Theory of oscillations of the impedance of a metal plate in the case of diffuse reflection of carriers

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A method is proposed for solving a system of integrodifferential equations of the Wiener-Hopf type describing propagation of radiowaves in a metal in the case of diffuse reflection of carriers from the surface. It is based on separation of the complete system of equations into two much simpler systems, one of which describes the distribution of the long-wavelength component of the field and the other that of the short-wavelength component. The method is used to solve the problem of penetration of a radiofrequency field into a compensated metal subjected to a magnetic field perpendicular to the surface. The case of open orbits is dealt with. A system of equations with a long-wavelength component of the field is solved. Calculations are made of a complete distribution of the field and of the surface impedance tensor for a semi-infinite metal. This distribution is used to find the impedance of a plate in the antisymmetric excitation case. The presence of open orbits gives rise to the following effects: one of the diagonal elements of the impedance tensor is independent of the magnetic field and corresponds to the anomalous skin effect; the oscillatory components of this element are very small; the real part of the second diagonal element changes only slightly, whereas both the smooth and oscillatory components of its imaginary part rise considerably.

The Doppler-shifted cyclotron resonance (DSCR) in metals with closed carrier orbits has already been studied thoroughly both theoretically and experimentally. However, this is not true of the DSCR in compensated metals with open orbits. Studies of such metals have revealed[1,2] that the oscillations have a much smaller amplitude for one of the linear polarizations than for the other. This has not yet been explained. Moreover, there is no agreed view on the nature of the observed oscillations. Naberezhnykh and Eremenko[3] investigated the impedance of cadmium in the geometry with open orbits and they attributed the oscillations to dopplerons. On the other hand, Berset *et al.*[2] reported observations of the Gantmakher-Kaner (GK) oscillations because carriers with open orbits responsible for strong collisionless absorption should prevent doppleron propagation.

A theory of the skin effect and of the DSCR in a compensated metal with open orbits and specular carrier reflection was put forward in Ref. 4. It was shown that two different types of skin layer appear near the metal surface. A skin layer in which the electric field is directed along open orbits is anomalous and its depth is independent of the applied magnetic field H. However, the depth of the second skin layer is proportional to H^2. It was found that for both linear polarizations of the exciting field the amplitudes of the doppletron oscillations are equal to the amplitude of the GK oscillations. Hence, it was concluded that the reflection of carriers in the experiments reported in Refs. 1 and 2 was not specular and that the diffuse nature of the reflection plays a fundamental role. This role was demonstrated in Ref. 5, where the specularity coefficient was assumed to be arbitrary, but all the terms in the nonlocal conductivity with branch points were replaced with terms that had pole singularities. In the absence of branch points a system of integrodifferential equations which describes the field in a metal plate can be solved exactly and the solution is a sum of several exponential functions. In this model there is a doppleron and two different skin layers. It can be used to analyze how the nonspecularity of the reflection in the presence of two different skin layers results in a preferential enhancement of the doppleron oscillations for one of the linear polarizations. It was also shown there that this mechanism operates for all values of the specularity coefficient, with the exception of those close to unity. Nevertheless, the model of Ref. 5 suffers from several shortcomings. This model does not allow for collisionless absorption by carriers with open orbits or for the cyclotron absorption of carriers with closed orbits. Consequently, in principle, the GK oscillations do not appear in this model and the doppleron oscillations exist in a very wide range of magnetic fields.

This discussion demonstrates the urgent need for a theory of penetration of radiowaves into a metal plate with open orbits, which is characterized by a realistic nonlocal conductivity and diffusely reflecting surfaces. Such a theory is developed below. A difficulty is encountered because a system of integrodifferential equations cannot be solved in general for a segment. When the plate thickness exceeds the skin layer depth, the impedance of the plate can be found from the distribution of the field in a semi-infinite metal.[6] The problem of finding this distribution in the presence of open orbits reduces to a solution of a system of equations of the Wiener-Hopf type. A method for solving such a system in its general form is also unknown. However, in the range of magnetic fields exceeding the doppleron threshold there is a small physical parameter. This is manifested mathematically by the fact that all the branching points of the nonlocal conductivity and zeros of the dispersion equation are located in two widely separated regions. A method of separating the original system of equations into two much simpler systems for the long- and short-wavelength components of the field is developed for this situation in §1. The long-wavelength component is dealt with in §2 and the short-wavelength component—in §3. Next, the distribution of the field in a semi-

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infinite metal is used in §4 to obtain approximate expressions for the field distribution plate and for its impedance.

§1. SEPARATION OF THE LONG- AND SHORT-WAVELENGTH COMPONENTS

The distribution of an electric field $\mathscr{E}$ of frequency ω in a semiinfinite metal ($z>0$) under conditions of diffuse carrier reflection is described by the following system of equations:

$$\frac{d^2\mathscr{E}_\alpha(z)}{dz^2}+\frac{4\pi i\omega}{c^2}\sum_\beta\int_0^\infty dz'\sigma_{\alpha\beta}(z-z')\mathscr{E}_\beta(z')=0$$

$$(\alpha,\beta=x,y), \tag{1}$$

where $\sigma_{\alpha\beta}$ is the nonlocal conductivity tensor.

In the presence of open orbits, and also in the case of an inclined magnetic field **H** the system (1) cannot be separated into two independent equations and cannot be solved by the standard Wiener-Hopf method. Nevertheless, in the range of magnetic fields above the doppleron threshold it is possible to solve the system (1). In this range the field in a metal has two very different components. A short-wavelength component is associated with the DSCR and it varies over distances of the order of the extremal displacement u of carriers in one cyclotron period. A long-wave-length or skin component varies over much greater distances. In this situation it is possible to deduce much simpler systems of equations for the long- and short-wavelength components and these can then be solved.

