# Phenomenological description of relaxation processes in magnetic materials

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(Submitted 23 May 1984)
Zh. Eksp. Teor. Fiz. **87**, 1501–1508 (October 1984)

The relaxation terms in the equation of motion of the magnetic moment are generalized to the case in which an allowance is made for the spatial dispersion due to the exchange interaction. An investigation is made of the mechanism of exchange acceleration of relaxation of the magnetic moment. A calculation is made of the contribution made to the mobility of Bloch and Néel domain walls by the inhomogeneous exchange relaxation terms.

## 1. INTRODUCTION

Relaxation processes in a spin system govern a number of characteristics such as the ferromagnetic resonance (FMR) line width, the thresholds of parametric excitation of spin waves, and the width of the intensity peak of scattered neutrons. Two approaches are possible to these phenomena: macroscopic and microscopic. The former is based on the equation of motion of the magnetic moment with relaxation terms, the latter on a study of the processes of relaxation of spin waves.

The interest in relaxation processes has increased again recently because of intensive studies of the dynamics of domain walls. The domain wall mobility has been investigated by many authors (see, for example, Refs. 1 and 2). This wall mobility can be calculated on the basis of a microscopic analysis of the processes of collision of domain walls with spin waves, phonons, or other quasiparticles, as well as with impurities, dislocations, and other defects in a crystal.[3–5]

The domain wall mobility and the dependence of the steady-state domain wall velocity on an external magnetic field (external force) can also be calculated on the basis of the Landau-Lifshitz equation with dissipative terms. This approach makes it possible to find the dependence $v(H)$ in a wide range of external fields. It leads in a natural manner to the conclusion of the existence of limiting velocities both in simple single-lattice magnetic materials (as deduced by Walker—see Ref. 6) and in two-sublattice antiferromagnets or weak ferromagnets.[7,8]

A quantitative comparison of the relaxation constants in the Landau-Lifshitz equation obtained from the domain wall mobilities with the relaxation constants deduced from the FMR or antiferromagnetic resonance (AFMR) data shows that they differ quite considerably. The relaxation constants deduced from the domain wall mobilities are larger. This provided the stimulus for our review of the problem of a phenomenological description of relaxation processes in magnetic materials on the basis of the Landau-Lifshitz equation. It should be pointed out that the unsatisfactory features of the description of high-frequency properties of ferromagnets in the range of large wave vectors by the simplest relaxation term of the Landau-Lifshitz or Gilbert type has been pointed out already, for example, in Ref. 9.

We shall generalize the relaxation terms in the Landau-Lifshitz equation to allow for the spatial dispersion, i.e., we shall find the contribution made to the relaxation by the exchange interaction. The structure of these terms is such that the square of the magnetization is not conserved when the terms are included. It is shown that the relaxation terms of the $\gamma\mathbf{H}_e$ type, where $\mathbf{H}_e$ is the effective magnetic field, give rise to a two-stage relaxation process: a fast exchange-accelerated stage of relaxation of the magnetization is followed by a relatively slow relaxation of the magnetization to its equilibrium value. We shall calculate the domain wall mobilities for different types of the magnetization distribution in a domain wall, and we shall also find the FMR line widths and the damping factors of short-wavelength spin waves. We shall show that the relaxation constants governing the domain wall mobility are larger than the constants governing the FMR line width because of an allowance for the spatial dispersion in dissipative processes.

## 2. ALLOWANCE FOR THE SPATIAL DISPERSION IN THE RELAXATION TERMS OF THE EQUATION OF MOTION FOR THE MAGNETIZATION

It is known that the motion of the magnetic moment $\mathbf{M}$ of a ferromagnet is described by the Landau-Lifshitz equation[10]

$$\dot{\mathbf{M}}=g[\mathbf{M}\mathbf{H}_e]+(\lambda/M)[\mathbf{M}[\mathbf{M}\mathbf{H}_e]],\tag{1}$$

where $g$ is the gyromagnetic ratio; $\lambda$ is the relaxation constant; $\mathbf{H}_e$ is the effective magnetic field which is defined as the variational derivative of the internal energy $W$ of the ferromagnet:

