Measurement of the temperature dependence of the shear modulus of solid ^{4}He

V. L. Tsymbalenko

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The elastic modulus G' and the logarithmic decrement $\delta_{\rm He}$ have been measured, in the temperature range 0.40 to 1.75 K, on specimens of crystalline helium grown at a pressure ≈ 35 atm. The influence of the growth conditions and of thermal shocks could be studied and specimens with reproducible results could be chosen from the continuous recording of G' and $\delta_{\rm He}$. It was found that for such specimens G' decreases monotonically with temperature. This is interpreted as an increase in the mobility of defects with decreasing temperature. The $G'(T)$ and $\delta_{\rm He}(T)$ relations are analysed by the Granato-Lücke theory; the dislocation parameters of internal friction are obtained.

In the first experiments on internal friction in crystalline helium[1] at a frequency ≈ 80 kHz, only the logarithmic decrement $\delta_{\rm He}(T)$ was measured. It was found that the temperature dependence of the decrement has the form of a curve with a maximum $\delta_{\rm He} \approx 0.2$ at $T = 1.3$ K. These measurements have been supplemented in the present work by measurements of the temperature dependence of the shear modulus G'. The form of the defect mobility, responsible for the damping, can be determined from the results of such measurements.

There is also considerable interest in the continuous recording of $\delta_{\rm He}(T)$ and $G'(T)$. In the earlier experiments,[1,2] these quantities were measured at a relatively small number of points with considerable temperature intervals (up to 0.1 K). A narrow (in temperature) damping peak can be found by continuous recording, and time dependences can be followed in considerably more detail.

EXPERIMENTAL METHOD

Specimens of solid helium were grown by Shal'nikov's method[3] of oriented crystallization at constant pressure. The gaseous helium was purified beforehand by the thermomechanical effect (^{3}He impurity $\lesssim 10^{-6}$). The choice of growth rate and specimen cooling regime are discussed below.

The measuring cell is basically the same as was used before:[1] a quartz cylinder ($1_0 = 27$ mm, $D = 3.4$ mm) with a fundamental flexural mode of oscillation ($f_0 = 74.6$ kHz) was placed along the axis of a cylindrical container with a 0.5 mm gap between the walls. After crystallization, the composite quartz-helium vibrator formed was excited by a 120–250 μs duration radiopulse with repetition frequency ≈ 80 kHz. The measurement of the quality factor Q and the period of oscillation τ of the vibrator was carried out after the action of the pulse on the freely damped oscillations. The maximum deformation amplitude ε of the helium usually lay in the range 2×10^{-6}–2×10^{-7}. With this apparatus[4] Q and τ could be recorded continuously on a two-pen, two-coordinate chart recorder. The accuracy in measuring the quality factor was $\approx 2\%$ and in the period was 0.01 μs.

The temperature was measured by a carbon thermometer to an absolute accuracy $\approx 3\times 10^{-3}$ K. The temperature was stabilized with an accuracy $\sim 10^{-4}$ K by an electronic regulator.[5] An annealing cycle was carried out for the specimens for ≈ 30 min at a temperature 0.05 K lower than the melting temperature.

The most detailed study was made on specimens (28 experiments) grown at a pressure of 35 atm (molar volume $V_m = 20.35$ cm^3). Four experiments were carried out on specimens grown at 61 atm ($V_m = 19.1$ cm^3).

EXPERIMENTAL RESULTS

Unless otherwise stated, the results presented refer to specimens with $V_m = 20.35$ cm^3. The initial stage of the work consisted in the choice of growth rate and cooling-heating regime for the specimen. The first experiments showed that the growth rates of $\sim 10^{-3}$ cm $\cdot$ s^{-1}, used earlier,[1,2] were high. A number of damping peaks were observed in such specimens which changed in position and shape with time and also after heating. The scatter of points in the results obtained earlier (see Fig. 2 of Ref. 1) was evidently associated with this phenomenon. These processes are most clearly seen in Fig. 1, a, where results are shown of measurements on a specimen grown at the maximum rate of 4×10^{-3} cm $\cdot$ s^{-1}, before and after the first heating. We note that after four heating cycles the $Q(T)$ and $\tau(T)$ functions are still not established. These effects are absent in specimens grown at a rate $\leqslant 3\times 10^{-4}$ cm $\cdot$ s^{-1}.

