Color forces and the cumulative effect in scattering of hadrons by deuterons

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In double color charge exchange of an incident hadron on the various nucleons of a nucleus it is possible to diffractively excite a color dipole, the decay of which leads to emission of one of the nucleons of the target into the backward hemisphere. We have calculated the contribution of this mechanism to the cross section for the process $pd \to p_B X$, which turned out be very large in the hard part of the momentum spectrum $p_B \gtrsim 500$ MeV/c. We have analyzed in a quantum-mechanical approach the dependence of the significant longitudinal distances on the cumulative momentum. We discuss the predictions for production of dibaryon resonances with color separation, elastic pd backscattering, polarization effects, and other effects.

1. INTRODUCTION

Study of the dynamics of the peripheral interaction of hadrons encounters the principal unsolved problem of quantum chromodynamics—color confinement. Nevertheless the ideas of QCD in this region have turned out to be extremely fruitful and have led to the emergence of a definite phenomenological picture of the interaction. In hadron scattering color charge exchange occurs, and the structure, called color tube,[1–3] which connects the color objects that move apart with a large relative momentum stretches and is eventually broken as the result of formation in the color field of quark-antiquark pairs of gluons from the vacuum. The tube breaks up into individual colorless clusters with mass of the order of hadronic masses, the momentum spectrum of which forms a plateau on the rapidity scale. The time of hadronization, i.e., the process of complete breakup of the tube, is E/μ^2, where E is the energy of the incident hadron and μ is a characteristic mass associated with the parameters of the model. Since in a real experiment the detecting apparatus is located at macroscopic distances from the target, only the colorless products of the hadronization process reach the apparatus. For direct observation of the effects of color forces it would be necessary to place counters at a microscopic distance from the target, which of course is impossible. However, in the interaction of hadrons with nuclei there is a sequence of several nucleon targets separated from each other by a distance of the order of one fermi. At this distance the hadronization process is not yet complete (if $E/\mu^2 \gg R_A$) and the existence of color forces can be manifested in various observable effects. One of them is the emission of nucleons into the backward hemisphere, the so-called cumulative effect,[4] in hadron-nucleus collisions at high energies, and the present paper is devoted to this phenomenon.

In the second section we describe a mechanism, based on the action of color forces, of cumulative-nucleon production. The essence of the model is readily explained in terms of classical mechanics. In Section 3 we calculate the cross sections for the process $hd \to hp_B n$ and the inclusive reaction $hd \to p_B X$, where p_B is a proton emitted into the backward hemisphere. In Section 4 we consider the quantum-mechanical description of the present mechanism and show that in a certain region of the cumulative momenta $p_B \gtrsim 550$ MeV/c the process occurs mainly through formation of dibaryon resonances with separated color. We calculate the parameters of these resonances and show that they should be manifested in the form of peaks in the momentum spectrum of cumulative protons. In Section 5 we show that the mechanism considered gives an appreciable contribution also to the cross section for elastic pd backscattering. In the energy dependence of the cross section we expect peaks at an initial energy of several GeV due to dibaryon resonances with separated color. In Section 6 we discuss polarization effects in the cumulative process. In the last section we discuss proposals for experiments which are sensitive to the contribution of the mechanism under discussion.

2. PRODUCTION AND DECAY OF A COLOR DIPOLE IN HADRON-DEUTERON SCATTERING. CLASSICAL DISCUSSION

In Ref. 5 we proposed for the cumulative effect a mechanism based on diffractive excitation of a color dipole in the deuteron. Below we give a classical calculation of the contribution of this mechanism with correction of several inaccuracies.

In calculation of the dynamics of the process we shall make use of the color-tube model.[1–3] In this model it is assumed that the emission of colored objects occurs adiabatically, i.e., energetic bremsstrahlung is neglected and it is assumed that the color field carries only energy and does not have momentum. The tension coefficient of the tube $\varkappa$ can be estimated from the spectra of hadronic masses: $\varkappa = (2\pi\alpha'_R)^{-1} \approx 1$ GeV · fm^{-1}, where α'_R is the slope parameter of the Reggeon trajectories. In reality the effective tension of the color tube in a hadron-hadron interaction can differ appreciably from this value, since in such processes color octets are emitted, and not color triplets, and in addition the adiabatic approximation may turn out to be very crude. Below we have also used another parameter of the

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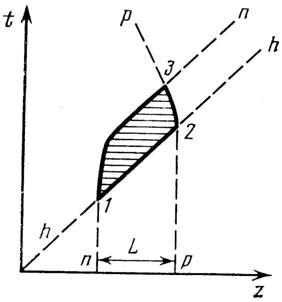

FIG. 1. Diagram of the longitudinal coordinate z as a function of the time t for the process $hd \to p_B hn$: the solid line shows the trajectories of colored objects and the dashed line shows the trajectories of colorless hadrons.

model—w—the probability of formation of a quark-antiquark pair from the vacuum in a color field per unit time per unit tube length. This quantity can be estimated either from the Schwinger formula[2] or from analysis of data on e^+e^- annihilation,[3] which gives $w \approx 2$ fm^{-2}.

The parameter w can be estimated also from the proton momentum spectrum in the reaction $pp \to pX$. In the target-fragmentation region the recoil proton has a momentum equal in order of magnitude to $p \approx \varkappa \tau$, where τ is the time from the moment of color charge exchange to the first rupture of the tube. The quantity τ is determined by the condition $\tau^2 w/2 \approx 1$ if it is assumed that the length of the color tube is $l \approx \tau$ (this is true only for small values of τ and $l \to m\varkappa$ as $\tau \to \infty$). On the other hand the momentum p is related to the Feynmann variable x by the equation $p = m(1-x^2)/2x$. Since the inelasticity coefficient is $\langle x \rangle \approx 0.5$, we have $\langle p \rangle \approx 1$GeV/$c$. From this we find $w \approx 2/\tau^2 \approx 2\varkappa^2/p^2 \approx 2$ fm^{-2}.

Let us now consider the interaction of a high-energy hadron with a pair of nucleons, as illustrated in Fig. 1. At point 1 color charge exchange of the incident hadron occurs on the first nucleon of the deuteron, after which both are color charged and the nucleon with mass m begins to be accelerated with an acceleration $\varkappa/m$. We note that as the velocity of the colored nucleon approaches the velocity of light the length of the color tube stops increasing, having reached a value $m/\varkappa \approx 1$ fm.

At point 2 the color-charged hadron charge-exchanges on the second nucleon and becomes colorless. The nucleon which is at point 2, after the second charge exchange, begins to be accelerated with an acceleration $-\varkappa/m$ in the direction opposite to the incident beam. It reaches its maximum momentum p_B at point 3, where there is a third color charge exchange, after which the nucleons are emitted in a colorless state. Generally speaking, the tube can be broken with a probability of the order of unity if L is large. We shall take into account this possibility below, and for the present we shall consider the pion-free process $hd \to p_B hn$.

The momentum p_B of the nucleon emitted into the backward hemisphere is easily found from the condition $\Delta p_h = \Delta E_h = \varkappa L$ for the incident hadron if its momentum is sufficiently large: $p_h \gg m$ and $p_h \gg \varkappa L$, where $L = z_2 - z_1$. From this condition it follows that

$$2(E-m)/(2m-p_L-E) = \varkappa L/m, \tag{1}$$

where $E = (m^2 + p_L^2 + p_T^2)^{1/2}$ is the energy of the cumulative nucleon. From this relation it is evident that with increase of the internucleon distance the momentum p_B increases and approaches a kinematic limit which at $\theta = 180°$ is $p_B^{\max} = 3m/4$. Note that the increase of the principal longitudinal distances with increase of p_B is consistent with the principles of quantum mechanics. Large momenta p_B correspond to formation of a color dipole with a large mass and consequently a large rms radius. Nevertheless we shall show in Section 4 that taking into account quantum effects can change the relation of the principal distances with the momentum p_B.

