unit cells, and $\boldsymbol{\sigma}_a^{(1)}$ and $\boldsymbol{\sigma}_a^{(2)}$ are Pauli matrices acting on the spin variables of the first and second atoms in the ath cell. We assume, as before, the exchange parameters J_{ab} and G_{ab} to belong-range and use the self-consistent-field method, but now in terms of a unit cell rather than of an individual spin. Namely, we seek the wave function of the ground state of the Hamiltonian (14) in the form of the product

$$\Psi=\prod_a\psi(\sigma_{az}^{(1)}, \sigma_{az}^{(2)}), \tag{15}$$

where $\psi(\sigma_{az}^{(1)}, \sigma_{az}^{(z)})$ is the sought two-spin wave function. We emphasize that for the onset of the nematic state in the system considered calls for vanishing of not only all the compo-

nents of the average total spin of the cell, but of each individual spin.

Averaging the Hamiltonian (14) over the function (15), we obtain the energy per unit cell

$$E=-J_1\langle\boldsymbol{\sigma}^{(1)}\rangle^2-J_2\langle\boldsymbol{\sigma}^{(2)}\rangle^2-J_3\langle\boldsymbol{\sigma}^{(1)}\rangle\langle\boldsymbol{\sigma}^{(2)}\rangle -G\langle q_{ik}\rangle^2-G'\langle p_{ik}\rangle^2-\tilde{G}\langle\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\rangle^2, \tag{16}$$

where the averaging is now over the two-spin wave function,

$$G=G_1+G_2,\quad G'=G_1-G_2,\quad \tilde{G}=G_1+G_2+G_3,$$

$$J_{1,2,3}=\sum_b J_{ab}^{(1,2,3)},\qquad G_{1,2,3}=\sum_b G_{ab}^{(1,2,3)},$$

$$q_{ik}={}^1/_2(\sigma_i^{(1)}\sigma_k^{(2)}+\sigma_k^{(1)}\sigma_i^{(2)})-{}^1/_3\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\delta_{ik},$$

$$p_{ik}={}^1/_2(\sigma_i^{(1)}\sigma_k^{(2)}-\sigma_k^{(1)}\sigma_i^{(2)}).$$

A two-spin-1/2 wave function is equivalent to an asymmetric spinor ψ^{mn} of rank 2. Putting

$$\psi^{mn}=\frac{1}{\sqrt{2}}\{(\boldsymbol{\sigma}\sigma_y)_{mn}\psi+(\sigma_y)_{mn}\psi_0\},$$

we arrive at a representation in which the wave function of an aggregate of a vector ψ and a scalar ψ_0; the two satisfy the normalization condition

$$|\psi_0|^2+|\boldsymbol{\psi}|^2=1.$$

The mean values in (16) are equal to

$$\begin{aligned}\langle\sigma_i^{(1)}\rangle&=\psi_0^*\psi_i+\psi_0\psi_i^*+ie_{ikl}\psi_l^*\psi_k,\\ \langle\sigma_i^{(2)}\rangle&=-\psi_0^*\psi_i-\psi_0\psi_i^*+ie_{ikl}\psi_l^*\psi_k,\\ \langle q_{ik}\rangle&=-\psi_i^*\psi_k-\psi_k^*\psi_i+{}^2/_3\delta_{ik}|\boldsymbol{\psi}|^2,\\ \langle p_{ik}\rangle&=ie_{ikl}(\psi_0^*\psi_l-\psi_l^*\psi_0),\quad \langle\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\rangle=1-4|\psi_0|^2.\end{aligned} \tag{17}$$

We choose the gauge of the wave function and of the coordinate system such as to satisfy the equalities

$$\psi_x=u,\quad \psi_y=iv,\quad \psi_z=0 \tag{18}$$

with positive u and real v. We set also $\psi_0=\xi+i\eta$.