To avoid thermal shock and the appearance of similar effects when measuring the temperature dependences, the range 1.2–1.9 K was covered in not less than 2 h. A cooling rate from 10^{-3} to 10^{-4} K $\cdot$ s^{-1} did not affect the form of the $Q(T)$ and $\tau(T)$ relations in the range 0.4–1.2 K. It is thus essential to grow a specimen at a rate $\leqslant 3\times 10^{-4}$ cm $\cdot$ s^{-1} and to observe the cooling-heating regime indicated, in order to obtain reproducible results.

An example of the $Q(T)$ and $\tau(T)$ traces for such a specimen is given in Fig. 1, b. The above conditions were satisfied in six experiments and reproducible results were obtained. The results obtained on these "good" specimens are described below. Direct examination of the structure of the specimens was not carried out. It is important that the distribution of defects established in the growth process does not vary during the time of an experiment, $\lesssim 14$ h. The internal friction for deformation $\varepsilon = 2\times 10^{-6}$ to 10^{-8} was amplitude-independent in all the specimens studied (among them

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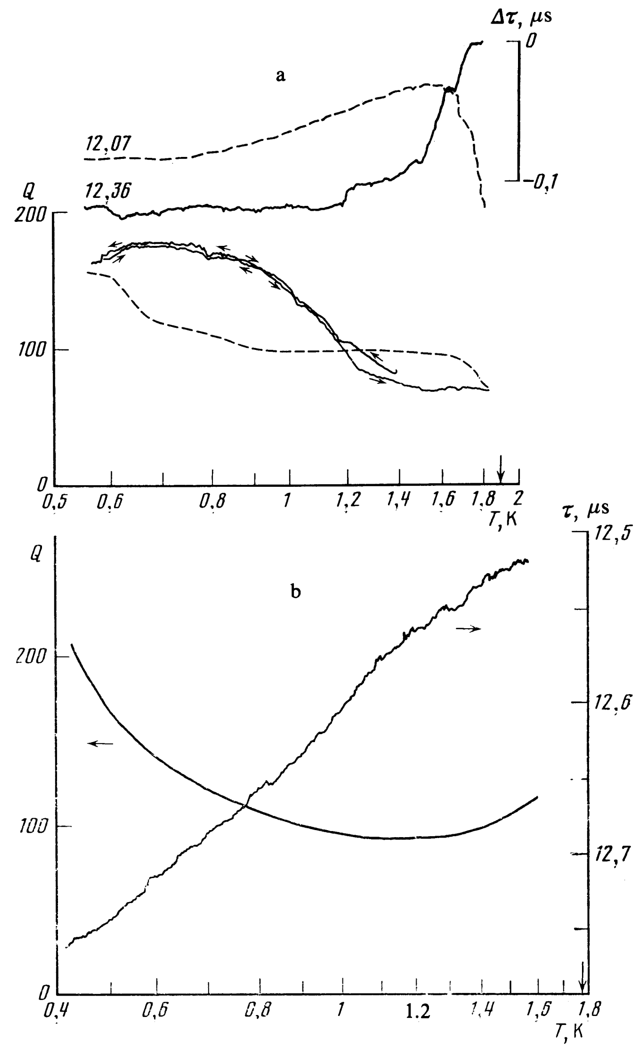

FIG. 1. Experimental traces of $Q(T)$ and $\tau(T)$. a) Specimen grown at the maximum rate. Solid lines—measurements immediately after the conclusion of growth; dashed lines—after the first anneal at $T = 1.82$ K. Curves of $\tau(T)$, against which are indicated the absolute values of the periods of oscillation, are shifted relative to one another; b) reproducible ("good") specimen. The vertical arrows indicate the melting temperatures.

also those with $V_m = 19.1\ \mathrm{cm}^3$), which agrees with the previous results.[1,2]

For determining the contribution of the helium to the damping and frequency of the vibrator, the problem of the oscillations of such a system was solved under the following assumptions.

1. The solid helium forms a cylindrical layer with an immobile outer surface, while the inner is firmly attached to the quartz.

