Free-free transitions of electrons in the presence of an electric field

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We consider the interaction between a strong electromagnetic wave and electrons localized in the space between a metal-vacuum interface and the electric retarding potential (the potential barrier). An expression is derived for the probability of passage of the electrons through this barrier as a result of multiphoton absorption. The possibility of experimentally observing this effect is discussed.

INTRODUCTION

The passage of an electron through a potential barrier in the presence of a strong electromagnetic field was considered by many workers in connection with various problems. In particular, following the paper by L. V. Keldysh,[1] many papers were devoted to multiphoton ionization of atoms (see, e.g. Refs. 2 and 3). This problem was investigated also in connection with the multiphoton photoeffect from crystals[4] and with the Franz-Keldysh effect in semiconductors.[5] In the case of multiphoton ionization of atoms, the investigation deals with the transition from the bound state of an electron to the continuous spectrum. One can, however, consider also a transition from a continuous spectrum with energy E into a continuum state with energy $E' > E$, i.e., the passage of an electron through a potential bounded in space, in the presence of a strong magnetic field. This problem can arise if the electrons move in a half-space bounded by a "potential wall" in the presence of a retarding electric field and a laser wave.[6] Electrons with initial energy $E \ll |eV_0|l$ (V_0 is the electric potential, e the electron charge, and l the barrier width) cannot overcome this barrier in the absence of an electromagnetic field. In the presence of such a field, however, multiphoton capture of electromagnetic-field quanta by the electron becomes possible from a state with energy $E \ll |eV_0|l$ to a state with energy $E' > |eV_0|l$. The present paper is devoted to a solution of this problem.

FORMULATION OF PROBLEM, ELECTRON WAVE FUNCTION

Let electrons move in a plane-polarized electromagnetic field along the x axis. An electromagnetic wave propagates perpendicular to the electron motion (along the z axis) and its vector potential $A(x,t)$ is directed along the x axis. The nonstationary Schrödinger equation that describes the electron motion in the electric field and in the laser wave is then

$$i\hbar\frac{\partial\Psi}{\partial t}=\hat{H}\Psi,$$
$$\hat{H}=\hat{H}_0+i\hbar\frac{e}{mc}A(x,t)\frac{\partial}{\partial x}+\frac{e^2}{2mc^2}A^2(x,t)-e\mathscr{E}'x, \tag{1}$$

where $H_0=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, m is the electron mass, and $\mathscr{E}'$ is the electric field strength.

We seek also for Eq. (1) a solution that goes over into the known Volkov solution when the field is turned off, and into a traveling electron wave in the absence of the electric field and of the laser wave. We neglect here and elsewhere the gradient terms of $A(x,t)$.[7]

We carry out a phase transformation of the wave function in Eq. (1):

$$\Psi(x,t)=\exp\left(\frac{ie}{\hbar c}\int A(x,t)\,dx\right)\tilde{\varphi}(x,t), \tag{2}$$

where $\tilde{\varphi}$ satisfies the equation

$$i\hbar\frac{\partial\tilde{\varphi}(x,t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{e}{c}\int A(x,t)\,dx-e\mathscr{E}'x\right]\tilde{\varphi}(x,t). \tag{3}$$

We get

$$i\hbar\frac{\partial\tilde{\varphi}(x,t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}-e\int\mathscr{E}(x,t)\,dx-e\mathscr{E}'x\right]\tilde{\varphi}(x,t), \tag{4}$$

where $\mathscr{E}$ is the laser-wave field strength. We recall that the gradient terms were left out of Eq. (1). The second term in the square brackets in the right-hand side of (4) can be excluded by using the Husimi transformation[8]:

$$\tilde{\varphi}(x,t)=\exp\left[i\int\dot{\eta}(x,t)\,dx_1+i\sigma(x,t)\right]\varphi(x_1,t), \tag{5}$$

where $x_1=x-\eta(x,t)$. We choose the functions η and σ such that the function φ satisfies the equation

$$i\frac{\partial\varphi(x_1,t)}{\partial t}=-\frac{1}{2}\frac{\partial^2\varphi(x_1,t)}{\partial x_1^2}-e\mathscr{E}'x_1\varphi. \tag{6}$$