We assume first that the constants G, G' and $\tilde{G}$ in (16) greatly exceed in absolute value the constants $J_{1,2,3}$. The system energy is then

$$E=-\frac{8G}{3}[(u^2+v^2)^2-3u^2v^2]-8G'(v^2\xi^2+u^2\eta^2) -\tilde{G}[1-4(\xi^2+\eta^2)]^2. \tag{19}$$

If the variable ξ is excluded with the aid of the normalization condition $\xi^2=1-u^2-v^2-\eta^2$, the resultant function $E(u^2, v^2, \eta^2)$ should be minimized in the region $0\leqslant u^2, v^2, \eta^2\leqslant 1$. The constants G, G' and $\tilde{G}$ should be regarded as positive to justify the choice of the wave function in the form (15) with a two-spin function that is the same for all unit cells. Since the second derivative

$$\frac{\partial^2}{\partial(u^2)^2}E(u^2, v^2, \eta^2)=-16\left(\frac{G}{3}+2\tilde{G}\right)$$

is negative everywhere, the minimum energy is reached at the boundary of the region considered. Moreover, for that part of the boundary where $\eta^2=1-u^2-v^2$ and $0<u^2$ and $v^2<1$ we have

$$\frac{\partial^2}{\partial(v^2)^2}E(u^2, v^2, 1-u^2-v^2)=-16\left(\frac{G}{3}+2\tilde{G}\right)<0,$$

so that the minimum can be reached only at those points of the region where at least one of the quantities u, v, or η is zero. Simple analysis of (19) shows that three types of state can correspond to the energy minimum.

1. If $3\tilde{G}>\max(G, G')$, the energy is a minimum at $u=v=0$. In this case $\psi=0$ and $|\psi_0|=1$, so that we are dealing with a nonmagnetic state in which the spin symmetry $O(3)$ is not broken.

2. At $3G'-2G<3\tilde{G}<G$ the energy reaches its minimum in two states: $u=1$, $v=\xi=\eta=0$ and $v=1$, $u=\xi=\eta=0$. In both states

$$\langle\boldsymbol{\sigma}^{(1)}\rangle=\langle\boldsymbol{\sigma}^{(2)}\rangle=\langle p_{ik}\rangle=0,$$

and the system is a spin nematic of type n. The order parameter $\langle q_{ik}\rangle=2(n_in_k-\delta_{ik}/3)$ corresponds to a director $\mathbf{n}$ directed along the z axis in the first state and along the axis in the second.

We consider now the total energy (16) under conditions (18) near the state $u=1$ as a function of small v, ξ, and η. Expanding up to quantities of second order of smallness, we get

$$E=-{}^8/_3G-\tilde{G}+4v^2(2G-J_1-J_2-J_3)+4\xi^2({}^4/_3G+2\tilde{G}-J_1-J_2+J_3) +8\eta^2({}^2/_3)G+\tilde{G}-G'). \tag{20}$$

Thus, at parameters $J_{1,2,3}$ that are not too large in absolute value, the state considered corresponds as before to minimum energy.

Near the state with $\psi_x=1$, without the use of conditions (18), we obtain, accurate to quadratic terms,

$$\langle\sigma_x^{(1)}\rangle=-\langle\sigma_x^{(2)}\rangle=2\,\mathrm{Re}\,\psi_0,\quad \langle\sigma_y^{(1)}\rangle=\langle\sigma_y^{(2)}\rangle=-2\,\mathrm{Im}\,\psi_z,$$
$$\langle\sigma_z^{(1)}\rangle=\langle\sigma_z^{(2)}\rangle=2\,\mathrm{Im}\,\psi_y. \tag{21}$$

The mean value of the projection on the total spin $(1/2)(\sigma_x^{(1)}+\sigma_x^{(2)})$ on the x axis contains no linear terms. It follows hence, as before, that the state considered is characterized by a zero longitudinal susceptibility. The transverse susceptibility can be easily calculated by noting that by virtue of (20) the conditions (18) remain in force also in the presence of a weak field directed along the z axis. With the aid of (20) and (21) we get

$$\chi_\perp=\gamma^2/2(2G-J_1-J_2-J_3). \tag{22}$$

3. At $3\tilde{G}<\min(G', 3G'-2G)$ the energy (19) is a minimum for states with $v=\xi=0$, $u=\cos\theta$, and $\eta=\sin\theta$, where

$$\cos^2\theta=\frac{3(G'-3\tilde{G})}{(3G'-2G-3\tilde{G})+3(G'-3\tilde{G})}. \tag{23}$$

The same energy is possessed by states with $u=\eta=0$,

$\xi = \sin\theta$, and $v = \cos\theta$. In all these states

$$\langle \boldsymbol{\sigma}^{(1)} \rangle = \langle \boldsymbol{\sigma}^{(2)} \rangle = 0.$$

If the pseudovector **p** is defined by the inequality

$$\langle p_{ik} \rangle = |\sin 2\theta| e_{ikl} p_l,$$

we get $|\mathbf{p}| = 1$,

$$\langle q_{ik} \rangle = -2\cos^2\theta (p_i p_k - {}^1/_3 \delta_{ik}),$$

and at $v = \xi = 0$ and $u = \eta = 0$ we have respectively $p_x = \text{sign}(\sin 2\theta)$ and $p_y = -\text{sign}(\sin 2\theta)$. The system symmetry corresponds thus to a spin nematic with director **p**.