2. The shear modulus is complex: $G^*(\omega) = G' + iG''$.

3. Edge effects (correction of the order of 0.01) and the anisotropy of G^* were not taken into account.

It follows from the solution that:

$$\delta_{\mathrm{He}} = \alpha\delta/[1-(\tau/\tau_0)^2], \qquad G' \sim (\tau_0/\tau)^2 - 1, \tag{1}$$

where τ and τ_0 are the periods of oscillation of the compound vibrator and of the free quartz respectively, α is a factor taking the inertia of the helium in the gap into account ($\alpha \approx 0.6$ in the present experiments). We note that the formula used previously to evaluate δ_{He} from δ gives values double the logarithmic decrement, i.e., the previous values[1,2] of δ_{He} must be halved in order to agree with the generally accepted determination of $\delta_{\mathrm{He}} = \pi G''/G'$.

Experimental results analyzed according to Eq. (1) are shown in Fig. 2. The shear modulus (Fig. 2, a) falls monotonically with decreasing temperature. Such a form of $G'(T)$ was also observed on specimens with defects (growth rate 7×10^{-4} to $10^{-3}\ \mathrm{cm\cdot s^{-1}}$). The relative change in shear modulus is large: $\Delta G'/G' \sim 0.25$. It is therefore essential to take into account the temperature dependence of the scaling factor $1-(\tau/\tau_0)^2$ when evaluating δ_{He}. The results of the previous work[1,2] were analyzed with a constant coefficient $1-(\tau/\tau_0)^2$, so that the shape of the $\delta_{\mathrm{He}}(T)$ curve is distorted. The results of measurements of $\delta_{\mathrm{He}}(T)$ with these corrections applied are shown in Fig. 2, b. It can be seen that the $\delta_{\mathrm{He}}(T)$ curves are smooth and have a characteristic broad maximum at $T \approx 1.1$ K. The position of the maximum was shifted in the direction of lower temperatures in comparison with untreated results (including the earlier results). The values of $\delta_{\mathrm{He}}(T)$ agree with previous measurements as to order of magnitude.

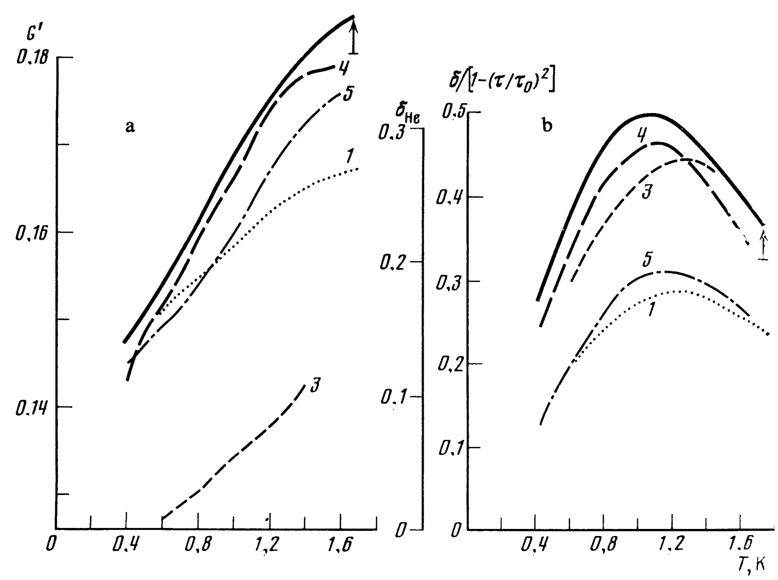

FIG. 2. Temperature dependence: a) of the shear modulus, b) of the decrement. The specimen numbers are indicated against the curves. The full lines are calculated according to Eqs. (4), (5) for specimen No. 4. For clarity, curves are shifted by the amounts shown by the arrows.

The $G'(T)$ and $\delta_{He}(T)$ dependences for specimens grown at a pressure of 61 atm are similar to those shown in Fig. 2. The shear modulus decreases monotonically with decreasing temperature and $\Delta G'/G' \approx 0.3$ between 2.5 and 0.4 K. The damping has a maximum at $T \approx 2.1$ K, below which it falls monotonically with temperature. The order of magnitude is $\delta_{He}^{max} \approx 0.3$.