The equations for η and σ are then

$$\ddot{\eta}=e\mathscr{E}(x,t), \tag{7}$$

$$\dot{\sigma}={}^1/_2\dot{\eta}^2+e\mathscr{E}(x,t)\eta+e\mathscr{E}'\eta. \tag{8}$$

The system (7), (8) has a simple solution, if $\mathscr{E}(x,t)$ is chosen in the form

$$\mathscr{E}(x,t)=\mathscr{E}(x)\cos\omega t \tag{9}$$

(here ω is the laser-wave frequency). Integrating the system of differential equations (7) and (8) we obtain

$$\eta=-\frac{e\mathscr{E}(x)}{\omega^2}\cos\omega t+Ct+B, \tag{10}$$

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$$\sigma=-\frac{3}{8\omega}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2\sin 2\omega t-\frac{1}{4}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2 t+\frac{1}{2}C^2t+\frac{e\mathscr{E}(x)}{\omega}B\sin\omega t-C\frac{e\mathscr{E}(x)}{\omega^2}\cos\omega t+D. \quad (11)$$

We turn now to Eq. (6). We express $\varphi(x_1,t)$ as

$$\varphi(x_1,t)=\exp(-iEt)\Phi(x_1), \quad (12)$$

where $\Phi(x_1)$ is a certain function of x_1 only. The result is an equation for $\Phi(x_1)$ in the form

$$\frac{1}{2}\frac{d^2\Phi(x_1)}{dx_1^2}+(e\mathscr{E}'x_1+E)\Phi(x_1)=0. \quad (13)$$

The solution of (13) is

$$\Phi(x_1)=C_1\,\mathrm{Ai}(\xi(x_1)),\quad \xi(x_1)=(x_1+E/e\mathscr{E}')(2me\mathscr{E}')^{1/3}, \quad (14)$$

where $\mathrm{Ai}[\xi(x_1)]$ is an Airy function and C_1 is a normalization constant.

Using now relations (5), (10), (11), (12), (14), and (2) we obtain an expression for $\Psi(x,t)$:

$$\Psi(x,t)=C_1\exp\left\{-iEt+\frac{1}{8}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2\frac{1}{\omega}\sin 2\omega t-\frac{i}{4}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2 t\right\}\mathrm{Ai}(\xi(x_1)). \quad (15)$$

A similar relation was obtained by another method in Ref. 5. The constants C, B, and D are chosen to satisfy the condition that the wave function obtained become the Volkov function[9] when the field is turned off. The function (15) satisfies this condition at $C=B=D=0$. The quantity x_1 in (15) is given by

$$x_1=x+\frac{E}{e\mathscr{E}'}+\frac{e\mathscr{E}(x)}{\omega^2}\cos\omega t. \quad (16)$$

We emphasize that the solution obtained in this manner for the Schrödinger equation is a particular solution and goes over, when the fields are turned off, into a plane traveling wave with momentum p and energy $E=p_2/2m$. As $t\to-\infty$ (the instant when the electromagnetic field is turned off) the general solution of (1) should transform into the wave function $\Psi_E^{(0)}(x,t)$ of the motion of an electron of energy E in a uniform electric field. For an arbitrary instant of time, this solution should be represented by the superposition

$$\Psi(x,t)=\Psi_E^{(0)}(x,t)+\sum_{E'}\alpha_{EE'}(t)\Psi_{E'}(x,t), \quad (17)$$

where $\alpha_{EE'}$ is the amplitude of the transition of the electron from the state E into the state E'. Substituting (17) in Eq. (1) we obtain (in the usual units)

$$i\hbar\left[\frac{\partial\Psi_E^{(0)}(x,t)}{\partial t}+\sum_{E'}\dot\alpha_{EE'}(t)\Psi_{E'}(x,t)+\sum_{E'}\alpha_{EE'}(t)\frac{\partial\Psi_{E'}(x,t)}{\partial t}\right]$$
$$=\hat H_0\Psi_E^{(0)}(x,t)+\hat H_0\sum_{E'}\alpha_{EE'}(t)\Psi_{E'}(x,t)+\hat V\Psi_E^{(0)}(x,t)+\hat V\sum_{E'}\alpha_{EE'}(t)\Psi_{E'}(x,t), \quad (18)$$

where

$$\hat V=i\hbar\frac{e}{mc}A(x,t)\nabla+i\hbar\frac{e}{2mc}\nabla A(x,t)+\frac{e^2}{2mc^2}A^2(x,t)-e\mathscr{E}'x. \quad (19)$$