If we assume that the field $\mathscr{E}(z)$ is zero where $z<0$, we can write down the system (1) in the Fourier representation

$$[q^2-i\xi s(q)]\mathscr{E}_q=-\mathscr{E}'-iq\mathscr{E}-\mathscr{F}_q, \tag{2}$$

where

$$\mathscr{E}(\zeta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathscr{E}_q e^{iq\zeta}dq,\qquad \sigma(\zeta)=\frac{n_1ec}{2\pi H}\int_{-\infty}^{\infty}s(q)e^{iq\zeta}dq, \tag{3}$$

$\xi=\omega n_1eu^2/\pi cH$; $\zeta=2\pi z/u$ and $q=ku/2\pi$ are the dimensionless coordinate and wave vector; n_1 is the density of electrons with closed orbits; $\mathscr{E}$ and $\mathscr{E}'$ are the values of the field and its derivative with respect to ζ on the surface $Z=0$. Although for the sake of simplicity we have dropped from Eqs. (2) and (3) the tensor indices, we must remember that both $\mathscr{E}$ and $\mathscr{F}$ are vectors, whereas σ, s, and the expression enclosed in the brackets in Eq. (2) are tensors. By definition, the function $\mathscr{E}_q$ is regular in the lower half-plane and decreases at high values of q in proportion to $1/q$, whereas the function $\mathscr{F}_q$ is regular in the upper half-plane.

The conductivity component $s_R(q)$ associated with the DSCR changes significantly for values of q of the order of unity and it can be represented in the form

$$s_R(q)=s_R(0)+\beta q^2+\Delta s(q),\qquad \beta=ds_R/dq^2|_{q=0}. \tag{4}$$

The tensors β and Δs have the following properties: their off-diagonal parts are antisymmetric and the diagonal parts are proportional to a unit tensor. The tensor Δs has branch points $q\approx\pm1$ corresponding to the collisionless cyclotron absorption threshold (DSCR) and in the limit $q\to0$ it approaches zero proportionally to q^4. It is natural to call the quantity

$$s_L(q)=s(q)-\Delta s(q)$$

the long-wavelength conductivity. This quantity has branch points in the range $|q|\ll1$ (these points are associated with the collisionless absorption by carriers with open orbits) and it has no singularities in the range $|q|\gtrsim1$.

Using the tensors s_L and $\alpha=1-i\xi\beta$, we can express the quantity in square brackets in Eq. (2) as a product:

$$q^2-i\xi s(q)=D_R(q)D_L(q), \tag{5}$$

$$D_R(q)=(q^2-i\xi s)(q^2-i\xi s_L)^{-1}\alpha,\qquad D_L(q)=\alpha^{-1}(q^2-i\xi s_L). \tag{6}$$

The tensor D_R has branch points associated with the DSCR and the tensor $|D_R|=0$ has a doppleron root, the value of which is of the order of unity. In the range of low values of q the tensor D_R has no singularities and for $q\to0$ it tends to a unit tensor. Therefore, this tensor is associated with the short-wavelength component of the field. The tensor D_L has the same branch points as $s_L(q)$ and the equation $|D_L|=0$ has roots corresponding to the skin component, which also lie in the range $|q|\ll1$. If $|q|\gtrsim1$, the tensor D_L has no singularities and it reduces to q^2. This tensor is associated with the long-wavelength component.

It follows from the properties of the tensors β and Δs that D_R can be diagonalized approximately in the case of a circular polarization and conversion to this case involves the use of the matrix

$$b=\begin{pmatrix}1 & i\\ 1 & -i\end{pmatrix}.$$

The diagonal tensor can be factorized, i.e., it can be represented in the form of a product of two tensors, one of which is nonsingular in the upper half-plane (index 1) and the other in the lower half-plane (index 2). This is easily done if each of the tensor elements is represented as a product of a function which is regular and has no zeros in the upper half-plane and a function which is regular and has no zeros in the lower half-plane. We shall identify the tensors for circular polarizations by a bar ($\bar{a}=bab^{-1}$) and then the tensor D_R can be represented in the form

$$D_R(q)=b^{-1}\bar\psi_1\bar\psi_2^{-1}b. \tag{7}$$

Bearing in mind that the tensor $\bar\alpha$ is diagonal, we can now determine the tensor $\sqrt{\bar\alpha}$, the squares of the elements of which are equal to the corresponding elements of $\bar\alpha$. Obviously, $\sqrt{\bar\alpha}$ commutes with $\bar\psi_1$ and $\bar\psi_2$. Therefore, Eq. (7) can be converted to the form $D_R(q)=\alpha\varphi_1\varphi_2^{-1}$, where

$$\varphi_1=b^{-1}(\sqrt{\bar\alpha})^{-1}\bar\psi_1 b,\qquad \varphi_2^{-1}=b^{-1}(\sqrt{\bar\alpha}\,\bar\psi_2)^{-1}b.$$

We shall now assume that we have been able to factorize the tensor D_L:

$$D_L(q)=(q^2+\eta^2)\tau_1\tau_2^{-1}, \tag{8}$$

where η is an arbitrary constant. Then, Eq. (5) becomes

$$q^2-i\xi s(q)=(q^2+\eta^2)\alpha\varphi_1\varphi_2^{-1}\tau_1\tau_2^{-1}. \tag{9}$$

We shall now use the above-mentioned properties of D_L and D_R, because of which the tensors τ are converted into unit tensors in the range $|q|\gtrsim1$, whereas the same happens to the tensors φ in the range $|q|\ll1$. It therefore follows that the tensors φ commute with the tensors τ. This is an important property and it is a consequence of the large difference between the wavelength of the two components; this property makes it possible to factorize the product $D_R D_L$ and—in the final analysis—to solve the system of the Wiener-Hopf equations. In fact, if in Eq. (9) we transpose φ_1 and φ_2^{-1} with τ_i, we can represent Eq. (2) in the form

$$(q-i\eta)\varphi_2^{-1}\tau_2^{-1}\mathscr{E}_q=-(q+i\eta)^{-1}\varphi_1^{-1}\tau_1^{-1}\alpha^{-1}(\mathscr{E}'+iq\mathscr{E}+\mathscr{F}_q). \tag{10}$$