DISCUSSION OF THE RESULTS

The observed damping is produced by the motion of intrinsic (in view of the high purity) defects in the specimen in the mechanical stress field. This phenomenon has a relaxation nature, which is confirmed by the absence of an amplitude dependence over a wide range of magnitudes of the deformation. The theory of mechanical relaxation[6] gives the following expressions for G'' and G':

$$G' = G_U - \frac{G_U - G_R}{1 + (\omega\tau_p)^2}, \quad G'' = (G_U - G_R)\frac{\omega\tau_r}{1 + (\omega\tau_r)^2}, \quad (2)$$

where G_U and G_R are respectively the nonrelaxation and relaxation shear moduli, τ_r is the relaxation time and ω is the measuring frequency. It follows from these relations (see Fig. 2): a) that we have $\omega\tau_r \approx 1$ ($\tau_r \approx 2\times 10^{-6}$ s) at a temperature ≈ 1.1 K; b) that τ_r decreases monotonically with decreasing temperature. This indicates that the mobility of defects which are responsible for the damping grows with decreasing temperature.

There is usually a distribution of τ_r values in a solid, and the expressions for δ and G' occur as an integral of Eqs. (2) over the spectrum. This fact does not change conclusion (b) qualitatively; however, the form of the distribution function has now to be taken into account to find the numerical value of τ_r at the damping maximum (in the Granato-Lücke theory,[6] for example, $\omega\tau_r \approx 0.084$ at the maximum).

The temperature dependence of τ_r is of greatest interest. To derive this relation from our experimental results, a model has to be chosen for internal friction by a specific type of defect. Since there is no theory of the dynamics of defects in quantum crystals (except for vacancies, for which[7] $\delta_{vac} \lesssim 10^{-3}$), the choice of a model has to be made by analogy with the features of internal friction in classical solids. Two mechanisms can give the observed large value of logarithmic decrement: grain-boundary relaxation and oscillations of dislocation segments (Granato-Lücke theory). The damping due to the first mechanism is observed near the melting point[6] at frequencies ~ 1 Hz. The temperature variation of τ_r is an activated dependence: $\tau_r \sim \exp(\Delta/T)$. It is, therefore, little likely that the phenomena observed in crystalline helium are due to this mechanism.

Analysis of the $\delta_{He}(T)$ and $G'(T)$ relations was carried out within the framework of the Granato-Lücke theory[6] (see also Ref. 8), where the following expressions are obtained for G' and δ:

$$\frac{G_U - G'}{G_U} = \frac{4(1-\nu)}{\pi^3}\Omega\Lambda l^2 \frac{1-(\omega/\omega_0)^2}{[1-(\omega/\omega_0)^2]^2 + (\omega\tau_r)^2},$$

$$\omega_0 = \frac{1}{l}\left[\frac{2G_U}{(1-\nu)\rho}\right]^{1/2},$$

$$\delta = \frac{4(1-\nu)}{\pi^2}\Omega\Lambda l^2 \frac{\omega\tau_p}{[1-(\omega/\omega_0)^2]^2 + (\omega\tau_r)^2},$$

$$\tau_p = \frac{Bl^2(1-\nu)}{2\pi G_U b^2}, \quad B = gT^n, \quad (3)$$

where B is the damping constant, l is the length of the dislocation segment, ρ is the density, Λ the concentration of dislocations, Ω is an orientational factor, $\nu \approx 0.3$ is Poisson's ratio and b is the burgers vector. The total damping and the modulus defect were found by integration over the distribution function of segment lengths, which was taken as exponential.[9] We note that averaging over the distribution function was not taken into account before.[1] This led, in particular, to a reduction in the value of n.

The final relations, in a form convenient for numerical integration are:

$$\delta = \frac{4(1-\nu)}{\pi^2}\Omega\Lambda L^2\omega\tau_r \int_0^\infty \frac{x^5 e^{-x}\,dx}{[1-(\omega/\omega_0)^2x^2]^2 + (\omega\tau_p)^2x^4}, \quad (4)$$

$$\frac{G_U - G'}{G_U} = \frac{4(1-\nu)}{\pi^3}\Omega\Lambda L^2 \int_0^\infty \frac{x^3[1-(\omega/\omega_0)^2x^2]e^{-x}\,dx}{[1-(\omega/\omega_0)^2x^2]^2 + (\omega\tau_r)^2x^4}, \quad (5)$$

where L is the average length of a dislocation loop; L is substituted for the parameter 1 in the expressions for ω_0 and τ_r.