Recognizing that

$$i\hbar\frac{\partial\Psi_E^{(0)}(x,t)}{\partial t}=\hat H_0\Psi_E^{(0)}(x,t)-e\mathscr{E}'x\Psi_E^{(0)}(x,t) \quad (20)$$

and $\Psi_{E'}(x,t)$ satisfies Eq. (1), we get from (18) the following equation for $\alpha_{EE'}(t)$ [using also the orthogonality of $\Psi_{E'}(x,t)$]

$$\alpha_{EE'}(t)=\int_{-\infty}^{t}\int_{0}^{l}\Psi_{E'}^{*}(x,t)\frac{e}{mc}A(x,t)\nabla\Psi_E^{(0)}(x,t)\,dx\,dt-\frac{i}{\hbar}\int_{-\infty}^{t}\int_{0}^{l}\Psi_{E'}^{*}(x,t)\frac{e^2}{2mc^2}A^2(x,t)\Psi_E^{(0)}(x,t)\,dx\,dt. \quad (21)$$

The differential probability $W_n^{(1)}$ of the multiphoton process can then be written in the form

$$dW_n^{(1)}=|\alpha_{EE'}(\infty)|^2 g(E')\,dE'. \quad (22)$$

Here $g(E')$ is the density of states:

$$g(E')=L'(2mE')^{1/2}/2\pi\hbar E', \quad (23)$$

and L' is the normalization length.

We take explicit account of the dependence of $A(x,t)$ on x:

$$A(x,t)=-\mathscr{E}\lambda\exp[-(x-x_0)^2/2d^2]\sin\omega t,$$

where d is the width of the laser beam and x_0 is the position of the maximum of the laser-wave field. We express the wave function $\Psi_E^{(0)}(x,t)$ in (21) in the form

$$\Psi_E^{(0)}(x,t)=\Psi_E^{(0)}(x)e^{-iEt},$$

where

$$\Psi_E^{(0)}(x)=\frac{1}{L^{1/2}}\sin\left[\frac{2}{3}\left(\frac{E-e\mathscr{E}'x}{e\mathscr{E}'l_0}\right)^{3/2}+\frac{\pi}{4}\right],\quad x<x',$$
$$\Psi_E^{(0)}(x)=\frac{1}{2L^{1/2}}\exp\left[-\frac{2}{3}\left(\frac{e\mathscr{E}'x-E}{e\mathscr{E}'l_0}\right)^{3/2}\right],\quad x>x'. \quad (24)$$

Here $x'=E/e\mathscr{E}'$ is the turning point, L the normalization length, and $l_0=(\hbar^2/2me\mathscr{E}')^{1/3}$ the field length.

For the potential considered, Eq. (24) is valid down to x values close to the point $x=0$. We assume that the electron motion at $x\leqslant 0$ is bonded by a "potential wall." In this case

$$\Psi_E(0)=0. \quad (25)$$

The wave function (24) satisfies the boundary condition (25) if the following phase relation is satisfied

$$^2/_3(E/e\mathscr{E}'l_0)^{3/2}+\pi/4=\pi(n_0+1),\quad n_0=0,1,2,\ldots \quad (26)$$

or

$$n_0=\pi^{-1}[^2/_3(E/e\mathscr{E}'l_0)^{3/2}-3\pi/4].$$

We assume further that the final state of the electron corresponds to above-barrier motion: $E'>e\mathscr{E}'l$. Neglecting the reflected wave and assuming that $E'/e\mathscr{E}'\gg(e\mathscr{E}/m\omega^2)\cos\omega t$, we get from (15)

$$\Psi_{E'}(x,t)=\frac{1}{(L')^{1/2}}\exp\left[i\frac{p'}{\hbar}x-\frac{i}{4}\frac{p'}{\hbar}\left(\frac{e\mathscr{E}'x}{E'}\right)^2\frac{E'}{e\mathscr{E}'}-\ldots\right]$$

$$\times\exp\left\{\frac{i}{8m\omega}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2\sin 2\omega t-\frac{i}{4m}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2 t+\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\cos\omega t-i\frac{E'}{\hbar}t\right\}. \tag{27}$$