3. THE REACTION $hd \to p_B X$

The contribution of color-dipole production to the cross section for the reaction $hd \to p_B hn$ can be written as follows:

$$\frac{d\sigma}{dp_L d^2 p_T} = \frac{1}{8}\,\frac{\beta\alpha_s^2 B}{\pi} \exp(-Bp_T^2)\,(\sigma_{in}^{hN})^2 |\psi_d(L')|^2 D(L) \frac{dL}{dp_B}. \tag{2}$$

The cross section for color charge exchange on the first nucleon is σ_{in}^{hN}. The probability of color charge exchange on the second nucleon is suppressed by a small factor of the Glauber-correction type and is equal to $\sigma_{in}^{hN}|\psi_d(L)|^2 dL$. Note that the length of the color tube is less than the distance between the centers of the nucleons, which is taken into account in Eq. (2) in the argument of the deuteron wave function $L' = L + R_0$, where $R_0 \approx 0.5$ fm is the radius of the nucleon core.

The probability that during the entire process no quark-antiquark pair is produced from the vacuum is taken into account in Eq. (2) by the factor

$$D(L) = \exp\left(-w\int dl\,dt\right), \tag{3}$$

where $\int dldt$ is equal to the area of the hatched region in Fig. 1.

Note that for $L \gg m/\varkappa$ $D(L) \approx \exp(-wLm/\varkappa)$. The coefficient 1/8 in Eq. (2) takes into account the relative probability of emission of nucleons in a colorless state. Since the first two color charge exchanges "selected" a deuteron configuration with the nucleons at one impact distance, the probability of the third color charge exchange does not contain an additional smallness of the Glauber-correction type, but gives only a factor $\beta\alpha_s^2$, where $\alpha_s = g^2/4\pi$ and the numerical coefficient β is estimated below.

The dependence of thr cross section (2) on the transverse component of the momentum p_B of the proton emitted backwards is represented at small p_T in Gaussian form with a slope B.

In order to estimate the quantities β and B more accurately, let us consider the Feynman graphs in Fig. 2, which describes this process, in which the color charge exchange is modeled by gluon exchange. Naturally the calculation of such a graph cannot take into account confinement effects, but it is natural to assume that these effects do not influence the total cross section of the reaction, but only change the shape of the proton momentum spectrum. Therefore the

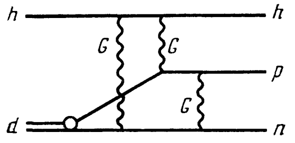

FIG. 2. Feynman diagram with three-gluon exchange in the process $hd \to p_B hn$.

contribution of the graph in Fig. 2 to the total cross section for the reaction $hd \to hpn$ can be equated to the integral of the expression (2) (after omitting in the latter the suppression factor $D(L)$) and in this way we can find the parameters β and B.

In the Appendix we have calculated the contribution of the three-gluon graph, which is shown in Fig. 2, and find $B \approx 13\,(\mathrm{GeV}/c)^2$, $\beta \approx 0.17$.

In Fig. 3 we have shown a calculation in accordance with Eq. (2) of the momentum spectrum of protons in the reaction $pd \to p_B\, pn$ emitted at 180°. In order to demonstrate the sensitivity of the results to the parameters, the calculations were carried out for values $\varkappa = 1$ and 2 GeV/fm and $\tilde{w} = 2$ and 3, where $\tilde{w} = wm^2/\varkappa^2$ is a dimensionless parameter which is convenient to introduce instead of w. In order to see the effect of the factor $D(L)$, we have plotted also the curve with $w = 0$. In the calculation we used the Hamada-Johnston wave function.[6] It is evident that for a momentum $p_B \approx 500$ MeV/c the spectrum has a maximum. To this we must add the contributions of other mechanisms: the spectator mechanism,[7] which is dominant in the region of small momenta, the isobar mechanism,[8] which is dominant at low energies of the incident hadron, and so forth.

Experimental data at high energies exist so far only for the inclusive process $pd \to p_B X$.[9] Since the process is diffractive, the excitation of the incident hadron, which does not influence the shape of the spectrum of protons emitted backwards, can be taken into account approximately by multiplying the expression (2) by the shower enhancement factor $C_h = 1 + \sigma^{hN}_{\mathrm{diff}}/\sigma^{hN}_{el}$. For nucleons $C_N \approx 1.4$ and for pions $C_\pi \approx 1.6$.

The main channel of decay of the color dipole is the formation of $q\bar{q}$ pairs from the vacuum, i.e., division of the color dipole into several dipoles of smaller mass. This circumstance leads to a significant decrease of the cumulative momentum. Therefore taking into account pion production should not greatly change the estimate of the cross section in the energetic part of the cumulative-proton momentum spectrum (estimates made with a classical approach confirm this). The results of the calculation are shown in Fig. 4. In the region of small momenta $p_B \lesssim 400$ MeV/c the main contribution is evidently from the Fermi motion of the nucleons in the deuteron. In Fig. 4 we have shown the contribution of the spectator mechanism.[7] Also in that figure we have given the experimental data[9] for the reaction $pd \to p_B X$ obtained at an initial momentum 8.9 GeV/c. From the comparison it can be seen that the mechanism proposed here describes in order of magnitude the experimental data at momenta $p_B \gtrsim 500$ MeV/c.

4. QUANTUM-MECHANICAL APPROACH

The main properties of the quantum-mechanical approach can be studied in the example of a one-dimensional nonrelativistic problem. In part 3 of this section the results are generalized to a more realistic case.

1. Scattering of two particles. Dibaryon resonances

We shall consider first a system of two particles, each of which can be found in two states—white and colored, which are denoted respectively as $\binom{0}{1}$ and $\binom{1}{0}$. The Hamiltonian of this system we shall write in the form

$$H = p_1^2/2m + p_2^2/2m + \Pi_1\Pi_2 V(x_1 - x_2) + \sigma_1\sigma_2 v(x_1 - x_2). \quad (4)$$

Here p_1 and p_2 are the particle-momentum operators $\Pi_i = \left(\begin{smallmatrix}1&0\\0&0\end{smallmatrix}\right)_i$ is the projection operator on the color state of ith particle, and $V(x)$ is the confinement potential for colored

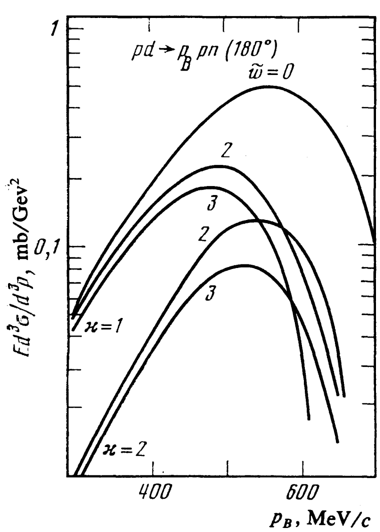

FIG. 3. Cross section for the reaction $pd \to p_B pn$ calculated for various values of the parameters $\mathscr{H}$ and $\tilde{w}$.

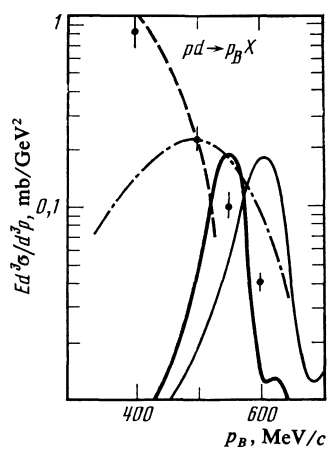

FIG. 4. Cross section for the reaction $pd \to p_B x$: the points are the data from Ref. 9 at $p_{\mathrm{lab}} = 8.9$ GeV/c; the dash-dot curve is the cross section for $pd \to p_B pn$ calculated from Eq. (2) with $\mathscr{H} = 1$ GeV/F and $\tilde{w} = 2$; the heavy solid line is the contribution (35) to the cross section at $\theta = 180°$; the thin solid line is for $\theta = 140°$; the dashed curve is the contribution of the spectator mechanism.

particles with relative distance x. Note that for a colored string we have $V(x) = \varkappa|x|$; $\sigma_i = \binom{0\,1}{1\,0}$ is an operator changing the color state of the particle. The last term in Eq. (4) corresponds to exchange of color between particles. For simplicity we shall take $v(x)$ in the form

$$v(x_1-x_2)=\alpha\delta(x_1-x_2). \tag{5}$$

The sum of the first three terms of expression (4) can be considered as a Hamiltonian H_0 acting in two orthogonal subspaces: *i*) a system of white noninteracting particles, and *ii*) a system of colored particles interacting with a potential $V(x_1-x_2)$. The last term in (4), which describes the color charge exchange, mixes these subspaces. We shall consider it as a perturbation.