The experimental $\delta_{He}(T)$ curve was approximated by the theoretical Eq. (4) by choosing ω_0, gL^2 and n($\Omega\Lambda L^2$ is a scaling factor which is found from the value of the maximum damping). We note that the $\delta_{He}(T)$ curve cannot be approximated by Eq. (4) within the limits of experimental accuracy by choosing only the two parameters gL^2 and n (i.e., neglecting the inertia of the dislocation core, $\omega_0 = 0$). A possible explanation is that inertial effects provide an appreciable contribution to the damping at low temperature. Iwasa *et al.*[10] remarked on the necessity of taking this effect into account when studying the propagation of ultrasonics in solid ^{4}He. The results of the analysis with the inertia of dislocations taken into account are presented later.

The value of the exponent n, obtained on six specimens, lies in the range 2.82–3.26; it is suggested later that n is exactly equal to three in all specimens. Such a temperature dependence of the damping constant is characteristic for the flutter-effect mechanism.[11] The parameters of dislocation internal friction, obtained by the analysis described with $n = 3$ are given in Table I. These values were then substituted in Eq. (5) and the $G'(T)$ variation was found by numerical integration. The results of the calculations for specimen No. 4 are shown in Fig. 2. It can be seen that the agreement between the calculated $G'(T)$ variation and experiment is good. This confirms the reliability of the choice of model.

The internal friction in crystalline helium with $V_m = 20.3$ cm^3 was studied using ultrasonic methods by: Wanner *et al.*,[12] Iwasa *et al.*[10] and by Tsuruoka and Hiki.[13] Wanner *et al.* obtained the following values for the parameters: $n = 1.45$–2.1, $\Lambda = 0.7$–4×10^5 cm^{-2}, $L = 6$–11×10^{-4} cm, $g_{ult} = 2.2$–3.2×10^{-7} cgs units. Iwasa *et al.* deduced the values $n = 3$, $\Omega\Lambda = 2.6$–11.8×10^4, $L = 3.9$–6.1×10^{-4}, $g_{ult} = 2$–5×10^{-7} cgs units. Our results agree well with these in the form of the temperature dependence

TABLE I.

Specimen No.	$\Omega\Lambda L^2$	L, 10^{-2} cm	$\Omega\Lambda$, 10^3 cm^{-2}	g, 10^{-16} cgs units K^{-3}
1	0,38	1,0	3,8	8,7
2	0,42	1,0	4,2	10,3
3	0,59	1,1	4,9	7,0
4	0,63	0,96	6,8	16,3
5	0,43	0,90	5,3	13,5

$B(T)$; the magnitude of the exponent agrees with the results of Iwasa *et al.*[10] The values of the dislocation density and average loop length differ by an order of magnitude (see Table I). This difference can be explained by a difference in the structure of the specimens. However, the parameter g determined from our measurements is two orders of magnitude less than the values obtained from ultrasonic measurements. Calculation from the flutter effect theory[11] gives a value $g_{\text{theor}} = 1.6\times 10^{-7}$ cgs units, agreeing with results of experiments in the megahertz frequency region and differs appreciably from our results.

Tsuruoka and Hiki[13] measured the frequency (5–50 MHz) and temperature (1.38–1.70 K) dependences of the logarithmic decrement for specimens with $V_m = 20.5$ cm^3. The main results are as follows: a maximum is observed in the frequency dependence of the damping at a frequency $f_{\max} \sim 15$ MHz; the value is $\delta_{\text{He}}^{\max} \approx 0.002$; $f_{\max}$ and $\delta_{\text{He}}^{\max}$ are independent of temperature in the range studied; as the specimen temperature is changed in a stepwise fashion, the value $f_{\max}$ is established within a time ≈ 30 min. We note that these results differ considerably from those of Iwasa *et al.*[10] and of Wanner *et al.*[12] Our results ($\delta_{\text{He}}^{\max} \sim 0.3$, $f_{\max}(1.1\ \text{K}) \approx 80$ kHz) do not agree with the values obtained in these authors' works.

Experiments were carried out to study the time dependences of δ_{He} and G', in which the specimen's temperature was changed by 0.1–0.15 K in ≈ 3 min and then held constant during 3 h. The decrement and shear modulus were recorded continuously on the chart recorder. At temperatures 1.8–1.2 K, temperature jumps led, in a container of our geometry, as noted above, to plastic deformation of the specimen and to a change of δ_{He} and G' with time. The actual time variation of δ_{He} and G' depended on the magnitude of the temperature jump, its sign and the initial temperature; it was not possible to obtain completely reproducible results. In a number of experiments the decrement and modulus did not reach a stationary value during the period of the measurements. Within the accuracy of measurement, the decrement and modulus remained constant after a stepwise change of temperature as described, in the range 1.2–0.45 K.