We calculate first in the expression for $\alpha_{EE'}(\infty)$ the integral with respect to t:

$$I_1=\frac{e\mathscr{E}(x)\lambda}{mc}\int_{-\infty}^{+\infty}(\sin\omega t)\exp\left\{-i\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\cos\omega t-i\frac{E-E'}{\hbar}t-\frac{i}{8m\omega}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2\sin 2\omega t\right\}dt-i\frac{(e\mathscr{E}(x)\lambda)^2}{2mc^2}\int_{-\infty}^{+\infty}(\sin^2\omega t)$$

$$\times\exp\left\{-i\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\cos\omega t-i\frac{E-E'}{\hbar}t-i\frac{1}{8m\omega}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2\sin 2\omega t\right\}dt. \tag{28}$$

The integrals in (28) are calculated in Appendix 1. As a result we get

$$I_1=\frac{e\mathscr{E}(x)\lambda}{mc}J_n'\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\delta(E'-E-n\hbar\omega), \tag{29}$$

where J_n is a Bessel function. We have taken into account here only absorption processes, since the initial energy $E\ll\hbar\omega$. We now calculate the integral with respect to x in the amplitude $\alpha_{EE'}(\infty)$. The calculation is given in Appendix 2. As a result we have two cases: 1) $e\mathscr{E}p'/m\omega^2\hbar\ll 1$,

$$\alpha_{EE'}(\infty)\approx\frac{p}{p'^2(LL')^{1/2}}n\hbar\omega J_n\left(\frac{e\mathscr{E}_0p'}{m\omega^2\hbar}\right)\delta(E'-E-n\hbar\omega), \tag{30}$$

where

$\mathscr{E}_0=\mathscr{E}\exp(-x_0^2/2d^2)$,

and in case 2) $e\mathscr{E}_0p'/m\omega^2\hbar\gg 1$

$$\alpha_{EE'}(\infty)=\frac{p}{p'}\frac{e\mathscr{E}_0\lambda}{mc(LL')^{1/2}}J_n'\left(\frac{e\mathscr{E}_0p'}{m\omega^2\hbar}\right)\delta(E'-E-n\hbar\omega). \tag{30'}$$

Using relation (23) and integrating with respect to E', we obtain the probability per unit time:

$$W_n^{(1)}\approx\frac{e\mathscr{E}'}{p'}J_n^2\left(\frac{e\mathscr{E}_0p'}{m\omega^2\hbar}\right) \tag{31}$$

for the case when $e\mathscr{E}_0p'/m\omega^2\hbar\ll 1$, and

$$W_n^{(1)}\approx\frac{e\mathscr{E}'}{p'}\frac{(e\mathscr{E}_0\lambda)^2}{mc^2E'}\left[J_n'\left(\frac{e\mathscr{E}_0p'}{m\omega^2\hbar}\right)\right]^2 \tag{31'}$$

for the case when $e\mathscr{E}_0p'/m\omega^2\hbar\gg 1$.

Equations (30) and (31′) are valid so long as the gradient terms, which are estimated in the Appendix, can be neglected. In the derivation of (31) and (31′) we used a quasiclassical normalization condition for the initial state of the electron, viz., the wave function is localized between the potential wall and the turning point in the electric field.

CONCLUSION

The effect considered here is similar to the multiphoton photoeffect from metals.[10] In contrast to the photoeffect, however, the probability of multiphoton absorption by electrons emitted from a cathode is determined by the potential-barrier parameters and by the electron momentum in the final state. The experimental setup for the observation of this effect can be similar to that described in Ref. 6, where a vacuum-tube-like device was used and the beam of a pulsed neodymium laser passed between the cathode and the retarding grid.