$$\mathbf{H}_e(\mathbf{x})=-\delta W/\delta\mathbf{M}(\mathbf{x}).\tag{2}$$

In the case of a uniaxial crystal, this energy is

$$W=\int\left\{\frac{1}{2}\alpha_{mn}\frac{\partial\mathbf{M}}{\partial x_n}\frac{\partial\mathbf{M}}{\partial x_m}-\frac{1}{2}\beta M_z^2+\frac{\mathbf{H}_m^2}{8\pi}-\mathbf{M}\mathbf{H}_0+\frac{1}{2}f(M^2)\right\}d^3x,\tag{3}$$

where $\alpha_{nm}$ are the exchange constants; $\beta$ is the anisotropy constant ($\beta>0$); $\mathbf{H}_m$ is the magnetic field of the dipole interaction; $\mathbf{H}_0$ is the external magnetic field; $f(M^2)$ is the energy density of the homogeneous exchange interaction. The order-of-magnitude expressions are $\alpha\sim(T_c/\mu M_0)a^2$, $\beta\sim 1$, and $f(M^2)\sim(T_c/\mu M_0)M^2$, where $T_c$ is the Curie temperature, $\mu$ is the Bohr magneton, $a$ is the lattice constant, and $M_0$ is the equilibrium value of the magnetic moment. It should be pointed out that $(T_c/\mu M_0)\sim 10^3$ if $T_c\sim 10^2$ K and $M_0\sim 10^3$ G.

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Using Eq. (2), we can find the effective magnetic field

$$\mathbf{H}_e=-f'(M^2)\mathbf{M}+(\nabla\alpha\nabla)\mathbf{M}+\beta\mathbf{n}(\mathbf{n}\mathbf{M})+\mathbf{H}_m+\mathbf{H}_0. \tag{4}$$

Here, a prime of the function $f$ denotes differentiation with respect to the argument of the function and

$$(\nabla\alpha\nabla)=\alpha_{nm}\frac{\partial}{\partial x_n}\frac{\partial}{\partial x_m}.$$

The relaxation term in the equation of motion (1) can be represented also in the form proposed by Gilbert:

$$\dot{\mathbf{M}}=g_1[\mathbf{M}\mathbf{H}_e]+(\nu/M)[\mathbf{M}\dot{\mathbf{M}}]. \tag{5}$$

Solving this equation for $\dot{M}$, we obtain Eq. (1) where[11]

$$g=g_1/(1+\nu^2),\quad \lambda=g\nu. \tag{6}$$

It therefore follows that Eqs. (1) and (5) are equivalent apart from the notation. Both equations, (1) and (5), contain the integral of motion $M^2$ and the relaxation terms do not include the derivatives with respect to the coordinates, i.e., they do not allow for the spatial dispersion. The relaxation terms in Eqs. (1) and (5) are of the relativistic origin, as pointed out in Ref. 10. In fact, we have

$$\dot{W}=-\int \dot{\mathbf{M}}\mathbf{H}_e d^3x. \tag{7}$$

Using Eq. (5), we can rewrite this relationship in the form

$$\dot{W}=(\nu/gM_0)\int \dot{\mathbf{M}}^2 d^3x. \tag{8}$$

Hence, it follows that energy dissipation occurs also in the case when the magnetization precession is homogeneous. As soon as the relativistic interaction results in relaxation of the homogeneous magnetization, the value of $\nu$ becomes of the relativistic origin.