For scattering of two white particles we shall write the T matrix in the form of a peturbation-theory series

$$T=vG_0v+vG_0vG_0vG_0v+\ldots, \tag{6}$$

which is shown graphically in Fig. 5. Here $G_0 \equiv G_0(\mathscr{E}+i0)$, where $\mathscr{E}$ is the total energy of the particle system and $G_0(z)=(z-H_0)^{-1}$ is the resolvent of the operator H_0; G_0 can be represented in the form of an orthogonal sum of resolvents acting in the two subspaces:

$$G_0=(1-\Pi_1\Pi_2)G_f+\Pi_1\Pi_2G_c. \tag{7}$$

Here

$$G_c(\mathscr{E})=\sum_n\int\frac{dP}{2\pi}\frac{|P,n\rangle\langle P,n|}{\mathscr{E}-P^2/2M-E_n}; \tag{8}$$

G_f is the resolvent of the free motion; $|P,n\rangle$ is the eigenstate of the Hamiltonian

$$H_0^{conf}=\frac{p_1^2}{2m}+\frac{p_2^2}{2m}+V(x_1-x_2)$$

with total momentum P and energy in the c.m.s. E_n; the total mass of the system is $M=2m$.

The matrix element of the operator $G_c(\mathscr{E})$ reduces to the form

$$\langle x_1', x_2'|G_c(\mathscr{E})|x_1, x_2\rangle=\int\frac{dP}{2\pi}e^{iP(X'-X)}g_c(E;x',x), \tag{9}$$

where $x=x_1-x_2$, $X=(x_1+x_2)/2$, $E=\mathscr{E}-P^2/2M$, and g_c is the resolvent of the Hamiltonian of the relative motion

$$g_c(E;x',x)=\sum_n\frac{\varphi_n(x')\varphi_n^*(x)}{E-E_n}. \tag{10}$$

The resolvent of the motion $g_f(E)$ has the form

$$g_f(E;x',x)=\frac{-i\mu}{(2\mu E)^{1/2}}\exp\left(i(2\mu E)^{1/2}|x'-x|\right), \tag{11}$$

where $\mu=m/2$ is the reduced mass. The series

$$vg_cv+vg_cvg_fvg_cv+\ldots \tag{12}$$

(which is illustrated in Fig. 5) is easily summed and the amplitude of the reflection is obtained in the form

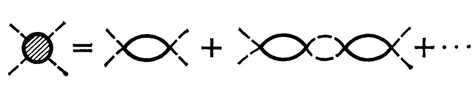

FIG. 5. Expansion of the amplitude for elastic scattering of two colorless particles in series of color charge exchanges.

$$A=\alpha^2\mu g_c/(ik-\alpha^2\mu g_c), \tag{13}$$

where k is the momentum of the particles in the c.m.s. and

$$g_c\equiv g_c\left(\frac{k^2}{2\mu}+i0;0,0\right)=\sum_n\frac{|\varphi_n(0)|^2}{k^2/2\mu-E_n+i0}.$$

We note that in investigation of the poles of expression (13) one can show that the system of two particles has one state with negative energy and a set of dibaryon resonances in the $N_c\,N_c$ system with energy $E=E_n-i\Gamma_n^{el}/2$, where

$$\Gamma_n^{el}\approx\alpha^2|\varphi_n(0)|^2m/k. \tag{14}$$

The resonance width Γ_n^{el} is due to the possibility of decolorization and decay to the channel NN.

In the case of a linear potential $V(x)=\varkappa|x|$ the wave functions of the resonances have the form

$$\varphi_n(x)=(\varepsilon/2a_n')^{1/2}\,\mathrm{Ai}\,(\varepsilon|x|-a_n')/\mathrm{Ai}\,(-a_n'), \tag{15}$$

where $\varepsilon=(2\mu\varkappa)^{1/3}$ $\mathrm{Ai}(y)$ is the Airy function, and $-a_n'$ are the zeros of the derivative of the Airy function, $\mathrm{Ai}'(-a_n')=0$. The energy spectrum of the resonances in this case has the form

$$E_n=a_n'(\varkappa^2/m)^{1/3}. \tag{16}$$

The possibility of production of $q\bar{q}$ pairs in the tube can be taken into account by introducing an imaginary part of the potential by means of the substitution $\varkappa\to\varkappa-iw/2$, where w is the probability density of production of $q\bar{q}$ pairs which was introduced in Section 2. Carrying out this substitution in Eq. (16), we find the combined width of multiparticle decays

$$\Gamma_n^{in}\approx2wE_n/3\varkappa. \tag{17}$$

When this is taken into account it is necessary in Eq. (10) to make the substitution $E_n\to E_n-i\Gamma_n^t/2$. This approximation is valid only for the condition $\Gamma_n^t \ll E_{n+1}-E_n$, which is not satisfied for large values of n. This condition means that the lifetime of the resonance must exceed the time of rotation in the classical orbit. However, for heavy states the probability of breaking of the string is so great that the concept of a resonance loses its meaning.

Nevertheless in the case of a linear potential we can obtain an exact expression for the propagation function

$$g_c(E;\ x,\ 0)=(\mu/\varepsilon)\,\mathrm{Ai}(\varepsilon|x|-\varepsilon E/\varkappa)/\mathrm{Ai}'(-\varepsilon E/\varkappa). \tag{18}$$

If at high energies we use the asymptotic behavior of the Airy function,[10] then this expression acquires the form (11), i.e., the propagator of the free motion. This result has a more general nature and can be understood in the following way. If we represent the propagation function of the particles in the form of a sum over their trajectories of motion, then in the region of a pole the contribution of the classical trajectories for which the Bohr condition is satisified is enhanced as the result of multiple crossing of these trajectories. The possibility of production of $q\bar{q}$ pairs from the vacuum, i.e., the removal into the other subspace, limits the time of motion of a particle along the trajectory. With increase of the energy E the length of the trajectories increases, and the particle is able to travel without breakup along the shortest of them.

Therefore, the influence of the potential can be neglected, i.e., the propagation function goes over into the free function.

2. Dual properties of the scattering amplitude

The amplitude (13) has the property of duality. At high energies, as we have already mentioned, the propagation function g_c can be replaced by the free function, which is the simplified model considered above corresponds to pomeron exchange. On the other hand the amplitude at low energies has a resonance behavior, which can be seen from the representation (10) for g_c.

It is interesting that the duality property exists also on the average. Actually the contribution of dibaryon resonances, averaged over an interval ΔE, to the imaginary part of the elastic NN scattering amplitude is

$$\overline{\operatorname{Im} A}=\left(\frac{dE_n}{dn}\right)^{-1}\int dE\,\frac{\Gamma_n^{el}\Gamma_n^{t}}{(E-E_n)^2+(\Gamma_n^{t}/2)^2}\,\frac{1}{2}\left(\frac{2E}{\mu}\right)^{1/2}$$

$$=\pi\alpha^2\left(\frac{dE_n}{dn}\right)^{-1}|\varphi_n(0)|^2. \tag{19}$$

If we use the asymptotic form of the amplitude, i.e., if we replace the propagator g_c by the free propagator, then we obtain

$$\operatorname{Im} A=\alpha^2(\mu/2E)^{1/2}. \tag{20}$$

Equations (19) and (20) coincide if one has the relation

$$\varphi_n^2(0)\pi\left(\frac{dE_n}{dn}\right)^{-1}\left(\frac{2E_n}{\mu}\right)^{1/2}\approx 1. \tag{21}$$

This relation is satisfied approximately for various confinement potentials and has a ready physical interpretation. By means of the quasiclassical relations

$$dE_n/dn=2\pi/T_{\rm cl},\quad (2E_n/\mu)^{1/2}=v_{\rm cl}(0),$$

where $v_{\rm cl}(x)$ and $T_{\rm cl}$ are the velocity and period of rotation in the classical trajectory, we can rewrite (21) in the form

$$|\varphi_n(0)|^2\,dx=\frac{dx/v_{\rm cl}(0)}{T_{\rm cl}/2}.$$

This relation signifies equality of the quantum-mechanical and classical probabilities of finding a particle in the interval dx.

