Induction of intermediate ordered phases in Heisenberg magnets by fluctuational exchange inversion

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The critical behavior of an n-component Heisenberg magnet is considered for the case of alternating interactions (e.g., ferromagnetic and antiferromagnetic). It is shown that instead of a continuous transition to the order phase here there is a first-order transition; depending on the amplitude and radii of the exchange integrals, this transition can occur to a phase which is unfavorable from the standpoint of Landau theory but which is stabilized by critical fluctuations.

1. INTRODUCTION

There are many magnetic materials in which an order-disorder transition is followed by a sequence of order-order phase transitions as the temperature is lowered futher (see Refs. 1 and 2). An explanation has been given for this effect based on the assumption of an inversion of the exchange interaction due to thermal expansion of the lattice.[3–4] However, such a mechanism applies only to a small number of magnets having high critical temperatures and large compressibilities. Other explanations have been based on allowance for a complex topology of the Fermi surface,[5] an unusual temperature dependence of the magnetic crystallographic anistropy,[6] and a non-Heisenberg character of the exchange.[7]

In the present paper we propose a new mechanism which can give rise to one or several order-order transitions after the transition from the disordered phase: the strong fluctuational protraction of the transition to the low-temperature phase. This mechanism works even in the isotropic case and is based on allowance for a possible spherical spatial dispersion of the interaction between magnetic atoms.

In principle, the final conclusions are independent of the specific choice of the sequence of transitions between different magnetic phases. The transitions could be either transitions from a ferromagnetic state to an antiferromagnetic state (and vice versa) or, more generally, transitions between arbitrary magnetic structures. In what follows we shall for the sake of definiteness consider a sequence of paramagnetic-antiferromagnetic-ferromagnetic (PM-AFM-FM) transitions.

2. DERIVATION OF THE GINZBURG-LANDAU FUNCTIONAL

Let us assume that the total exchange integral $J_{\mathbf{11}'}$ (the vector $\mathbf{1}$ enumerates the lattice sites) for some reason (e.g., the superposition of different exchange mechanisms) consists of a strong ferromagnetic part $J^f_{\mathbf{11}'}$ and a weaker antiferromagnetic part $J^a_{\mathbf{11}'}$ with $\Sigma_l\,(J^f_{\mathbf{11}'} - J_{\mathbf{11}'}) > 0$ (this corresponds in the $\mathbf{q}$ representation to the circumstance that the function $J_\mathbf{q}$ has a maximum both at the center of the Brillouin zone and at singular points on its boundary, with the $\mathbf{q} = 0$ maximum of $J_\mathbf{q}$ being the largest), so that the FM state is realized at $T = 0$. We shall show, however, that this situation is mathematically equivalent to the case of coupled fluctuating fields.[8–12] Thus the character of the phase transition is altered, and depending on the relationship of the radii of the FM and AFM interactions, a transition to an AFM state can occur first as the temperature is lowered from the PM phase.

Starting from the Heisenberg Hamiltonian

$$H = -\frac{1}{2}\sum_{\mathbf{11}'} J_{\mathbf{11}'}\mathbf{S}_\mathbf{1}\mathbf{S}_{\mathbf{1}'}$$

and assuming for simplicity that the magnetic moments are classical ($|\mathbf{S}_1|1$), we can write the partition function of the system in the form

$$Z = c_0^{-1}\int\prod_\mathbf{1} d\mathbf{S}_\mathbf{1}\int\prod_\mathbf{1} d\varphi_\mathbf{1} \times\exp\left[-\frac{1}{2T}\sum_{\mathbf{11}'}\varphi_\mathbf{1}J^{-}_{\mathbf{11}'}\varphi_{\mathbf{1}'} + \frac{1}{T}\sum_\mathbf{1}\varphi_\mathbf{1}\mathbf{s}_\mathbf{1}\right], \qquad (1)$$

where the matrix $J^{-1}_{\mathbf{11}'}$ is the inverse of the matrix $J_{\mathbf{11}'}$:

$$\sum_{\bar{\mathbf{1}}} J^{-1}_{\mathbf{1}\bar{\mathbf{1}}}J_{\bar{\mathbf{1}}\mathbf{1}'} = \delta_{\mathbf{11}'},$$

$$c_0 = \int\prod_\mathbf{1} d\varphi_\mathbf{1}\exp\left[-\frac{1}{2T}\sum_{\mathbf{11}'}\varphi_\mathbf{1}J^{-1}_{\mathbf{11}'}\varphi_{\mathbf{1}'}\right].$$