We shall show how to allow for the spatial dispersion in the relaxation terms. Inclusion of these terms in the case of weakly inhomogeneous states is justified only when they are governed by the exchange interactions. We shall denote them by $\mathbf{R}$ and write down the equation of motion for the magnetization in the form

$$\dot{\mathbf{M}}=g_1[\mathbf{M}\mathbf{H}_e]+(\nu/M)[\mathbf{M}\dot{\mathbf{M}}]+\mathbf{R}. \tag{9}$$

The quantity $\mathbf{R}$ should be in the form of a divergence of a tensor. This is due to the fact that the exchange approximation conserves each of the components of the magnetic moment of a body

$$\mathfrak{M}=\int \mathbf{M}d^3x \tag{10}$$

and the Landau-Lifshitz equation should be in the form of the law of conservation

$$\frac{\partial M_i}{\partial t}+\frac{\partial \Pi_{ik}}{\partial x_k}=\frac{\partial \Pi_{ik}^R}{\partial x_k}=R_i, \tag{11}$$

where $\Pi_{ik}$ and $\Pi_{ik}^R$ are the dynamic and dissipative fluxes of the $i$th component of the magnetic moment per unit time through a unit area which is orthogonal to the $k$ th axis. The tensor $\Pi_{ik}$ is[12]:

$$\Pi_{ik}=\alpha_{ks}\varepsilon_{inm}M_n\frac{\partial M_m}{\partial x_s}. \tag{12}$$

We shall find $\Pi_{ik}^R$ by substituting $\dot{\mathbf{M}}$ from Eq. (11) into Eq. (7). We then obtain

$$\dot{W}=-\int\frac{\partial \Pi_{ik}^R}{\partial x_k}H_{ei}d^3x$$

or

$$\dot{W}=\int \Pi_{ik}^R\frac{\partial H_{ei}}{\partial x_k}d^3x. \tag{13}$$

The dissipative flux and the effective magnetic field $H_e$ vanish in the equilibrium state. Expanding $\Pi_{ik}^R$ as a series in powers of $\delta H_{ei}/\delta x_k$, we obtain

$$\Pi_{ik}^R=-\nu_{ik,mn}\frac{\partial H_{en}}{\partial x_m} \tag{14}$$

and

$$R_i=-\nu_{ik,mn}\frac{\partial^2 H_{en}}{\partial x_m\partial x_k}. \tag{15}$$

In the determination of the nature of $\mathbf{R}$ we have used the law of conservation of the total magnetic moment. A further simplification of the nature of $\mathbf{R}$ is obtained if we note that in the case of homogeneous rotation the vector $\mathbf{R}$ should transform in the same way as $\mathbf{M}$ or, which is equivalent, as $\mathbf{H}_e$, since in the exchange approximation the rotation of a spin system as a whole does not alter its dynamic or static properties. Hence, it follows that

$$\nu_{ik,mn}=gM\nu_{mk}^e\delta_{in} \tag{16}$$

and

$$R_i=-gM\nu_{mk}^e\frac{\partial^2 H_{ei}}{\partial x_m\partial x_k}=-gM(\nabla\nu^e\nabla)H_{ei}. \tag{17}$$

It is important to point out that $\mathbf{MR}\neq 0$. This means that an allowance for the relaxation terms of the exchange origin removes the integral of motion $M^2$.

The relaxation term (17) describes the establishment of a homogeneous distribution of the magnetization in a body and the rate of relaxation increases on increase in the local magnetization inhomogeneity. Since the components of the magnetic field then migrate from the regions where they are larger to those where they are smaller, it naturally follows that the local magnetization $\mathbf{M}$ is not conserved. It is sometimes convenient to know the expression for $\mathbf{R}$ not in terms of $\mathbf{H}_e$ but in terms of $\dot{\mathbf{M}}$. We can obtain such an expression by deriving $\mathbf{H}_e$ from Eq. (9) ignoring dissipation:

$$g\mathbf{H}_{e\perp}=\frac{[\dot{\mathbf{M}}\mathbf{M}]}{M^2},\quad \mathbf{H}_\perp=\mathbf{H}-\frac{\mathbf{M}(\mathbf{MH})}{M^2}. \tag{18}$$