Let us consider states close to the state with $|p_x| = 1$. At small v and ξ the first three terms of (16) are of second order of smallness, as follows from (17), therefore the character of the equilibrium state does not change at exchange constants $J_{1,2,3}$ that are not too large in absolute value.

The magnetic susceptibility that is longitudinal in p is likewise zero in this case, since $\langle \sigma_x^{(1)} + \sigma_x^{(2)} \rangle$ does not contain terms linear in the deviations of the wave-function components from values corresponding to a state without a field. The z-component of the total spin is now

$$ {}^1/_2 \langle \sigma_z^{(1)} + \sigma_z^{(2)} \rangle = 2\cos\theta v, \tag{24}$$

so that a nonzero v appears in a magnetic field directed along the z axis. Expanding the energy (16) near the equilibrium value in powers of v at $\xi = 0$, we find

$$E(v) - E(0) = 8v^2 \left\{ \cos^2\theta \left(\frac{G}{3} + G' - 4\tilde{G} - J_3 \right) - {}^1/_2 (J_1 + J_2 - J_3) + 3\tilde{G} \right\}. \tag{25}$$

From (24) and (25) we obtain the following expression for the transverse susceptibility of a **p**-nematic:

$$\chi_\perp = {}^1/_4 \gamma^2 \cos^2\theta \left\{ \cos\theta \left(\frac{G}{3} + G' - 4\tilde{G} - J_3 \right) - {}^1/_2 (J_1 + J_2 - J_3) + 3\tilde{G} \right\}^{-1}. \tag{26}$$

If $3G' - 2G - 3\tilde{G} \to 0$, it can be seen from (23) that $\cos^2\theta \to 1$ and the **p**-nematic is transformed into an **n**-nematic. Equation (26) is then transformed into (22).

The foregoing examples show that magnetic ordering with the symmetry of a spin-nematic should be sought for in experiments on substances in which, for one reason or another, the many-particle exchange is not small compared with the usual exchange. Since the magnetic properties of spin nematics are the same as those of antiferromagnets, they can be distinguished only by using neutron scattering or NMR, methods that can determine the average microscopic spin density.

It is of particular interest to discuss here the possibility of nematic ordering in an exchange nuclear magnet, solid ^{3}He. At present it is universally accepted (see Refs. 7 and 8) that the principal mechanism of exchange interaction of nuclear spins in solid ^{3}He is four-particle cyclic exchange, which exactly contributes to nematic ordering. On the other hand, the conclusion that antiferromagnetic ordering was observed in solid ^{3}He at temperatures below 1 mK was based either on a study of macroscopic magnetic properties, or on an investigation of the resonant properties of the systemat frequencies much lower than the exchange frequencies.[7,8] It is clear from our present results these properties are typical of antiferromagnets to the same degree as for spin nematics. It can therefore not be excluded that what is realized is not a structure of the *uudd* type but a nematic structure in which the director **p** behaves under spatial transformation like the antiferromagnetic vector **1** in the *uudd* structure. Since we are dealing here with ordering of nuclear spins, the only method capable of unambiguously answering this question is investigation of neutron scattering.

We note in conclusion that a specific example of nematic spin ordering is the spin system of superfluid ^{3}He-*A*. The pair spin correlator (1) in ^{3}He-*A* is anisotropic and has the structure discussed above and determined by the spin director. The distinquishing feature is that ^{3}He-*A* is not invariant to time reversal. The equality $\langle \mathbf{s}(\mathbf{r}) \rangle = 0$ holds in it by virtue of the homogeneity and of the absence of ferromagnetism.

We are grateful to A. S. Borovik-Romanov, I. E. Dzyaloshinskiĭ, Ya. B. Zel'dovich, V. I. Marchenko, L. P. Pitaevskiĭ, and L. A. Prozorova for a helpful discussion of the work and for valuable remarks.

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Translated by J. G. Adashko