After integration over angles relation (1) becomes

$$Z = c_0^{-1}\int\prod_\mathbf{1} d\varphi_\mathbf{1}\exp\left\{-\frac{1}{2T}\sum_{\mathbf{1}\bar{\mathbf{1}}'}\varphi_\mathbf{1}J^{-1}_{\mathbf{11}'}\varphi_{\mathbf{1}'} + \sum_\mathbf{1}\ln\left[\left(\frac{2T}{\varphi_\mathbf{1}}\right)^{n/2-1}\Gamma\left(\frac{n}{2}\right)I_{n/2-1}\left(\frac{\varphi_\mathbf{1}}{T}\right)\right]\right\},$$

where $\Gamma(n/2)$ is the gamma function, $I_\nu(x)$ is a modified Bessel function, and n is the number of components of the vector $\mathbf{S}_1$. Transforming to the Fourier components and expanding the argument of the exponential in these components we get

$$Z = c_0^{-1}\int\prod_\mathbf{q} d\varphi_\mathbf{q}\exp\left[-\frac{1}{2T}\sum_\mathbf{q}\left(J_\mathbf{q}^{-1} - \frac{T^{-1}}{n}\right)|\varphi_\mathbf{q}|^2 - \frac{1}{4n^2(n+2)NT^4}\sum_{\mathbf{q}_1,\ldots,\mathbf{q}_3}\varphi_{\mathbf{q}_1}\varphi_{\mathbf{q}_2}\varphi_{\mathbf{q}_3}\varphi_{-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3}\right]. \qquad (2)$$

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Relation (2) implies that, thermal fluctuations aside, the paramagnetic phase first loses thermodynamic stability for certain modes φ_{q_i} at the bare critical temperature $T_0 = J_{\mathbf{q}_i}/n$, where $\mathbf{q}_i$ are the wave vectors corresponding to equivalent (with respect to the symmetry group of the crystal lattice) maxima of the Fourier transform of the total exchange integrals $J_{\mathbf{q}}$. If $J_{\mathbf{q}}$ has an absolute maximum at $\mathbf{q} = 0$, the resulting magnetic ordering is homogeneous and the FM structure is realized, while if the maxima of $J_{\mathbf{q}}$ lie at singular (in the group-theoretical sense) points $\mathbf{q}_{0i}$ of the Brillouin zone boundary, then the structure turns out to be antiferromagnetic. If the maxima occur at aribtrary positions $\mathbf{q}_i$ within the Brillouin zone, a complex and, generally speaking, incommensurate structure can arise. In our case $J_{\mathbf{q}}$ has maxima both at $\mathbf{q} = 0$ and at the zone boundary $\mathbf{q} = \mathbf{q}_{0i}$, with the ferromagnetic part of the interaction, in accordance with the assumption made earlier, being stronger than the antiferromagnetic part, i.e., $J_{\mathbf{q}=0} > J_{\mathbf{q}=\mathbf{q}_{0i}}$, so that mean field theory would give a second-order phase transition from the PM to the FM state at $T_{10} = J_{\mathbf{q}=0}/n$ However, a systematic allowance for critical fluctuations qualitatively alters the transition picture, which cannot in principle be described by Landau theory.

Near the point $T_0 = J_{\mathbf{q}i}/n$ the coefficient $(J_{\mathbf{q}i}^{-1} - 1/nT)$ of the terms $\sim |\varphi_{\mathbf{q}i}|^2$ changes sign, and so the coefficient becomes very small for the modes $\varphi_{\mathbf{q}}$ with wave vectors $\mathbf{q}$ lying inside certain small neighborhoods λ of the vectors $\mathbf{q}_i$. These modes fluctuate strongly. Let us therefore single out in functional (2) the important modes for the FM and AFM transitions, viz., those with wave vectors lying near the points $\mathbf{q} = 0$ and $\mathbf{q} = \mathbf{q}_{0i}$, respectively. To do this, let us surround these points in the first Brillouin zone by neighborhoods of radius λ (for simplicity we assume all the λ are equal, although this is not essential) and integrate relation (2) over the modes with wave vectors lying outside these neighborhoods. This, of course, will lead to the same renormalization of the bare constants for the resulting FM and AFM parts. Such renormalizations are unimportant for our present purposes and will be ignored from now on. Then the local magnetization of the system (in the case of a two-sublattice antiferromagnet) is given by the relation

$$\varphi_{\mathbf{l}} = \varphi_{\mathbf{q}=0} + \sum_j \varphi_{\mathbf{q}_{0j}} \cos \mathbf{q}_{0j}\mathbf{l}.$$