It is this part that determines the magnetization dynamics [see Eq. (1)]. Therefore, the dissipation in the dynamic processes can be described by the relaxation term

$$\mathbf{R}=-M(\nabla\nu^e\nabla)[\dot{\mathbf{M}}\mathbf{M}]/M^2. \tag{19}$$

We shall now write down explicitly the equation of motion of the magnetic moment allowing for the relaxation terms of the relativistic and exchange origins:

$$\dot{\mathbf{M}}=g[\mathbf{M}\mathbf{H}_e]+(\nu/M)[\mathbf{M}\dot{\mathbf{M}}]-gM(\nabla\nu^e\nabla)\mathbf{H}_e \tag{20}$$

or

$$\dot{\mathbf{M}}=g[\mathbf{M}\mathbf{H}_e]+M\{\nu+(\nabla\nu^e\nabla)\}([\mathbf{M}\dot{\mathbf{M}}]/M^2). \tag{21}$$

The dissipative function $F$ is

$$F=-\frac{1}{2}\dot{W}=\frac{1}{2}\int d^3x\left\{\nu\dot{\mathbf{M}}^2+\nu_{nm}^e\frac{\partial \mathbf{H}_e}{\partial x_n}\frac{\partial \mathbf{H}_e}{\partial x_m}\right\} \tag{22}$$

or

$$F=\frac{1}{2}\int d^3x\left\{\nu\dot{\mathbf{M}}^2+\nu_{nm}^e\frac{\partial}{\partial x_n}\frac{(\mathbf{M}\dot{\mathbf{M}})}{M^2}\frac{\partial}{\partial x_m}\frac{[\mathbf{M}\dot{\mathbf{M}}]}{M^2}\right\}. \tag{22'}$$

We shall now consider the problem of the relaxation terms in the Landau-Lifshitz equation using a system of the Onsager equations.[13] Since $T\dot{\sigma} = -\dot{W}$ ($\sigma$ is the entropy of the system), it follows from Eq. (7) that the quantities describing the quasiequilibrium state are the components of the magnetic moment $\mathbf{M}$ and the generalized forces are the components of the effective magnetic field $\mathbf{H}_e$. After allowance for the spatial dispersion the system of the Onsager equations becomes

$$\dot{M}_i(\mathbf{x}, t) = \int \gamma_{ik}(\mathbf{x}-\mathbf{x}', M) H_{ek}(\mathbf{x}', t)\, d^3x'. \tag{23}$$

In the case of a weak spatial dispersion, when $H_e(x,t)$ varies slowly with the coordinate, we have[1)]

$$\dot{M}_i(\mathbf{x}, t) = \gamma_{ik}(M) H_{ek} + \gamma_{ik,nm} \frac{\partial^2 H_{ek}}{\partial x_n \partial x_m}, \tag{24}$$

where

$$\gamma_{ik}(\mathbf{M}) = \int \gamma_{ik}(\mathbf{x}, \mathbf{M})\, d^3x, \qquad \gamma_{ik,nm}(\mathbf{M}) = \int \gamma_{ik}(\mathbf{x}, \mathbf{M})\, x_n x_m d^3x. \tag{25}$$

It follows from the definition that the tensor $\gamma_{ik,nm}$ is symmetric in respect of the pair of the last indices. We can easily see that the energy dissipation is governed by $\gamma^s_{ik}$ and $\gamma^s_{ik,nm}$ which are the parts of the tensors $\gamma_{ik}$ and $\gamma_{ik,nm}$ symmetric in respect of the indices $i$ and $k$. The parts of the tensors $\gamma_{ik}$ and $\gamma_{ik,nm}$ antisymmetric in respect of the same indices, govern the dynamics of $M$. We shall allow for the contribution made to the dynamics only by the antisymmetric part $\gamma_{ik}$, because the contribution of $\gamma_{ik,nm}$ contains an additional small parameter due to weak inhomogeneities. The structure of the dynamic part of the equation of motion of the magnetization has been discussed on many occasions (see, for example, Ref. 14) and can be represented in the form

$$\gamma_{ik}{}^a = -g\varepsilon_{ikn} M_n. \tag{26}$$

The same considerations which were used to simplify the tensor $\nu_{ik,nm}$ allow us to determine $\gamma^s_{ik,nm}$ in the form

$$\gamma_{ik,nm} = -\gamma_{nm}{}^e \delta_{ik}. \tag{27}$$

Using Eqs. (26) and (27), we can rewrite Eq. (25) in the form[2)]

$$\dot{\mathbf{M}} = g[\mathbf{M}\mathbf{H}_e] + \hat{\gamma}\mathbf{H}_e - (\nabla \gamma^e \nabla)\mathbf{H}_e. \tag{28}$$