Hikata *et al.*[14] studied the propagation of large amplitude ultrasound ($f = 10$ MHz) at $T = 1.67$ K in specimens grown at a pressure of 32.5 atm. According to their results, the logarithmic decrement is independent of amplitude up to values of $\varepsilon \lesssim 10^{-4}$, which agrees with our results.

Measurements have been carried out in the low frequency region by Andronishkavili *et al.*[15] ($f \sim 500$ MHz) and by Paalanen *et al.*[16] ($f = 331$ Hz). Both groups obtained the solid helium specimens by the blocked capillary method. The solid helium is plastically deformed in the process of growth of a specimen by such a method, and the magnitudes of the residual stresses, their distribution and the uniformity of the specimen depend strongly on the geometry of the container, the cooling regime, the annealing, etc. The structure of the specimens studied by Andronishkavili *et al.*[15] and by Paalanen *et al.*[16] therefore differ appreciably from our "good" specimens, and it is difficult to make a direct comparison of the results. Nevertheless, we note that Paalanen *et al.* deduced that in pure helium specimens the elastic modulus is independent of temperature in the range 0.05–2 K, while the decrement (in other specimens) increases monotonically with temperature. Within the framework of the Granato-Lücke theory they obtained for a specimen with $V_m = 19.72$ cm^3 that $BL^2 = 1.3\times 10^{-13}\, T^2$ cgs units, and this value agrees with our results at $T \approx 1$ K.

As can be seen from this brief discussion of results obtained at other frequencies, our results qualitatively agree best of all with those of Refs. 10, 13, 14 and 16. However, the numerical values of the parameters Λ, L, and g differ greatly. The differences in dislocation density (and in the loop length) can be explained by a difference in the structure of the specimens. To explain the difference between g and g_{ult} ($g/g_{\text{ult}} \sim 10^{-2}$) it must be assumed either that the properties of different types of dislocations are being studied at different frequencies so that a comparison cannot be made, or that the phonon friction is anomalously small in the kilohertz region.

Another possible explanation of the observed $\delta_{\text{He}}(T)$ variation is that the temperature dependence of the damping constant is not described by a simple power law $B \sim T^3$. The $\delta_{\text{He}}(T)$ curve can then approximate to Eq. (4) without the inertial term ($\omega_0 = 0$), if it is assumed that $B \propto T^3$ for $T > 1.4$ K and $B \propto T^2$ for $T < 0.7$ K. In that case, only the product gL^2 can be determined from our experiments, and this lies in the range $(0.8\text{–}1.5)\times 10^{-13}$ cgs units and agrees well with other results.[10,13,16] If it is further assumed that $g \sim g_{\text{ult}}$, then the average loop length $L \sim 10^{-3}$ cm and the influence of inertial effects in the temperature range studied is small.

Which of the suggested explanations is valid cannot be determined from our results. For an experimental resolution of this question, measurements of δ_{He} and G' must be carried out down to a temperature ~ 20 mK. If the first model considered is true, then inertial effects will make an ever increasing contribution as the temperature is lowered, and this will lead at sufficiently low temperature to $\delta_{\text{He}} = \text{const} \neq 0$. For the parameters given in Table I, the arrival of the decrement at a constant value $\delta_{\text{He}}(T\to 0) \approx 0.1$ takes place at $T \approx 50$ mK. In the second model the decrement decreases monotonically according to the law $\delta_{\text{He}} \propto T^2$ as $T \to 0$. Inertial effects, of course, make their own contribution in this case, but it is apparent at appreciably lower temperatures.

It has thus been possible, by measurements of the elastic modulus and damping by a new method, to establish that as the temperature is lowered the mobility of the defects responsible for damping increases. This is a strong argument in favor of a dislocation mechanism for internal friction. The combined measurements of δ_{He} and G' have led to an appreciable refinement of the temperature dependence $\delta_{He}(T)$ which has made it possible to analyze the results according to the Granato-Lücke theory with great accuracy.

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