Let us now expand $J_{\mathbf{q}}$ about the points $\mathbf{q} = 0$ and $\mathbf{q} = \mathbf{q}_{0i}$ and make the substitutions

$$m_f(\mathbf{q}) = (n_0/TJ_{\mathbf{q}=0})^{1/2}\varphi_{\mathbf{q}}\big|_{|\mathbf{q}|<\lambda}$$

and

$$m_a(\mathbf{q}) = (n_0/TJ_{\mathbf{q}_{0i}})^{1/2}\varphi_{\mathbf{q}}\big|_{|\mathbf{q}-\mathbf{q}_{0i}|<\lambda}$$

(n_0 is the number of spins per unit volume). The sums over the neighorhoods of of the points $\mathbf{q}_{0i}$ can be reduced to a single sum over a spherical region of radius λ at the center of the Brillouin zone (shown schematically for a planar square lattice in Fig. 1).

As a result, the functional in the argument of the exponential of relation (2) assumes the standard Ginzburg–Landau form for coupled vector fields:

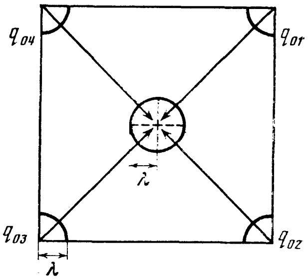

FIG. 1. Contraction of the neighborhoods $|\mathbf{q} - \mathbf{q}_{0i}| < \lambda$ of the points $\mathbf{q}_{0i}$ to the center of the Brillouin zone.

$$H = \frac{1}{2}\int d\mathbf{r}\Big[\tau_f \mathbf{m}_f^2(\mathbf{r}) + \tau_a \mathbf{m}_a^2(\mathbf{r}) + c_f(\nabla \mathbf{m}_f(\mathbf{r}))^2 + c_a(\nabla \mathbf{m}_a(\mathbf{r}))^2 + \frac{1}{4} g_f \mathbf{m}_f^4(\mathbf{r}) + \frac{1}{4} g_a \mathbf{m}_a^4(\mathbf{r}) + \frac{1}{2}\tilde{v}\mathbf{m}_f^2(\mathbf{r})\mathbf{m}_a^2(\mathbf{r}) + \tilde{w}(\mathbf{m}_f(\mathbf{r})\mathbf{m}_a(\mathbf{r}))^2\Big], \quad (3)$$

where

$$\tau_{f,a} = (T - T_{f0,a0})/T, \quad g_f = 2J_{\mathbf{q}=0}^2/n^2(n+2)n_0T^2,$$

$$g_a = 2J_{\mathbf{q}_{0i}}^2/n^2(n+2)n_0T^2, \quad \tilde{v} = \tilde{w} = 2J_{\mathbf{q}=0}J_{\mathbf{q}_{0i}}/n^2(n+2)n_0T^2,$$

and the coefficients

$$c_{f,a} = {}^1/_2 \int d\mathbf{r}\, r^2 J_{f,a} \Big/ \int d\mathbf{r} J_{f,a}$$

are the squares of the decay radii of the corresponding exchange integrals.

We note that the resulting Hamiltonian (3) is, in essence, the Hamiltonian of a two-sublattice antiferromagnet written in terms of the ferromagnetism ($\mathbf{m}_f = \mathbf{m}_1 + \mathbf{m}_2$) and antiferromagnetism ($\mathbf{m}_a = \mathbf{m}_1\mathbf{m}_2$) vectors, where $\mathbf{m}_1$ and $\mathbf{m}_2$ are the magnetization vectors of the corresponding sublattices. However, in contrast to an ordinary antiferromagnet, where τ_f is always positive, here τ_f can change sign because of the maximum of $J_{\mathbf{q}}$ at the Brillouin zone center.