The first and last terms on the right-hand side of this equation are identical with the corresponding terms in Eq. (20) and we shall not discuss them further. The second term[9]

$$\mathbf{R}_1 = \hat{\gamma}\mathbf{H}_e \tag{29}$$

differs from the corresponding relaxation terms in Eq. (20). The relaxation tensor $\hat{\gamma} \equiv \hat{\gamma}^s$ is of the relativistic origin, since it describes the relaxation of $\mathbf{M}$ also in the case of homogeneous oscillations. An interesting feature of $\mathbf{R}_1$ is the fact that $\mathbf{R}_1$ results in an exchange acceleration of the relaxation of the magnetic moment, whereas the relaxation terms

$$(\nu/M)[\mathbf{M}\dot{\mathbf{M}}] \text{ и } (\lambda/M)[\mathbf{M}[\mathbf{M}\mathbf{H}_e]],$$

occurring in Eqs. (1) and (5) do not describe at all the magnetization relaxation process. We can demonstrate that this is true by noting that, apart from linear terms, the magnetization $\mathbf{M} = \mathbf{M}_0 + \mathbf{m}$ is given by

$$\mathbf{M}^2 = \mathbf{M}_0^2 + 2\mathbf{M}_0\mathbf{m},$$

and, for the sake of simplicity, assuming that the tensor $\gamma$ is diagonal. We then have

$$\mathbf{M}_0\dot{\mathbf{m}} = \gamma \mathbf{M}_0 \mathbf{H}_e(\mathbf{m}). \tag{30}$$

Using Eq. (4) for $\mathbf{H}_e(\mathbf{m})$ in the principal exchange approximation, we obtain

$$\mathbf{H}_e(\mathbf{m}) = -2f''(M_0^2)\mathbf{M}_0(\mathbf{m}\mathbf{M}_0). \tag{31}$$

In this formula the symbol $f''$ denotes the second derivative of $f(M^2)$ with respect to its argument. Substituting Eq. (31) into Eq. (30), we obtain

$$\dot{m}_\parallel = -(1/\tau_e) m_\parallel, \tag{32}$$

where

$$1/\tau_e = 2\gamma f''(M_0^2) M_0^2. \tag{33}$$

Assuming that $f'' \sim f/M^4$ and estimating $f$ from $f \sim (T_c/\mu M_0) M^2$, we obtain the following order-of-magnitude estimate

$$\tau_e \sim \frac{1}{\gamma}\left(\frac{\mu M_0}{T_c}\right) = \tau_\perp\left(\frac{\mu M_0}{T_c}\right). \tag{34}$$

The time dependence of the part $\mathbf{m}$ transverse to $\mathbf{M}_0$ (without allowance for the spatial dispersion of the relaxation) is given by

$$\dot{\mathbf{m}}_\perp = g[\mathbf{M}_0\mathbf{H}_e(\mathbf{m})] + \gamma\xi\mathbf{m}_\perp, \quad \xi \approx 1. \tag{35}$$

Hence, we can see that $\tau_\perp = 1/\gamma$ is the relaxation time of the transverse components of the homogeneous magnetization. It is clear from Eq. (34) that the relaxation time of the longitudinal component $\mathbf{m}_\parallel$ is reduced by a factor $(\mu M_0/T_c)$, compared with the relaxation time of the transverse component $m_\perp$. Since $\tau_e$ governs the relaxation time of the magnetic moment $\mathbf{M}$, it follows that Eqs. (33) and (34) are evidence of the exchange acceleration of the process of relaxation of the quantity $\mathbf{M}$ compared with the relaxation of $\mathbf{M}$ to its equilibrium direction if the relaxation is described by the term $\mathbf{R}_1 = \gamma\mathbf{H}_e$. This relaxation mechanism is deduced in Ref. 9 on the basis of a microscopic analysis. We can show that an allowance for the off-diagonal elements of the tensor $\gamma^s_{ik}$ does not alter the conclusions reached on the relaxation times of $\mathbf{m}_\perp$ and $\mathbf{m}_\parallel$.