One can hope that the Pomeron-dibaryon resonance duality is preserved also in a more realisitc approach. We note that in meson-nucleon scattering the pomeron corresponds to 5-quark resonances with separated color.

3. Scattering of three particles

The Hamiltonian (4) is easily generalized to the case of several particles:

$$H=\sum_i\frac{p_i^2}{2m}+\sum_{i<j}\Pi_i\Pi_jV(x_i-x_j)+\sum_{i<j}\sigma_i\sigma_jv(x_i-x_j). \tag{22}$$

We shall discuss the amplitude of scattering of three white particles $1+2+3\to 1'+2'+3'$. The Hamiltonian H_0 which consists of the first two terms of (22) does not mix the orthogonal subspaces of the states, in which all particles are white, and in which particle 3 is white and free while particles 1 and 2 are colored and interact with the potential $V(x_1-x_2)$. We shall again assume that the complete solution of the corresponding two-particle problem is known, and the last term in (22) will be regarded as a perturbation.

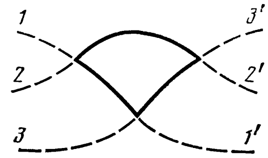

FIG. 6. Amplitude of elastic scattering of three colorless particles with participation of all particles in lowest order in the coupling constant.

In the lowest order in α the amplitude for scattering with participation of all three particles is illustrated in Fig. 6. This is the analog of the diagram of Fig. 2 which describes the cumulative process.

We need the three-particle resolvent which acts in the subspace where particles 1 and 2 are colored and 3 is white. The matrix elements of the resolvent are simply expressed in terms of g_c, which was determined in (10):

$$\langle x_1', x_2', x_3'|G_0^{(12)}(E+i0)|x_1, x_2, x_3\rangle$$

$$=\int\frac{dp_3}{2\pi}\frac{dP_{12}}{2\pi}\exp[ip_3(x_3'-x_3)]$$

$$\times\exp[iP_{12}(X_{12}'-X_{12})]g_c(E_{12}+i0; x_{12}', x_{12}),$$

where

$$X_{12}=(x_1+x_2)/2,\quad x_{12}=x_1-x_2,$$

$$E_{12}=E-p_3^2/2m-P_{12}^2/2M\quad (M=2m)$$

is the energy of relative motion of the system (1–2) (here p_1, p_2, and p_3 are the particle momenta and $P_{12}=p_1+p_2$). The amplitude corresponding to Fig. 6 is written as

$$A\equiv\langle p_1', p_2', p_3'|T|p_1, p_2, p_3\rangle$$

$$=\langle p_1', p_2', p_3'|v^{(12)}G_0^{(12)}(E+i0)v^{(13)}G_0^{(23)}$$

$$\times(E+i0)v^{(23)}|p_1, p_2, p_3\rangle,$$

where

$$v^{(12)}\equiv v(x_1-x_2)=\alpha\delta(x_1-x_2);\quad 2mE=\sum_{i=1}^{3}p_i^2=\sum_{i=1}^{3}p_i'^2.$$

Going over to the coordinate representation, after simple calculations we obtain

$$A=\alpha^3\int dx\, e^{ix(p_3-p_1')/2}g_c(E_{23}'; 0, x)g_c(E_{12}; x, 0).$$

From this we obtain the scattering amplitude in the case in which particle 1 is scattered by a stationary "deuteron"—the bound state of particles 2 and 3:

$$A_d\equiv\langle p_1', p_2', p_3'|T|p_1, d\rangle$$

$$=\alpha^3\int\frac{dq}{2\pi}dx\,\bar{\psi}_d(q)e^{ix(q-p_1')/2}g_c(E_{23}'; 0, x)g_c(E_{12}; x, 0), \tag{23}$$

where $p_3=q$, $p_2=-q$, and $\bar{\psi}_d(q)$ is the wave function of the "deuteron" in the momentum representation. We shall make several approximations in (23). We shall assume that the incident particle is very fast: $p_1\gg q$, p_2', p_3', and that here $p_1+p_1'\ll q$, p_2', p_3'. Then, neglecting this difference, we have $p_3'=-p_2'\equiv -p_B$. The last factor in (23) can be replaced by the free propagation function, which we find from Eq. (18) using the asymptotic behavior of the Airy function:

$$g_c(E_{12}+i0;x,0)\to\frac{-im}{p_1}\exp\left\{\frac{i}{2}(p_1+q)|x|\right\}\times\exp\left(-\frac{1}{4}wx^2\frac{\mu}{p_1}\right). \tag{24}$$

The appearance of the last factor is due to the inclusion of the imaginary part of $\varkappa$ in Eq. (18) and corresponds to the factor $D(L)$ which takes into account the possibility of breaking of the string in Eq. (2).

Then from Eq. (23) we obtain

$$A_d=-\frac{im\alpha^3}{p_1}\int_0^\infty dx\,\psi_d(x)g_c(E_{23}';0,x)\exp\left(-\frac{w\mu}{4p_1}x^2\right), \tag{25}$$

where $E_{23}'=p_B^2/m$. The physical meaning of Eq. (25) is clear: the potential V accelerates and brings closer together particles 2 and 3 from an initial relative distance x down to zero, and in so doing gives them momenta $\pm p_B$.

If we substitute the expansion (10) into Eq. (25), then the overlap integral of the wave functions of the resonances and the deuteron appears. However, this approach does not mean that states with spatially separated color are present with appreciable weight in the deuteron. The color is carried by the incident hadron, and the amplitude for formation of a color dipole with dimension x is proportional to the deuteron wave function $\psi_d(x)$.

The main difference of Eq. (5) from the classical expression (2) is that in (25) there is no unique correspondence between the value of the cumulative momentum p_B (i.e., the energy E_{23}') and the distance x which has been "prepared" in the deuteron.

We shall investigate the dependence on p_B of the principal longitudinal distances x in Eq. (25). If the momentum p_B is sufficiently large, it is possible to use the quasiclassical approximation:

$$g_c(E_{23}';0,x)=\frac{-im}{2[q(0)q(x)]^{1/2}}\times\exp\left\{i\int_0^x q(y)dy-\frac{1}{2}\int_0^T W[y(t)]dt\right\}, \tag{26}$$

where

$$q(x)=[p_B^2-mV(x)]^{1/2}. \tag{27}$$

The second term in the argument of the exponential takes into account the possibility of breaking of the string and appears with use of a complex potential $V(x)-(i/2)W(x)$. The function $y(t)$ is a solution of the classical equation of motion

$$dy/dt=[2\mu(E_{23}'-V(y))]^{1/2}/\mu$$

with the boundary condition $y(0)=x$. The integral over time $T=T(x)$ is determined by the requirement $y(T)=0$. For a string with a potential $(\varkappa-iw/2)|x|$ we have

$$\frac{1}{2}\int_0^T W[y(t)]dt=\frac{w}{3}\left(\frac{m}{\varkappa}\right)^{1/2}x^{3/2}.$$

We shall evaluate the integral (25) by the stationary-phase method:

$$A_d\approx\left(\frac{-im}{p_1}\right)\frac{-im}{2[q(0)q(x)]^{1/2}}\times\left(-\frac{1}{2\pi}\frac{\partial q(x)}{\partial x}\right)^{-1/2}\psi_d(x)D^{1/2}(x)\big|_{x=L(p_B)}, \tag{28}$$

where the value of $x=L(p_B)$ determined by the stationary-phase condition is

$$q(x,p_B)|_{x=L(p_B)}=0,$$

and is the classical distance corresponding to Eq. (1).

The function $D(x)$ in (28) includes the exponential factors from (25) and (26) which take into account breakings of the string; $D(x)$ coincides with the classical expression (3) for the nonrelativistic case. From this we have

$$q(x)\frac{\partial q}{\partial x}\Big|_{x=L}=q(x)\frac{\partial q}{\partial p_B}\Big|_{x=L}\left(\frac{dL}{dp_B}\right)^{-1}=q(0)\left(\frac{dL}{dp_B}\right)^{-1}.$$

Substituting this expression into (28), we obtain finally

$$A_d\approx-\frac{m^2}{p_1p_B}\left(\frac{\pi}{2}\right)^{1/2}\psi_d(L)\left(\frac{dL}{dp_B}\right)^{1/2}D^{1/2}(L). \tag{29}$$

This evaluation of the expression (25) corresponds to Eq. (2).