3. RENORMALIZATION GROUP EQUATIONS

The critical dynamics of the system under discussion is governed by the evolution of the bare constants g_f, g_a, $\tilde{v}$, and $\tilde{w}$ as $T \to T_c$. The "dressing" of these constants is described by the renormalization group equations. After awkward but standard calculations in the framework of the ε expansion,[13] we obtain, with accuracy to terms $\sim \varepsilon^2$, the following equations for the charges in functional (3):

$$\dot{u}_f = \varepsilon u_f - \left[\frac{(n+8)}{2} u_f^2 + \frac{nv^2}{2} + 2vw + 2w^2\right],$$

$$\dot{u}_a = \varepsilon u_a - \left[\frac{(n+8)}{2} u_a^2 + \frac{nv^2}{2} + 2vw + 2w^2\right], \quad (4)$$

$$\dot{v} = \varepsilon v - \frac{1}{2}[(u_f + u_a)((n+2)v + 2w) + 4v^2 + 4w^2],$$

$$\dot{w} = \varepsilon w - w[(u_f + u_a) + 4v + (n+2)w].$$

Here we have introduced the notation

$$v=\tilde{v}\bar{r}^4/c_fc_a, \quad w=\tilde{w}\bar{r}^4/c_fc_a, \quad u_{f,a}=g_{f,a}\bar{r}^4/c_{f,a}^2,$$

where r is the avrage distance between magnetic atoms and n is the number of components of the order parameter. System (4) has the following fixed points:

$$\mu_1^*: \; u_f^*=u_a^*=v^*=w^*=0;$$

$$\mu_2^*: \; u_f^*=\frac{2\varepsilon}{(n+8)}, \quad u_a^*=v^*=w^*=0;$$

$$\mu_3^*: \; u_a^*=\frac{2\varepsilon}{(n+8)}, \quad u_f^*=v^*=w^*=0;$$

$$\mu_4^*: \; u_f^*=u_a^*=\frac{2\varepsilon}{(n+8)}, \quad v^*=w^*=0;$$

$$\mu_5^*: \; u_f^*=u_a^*=v^*=\frac{\varepsilon}{n+4}, \quad w^*=0;$$

$$\mu_6^*: \; u_f^*=u_a^*=\frac{n\varepsilon}{(n^2+8)}, \quad v^*=\frac{8-2n}{2(n^2+8)}\varepsilon, \quad w^*=0;$$

$$\mu_7^*: \; u_f^*=u_a^*=v^*=\frac{2\varepsilon}{(n^2+8)}, \quad w^*=\frac{(n-2)}{(n^2+8)}\varepsilon;$$

$$\mu_8^*: \; u_f^*=u_a^*=v^*=w^*=\frac{\varepsilon}{n+8}.$$

In addition, for $n = 2$ there is another fixed point:

$$\mu_9^*: \; u_f^*=u_a^*=\varepsilon/10, \quad v^*=3\varepsilon/10, \quad w^*=-\varepsilon/10.$$

Of all these fixed points, only μ_7^* is stable (attractive). The bare parameters of Hamiltonian (3) form a hypersurface, specified by the two conditions $w=(u_a u_f)^{1/2}$, $v=(u_a u_f)^{1/2}=w$, in the space $\mu(u_f, u_a, v, w)$. In the framework of mean field theory the equality of the free energies of the FM and AFM phases occurs at $g_f = g_a$, so that phase boundary between them on the set of parameters μ is given by the condition $u_f c_f^2 = u_a c_a^2$. The structure of the ordered phase is determined not by the dressed charges (u_f, u_a, v, w) themselves, but by their ratios. Therefore, we rewrite system of equations (4) for the ratios $x = u_f/w$, $y = u_a/w$, $z = v/w$ in the form

$$\begin{aligned}
\frac{\dot{x}}{w} &= -\frac{n+6}{2}x^2+xy+4xz-\frac{nz^2}{2}+(n+2)x-2z-2,\\
\frac{\dot{y}}{w} &= -\frac{n+6}{2}y^2+xy+4yz-\frac{nz^2}{2}+(n+2)x-2z-2,\\
\frac{\dot{z}}{w} &= -\frac{n}{2}z(x+y)+2z^2+(n+2)z-(x+y)-2.
\end{aligned} \tag{5}$$

Fixed points $\mu_1^*, \ldots, \mu_6^*$ are absent in the space (x, y, z), while points μ_7^* and μ_8^* go over to the points $\mu_{\rm I}^*(1,1,1,)$ and $\mu_{\rm II}^*$ (ν,ν,ν), where $\nu = 2/(n-2)$. The set of bare values of the parameters x, y, and z form a hyperbola $xy = 1$ in the plane $z = 1$, and the boundary surface between the two phases is the plane $xc_f^2 = yc_a^2$.