The left-hand side of Eq. (10) is regular in the lower half-plane, whereas the right-hand side is regular in the upper half-plane. Therefore, both of them are equal to a constant vector $-iA\alpha^{-1}\mathscr{E}$, where

$$A=\lim_{q\to\infty}\varphi_1^{-1}(q)=b^{-1}\sqrt{\bar{\alpha}}\,b$$

(in the limit $q\to\infty$, the tensor $\bar{\psi}_1$ tends to a unit tensor). We then obtain

$$\mathscr{E}_q=-\frac{i}{q-i\eta}\tau_2(q)\varphi_2(q)A\alpha^{-1}\mathscr{E}=-\frac{i}{q-i\eta}\tau_2(q)\psi_2(q)\mathscr{E},$$

and the expression for the field becomes

$$\mathscr{E}(\zeta)=\frac{1}{2\pi i}\int_{-\infty-i\delta}^{\infty-i\delta}\frac{dq}{q-i\eta}e^{iq\zeta}\tau_2(q)b^{-1}\bar{\psi}_2(q)b\mathscr{E}, \tag{11}$$

where δ is a small constant.

We shall obtain a general expression for the impedance tensor of a semi-infinite metal. We shall write down the impedance Z in the form

$$Z=\frac{2\omega u}{c^2}T(0), \tag{12}$$

where the tensor $T(\zeta)$ is given by the relationship

$$i\mathscr{E}(\zeta)=T(\zeta)\mathscr{E}'. \tag{13}$$

In the calculation of Z we have to know the behavior of τ_2 and $\bar{\psi}_2$ in the limit $q\to\infty$. Using the definition of Eq. (7), we can represent $\bar{\psi}_2$ in the form

$$\bar{\psi}_2(q)=\exp\left[\frac{1}{2\pi i}\int_{-\infty+i\mu}^{\infty+i\mu}\frac{dz}{z-q}\ln(\bar{\alpha}-i\xi\overline{\Delta s}\bar{D}_L^{-1})\right]. \tag{14}$$

It follows from Eq. (14) that in the limit $q\to\infty$, we have

$$\bar{\psi}_2(q)\to 1+\frac{1}{q}\bar{M}, \tag{15}$$

where

$$\bar{M}=-\frac{1}{2\pi i}\int_{-\infty}^{\infty}dz\ln\left(\bar{\alpha}-\frac{i\xi\overline{\Delta s}}{z^2}\right). \tag{16}$$

We shall later need also the quantity $\bar{\psi}_2(0)=(\sqrt{\bar{\alpha}})^{-1}$. The tensor τ_2 in the limit $q\to\infty$ will be represented in the form analogous to Eq. (15):

$$\frac{q}{q-i\eta}\tau_2(q)\to 1+\frac{1}{q}K. \tag{17}$$

It follows from Eqs. (11), (15), and (17) that

$$\mathscr{E}'=i(K+b^{-1}\bar{M}b)\mathscr{E}, \tag{18}$$

which is used to find the impedance. The first term on the right-hand side of Eq. (18) is due to the long-wavelength component, whereas the second term is due to the short-wavelength component. Therefore, in the case of diffuse reflection of carriers these components make additive contributions to the reciprocal impedance tensor Z^{-1}, whereas in the specular reflection case they contribute to the impedance tensor Z.

It should be pointed out that a relationship between $\mathscr{E}'$ and $\mathscr{E}$ formally similar to Eq. (18) is given in Ref. 7. However, in Ref. 7 instead of K we have $T^{-1}(0)$ for the specular reflection of carriers and instead of the second term we have its asymptotic expression for strong magnetic fields. The relationship given in Ref. 7 is identical with Eq. (18) only in strong fields ($\xi\ll 1$) and only if there are no open orbits and no magnetic Landau damping.

Since all the singularities of D_L lie in the range $|q|\ll 1$, whereas the singularities of D_R lie in the range $|q|\sim 1$, we can represent Eq. (11) as a sum of the term

$$E(\zeta)=\frac{1}{2\pi i}\int_{-\infty-i\delta}^{\infty-i\delta}\frac{dq}{q-i\eta}e^{iq\zeta}\tau_2(q)b^{-1}\bar{\psi}_2(0)b\mathscr{E}, \tag{19}$$

which is due to small values of q, and a term

$$e(\zeta)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dq}{q-i\delta}e^{iq\zeta}b^{-1}\bar{\psi}_2(q)b\mathscr{E}, \tag{20}$$

which describes the contribution when $q\gtrsim 1$.

It follows from Eq. (19) that the long-wavelength component and its derivative on the surface are

$$E=b^{-1}(\sqrt{\bar{\alpha}})^{-1}b\mathscr{E},\qquad E'=iKb^{-1}(\sqrt{\bar{\alpha}})^{-1}b\mathscr{E}. \tag{21}$$

Therefore, the tensor K occurs in the relationship between the long-wavelength component and its derivative on the surface:

$$E'=iKE. \tag{22}$$

It should be noted that the impedance in relatively weak fields, when the tensor α is far from unity, cannot be calculated using Eqs. (19) or (20), because the terms proportional to K are omitted from Eq. (20).

In the calculation of the long-wavelength component it is preferable to avoid diagonalization and subsequent factorization of the tensor D_L by solving directly a system of equations which is satisfied by the components of the tensor E_q. It follows from Eqs. (8), (19), (17), and (21) that this system is

$$D_L(q)E_q=-E'-iqE-F_q, \tag{23}$$

where the required function E_q is regular in the lower half-plane and F_q is regular in the upper half-plane. The system (23) differs fundamentally from the initial system (2) because $D_L(q)$ has no singularities associated with the DSCR.