We shall conclude this section by giving the dissipative function $F$. If $\gamma^s_{ik} = \gamma\delta_{ik}$ and $\gamma^e_{ik} = \gamma^e\delta_{ik}$, we find that

$$F = \frac{1}{2}\int \left\{ \gamma \mathbf{H}_e^2 + \gamma^e \left(\frac{\partial \mathbf{H}_e}{\partial x_i}\right)^2 \right\} d^3x. \tag{36}$$

We can determine the relaxation constants $\gamma$ and $\gamma^e$ if we know the FMR line width and the mobility of domain walls, for example, of the Bloch walls. Naturally, in the general case the number of the relaxation constants is more than two and we can determine them if we have, in addition to the experimental data obtained under various conditions for the FMR line width, also the data on the mobility of domain walls of different symmetries.

## 3. MOBILITY OF DOMAIN WALLS, FERROMAGNETIC RESONANCE LINE WIDTH, EXCHANGE DAMPING OF SPIN WAVES

We shall show that the relaxation constants $\nu$ and $\gamma$ occur in the expressions for the FMR line width and for the domain wall mobility. We shall consider the specific case of a uniaxial ferromagnet with the easy-axis magnetic anisotropy. Its internal energy is given by Eq. (3). The FMR line width for an ellipsoid with the revolution axis parallel to the anisotropy axis is easily calculated from Eq. (5) and it is given by [9,11]:

$$\Delta H=(H_A+H_0-4\pi MN_3)\nu, \tag{37}$$

where $H_A=\beta M$ and $N_3$ is the demagnetization factor along the $z$ axis.

We shall now calculate the mobility of a Bloch domain wall. We shall assume that the wall is parallel to the $ZY$ plane, i.e., that $\varphi=\pi/2$, and also that $\theta=\theta(x-vt)$ ($\varphi$ and $\theta$ are the azimuthal and polar angles of the vector **M**). The distribution in a domain wall is given by [10]

$$x_0\theta_x=\sin\theta; \tag{38}$$

here, $x_0$ is the thickness of a Bloch domain wall; $x_0^2=\alpha/\beta$; $\theta_x=d\theta/dx$. We can easily see that if $\theta=\theta(x-vt)$ and $\varphi=\pi/2$, it follows from Eq. (22′) that

$$F=(Mv^2/2g)S\int[\nu\theta_x^2+\nu^e\theta_{xx}^2]dx, \tag{39}$$

where $S$ is the domain wall area. Using Eq. (38), we now obtain

$$F=(Mv^2/gx_0)S[\nu+(\nu^e/3x_0^2)]. \tag{40}$$

We shall apply a standard procedure to find the friction force and the mobility of a domain wall using the relationship $\eta\mathbf{F}_{\rm fr}=-S\mathbf{v}$. Bearing in mind that the work done by the friction force per unit time is

$$\mathbf{F}_{\rm fr}\mathbf{v}=-(Sv^2/\eta)=-2F$$

we find that the Bloch domain wall mobility is

$$\eta_0^{-1}=(2M/gx_0)[\nu+(\nu^e/3x_0^2)]. \tag{41}$$

In the case of a Néel domain wall parallel to the $ZY$ plane, we have

$$\varphi=0,\ \theta=\theta(x-vt),\quad x_1\theta_x=\sin\theta, \tag{42}$$

where $x_1$ is the thickness of this wall and $x_1^2=\alpha/(\beta+4\pi)$. Using these formulas and Eq. (22′), we find that

$$F=(M/gx_1)v^2S[\nu+(\nu^e/3x_1^2)].$$

Hence, the mobility of Néel domain walls is given by

$$\eta_1^{-1}=(2M/gx_1)[\nu+(\nu^e/3x_1^2)]. \tag{43}$$

Comparing Eqs. (21) and (19) and bearing in mind that $x_1<x_0$, we can see that the mobility of Bloch domain walls is higher than the mobility of Néel walls: $\eta_0>\eta_1$. If we know $\Delta H$, $\eta_0$, and $\eta_1$, we can readily find $\nu$ and $\nu^e$.