4. EVOLUTION OF THE INVARIANT CHARGES IN THE CRITICAL REGION

The study of system (5) is particularly simple and clear for $n = 1$ when there is no term of the form $\tilde{w}(m_f m_a)^2$ in Hamiltonian (3). Hamiltonian (3) then reduces to the analogous Hamiltonian for the $n = 1$ case by the formal replacement $\tilde{v} + 2\tilde{w} \to \tilde{v}$ (or $\tilde{w} \to 0$). Now the points μ_7^*, μ_8^*, and μ_9^* are absent and the point μ_5^* is stable. We note that in the approximation to first order in ε the point μ_6^* has an exponent of zero in addition to the negative exponents, but a more rigorous examination shows that this point is a saddle. Thus, for $n = 1$ the number of charges is reduced, and system (5) becomes

$$\begin{aligned}
\frac{\dot{x}}{v} &= \frac{1}{2}[-6x^2+3xy+4x-1],\\
\frac{\dot{y}}{v} &= \frac{1}{2}[-6y^2+3xy+4y-1],
\end{aligned} \tag{6}$$

where $x = u_f/v$, $y = u_a/v$, and the fixed points $\mu_{\rm I}^*$ and $\mu_{\rm II}^*$ go over to $\mu_a^*(1/3, 1/3)$ and $\mu_b^*(1,1)$, respectively. The pattern of flow lines for system (6) is shown in Fig. 2a. A numerical analysis shows that the separatrix of the family of critical surfaces of the stable fixed point μ_b^* touches the initial-condition hyperbola $(xy = 1/9)$ at a single point μ_a^*, so that, with the exception of this point, the flow lines, originating from the hyperbola $xy = 1/9$, leave the stability region of Hamiltonian (3) with respect to one of the parameters u_a or u_f. Thus, at all values of the bare parameters (with the exception of the point $u_f = u_a$) the effective Hamiltonian loses stability in the critical region, and the phase transition to the ordered state turns out to be of first order.

We also note that below the point μ_a^* the straight line $u_f = u_a$ "repels" the flow lines and so cannot intersect them. Since $J_{\mathbf{q}=0} > J_{\mathbf{q}=\mathbf{q}0i}$ (in accordance with the initial assumption) and, consequently, $g_f > g_a$, the bare values of x and y on the curve $xy = 1/9$ should always be chosen to the right of the phase-equilibrium line $u_f c_f^2 = u_a c_a^2$. Because the straight line $xc_f^2 = yc_a^2$ $(u_f c_f^2 = u_a c_a^2)$, depending on the value of the ratio c_a/c_f, can occur either above or below the bisectrix $x = y$, two fundamentally different cases are possible for the trajectories $(x(t), y(t))$. The first case occurs under the inequality $g_f c_a^2/g_a c_f^2 > 1$ for arbitrary values of the ratio c_a/c_f. Here the flow lines, beginning in the stability region of the FM phase, go outside the stability boundary of the AFM Hamiltonian (Fig. 2a, b), so that a first-order phase transition occurs to an AFM state which is stabilized by strong ferromagnetic fluctuations. The second case is specified by

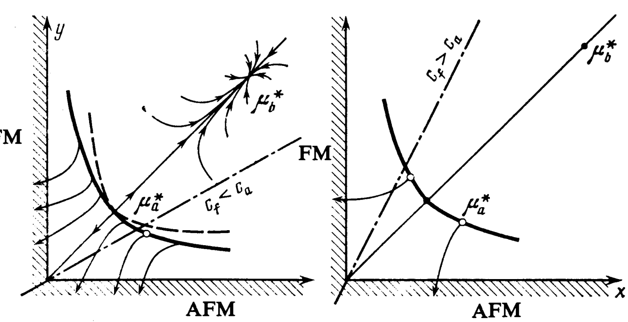

FIG. 2. a) Pattern of flow lines for $n = 1$. The solid line is that set of bare paraameters, the dashed line is the sparatrix, and the dot-and-dash line is the phase boundary. Straight lines $x = 0$ and $y = 0$ are the stability boundaries of the FM and AFM Hamiltonians, respectively. The circlet shows the origin of the flow line of $g_f > g_a$ $(c_f < c_a)$. b) Possible variations of the origin of the flow lines: 1) for $g_f c_a^2/g_a c_f^2 < 1$,2) for $g_f c_a^2/g_a c_f^2 > 1$.

the inequality $g_f c_f^2/g_a c_f^2 < 1$ and can occur only under the condition $c_a/c_f < 1$. In this situation the flow lines begin in the region corresponding to the mean field FM Hamiltonian and end at its stability boundary (Fig. 2b). The presence of fluctuations of the AFM phase in the system causes the second-order transition to the FM state to be replaced by a first-order transition and is not accompanied by inversion of the transitions.