The situation in respect of the short-wavelength component is simpler because the tensor $\bar{D}_R$ is diagonal. This means that even in the presence of open orbits the system of equations for the short-wavelength component can be separated into two independent equations for the components with the plus and minus polarizations, which can then be solved by the standard Wiener-Hopf method.

§2. LONG-WAVELENGTH COMPONENT

In this and the next sections we shall consider the model of a compensated metal with open orbits proposed in Ref. 4. In this model the electron part of the Fermi surface is a corrugated cylinder parallel to the p_z axis, whereas the hole part consists of two flat cylinders parallel to the p_x and p_z axes; a magnetic field H and the normal to the metal surface are directed along the z axis. A part of the nonlocal conductivity corresponding to the corrugated cylinder has a square-root singularity associated with the DSCR. Singularities of two types are encountered in the conductivity of real metals: a square-root singularity when the DSCR is due to carriers with finite orbits and a singularity of the weak logarithmic type ($x\ln x$) when the resonance is due to electrons at a limiting point. An analysis of the square-root singularity is of considerable interest because in this case the resonance is stronger and the associated impedance oscillations are observed in a wide range of magnetic fields. The part of the conductivity associated with open orbits (corresponding to the cylinder parallel to the p_x axis) varies as $1/|q|$ right up to very small values of q, i.e., it behaves exactly as in the case of real metals. In this connection it must be stressed that the structure of a radiofreuency field in a metal is governed by singularities of its nonlocal conductivity and is not very sensitive to details of the Fermi surface shape. Therefore, the model in question exhibits the principal properties characteristic of the propagation of radiowaves in several metals.

When the fraction of the open orbits is not too small, the long-wavelength component can be separated only in the case

$$\xi \equiv 2(H_L/H)^3 \ll 1, \tag{24}$$

i.e., this can be done in the range of fields H considerably higher than the doppleron threshold field H_L. In this range we can ignore the value of β, the value of α reduces to unity, and the elements of the tensor s_L become (see Ref. 4)

$$s_{xx}=a\gamma_0, \quad s_{yx}=-s_{xy}=a, \quad s_{yy}=s_{xx}+as_0, \quad s_0=b_0/(q^2+\gamma^2)^{1/2}, \tag{25}$$

where

$$\gamma_0=\gamma_1/a+(1-a)\gamma_3/a, \quad \gamma=b_0\gamma_2, \quad b_0=m_1v_1/m_2v_2,$$
$$\gamma_i=\nu_i/\omega_{ci}, \quad \omega_{ci}=eH/m_ic \quad (i=1,2,3),$$

a is the fraction of holes with open orbits; m_i, v_i, and μ_i are the cyclotron mass, the maximum value of the z component of the velocity, and the frequency of collisions of carriers belonging to the ith group; the index 1 refers to electrons, whereas the index 2 refers to holes with open orbits, and the index 3 to holes with closed orbits. Consequently, the system (23) expressed in terms of Cartesian coordinates becomes

$$(q^2-q_0^2)E_{qx}+i\xi_0E_{qy}=-E_x'-iqE_x, \tag{26}$$

$$-i\xi_0E_{qx}+(q^2-q_0^2-i\xi_0s_0)E_{qy}=-E_y'-iqE_y-F_{qy}, \tag{27}$$

where

$$\xi_0=a\xi, \quad q_0=(1+i)(\xi_0\gamma_0/2)^{1/2}.$$

It is important to note that Eq. (26) contains only the local elements of the tensor s_L which do not have branch points, so that they do not contain the function F_{qx}. Therefore, we can eliminate E_{qx} from Eqs. (26) and (27), and then E_{qy} is described by the equation

$$D(q)E_{qy}=-(E_y'+iqE_y+F_{qy})-(E_x'+iqE_x)i\xi_0/(q^2-q_0^2), \tag{28}$$

where

$$D(q)=q^2-i\xi_0(\gamma_0+b_0/(q^2+\gamma^2)^{1/2})-\xi_0^2/(q^2-q_0^2). \tag{29}$$

Equation (28) can be solved by a combination of the methods that are used to deal with boundary-value problems involving functions of the complex variable.

We shall represent the function (29) in the form

$$D(q)=(q^2+\eta^2)t_1(q)/t_2(q), \tag{30}$$

where

$$t_{1,2}(q)=\exp\left\{\frac{1}{2\pi i}\int_{-\infty\mp i\mu}^{\infty\mp i\mu}\frac{dz}{z-q}\ln\frac{D(z)}{z^2+\eta^2}\right\}, \tag{31}$$

and we shall write down Eq. (28) in the form

$$\frac{q-i\eta}{t_2(q)}E_{qy}$$
$$=-\frac{1}{(q+i\eta)t_1(q)}\left[(E_y'+iqE_y+F_{qy})+(E_x'+iqE_x)\frac{i\xi_0}{q^2-q_0^2}\right]. \tag{32}$$

The second term on the right-hand side of Eq. (32), which we shall denote by G, can be represented by the difference $G(q) = G_1(q) - G_2(q)$, where

$$G_{1,2}(q)=\frac{1}{2\pi i}\int_{-\infty\mp i\mu}^{\infty\mp i\mu}\frac{dz}{z-q}G(z). \tag{33}$$

Equation (32) can now be written in the form

$$\frac{q-i\eta}{t_2(q)}E_{qy}+G_2(q)=-\frac{E_y'+iqE_y+F_{qy}}{(q+i\eta)t_1(q)}+G_1(q),$$

where the left-hand side is regular in the lower half-plane and the right-hand side in the upper half-plane. Consequently, both are equal to the constant $-iE_y$. Therefore, E_{qy} is described by the expression

$$E_{qy}=-\frac{t_2(q)}{q-i\eta}[iE_y+G_2(q)]. \tag{34}$$

In view of the analytic properties of t_1, the integral (33) governing G_2 is easily calculated by deforming the contour to the upper half-plane, which gives

$$G_2(q)=\frac{C}{q-q_0}(E_x'+iq_0E_x), \quad C=\frac{i\xi_0}{2q_0}(q_0+i\eta)^{-1}t_1^{-1}(q_0). \tag{35}$$