Thus, in the quasiclassical approximation the principal longitudinal distances in (25) increase with increase of p_B. However, from Eqs. (25) and (27) it is evident that this increase is limited as the result of the falloff of the wave function $\psi_d(x)$ and the factor $D^{1/2}(x)$ at large x. For real values of the parameters the integral (25) is cut off by the factor $D^{1/2}(x)$ at distances $x\approx\varkappa/wm\approx 1$–2 fm, which corresponds approximately to $p_B\approx 0.5$ GeV/c. At larger values of p_B the stationary-phase approximation no longer works. With further increase of p_B the meaning of Eq. (25) can be interpreted as follows. The energy E_{23}' is made up of the work of the color forces $\sim\varkappa^2/wm$ and the nucleon kinetic energy which has been "prepared" in the deuteron. However, if the latter becomes too large, the two-nucleon interpretation of the deuteron wave function loses its meaning and it is necessary to take into account the quark structure of the nucleons.

Note that in the spectator mechanism[7] the entire cumulative momentum must be prepared beforehand in the deuteron. Therefore the region of applicability of the present mechanism extends to much larger energies E_{23}' than that of the spectator mechanism.

4. Generalization to a realistic case

The problem considered above contains all of the main features of the quantum-mechanical approach. However, it was solved in the one-dimensional case, in the nonrelativistic approximation, with a δ-shaped exchange potential $v(r)$ with an extremely simplified color structure. We shall generalize the problem to a more realistic case.

We shall assume that the color state has eight components (a color octet). Here the generalized potential in the Hamiltonian (4) has the form $\delta_{ab}v(r)/8^{1/2}$, where $a,b=1,\dots,8$.

The wave functions (15) of dibaryon resonances in the s state are modified as follows:

$$\varphi_n^{ab}(r)=\frac{1}{\sqrt{8}}\delta^{ab}\varphi_n(r)=\frac{\delta^{ab}}{8^{1/2}}\left(\frac{\varepsilon}{4\pi}\right)^{1/2}\frac{\mathrm{Ai}(\varepsilon r-a_n)}{r\,\mathrm{Ai}'(-a_n)}, \tag{30}$$

where $a_n = 2.3, 4.1, 5.5 \ldots$ are the zeros of the Airy function: $\mathrm{Ai}(-a_n) = 0$.

We note that instead of the potential $V(r) = \varkappa r$ it is possible to use a more realistic potential

$$V(r)=\begin{cases}\infty & \text{for} \quad r<R_0\\ \varkappa(r-R_0) & \text{for} \quad r>R_0.\end{cases}$$

Here we have taken into account the repulsive nucleon core with $R_0 \approx 0.5$ fm. The corresponding modification of the wave function (30) is very simple: it is sufficient in the argument of the Airy function to make the substitution $r \to r - R_0$, assuming $\varphi_n = 0$ at $r < R_0$. The masses of the corresponding resonances are

$$M_n \approx 2m + a_n(\varkappa^2/m)^{1/3}. \tag{31}$$

It can be seen that even the first dibaryon resonance with separated color has a large mass of about 3 GeV/c^2. The width of the decay into the multiparticle channels (which is close to the total width) is given as before by Eq. (17) and for the first resonance amounts to about 200 MeV. We note that these mass and width values are preliminary since the values of $\varkappa$ and w are poorly known.

The expression (14) for the decay width of the resonance into two nucleons is replaced by

$$\Gamma_n^{el} = \frac{mQ_n}{2\pi}\left|\int d^3r v(r)\varphi_n(r)e^{i\mathbf{Q}_n\mathbf{r}}\right|^2, \tag{32}$$

where $Q_n = (M_n^2 - 4m^2)^{1/2}/2$ is the nucleon momentum in the c.m.s. Let us estimate the value of Γ_n^{el}. We shall take the potential $v(r)$ in the form

$$v(r) = v(0)\exp(-r^2/4B). \tag{33}$$

The parameters $v(0)$ and B can be found by calculating the NN scattering cross section. The parameter B turns out to be equal to the slope parameter of the NN elastic scattering cross section $B \approx 10\ (\mathrm{GeV}/c)^{-2}$, and

$$v(0) = (2\sigma_{in})^{1/2}/4\pi B \approx 0{,}1\ \mathrm{GeV}. \tag{34}$$

Here we have taken into account that the color exchange is not scalar, but vector, which provides a total cross section which does not depend on the energy. Then from (32) we find for the first resonance $\Gamma_1^{el} \approx 10$ MeV. It is evident that $\Gamma^{el} \ll \Gamma^{in}$. We note that Γ_n^{el} is calculated much less reliably than Γ_n^{in}, since the decay into $2N$ occurs as the result of color charge exchange inside the resonance, i.e., it is important to take into account the quark structure of the nucleon. In addition, as can be seen from Eq. (32) Γ^{el} depends exponentially on $\varkappa$ and with increase of $\varkappa$ by a factor of two Γ^{el} increases by almost an order of magnitude.

It must also be emphasized that the parameters of the potential $v(r)$ depend on the energy. The estimate (34) applies to high energies, and at energies of the order of the masses of the lower dibaryon resonances $v(r)$ can differ significantly from (34). This corresponds to the fact that in the color charge-exchange amplitude it is necessary to take into account quark exchanges in addition to gluon exchange. At low energies these corrections can increase $v^2(0)$ by a factor of the order of 2.

The cross section for the reaction $hd \to p_B nh$ takes the form (compare with Eq. (25))

$$E_B\frac{d^3\sigma}{d^3p_B} = \frac{2(\sigma_{in}^{hN})^2B}{\pi Q} \times\left|\sum_n \frac{(\Gamma_n^{el})^{1/2}}{M-M_n+i\Gamma_n^t/2}\int_0^\infty dz\psi_d(z)\varphi_n(z)D^{1/2}(z)\right|^2. \tag{35}$$

Here $Q = m(\alpha - 1)/[\alpha(2-\alpha)]^{1/2}$ is the relative momentum of the nucleon pair in the c.m.s.; $M = 2m/[\alpha(2-\alpha)]^{1/2}$ is the effective mass of the pair; $\alpha = (E_B + p_B^L)/m$ is the light-front variable. Here we have introduced the combinatorial factor 4 which takes into account permutation of the nucleons.

Figure 4 shows the results of calculations carried out according to Eq. (35) with inclusion of the shower enhancement coefficient for proton emission angles 180 and 140°. The value of Γ_n^{el} which determines the value of the cross section (35), as we mentioned above, was calculated very unreliably. In the calculation we set $\Gamma_1^{el} = 30$ MeV. It can be seen that the curve for 180° is in good agreement with the result of the classical approach. From comparison of the curves for different angles it is evident that the angular dependence of the cross section changes considerably with change of p_B. Generally speaking, the differential cross section has the following scaling behavior: for a fixed value of α the cross section does not depend on the angle. It is easy to verify that this property is present also in the spectator mechanism for scattering by a deuteron, and consequently, the relative contribution of the two mechanisms does not depend on the scattering angle. We note also that since the masses and widths of the dibaryon resonances have been calculated approximately, the real position of the peaks in the momentum spectrum can differ from those in Fig. 4.

The general normalization of the cross section also was determined with a large uncertainty as the result of poor knowledge of the parameters of the theory. Nevertheless from Fig. 4 we can conclude that the results of the calculation agree in order of magnitude with the experimental data for $p_B \gtrsim 550$ MeV/c.