For $n \neq 1$ the picture described above remains qualitatively the same. The flow lines start from the hyperbola $xy = 1$ in the plane $z = 1$, break downward ($\dot{z}|_{xy=1, z=1} < 0$), and continue beyond the planes $x = y$ and $z = 0$. The bare values of x and y should be chosen to the right of the plane $xc_f^2 = yc_a^2$ which delimits the stability regions of the AFM and FM phases. Figure 2 in this case can be regarded as a projection of the flow-line pattern described above onto the $z = \text{const}$ plane, so that the above analysis of the possible situations for $n = 1$ goes over in its entirety to the case $n \neq 1$.

Of particular interest is the study of the flow lines which depart from the immediate vicinity of the saddle-type fixed point $\mu_{\rm I}^*$, in which the set of bare parameters touches the separatrix of the stable point $\mu_{\rm II}^*$. We note that the strict equalities $z = 1$ and $xy = 1$ for the set of initial values is partly a consequence of the approximation made in deriving Hamiltonian (3). In particular, in the integration over all the modes lying outside the neighborhoods of the points $\mathbf{q} = 0$ and $\mathbf{q} = \mathbf{q}_{0i}$ at which $J_\mathbf{q}$ has maxima, the bare values of the parameters are renormalized in the same way only in the lowest order of perturbation theory. In the higher orders this renormalization contains corrections which depend explicitly on $\mathbf{q}$, and the relations $z = 1$ and $xy = 1$ are violated. This causes a certain deformation of the set of initial conditions, and as a result the region in which this set is intersected by the separatrix can either widen or vanish entirely. In the case of a widening intersection region there arises a set of bare parameters (x, y) from which the flow lines arrive at fixed point $\mu_{\rm II}^*$, corresponding to a second-order transition to a magnetic structure described by a Heisenberg Hamiltonian with a two-component order parameter. Thus, at the phase transition point in this case an asymptotic symmetry[8–11,14,15] arises, so that FM and AFM ordering can arise in the system with equal probability. To answer the question of which structure is realized in an actual experiment it is necessary to know the character of the infinitesimal fields which break the asymptotic symmetry. Here the situation is exactly as in the case of an ordinary isotropic ferromagnet, when the turning on of an infinitesimal external magnetic field determines the direction of the magnetization below the phase transition point. Since the function $J(\mathbf{q})$ can in general be altered by an external applied pressure or by impurities, it is in principle possible to obtain either an improvement in the conditions for the second-order transition to the ordered phase or a degradation of the conditions to the point where such a transition is completely impossible.

5. INVERSION OF THE TRANSITIONS

We have thus seen that in the situation discussed above, as in Refs. 16–20, allowance for critical fluctuations lead to a stituation in which the system can undergo a transition from the disordered phase to a state (AFM) which is unfavorble in the framework of Landau theory. The analysis given in Ref. 21 justifies the assumption that this conclusion does not rely on the fourth-order form (3) for the free energy and that a more exact calculation of the free energies of the FM and AFM phases will not change the results in a qualitative way. In addition, if the relationship between the bare values $J_{\mathbf{q}=0}$ and $J_{\mathbf{q}_{0i}}$ or (c_f and c_a) changes under external influences (e.g., pressure), then the P-T diagram exhibits characteristic "breaks" analogous to those found in Refs. 16–20.