Consequently, the expression for the components of the field $E_y(\zeta)$ can be written in the form

$$E_y(\zeta)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}dq\,e^{iq\zeta}\frac{t_2(q)}{q-i\eta}\left[iE_y+\frac{C}{q-q_0}(E_x'+iq_0E_x)\right]. \tag{36}$$

Next, substituting E_{qy} in Eq. (26), we find E_{qx} and the corresponding component of the field:

$$E_x(\zeta)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{dq}{q^2-q_0^2}e^{iq\zeta}(E_x'+iqE_x+i\xi_0E_{qy}). \tag{37}$$

Differentiating Eqs. (36) and (37) with respect to ζ and then allowing ζ to approach zero, we obtain two algebraic equations which enable us to determine the tensor K [see Eq. (22)]:

$$E_y'=iQE_y+C(E_x'+iq_0E_x),$$
$$E_x'=C^2E_x'+iq_0(1+C^2)E_x-2iq_0CE_y. \tag{38}$$

The integral of the third term in Eq. (37) can be calculated if we first assume that $\zeta = 0$, then close the contour in the lower half-plane, and use the relationship $t_2(-q_0) = 1/t_1(q_0)$. In the calculation of E'_y we must allow for the fact that in the limit $q \to \infty$, we have

$$\frac{q}{q-i\eta} t_2(q) \to 1 + \frac{Q}{q},$$

where

$$Q = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dq \ln\frac{D(q)}{q^2}. \tag{39}$$

If the fraction of the open orbits is not too small, the quantity Q corresponds to the anomalous effect and it is given by

$$Q = (3^{-1/2}+i)(b_0\xi_0)^{1/3}. \tag{40}$$

If Eqs. (36) and (37) are used to calculate E_y and E_x, the first of these equations changes to an identity and the second becomes equivalent to the second expression in Eq. (38).

Solving the system (37), we find that

$$K_{xx} = q_0(1+C^2)/(1-C^2), \quad K_{yx} = -K_{xy} = 2q_0C/(1-C^2),$$
$$K_{yy} = Q - 2q_0C^2/(1-C^2). \tag{41}$$

If $|C| \ll 1$, the tensor K is practically diagonal. We then have $K_{xx} = q_0$ and $K_{yy} = Q$, i.e., the normal skin effect appears for the x polarization and the anomalous effect for the y polarization. This is precisely the situation in the range of magnetic fields such that $\xi_0 \ll b_0^2\gamma_0$ (although in the vicinity of the doppleron threshold we have $\xi_0 \sim 1$ and $\gamma_0 \ll 1$, and in sufficiently strong fields the value of ξ_0 becomes less than γ_0, since $\xi_0 \propto H^{-3}$ and $\gamma_0 \propto H^{-1}$). In the range of fields where

$$b_0^2\gamma_0 \ll \xi_0 \ll 1, \tag{42}$$

the quantity C becomes close to unity and it can be written conveniently in the form

$$C = e^{-q_0 J}, \quad J = \frac{b_0}{\pi\xi_0}\ln\frac{e^2\xi_0}{ib_0^2(\gamma_0+e\gamma_2)}. \tag{43}$$

The elementa of the tensor K are then

$$K_{xx} = K_{yx} = -K_{xy} = J^{-1}, \quad K_{yy} = Q. \tag{44}$$

In principle, the formulas (36) and (37) allow us to calculate the field distribution in a semi-infinite metal. Each of the components $E_x(\zeta)$ and $E_y(\zeta)$ represents a sum of two terms, one of which is proportional to E'_x and the other to E'_y:

$$E_\alpha(\zeta) = -iP_{\alpha\beta}(\zeta)E'_\beta. \tag{45}$$

We shall use only the element $P_{xx}(\zeta)$ which is the largest. Using Eqs. (29)–(32), (33)–(35), (37), and (43)–(44), we can transform the expression for this element in the range defined by Eq. (42) to

$$P_{xx}(\zeta) \approx \frac{b_0}{\pi\xi_0}\int_{-\infty}^{\infty} dq\left(\frac{1}{(q^2+\gamma^2)^{1/2}+b_0\gamma_0} - \frac{1}{(q^2+\gamma^2)^{1/2}-i\xi_0/b_0}\right)e^{iq\zeta+\Omega(q)}, \tag{46}$$

where

$$\Omega(q) = \frac{1}{2\pi i}\int_{-\infty-i\mu}^{\infty-i\mu}\frac{ds}{s-q}\ln\left(1 - \frac{b_0}{i\xi_0}(s^2+\gamma^2)^{1/2} + \frac{b_0\gamma_0}{(s^2+\gamma^2)^{1/2}}\right). \tag{47}$$

In Eq. (46) we must separate the exponential term associated with the pole at the point $q = i\xi_0/b_0$. The rest of the integral cannot be represented in an analytic form for an arbitrary value of ζ. Calculating the function P_{xx} for values of ζ satisfying the inequalities $b_0\xi_0^{-1} \ll \zeta \ll b_0^{-1}\gamma_0^{-1}$ and bearing in mind that $P_{xx}(0) = J$ and $P'_{xx}(0) = i$, we can write down the following expression:

$$P_{xx}(\zeta) \approx \frac{2^{1/2}b_0}{i\xi_0}\exp\left[-\frac{\xi_0\zeta}{b_0} + \frac{\pi i}{8}\right]$$
$$-\frac{b_0}{\pi\xi_0}\ln\left[\frac{e^C b_0^2(\gamma_0+\gamma_2)}{\xi_0}\left(\frac{\xi_0\zeta}{b_0}+A\right)\frac{b_0^2+\xi_0^2\zeta^2}{Bb_0^2+\xi_0^2\zeta^2}\right], \tag{48}$$

where

$$A = \frac{i}{\pi}(1-2^{1/2}e^{\pi i/8})^{-1},$$

$$B = -iA\frac{\gamma_0+\gamma_2}{\gamma_0+e\gamma_2}\exp(2+C+i\pi 2^{1/2}e^{\pi i/8}).$$

Here, C is the Euler constant. The formula (48) desribes approximately the distribution of the long-wavelength component also for $\zeta \sim b_0/\xi_0$.