We shall now give the expression for the damping of spin waves with a vector **k** considered in the exchange approximation. Assuming that

$$\mathbf{M}=\mathbf{M}_0+[\mathbf{m}(\mathbf{k})\exp\{i(\mathbf{kr}-\omega(\mathbf{k})t\}+\text{c.c.}]$$

and ignoring the relativistic terms, we find from Eqs. (9) and (12) that

$$F=(\nu^e/gM_0)k^2\omega(\mathbf{k})|\mathbf{m}(\mathbf{k})|^2V,\qquad W=(\omega(\mathbf{k})/gM_0)|\mathbf{m}(\mathbf{k})|^2V, \tag{44}$$

where $V$ is the volume of the investigated crystal. Hence, we readily find the damping factor of spin waves

$$\gamma^e(k)=F/2W={}^1/_2\nu^ek^2\omega(k)={}^1/_2\nu^egM_0\alpha k^4. \tag{45}$$

This result is in full agreement with a microscopic calculation of the exchange damping of spin waves.[15,16] We shall now consider the dissipative function (36) or Eq. (28). In this case the FMR line width and the damping factor of spin waves considered in the exchange approximation are described by

$$\Delta H=(H_A+H_0-4\pi N_3M)(\gamma/gM_0), \tag{46}$$

$$\gamma(k)=\gamma^ek^2\omega(k)/2(gM_0)^2. \tag{47}$$

Without stopping at simple steps, we shall give the final expressions for the domain wall mobilities. Using Eqs. (36) and (18) for the mobility of a Bloch domain wall in a uniaxial ferromagnet, we obtain

$$\eta_0^{-1}=(2/x_0g^2)[\gamma+(\gamma^e/3x_0^2)]. \tag{48}$$

Similarly, the mobility of a Néel domain wall is

$$\eta_1^{-1}=(2/x_1g^2)[\gamma+(\gamma^e/3x_1^2)]. \tag{49}$$

The expressions (48) and (49) reduce to Eqs. (41) and (43) if we make the substitution $\gamma=gM\nu$.

We shall conclude by noting that—as is demonstrated by the formulas (40), (43), (48), and (49) for the domain wall mobilities and by the formulas (37) and (46) for the FMR line width—the effective relaxation constant deduced from the domain wall mobility

$$\eta^{-1}=2\nu_{eff}M/gx=2\gamma_{eff}/g^2x, \tag{50}$$

where

$$\nu_{eff}=\nu+\nu^e/3x^2,\ \gamma_{eff}=\gamma+\gamma^e/3x^2 \tag{51}$$

and $x$ is the thickness of the domain wall in question, is greater than the relaxation constants $\nu$ or $\gamma$ deduced from the ferromagnetic resonance data.

We shall now compare the relaxation constants governed by the inhomogeneous exchange interactions and the homogeneous relativistic interactions. Since these constants are proportional to the probabilities of the scattering processes governed by the relevant energies, it follows that $\gamma\approx\beta$, $\gamma^e\sim\alpha$, and the ratio $\gamma x^2/\gamma^e$ is of the order of unity.

The author is grateful to B. A. Ivanov and I. V. Bar'yakhtar for discussing the results obtained.

[1)] We are assuming that the crystal in question has an inversion (symmetry) center, so that we can ignore the term $\gamma_{ik,n}(\delta H_{ek}/\delta x_n)$.

[2)] Equation (28) describes not only conventional spin waves, but also a relaxation mode associated with a change in the value of **M**.


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Translated by A. Tybulewicz