Observation of the effect considered in the present paper, requires rather high electromagnetic field strengths at the metal-vacuum interface, comparable with those indicated in Ref. 10. It appears that the electromagnetic field strengths at the cathode-vacuum interface in Ref. 6 were lower.

The multiphoton absorption effect considered above can be masked by inverse bremsstrahlung of electrons on molecules of the residual gas. Let us estimate the upper limit of the molecule density at which this inverse bremsstrahlung is negligible.

The probability, per unit time, of inverse bremsstrahlung of an electron with absorption of n field quanta, is[11,12]

$$W_n^{(2)}\approx\sigma_{el}vNJ_n^2\left(\frac{e\mathscr{E}_0p'}{m\omega^2\hbar}\right), \tag{32}$$

where v is the initial velocity, N is the density of the residual-gas molecules, and σ_{el} is the cross section for elastic scattering of the electrons by the molecules.

We then obtain in accordance with (31) and (32)

$$W_n^{(1)}/W_n^{(2)}\approx e\mathscr{E}'/p'\sigma_{el}vN. \tag{33}$$

Using $\sigma_{el}\sim 10^{-16}\,\text{cm}^2$, $v\sim 10^7$ cm/sec, $\mathscr{E}'\sim 1$ V/cm, and $p'\sim 10^8\,\text{cm}^{-1}$ we obtain $W_n^{(1)}/W_n^{(2)}\gg 1$ if $N\ll 10^{16}\,\text{cm}^{-3}$.

The effect considered above can thus predominate even in the case of very deep vacuum. We note that the presence of an electromagnetic-field gradient does not alter our conclusions, since an estimate of the corresponding terms in the Hamiltonian (1) leads to the inequality[14]

$$(e\mathscr{E}\lambda)^2x/mc^2d\ll e\mathscr{E}'d. \tag{34}$$

Even in the most unfavorable case when $x\sim d$ this inequality is satisfied with large margin for the fields used in Ref. 6.

The foregoing results are, strictly speaking, valid for specular reflection from a potential wall. Of course, another model of the potential energy of an electron incident on a metal from vacuum can also be considered, that of a deep potential well. This, however, will lead to an insignificant renormalization of the momentum p' in the multiplicand of the exponential, since the decisive factor in the derivation (31) and (31′) is the abrupt potential discontinuity at the metal-vacuum interface.

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APPENDIX 1

We calculate the following integrals:

$$I(r)=\int_{-\infty}^{+\infty}(\sin^r \omega t)\exp\left\{-i\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\cos\omega t - \frac{i(E-E')t}{\hbar}-\frac{i}{8m\omega}\left[\frac{e\mathscr{E}(x)}{\omega}\right]^2\sin 2\omega t\right\}dt.$$

Here $r=1$ and 2. We calculate the integral at $r=1$. The procedure for the integral at $r=2$ is similar. Before calculating the integral we use the expansion[13]

$$\exp\left[-i\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\cos\omega t\right]=\sum_{n=-\infty}^{\infty}J_n\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\times\exp\left[-in\omega t-in\frac{\pi}{2}\right],$$

$$\exp\left\{-\frac{i}{8}\left[\frac{e\mathscr{E}(x)}{m\omega^2}\right]^2\frac{1}{m\omega}\sin 2\omega t\right\}=\sum_{k=-\infty}^{+\infty}J_k\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)i^k\exp\left[-2ik\omega t-ik\frac{\pi}{2}\right].$$

Substituting these relations in the integral with respect to t, we have

$$\frac{1}{2i}\int_{-\infty}^{+\infty}[e^{i\omega t}-e^{-i\omega t}]\sum_{n=-\infty}^{+\infty}\sum_{k=-\infty}^{+\infty}J_n\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)J_k\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\times i^k\exp\left[-in\omega t-2ik\omega t-i\frac{\pi}{2}(k+n)\right]\exp[i(E'-E)t]dt$$

$$=\frac{1}{2i}\sum_n\sum_k i^k\exp\left[-i\frac{\pi}{2}(k+n)\right]J_n\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\times J_k\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\delta(E'-E-n\hbar\omega-2k\hbar\omega-\hbar\omega).$$