The first term in Eq. (48) corresponds to the skin root of the dispersion equation $q = i\xi_0/b_0$ referring to the x component of the field, whereas the second is an analog of the nonexponential Reiter-Sondheimer component. It is remarkable that at distances z smaller than al_2 (where l_2 is the distance traveled by holes with open orbits), the second term exhibits a logarithmic coordinate dependence. Therefore, in the range of fields described by Eq. (42) the presence of open orbits alters significantly not only the y but also the x long-wavelength component. The physical rason is as follows: holes with open orbits do not contribute to the Hall conductivity, so that compensation of the local Hall conductivities is disturbed ($s_{xy} \neq 0$). Consequently, the "effective" conductivity in the x direction depends on the nonlocal conductivity s_{yy}:

$$\tilde{s}_{xx} = s_{xx} + s_{xy}^2/s_{yy} \approx a(\gamma_0 + b_0^{-1}(q^2+\gamma^2)^{1/2}). \tag{49}$$

In the range defined by Eq. (42) the second term in Eq. (49) exceeds the first and this alters the structure of the x component. Conversely, in the range of strong fields where $\xi_0 \ll b_0^2\gamma_0$, the second term is small, the open orbits cease to play a significant role, and the nonlocal effects disappear:

$$P_{xx} = q_0^{-1}\exp(iq_0\zeta).$$

The last comment in connection with Eq. (45) is this. We must remember that in the experiments we set $\mathscr{E}'$, which is related to E' by

$$E' = iKE = iK\mathscr{E} = KT(0)\mathscr{E}'; \tag{50}$$

we have allowed here for the fact that, in the range of fields defined by Eq. (24), we have $\alpha = 1$ and it follows from the first expression in Eq. (21) that $E = \mathscr{E}$.

§3. SHORT-WAVELENGTH COMPONENT AND THE TOTAL IMPEDANCE OF A SEMI-INFINITE METAL

It follows from Eqs. (20) and (14) that the presence of open orbits gives rise to an additional collisional damping of dopplerons. In other respects the functional dependence of the short-wavelength component on the coordinate ξ remains unchanged. The distribution of the short-wavelength component in the case of a square-root singularity of the nonlocal conductivity in the absence of open orbits is found in Ref. 8. In this case the distribution of the Cartesian components $e(\zeta)$ can be written in the form

$$e(\zeta)=W(\zeta)\mathscr{E}, \tag{51}$$

where

$$W_{xx}(\zeta)=W_{yy}(\zeta)=\frac{1}{2}\left[e_{GK}(\zeta)-e_{GK}^{*}(\zeta)-\xi^{2}e^{iq_D\zeta}\right],$$
$$W_{xy}(\zeta)=-W_{xy}(\zeta)=\frac{i}{2}\left[e_{GK}(\zeta)+e_{GK}^{*}(\zeta)+\xi^{2}e^{iq_D\zeta}\right], \tag{52}$$

$$q_D=-1+\frac{1}{2}\xi^2+i\gamma_1+\frac{i}{2}ab_0\xi^3. \tag{53}$$

Here, q_D is the wave vector of a doppleron obtained allowing for the collisionless damping, the asterisk denotes complex conjugacy, and the Gantmakher-Kaner (GK) component at large distances from the surface ($\zeta\gg1$) is

$$e_{GK}(\zeta)=e^{(i-\gamma_1)\zeta}\frac{\xi^2}{2\pi}\int_0^\infty\frac{x^{1/2}\,dx}{x+1}\exp\left[\frac{i}{2}\xi^2\zeta(1+i\gamma_1)x\right]. \tag{54}$$

The tensor M defined by Eq. (16) is

$$\overline{M}=M=\xi/\pi. \tag{55}$$

We can now use Eqs. (18), (41), and (55) to calculate the tensor $T(0)$ which describes the impedance:

$$T_{xx}(0)=(M+q_0(1+C^2)/(1-C^2))^{-1},\quad T_{yy}(0)=1/Q,$$
$$T_{xy}(0)=-T_{yx}(0)=2q_0C/Q[(1-C^2)M+(1+C^2)q_0]. \tag{56}$$

It follows from Eq. (56) that the tensor $T(0)$ is practically diagonal. The element Z_{yy} is independent of the magnetic field and it corresponds to the anomalous skin effect due to carriers with open orbits. The element Z_{xx} is mainly due to carriers with close orbits and its magnitude depends strongly on H. In strong fields such that $\xi_0\ll b_0^2\gamma_0$, the quantity C is small, $T_{xx}(0)=q_0^{-1}$, and $Z_{zz}\propto H$. In this range of fields the normal skin effect in a magnetic field is observed for the x polarization. In the range defined by Eq. (42) the behavior of Z_{xx} is more complex. We now have

$$T_{xx}(0)=(M+1/J)^{-1}.$$

The quantity M is the contribution of the GK component and $1/J$ is the contribution of the x long-wavelength component. The term $1/J$ replaces the quantity q_0, which corresponds to the case of the absence of open orbits [in the range defined by Eq. (42) the value of q_0 is small compared with $1/J$]. The relationship between M and $1/J$ depends on the fraction a of the open orbits. If a is small, the open orbits have practically no influence on the surface resistance, but they increase the reactance. If $a\gtrsim1$, the surface resistance decreases. These characteristics of the behavior of the impedance of a semi-infinite metal should be detectable experimentally.