And so, under the condition $J_{\mathbf{q}=0}\, c_a/c_f\, J_{\mathbf{q}_{0i}} > 1$, as the temperature is lowered from the PM phase a first-order transition is obersved to an AFM state. This statre is stabilized solely by critical fluctuations, and therefore, outside the fluctuation region a FM ordering is established in the system in accordance with Landau theory. The AFM-FM transition will also be of first order. To estimate the temperature of the transition it is convenient to explicitly decompose the exchange integral $J_{\mathbf{l}\mathbf{l}'}$ into ferromagnetic $J^f_{\mathbf{l}\mathbf{l}'}$, and antiferromagnetic ($J^a_{\mathbf{l}\mathbf{l}'}$) parts and to write the partition function in the form

$$Z = C_0^{-1} \int \prod_{\mathbf{l}} d\psi_{\mathbf{l}}\, d\theta_{\mathbf{l}} \exp\Big\{ -\frac{1}{2T} \sum_{\mathbf{l}\mathbf{l}'} \big(J^{f^{-1}}_{\mathbf{l}\mathbf{l}'}\, \theta_{\mathbf{l}}\theta_{\mathbf{l}'} + J^{a^{-1}}_{\mathbf{l}\mathbf{l}'}\, \psi_{\mathbf{l}}\psi_{\mathbf{l}'} \big) + \sum_{\mathbf{l}} \ln\Big[\Big(\frac{2T}{(|\psi_{\mathbf{l}}+\theta_{\mathbf{l}}|)} \Big)^{n/2-1} \Gamma\Big(\frac{n}{2}\Big) I_{n/2-1}\Big(\frac{|\psi_{\mathbf{l}}+\theta_{\mathbf{l}}|}{T} \Big) \Big] \Big\}.$$

After the AFM ordering the integration over ψ_q can be approximately replaced by just the contribution of the condensed modes, which are given by the equation

$$\psi_{\mathbf{l}} = \sum_{\mathbf{l}'} J^a_{\mathbf{l}\mathbf{l}'} I_{n/2}(\psi_{\mathbf{l}}/T) / I_{n/2-1}(\psi_{\mathbf{l}}/T), \tag{7}$$

which corresponds to taking them into account in the framework of mean field theory. We note that for $n = 3$ equation (7) assumes the familiar form $\psi_{\mathbf{l}} = \Sigma_{\mathbf{l}'} J^a_{\mathbf{l}\mathbf{l}'} L(\psi_{\mathbf{l}}/T)$, where L is the Langevin function. The critical temperature of the AFM–FM transition can be estimated roughly from the condition that the free energies of the AFM and FM phases be equal: $F_{\rm FM} = F_{\rm AFM}$, where

$$F_{\rm FM} = -\frac{1}{2} (J^f_{\mathbf{q}=0})^{-1} \theta^2_{\mathbf{q}=0} + T \ln\Big[\Big(\frac{2T}{\theta_{\mathbf{q}=0}} \Big)^{n/2-1} \Gamma\Big(\frac{n}{2}\Big) I_{n/2-1}\Big(\frac{\theta_{\mathbf{q}=0}}{T} \Big) \Big],$$

$$F_{\rm AFM} = -\frac{1}{2} (J^a_{\mathbf{q}_{0i}})^{-1} |\psi_{\mathbf{q}_{0i}}|^2 + \frac{T}{N} \sum_{\mathbf{l}} \ln\Big[\Big(\frac{2T}{\psi_{\mathbf{q}_{0i}} \cos \mathbf{q}_{0i}\mathbf{l}} \Big)^{n/2-1} \Gamma\Big(\frac{n}{2}\Big) I_{n/2-1} \times \Big(\frac{\psi_{\mathbf{q}_{0i}} \cos \mathbf{q}_{0i}\mathbf{l}}{T} \Big) \Big].$$

Since $F_{\rm AFM}$ contains an even function in the logarithm and since $\cos \mathbf{q}_{0i}\mathbf{l}$ in the summation over $\mathbf{l}$ takes on the values ± 1, the last term in $F_{\rm AFM}$ is easily evaluated. As a result, we obtain the following expression for the critical temperature:

$$T_c^{\mathrm{FM}} = (J^f_{\mathbf{q}=0} - J^a_{\mathbf{q}_{0i}})/[(n-1)\ln(J^f_{\mathbf{q}=0}/J^a_{\mathbf{q}_{0i}})].$$

In conclusion we note that, genrally speaking, $J_{\mathbf{q}}$ can have several different local maxima. Their mutual inversion as a result of fluctuational renormalization can lead to a sequence of first-order phase transitions from one ordered phase to another.

The mechanism we have described can explain the order-order transitions in materials in which exchange inversion due to changes in the lattice parameters does not occur, and the sequence of magnetic transitions is not altered by an applied hydrostatic pressure.

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