Let $n+2k+1=n_0=\text{const}$, then $2k=n_0-n-1$ and the sum over k can be removed

$$\frac{1}{2i}\sum_n i^{\frac{n_0-1-n}{2}}\exp\left[-i\frac{\pi}{2}\left(\frac{n}{2}+\frac{n_0}{2}-\frac{1}{2}\right)\right]\cdot J_n\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\times J_{\frac{n_0-1-n}{2}}\left(\frac{1}{8}\left(\frac{e\mathscr{E}(x)}{\omega}\right)^2\frac{1}{m\omega}\right)\delta(E'-E-n\hbar\omega).$$

Let us determine the extremum of the expression

$$f=J_n\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)J_{\frac{n_0-1-n}{2}}\left(\frac{1}{8}\left(\frac{e\mathscr{E}(x)}{\omega}\right)^2\frac{1}{m\omega}\right),$$

under the assumption that the arguments are smaller than unity. Then

$$f\approx\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)^n\left[\frac{1}{8}\left(\frac{e\mathscr{E}(x)}{\omega}\right)\frac{1}{m\omega}\right]^{\frac{n_0-1-n}{2}}=x^n_*y^{\frac{n_0-1-n}{2}},$$

where

$$x_*=\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar},\quad y=\frac{1}{8}\left(\frac{e\mathscr{E}(x)}{\omega}\right)^2\frac{1}{m\omega}.$$

For the derivative f' we have

$$f'=nx_*^{n-1}y^{\frac{n_0-1-n}{2}}+x_*\frac{n_0-1-n}{2}y^{\frac{n_0-1-n}{2}-1}=0.$$

Hence

$$n=(n_0-1)(1-2y/x)^{-1}.$$

Since $y/x\ll 1$, we have $n\approx n_0-1$. The function f has thus an extremum at $n\approx n_0-1$ and is equal to

$$f\approx J_{n_0-1}\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right).$$

Thus,

$$I(1)\approx J_{n-1}\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\delta(E'-E-n\hbar\omega).$$

A similar calculation for $I(2)$ yields

$$I(2)\approx J_{n-2}\left(\frac{e\mathscr{E}(x)p'}{m\omega^2\hbar}\right)\delta(E'-E-n\hbar\omega).$$

APPENDIX 2

We present the calculation of the integral with respect to x in the amplitude $\alpha_{EE'}(\infty)$:

$$I_2=\int_0^\infty\exp\left[-i\frac{p'}{\hbar}x+\frac{i}{4}\frac{p'}{\hbar}\left(\frac{e\mathscr{E}'}{E'}\right)^2\frac{E'}{e\mathscr{E}'}x^2-\dots\right]\times\frac{\partial}{\partial x}[\Psi_E^{(0)}(x)]\exp\left[-\frac{(x-x_0)^2}{2d^2}n\right]dx.$$

We integrate I_2 by parts. Separating the most rapidly oscillating $\exp[-ip'x/\hbar]$, we have

$$I_2=-\frac{\hbar}{ip'}\int_0^\infty f(x)\frac{\partial}{\partial x}\Psi_E^{(0)}(x)d\exp\left(-i\frac{p'}{\hbar}x\right),$$

where

$$f(x)=\exp\left[\frac{i}{4}\frac{p'}{\hbar}\frac{e\mathscr{E}'}{E'}x^2-\dots\right]\exp\left[-\frac{(x-x_0)^2n}{2d^2}\right].$$

We ultimately obtain

$$I_2=-\frac{\hbar}{ip'}\left\{\exp\left(-i\frac{p'}{\hbar}x\right)f(x)\frac{\partial}{\partial x}\Psi_E^{(0)}(x)\Big|_0^\infty-\int_0^\infty\exp\left[-i\frac{p'}{\hbar}x\right]\times\left[f'(x)\frac{\partial}{\partial x}\Psi_E^{(0)}(x)+f(x)\frac{\partial^2}{\partial x^2}\Psi_E^{(0)}(x)\right]dx\right\}$$

$$\approx i\frac{p}{n'}(-1)^{n_0+1}\exp\left(-\frac{x_0^2n}{2d^2}\right).$$

The omitted integral is smaller by $e\mathscr{E}/Ep'$ than the retained term.

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