As we have already mentioned, the contribution of the present mechanism to the cross section for the cumulative process at high energies does not depend on the energy. However, at intermediate energies of a few GeV there is a specific dependence on energy. Indeed, the expression (23) for the amplitude of the process involves $g_c(E_{12})$—the propagation function of the system of the incident hadron and the target nucleon after their charge exchange. At high initial energies we replaced it by the free propagation function (26). However, at intermediate energies $g_c(E_{12})$, as can be seen from Eq. (10), has a resonance dependence on $E_{12} = (2mT_{\mathrm{kin}} + 4m^2)^{1/2}$, where T_{kin} is the kinetic energy of the incident hadron. If we assume that the mass of the first resonance is 3 GeV/c^2, then in the energy dependence of the cross section for a fixed value of p_B the first maximum should be observed at $T_{\mathrm{kin}} \approx 2.6$ GeV. In fact, just beginning with these energies, the mechanism under discussion gives a substantial contribution to the cumulative production of protons. We note also that in the case in which the incident hadron is a pion the resonance functions $g_c(E_{12})$ are not the dibaryon type, but

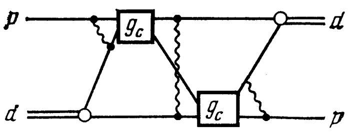

FIG. 7. Diagram describing the process of elastic pd scattering backward.

rather pion-nucleon five-quark resonances with separated color. The spectrum of excitations of these resonances is close to that for the dibaryon spectrum.

5. ELASTIC *pd* BACKSCATTERING

In the pion-free process $pd \to p_B pn$ at intermediate energies the proton and neutron which are emitted forward can have momenta comparable in magnitude and can form a bound state—a deuteron. Thus, the color-force mechanism contributes also to elastic pd backscattering. The corresponding diagram is shown in Fig. 7. The amplitude corresponding to this diagram has the form

$$A^{pd\to dp} = \int \bar{\psi}_d(q')v(x')g_c(E_{23}';x',z)v(x-y)g_c(E_{12};y,x)v(x)\bar{\psi}_d(q) \times \exp[ix'q_{32}'-iz(^1/_2P_{32}'-p_3)+iy(^1/_2P_{12}-p_1')+ixq_{12}]do, \quad (36)$$

where

$$q_{12}=(p_1-p_2)/2, \quad q_{32}'=(p_3'-p_2')/2;$$

$$P_{12}=p_1+p_2, \quad P_{32}'=p_3'+p_2';$$

$$p_3=p_d/2+q, \quad p_2=p_d/2-q, \quad p_1'=p_{d'}/2+q', \quad p_2'=p_{d}'/2-q';$$

$$do=d^3xd^3x'd^3zd^3yd^3qd^3q'.$$

If the propagation function g_c is represented in the form of a sum over resonances and if we neglect the momentum distribution of the nucleons in the deuteron, we can obtain the following expression for the cross section:

$$\frac{d\sigma}{d\Omega_{\text{c.m.s.}}}=\frac{25|\psi_d(0)|^4}{18Q^2} \times \left|\sum_{n,n'}\frac{(\Gamma_n^{el}\Gamma_{n'}^{el})^{1/2}F_{nn'}(p,p')}{(M-M_n+i\Gamma_n^t/2)(M-M_{n'}+i\Gamma_{n'}^t/2)}\right|^2, \quad (37)$$

where

$$F_{nn'}(p,p')=\int d^3rd^3r'\varphi_n(r)v(\mathbf{r}-\mathbf{r}')\varphi_{n'}(r') \times\exp[i(\mathbf{p}'+\mathbf{p}/2)\mathbf{r}/2-i(\mathbf{p}+\mathbf{p}'/2)\mathbf{r}'/2]. \quad (38)$$

Here $M^2 = 4m^2 + 2mT_{\text{kin}}$ and $Q^2 = mT_{\text{kin}}/2$, where T_{kin} is the kinetic energy of the incident proton in the laboratory system; p and p' are the initial and final momenta of the proton in the c.m.s. The combinatorial factor 25 takes into account permuation of the nucleons.

Equation (37) is readily interpreted (see Fig. 7). In collision of the incident proton with a target nucleon a dibaryon resonance is formed with a probability proprotional to Γ_n^{el}. Then the resonance is scattered backward by the second target nucleon by exchange of a colored nucleon. The amplitude of the latter process is described by Eq. (38). It is interesting that this scattering process occurs at a c.m.s. momentum equal to $\sim p/2$.

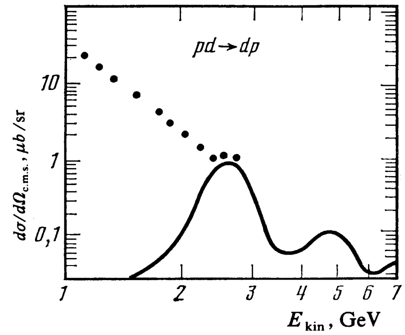

FIG. 8. Cross section for the reaction $pd \to dp$ at $\theta = 180°$: the points are data from Ref. 11, and the curve is a calculation on the basis of Eq. (37).

We shall evaluate the expression (38) for scattering angles close to 180° for $n = n' = 1$, using in the expression (30) for the Airy function the approximation

$$\text{Ai}\,(x-a_1)\approx 0.70x\exp(-0.29x^2),$$

where $a_1 = 2.34$ is the first zero of the function Ai$(-x)$.

From (38) we find

$$F_{11}(p,p')\approx v(0)(4\pi/\lambda)^{3/2}\exp[-p^2/16(\lambda-1/B)-9p_T'^2/64\lambda], \quad (39)$$

where $\lambda = 2\varepsilon\cdot 0.29 + 1/B$ (see Eqs. (15) and (33)).

For evaluation of the elastic pd-scattering cross section at 180° we shall neglect in the sum over n and n' in (37) the nondiagonal term and the dependence of $F_{nn'}$ on n. The results of the calculation with the parameters specified above are compared in Fig. 8 with the experimental data of Ref. 11. It should be noted that the normalization of the cross section has an additional dependence proportional to $\Gamma_{el}v^2(0)$ in comparison with the cross section for the reaction $pd \to p_B X$. For the values of these parameters which were fixed in the preceding section the cross section for the reaction $pd \to dp$ turns out to be underestimated by almost an order of magnitude. However, the values of these parameters, as we have already explained, are known with a very large uncertainty, within which it is possible to change the normalization of the calculation, as was done in Fig. 8.

As can be seen from Fig. 8, the experimentally observed change in the energy behavior of the curve at $\Gamma_{\text{kin}} \approx 2.5$ GeV can be due to the contribution of a dibaryon resonance with separated color with mass about 3 GeV/c^2. The contribution of this mechanism at lower energies, as can be seen from Fig. 8, is negligible. It is very important to obtain experimental data at higher energies.

6. POLARIZATION EFFECTS

Effects associated with the polarization of the incident particle in the reaction $hd \to p_B X$ at high energies are small in the present mechanism and fall off as a power of the energy. However, the polarization of the nucleons emitted backward does not approach zero with increase of the initial energy and in principle can be large. Nevertheless in the region of dominance of the first dibaryon resonances ($p_B \gtrsim 550$ MeV/c) the polarization of the cumulative nucleons is equal to zero if we can neglect the interference of the different resonances and also of the background. If the momentum is close to the kinematic limit, a source of polarization of cumulative protons is the interference of gluon and quark exchanges in the process of color charge exchange, which occurs at a finite energy. Therefore the polarization will depend on p_B but not on the incident energy. It is interesting that in this case also one can expect that the polarization is close to zero. In fact, in elastic NN scattering the polarization is due to interference of the imaginary pomeron amplitude without spin flip $\mathrm{Im} f^{P}_{++}$ with the real part of the contribution of leading Reggeons to the amplitude with spin flip $\mathrm{Re} f^{R}_{+-}$. The exchange degeneracy of the pairs of Reggeons $f-\omega$ and $\rho - A_2$ brings about a compensation of their contributions to $\mathrm{Im} f^R$ and an addition in $\mathrm{Re} f^R$, which leads to a large polarization. In the reaction considered here with color exchange $N_c\, N_c \to NN$ the polarization is due to interference of the real amplitude of gluon exchange without spin flip $\mathrm{Re} f^{G}_{++}$ with the imaginary part of the Reggeon amplitude with spin flip $\mathrm{Im} f^{R_c}_{+-}$, which is equal to zero if the colored Reggeons have exchange degeneracy. However, if the exchange degeneracy of the colored Reggeons is strongly broken, then in the protons emitted backward there can be a polarization at the level of several percent.[5] We note that the value of this polarization is equal to the azimuthal asymmetry of proton emission in the case in which the deuterons are polarized.