The tensor $T(\zeta)$ describing the complete distribution of the field in a semi-infinite metal and defined by Eq. (13) can be rewritten in the following form if we use Eqs. (45), (50), and (51):

$$T(\zeta)=[P(\zeta)K+W(\zeta)]T(0). \tag{57}$$

The presence of the factor $T(0)$ in Eq. (57) is of fundamental importance. In spite of the fact that the different elements of the tensor W are of the same order of magnitude [see Eq. (52)], the difference between $T_{xx}(0)$ and $T_{yy}(0)$ alters the ratio of the amplitudes of the short-wavelength components in different parts of the field in a metal. For example, if a doppleron is excited by a field $\mathscr{E}'_x$, it has a much larger amplitude than when it is excited by the field $\mathscr{E}'_y$ (to avoid misunderstanding, we recall that a doppleron traveling in a metal has a circular polarization). This applies also to the GK component. The effect is a consequence of the fact that in the case of diffuse reflection of carriers the amplitude of the short-wavelength component depends strongly on the depth of penetration of the long-wavelength component. In the presence of open orbits this penetration depth is very much greater for the x polarization than for the y polarization.

Our treatment applies to the specific case of the square-root singularity of the resonance part of the conductivity. A similar procedure can be applied also to the weakly logarithmic singularity, because in the range of strong fields defined by Eq. (24) the tensors M and W can be calculated quite easily.

§4. OSCILLATIONS OF THE IMPEDANCE OF A PLATE

A method representing a generalization of Ref. 6 can be used to obtain a rigorous expression for the impedance tensor in the range of fields H where the depth of the skin layer is less than the plate thickness d ("thick plate"). The depth of the skin layer $ub_0/2\pi\xi_0$ rises on increase in H and when it becomes generally invalid. In this range of fields the problem has a rigorous analytic solution only if the long-wavelength component is a natural mode of a metal (in the absence of the nonexponential term). This is not true in our case. Nevertheless, as in the case of a natural mode, we shall approximate the distribution of the long-wavelength part of the field in a plate by an antisymmetric combination of the relevant components in a semi-infinite metal:

$$E_a(\zeta)=[P(\zeta)-P(L-\zeta)]K\mathscr{E}, \tag{58}$$

where $L=2\pi d/u$ (it should be noted that the vector $\mathscr{E}$ is not a value of the electric field on the surface of the plate). For a thick plate the validity of Eq. (58) is self-evident. If the depth of penetration of the long-wavelength component is much greater than the plate thickness, the field $E_a(\zeta)$ becomes a linear function which is odd relative to the point $\zeta=L/2$, so that the explicit form of the distribution $P(\zeta)$ becomes unimportant. Therefore, the relationship (58) is rigorous also in this case. However, in the intermediate range of fields where $b_0\xi_0^{-1}\sim L$, Eq. (58) becomes of the interpolation type. It is clear that the error due to the use of Eq. (58) in this range does not exceed a value of the order of 10%. It should be stressed that only the expression for the x component of the distribution $E_a(\zeta)$ is obtained by interpolation. The depth of the anomalous skin layer for the y component is independent of H and it is usually small compared with the thickness of the samples used in experiments. Therefore, Eq. (58) for the y component is rigorous irrespective of the value of H.

It follows from Eq. (51) that a given field on the surface generates a definite short-wavelength component. In the range of strong fields $\xi \ll 1$ where the amplitude of the short-wavelength component is small, this remains true also of a plate:

$$e_a(\zeta)=[W(\zeta)-W(L-\zeta)]E_a(0) \quad (59)$$

(this is to be expected, because the short-wavelength component is formed over distances of the order of the displacement u of the resonance carriers, which is much less than the depth of the skin layer or the plate thickness). Adding Eqs. (58) and (59), we obtain the total field

$$\mathscr{E}_a(\zeta)=\{[P(\zeta)-P(L-\zeta)]+[W(\zeta)-W(L-\zeta)] \times[P(0)-P(L)]\}K\mathscr{E}.$$

The impedance tensor $Z^{(a)}$ of a metal plate is given by

$$4i\omega uc^{-2}\mathscr{E}_a(0)=Z^{(a)}\mathscr{E}_a'(0). \quad (60)$$

Having calculated $\mathscr{E}_a(0)$ and $\mathscr{E}_a'(0)$ and ignored the small terms $W(0)$ and $W(L)$, we find that

$$Z^{(a)}=\frac{4i\omega u}{c^2}\{[W'(0)+W'(L)]+[P'(0)+P'(L)] \times[P(0)-P(L)]^{-1}\}^{-1}. \quad (61)$$

In the approximation which is linear in respect of a small oscillatory quantity $W'(L)$, we have

$$Z^{(a)}=Z_0+\Delta Z, \quad (62)$$

$$\Delta Z=\frac{ic^2}{4\omega u}Z_0W'(L)Z_0, \quad (63)$$

$$Z_0=\frac{4\omega u}{c^2}\{-iW'(0)+[1-iP'(L)][P(0)-P(L)]^{-1}\}^{-1}. \quad (64)$$

An allowance is made in Eq. (64) for the fact that $P'(0)=i$. The first term in Eq. (62) represents a smooth part of the impedance which for a thick plate is equal to the impedance of a semi-infinite metal [this is easily shown if we bear in mind that $W'(0)=iM$ and $P^{-1}(0)=K$]. However, the second term describes oscillations of the impedance as a function of the magnetic field and these oscillations are due to the penetration of the short-wavelength components.

A direct calculation gives the following expression for the diagonal elements of the tensor Z_0:

$$Z_{0xx}=4\omega uc^{-2}[\xi/\pi+(1-iP_{xx}'(L))/(P_{xx}(0)-P_{xx}(L))]^{-1}, \quad (65)$$

$$Z_{0yy}=4\omega uc^{-2}Q. \quad (66)$$

We have allowed here for the fact that in the case of a plate of reasonable thickness we have the inequality $|Q|L\gg 1$, i.e., the depth of the anomalous skin layer is small compared with d. Therefore, naturally Z_{0yy} is independent of d.