7. DISCUSSION

A high-energy hadron interacting with a deuteron can transfer color from one nucleon to the other, converting the deuteron into a color dipole. The dipole, breaking up into colorless objects, can emit a nucleon into the backward hemisphere. This mechanism exploits directly the popular model of color strings, which reflects the space-time structure of hadron-hadron interactions in QCD. The study of hadron-nucleus interactions gives a unique possibility of verifying these ideas.

Note that the formation and decay of a color dipole kinematically recalls the mechanism considered in Ref. 8 with formation of a resonance in the intermediate state which, interacting with the second nucleon by the channel $N^*N \to NN$, as a result of the excess of mass forms a nucleon emitted into the backward hemisphere in the lab. At low momenta of the incident proton $p_{\rm lab} \approx 1.5$ GeV/c a large contribution to the cross section comes from formation of the Δ_{33} isobar,[8,12] which falls off with energy according to a power law. The contribution of diffraction excitations in the intermediate state does not depend on the initial energy. However, the cross section for diffraction dissociation summed over all final states is suppressed by about an order of magnitude in comparison with σ_{in}. This smallness enters into the cross section for the $pd \to p_B\, pn$ reaction quadratically, which at high energies makes the contribution of white intermediate states negligible in comparison with colored states.

Another mechanism whose contribution does not die out with increase of energy—the spectator mechanism[7]—has been mentioned above. This mechanism is dominant in the cross section for the reaction $pd \to p_B X$ in the soft part of the momentum spectrum $p_B \lesssim 500$ MeV/c. At high momenta p_B the calculations lose their meaning,[7] since at small internucleon distances it is impossible to use the two-nucleon wave function of the deuteron.

For the same reason there is a limitation of the region of applicability of the calculations described above, but this is much broader than for the spectator mechanism. A color dipole of large mass can be obtained not only by increase of the internal momentum which has been prepared in the deuteron, as in the spectator mechanism, but also as the result of the energy stored in the color tube.

The comparison made above between the calculations and the existing experimental data has permitted us only to conclude that the contribution of the mechanism considered corresponds in order of magnitude to the data on the cross section for the reaction $pd \to p_B X$ at $p_B \gtrsim 550$ MeV/c and the cross section for elastic pd scattering backwards at $T_{\rm kin} \gtrsim 2.5$ GeV. A significant fraction of the existing data on the cumulative effect has been obtained in nuclei with $A > 2$. Cascade multiplication of hadrons inside nuclei and the Fermi motion of the nucleon pair as a whole make it impossible to study any detailed effects in complex nuclei. Therefore it is necessary to recommend that the experimental study of the various processes be carried out in a deuterium target. Below we have given some examples of such processes.

1. One expects a change in the nature of the process $hd \to p_B X$ with increase of the momentum p_B. At $p_B \lesssim 500$ MeV/c the spectator mechanism is apparently dominant, and this is characterized by a high average multiplicity $\langle n \rangle_{hd} \approx \langle n \rangle_{hN}$ and a large loss of momentum by the leading particle $\langle x_F \rangle \approx 0.5$, where x_F is the Feynman variable.

If at $p_B \gtrsim 500$ MeV/c the contribution from decay of the color dipole is dominant, then since the process is diffractive the leading hadron should have the quantum numbers of the incident particle and a value of x_F in the diffraction region. Correspondingly the average multiplicity of the particles produced in this process is small.

From this description we can see if we separate the diffraction contribution in $hd \to p_B X$, considering for example the reaction $hd \to p_B\, hn$, then the background from the spectator mechanism will be suppressed.

2. The observation of peaks in the momentum spectrum of cumulative protons would be a serious argument not only in favor of the mechanism considered but also in favor of the existence of heavy dibaryon resonances with separated color. These resonances are analogous to giant resonances in nuclei, which are collective excitations of nucleons. It must be emphasized that the calculation of the spectrum of dibaryon resonances carried out above has in part an illustrative nature, since it did not take into account the quark structure of the nucleons, it used a linear confinement potential, and so forth. For example, allowance for the nucleon size in the potential $V(r)$ can change the mass of the dibaryon resonance by several hundred MeV. The values of the parameters $\varkappa$ and w also strongly influence the masses and widths of the resonances.

We note that the search for such dibaryon resonances in NN scattering is made difficult by the smallness of their production cross section. In fact, the contribution to the cross section of a dibaryon resonance $E=E_n$ is $(4\pi/k^2)\Gamma_n^{el}/\Gamma_n^{tot}$, which amounts to about 1% of δ_{NN}^{tot}.

Thus, the search for heavy dibaryon resonances in events with emission of energetic nucleons with $p_B \gtrsim 500$ MeV/c into the backward hemisphere appears optimal with respect to the signal-to-background ratio. In fact, at a high initial energy it is hard to find a mechanism capable of competing with the contribution of resonances (the spectator contribution can be suppressed by separation of diffraction).

As a target it is possible to use also light nuclei if in cumulative events one studies the distribution in the effective mass of a pair of protons, one of which is emitted into the backward hemisphere.

3. It is evident from Fig. 8 that it is important to obtain data on the cross section for elastic pd scattering backward at $T_{\rm kin} \gtrsim 3$ GeV. We note also that the mechanism considered contributes to the cross section for elastic πd scattering backward. In this case in the first and second nucleons of the deuteron there are excited not dibaryon resonances (see Fig. 7) but five-quark resonances with separated color of the type $\pi_c\ N_c$. Here the cross section for πd scattering backward is suppressed in comparison with the pd cross section by the combinatorial factor 4/25 which takes into account permutation of the nucleons and the smaller number of quarks in the pion.

4. It is important to note that the mechanism considered does not contribute to the production of cumulative nucleons in the deep inelastic scattering of leptons by deuterons. Therefore we should expect that

$$\sigma^{ld\to p_B X}/\sigma^{hd\to p_B X} \ll \sigma_{in}^{lN}/\sigma_{in}^{hN}.$$

In the case of dominance of the spectator mechanism the equality sign should appear in this relation. This statement does not extend to heavier nuclei in which the quark emitted by the lepton can, like the hadron, convert the pair of nucleons into a color dipole.

In conclusion we note that in dibaryon resonances there is an additional decay channel not considered above. The octet string can not only be broken transversely as the result of formation of quark-antiquark pairs from the vacuum. Generally speaking, it can split up longitudinally, into two triplet strings. This channel increases Γ_n^{in}, but it is not possible to estimate its contribution. We note that the limit $\Gamma_n^{in}/E_n \to \infty$ corresponds to the model of dual topological unitarization, where for simplicity is assumed that instead of the octet string two triplet strings always appear.

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APPENDIX

To simplify the notation we shall discuss a problem in which all three interacting particles A, B, and C are mesons consisting of quarks and antiquarks designated as $q_1\bar{q}_2$, $q_3\bar{q}_4$, and $q_5\bar{q}_6$. Particles B and C form a bound state on which particle A is scattered. We shall use the eikonal approximation in quantum chromodynamics, which is valid at large relative energies. In this approximation the scattering amplitude can be written in the form[13]

$$\begin{aligned} A(k_A, k_B, k_C) = 2s \int d^2x_1 \ldots d^2x_6 \psi^A_{i_1i_2}(x_1-x_2)\,[\psi^A_{j_2j_1}(x_1-x_2)]^* \\ \times \exp(-ik_A X_{12})\,\psi^B_{i_3i_4}(x_3-x_4)\,[\psi^B_{j_4j_3}(x_3-x_4)]^* \exp(-ik_B X_{34}) \\ \times \psi^C_{i_5i_6}(x_5-x_6)\,[\psi_{j_6j_5}(x_5-x_6)]^* \exp(-ik_C X_{56})\,\psi_d(X_{34}-X_{56}) \\ \times \exp\Big[-\frac{ig^2}{4}\sum_{a=1}^{8}\sum_{\alpha,\beta=1}^{6}{}' \, t^a(\alpha)\,t^a(\beta)\,V(x_\alpha-x_\beta)\Big]. \end{aligned} \quad \text{(A.1)}$$

Here x_α is the transverse coordinate of the αth quark (antiquark); k_A, k_B, and k_C are the transverse components of the momenta of particles A, B, and C after the interaction $\psi^A_{i_1i_2}(x_1-x_2)$ is the wave function of hadron A with quark color indices i_1 and i_2; $X_{\alpha\beta} = (x_\alpha + x_\beta/2$; $\psi_d(X_{34}-X_{56})$ is the wave function of the "deuteron"; the summation Σ' in the argument of the exponential is carried over all quark pairs belonging to different hadrons; the matrix is defined as

$$t^a(\alpha) = \begin{cases} \lambda^a(\alpha) & \text{for quarks } \alpha = 1, 3, 5 \\ -[\lambda^a(\alpha)]^T & \text{for antiquarks } \alpha = 2, 4, 6 \end{cases},$$

and $\lambda^a(\alpha)$ are the Gell-Mann matrices acting on the quark with number α; g is the QCD coupling constant, and

$$V(x) = \int \frac{d^2q}{(2\pi)^2} e^{iqx} \frac{1}{q^2}.$$

In Eq. (A.1) we understand also an implicit dependence on the longitudinal momenta of the quarks in the infinite-momentum frame. The eikonal approximation is valid if the distribution in the longitudinal momenta of the quarks in the hadron does not change during the interaction time. Since the hadrons are colorless, we have

$$\psi_{i_1i_2}(x_1-x_2) = (1/\sqrt{3})\,\delta_{i_1i_2}\psi^A(x_1-x_2).$$

The process illustrated in Fig. 2 corresponds to terms of third order in g^2 in the expansion of the exponential in Eq. (A.1). After summation over color indices and integration, the amplitude of the process reduces to the form

$$\begin{aligned} A(k_A, k_B, k_C) \\ = 16s(g^4/27) \int \frac{d^2p}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Phi(q_1, q_2)\,\Phi(-q_1, q_3) \\ \times \Phi(-q_2, q_3)\, q_1^{-2} q_2^{-2} q_3^{-2} \psi_d(p). \end{aligned} \quad \text{(A.2)}$$

Here we have used the notation

$$q_1 = q; \quad q_2 = k_A - q; \quad q_3 = k_B - q - p;$$
$$\Phi(k, q) = f(k+q) - f(k-q),$$

where

$$f(k) = \int d^2x \int_0^1 \frac{d\alpha}{4\pi\alpha(1-\alpha)} |\psi(x,\alpha)|^2 e^{ikx} \quad \text{(A.3)}$$

is the single-quark form factor of the hadron. The factor $[4\pi\alpha(1-\alpha)]^{-1}$ arises in the conversion form the system of reference in which the deuteron is at rest to the infinite-momentum system, in which the fraction of the total momentum carried by the quark is α. In Eq. (2) we have also used the notation

$$\psi_d(p) \equiv \psi_d(p, \alpha_N) = \int d^2x\, \psi_d(x, \alpha_N)\, e^{-ipx}.$$

In the case of baryon scattering the right-hand side of Eq.

(A.2) must be multiplied by $(3/2)^3$.

The dependence of the integrand of Eq. (A.2) on p is determined mainly by the function $\psi_d(p)$, which falls off rapidly with increase of p. Therefore all remaining factors in (A.2) can be determined for $p=0$. Then, after integration over the longitudinal component of p, we obtain the differential cross section in the form

$$\frac{d\sigma}{d^2k_A\,d^2k_B}=256\pi^2\alpha_s^6|A_0(k_A,k_B)|^2 \times\int\frac{d\alpha_N}{4\pi\alpha_N(1-\alpha_N)}|\psi_d(x=0,\alpha_N)|^2. \quad (A.4)$$

Here

$$A_0(k_A,k_B)=\int\frac{d^2q}{(2\pi)^2}\Phi(q_1,q_2)\Phi(-q_1,q_3) \times\Phi(-q_2,-q_3)q_1^{-2}q_2^{-2}q_3^{-2}. \quad (A.5)$$

The integral over α_N in (A.4) is

$$\int\frac{d\alpha_N}{4\pi\alpha_N(1-\alpha_N)}|\psi_d(x=0,\alpha_N)|^2=\int dL|\psi_d(x=0,L)|^2. \quad (A.6)$$

In the case $k_A=0$ Eq. (A.4) can be written as follows:

$$\left.\frac{d\sigma}{d^2k_A\,d^2k_B}\right|_{k_A=0}=\frac{2^{10}\alpha_s^6}{\mu^8}I(x)\int dL|\psi_d(L)|^2, \quad (A.7)$$

where μ is a parameter with dimensions of mass which enters into the nucleon form factor $f(k)=\mu^2/(k^2+\mu^2)$; $x=k_T^2/\mu^2$, and

$$I(x)=\frac{2}{(1+x)^2}\int_0^\infty dy\frac{(x-y)^2}{(1+y)^2(1+2x+2y)} \times\left[\frac{1+x+y}{|x^2-y^2|}-\frac{1}{[(1+x+y)^2-4xy]^{1/2}}\right].$$

We note that $I(0)=1$, and therefore

$$\left.\frac{d\sigma}{d^2k_A\,d^2k_B}\right|_{k_A=k_B=0}=\frac{2^{10}\alpha_s^6}{\mu^8}\int|\psi_d(L)|^2dL. \quad (A.8)$$

Numerical integration over k_B in (A.7) gives

$$\left.\frac{d\sigma}{d^2k_A}\right|_{k_A=0}=0{,}3\frac{2^{10}\alpha_s^6}{\mu^6}\int|\psi_d(L)|^2dL. \quad (A.9)$$

The slope parameter characterizing the dependence of the cross section on k_A can be defined as

$$B(k_A)=\frac{d}{dk_A^2}\left(\ln\frac{d\sigma}{dk_A^2}\right).$$

In contrast to the cross section, the quantity $B(k_A)$ found from (A.4) diverges logarithmically as $k_A\to0$. Instead of the procedure of cutting off the integrals by the effective mass of the gluon which was used in Ref. 5, we introduce the average value of the slope B defined as

$$B=\int d^2k_A\frac{d\sigma}{d^2k_A}\Big/\left.\left(\frac{d\sigma}{d^2k_A}\right)\right|_{k_A=0}. \quad (A.10)$$

We note that the momenta of the emitted particles k_A, k_B, and $-(k_A+k_B)$ enter into the expression (A.7) for the amplitude A symmetrically. Therefore, using the relation

$$\int d^2k_2\,d^2k_3\exp[-B_0(k_1^2+k_2^2+k_3^2)]\delta^2(k_1+k_2+k_3) =\frac{\pi}{2B_0}\exp\left(-\frac{3B_0}{2}k_1^2\right) \quad (A.11)$$

and the definition (A.10), we find

$$B=3B_0/2. \quad (A.12)$$

On the other hand, the value of B_0 can be found, using (A.11), from the ratio of expressions (A.8) and (A.9)

$$\frac{2B_0}{\pi}=\left(\frac{d\sigma}{d^2k_A\,d^2k_B}\right)_{k_A=k_B=0}\Big/\left(\frac{d\sigma}{d^2k_A}\right)_{k_A=0}=\frac{3{,}3}{\mu^2}. \quad (A.13)$$

We determine the values of α_s and μ from the elastic and rotal pp-scattering cross sections in the two-gluon approximation for the same parametrization of the form factor[14]:

$$\sigma_{tot}^{NN}=\frac{32\pi}{\mu^2}\alpha_s^2,\quad \frac{d\sigma_{el}^{NN}}{dt}=\frac{64\pi\alpha_s^4}{\mu^4}J(-t/\mu^2),$$

where

$$J(x)=(1+x)^{-2}\left[1+2x(1-x)^{-2}\ln\frac{4x}{(1+x)^2}\right].$$

Substituting the values $\sigma_{tot}^{NN}=40$ mb and $\sigma_{el}^{NN}=7$ mb, we obtain $\alpha_s=0.78$ and $\mu^2=0.62$ GeV^2. In this case from Eq. (A.10) we obtain the slope value $B=12.8$ GeV^{-2}.

For the coefficient β in Eq. (2) a comparison gives the value $\beta\approx0.17$.

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