We shall now consider the behavior of the element Z_{0xx}. The first term in the square brackets in Eq. (65) represents the contribution of the GK component, whereas the second represents the contribution of the long-wavelength component when an allowance is made for its multiple reflections from the plate surface. In the range of fields $\xi_0 \ll b_0^2\gamma_0$, when in the x polarization we have the normal skin effect, this term is

$$q_0(1+e^{iq_0L})(1-e^{iq_0L})^{-1}. \quad (67)$$

A change in the nature of the skin effect and the nonexponential long-wavelength component appear in fields $H\sim H_1$, corresponding to the condition $\xi_0\sim b_0^2\gamma_0$, i.e.,

$$H_1=H_L(\xi_{0L}/b_0^2\gamma_{0L})^{1/2}, \quad (68)$$

where ξ_{0L} and γ_{0L} are the values of ξ_0 and γ_0 at the doppleron threshold when $H=H_L$ (in the case of the square-root singularity of the resonance part of the conductivity we have $\xi_{0L}=2a$). When the fraction of the open orbits a is of the order of unity, the value of H_1 for real metals may be an order of magnitude (or more) greater than H_L. When a is reduced, the value of H_1 approaches H_L, i.e., the region where the open orbits have an influence on the structure $E_x(\zeta)$ becomes narrower.

We shall also introduce a field H_2 which corresponds to the condition $b_0\xi_0^{-1}=L$ and is related to H_1 by

$$H_2/H_1=(b_0\gamma_0L)^{1/2}=((n_1m_1v_1+n_3m_3v_3)d/n_2m_2v_2)^{1/2}\sim(d/al_2)^{1/2}, \quad (69)$$

where n_i is the density of carriers belonging to the ith group.

If $d>al_2$, the field H_2 is greater than H_1, i.e., the depth of the skin layer becomes comparable with the plate thickness in the region of the normal skin effect. This means that the formulas for the field distribution in the plate and, consequently, for the impedance are rigorous and not interpolated throughout the range $\xi\ll 1$.

If $d\leqslant al_2$, the element Z_{0xx} in the vicinity of the field H_2 is described only approximately by Eqs. (65) and (48). When the fraction of the open orbits is $a\sim 1$, the surface resistance maximum (Fisher and Kao effect[9]) also appears in teh vicinity of H_2. When the fraction of the open orbits is small, the maximum is located in higher fields, where the formula for the impedance again becomes rigorous. Moreover, in the latter case the formula for the surface resistance becomes rigorous also for $H\sim H_2$, because in this range of fields the first term in Eq. (65) is large compared with the second.

It remains to consider the changes which the presence of the open orbits induces in the oscillatory part of the impedance ΔZ. The doppleron oscillation sin relatively weak fields are suppressed because of the collisionless damping. Moreover, the change in the amplitude of the doppleron oscillaltions in the element

$$\Delta Z_{xx}=\frac{ic^2}{4\omega u}Z_{0xx}^2W_{xx}'(L) \quad (70)$$

is due to the change in the factor $|Z_{0xx}|^2$. The change in the amplitude of the GK oscillations of the surface resistance and reactance of the element ΔZ_{xx} are governed by the changes in the factors $\mathscr{R}_0^2-\mathscr{H}_0^2$ and $\mathscr{R}_0\mathscr{H}_0$, respectively; here, $\mathscr{R}_0=\mathrm{Re}Z_{0xx}$ and $\mathscr{H}_0=-\mathrm{Im}Z_{0xx}$. Finally, the amplitude of the oscillations of the nondiagonal elements ΔZ has now a small factor Z_{0yy}/Z_{0xx} and the element ΔZ_{yy} has a square of this factor.

We shall conclude by comparing the above theory with the experimental results reported in Ref. 5. All the characteristic features of the impedance tensor elements—the behavior of the smooth part $Z_{yy}^{(a)}$, the smallness of the oscillation amplitude of this element, the behavior of $\mathscr{R}_0$ (position and amplitude of the maximum) and $\mathscr{H}_0$, and doppleron and GK oscillations—are described correctly by our theory, i.e., the agreement between the theory and experiment is in fact semiquantitative. A quantitative comparison (particularly of the behavior of $\mathscr{H}_0$) would require detailed measurements of the smooth and oscillatory parts of the elements of the impedance tensor and simultaneous measurements of the distances traveled by resonance and nonresonance carriers. Such measurements have not yet been carried out.

[1]V. F. Gantmakher and É. A. Kaner, Zh. Eksp. Teor. Fiz. **48**, 1572 (1965) [Sov. Phys. JETP **21**, 1053 (1965)].

[2]M. Berset, W. M. MacInnes, and R. Huguenin, Helv. Phys. Acta **50**, 140 (1977).

[3]V. P. Naberezhnykh and T. M. Eremenko, Phys. Status Solidi *B* **58**, K175 (1973).

[4]I. F. Voloshin, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **80**, 2392 (1981) [Sov. Phys. JETP **53**, 1251 (1981)].

[5]I. F. Voloshin, N. A. Podlevskikh, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **85**, 373 (1983) [Sov. Phys. JETP **58**, 218 (1983)].

[6]I. F. Voloshin, S. V. Medvedev, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **71**, 1555 (1976) [Sov. Phys. JETP **44**, 814 (1976)].

[7]M. Ya. Azbel' and S. Ya. Rakhmanov, Zh. Eksp. Teor. Fiz. **57**, 295 (1969) [Sov. Phys. JETP **30**, 163 (1970)].

[8]I. F. Voloshin, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **72**, 735 (1977) [Sov. Phys. JETP **45**, 385 (1977)].

[9]H. Fischer and Y. H. Kao, Solid State Commun. **7**, 275 (1969).

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