A new mechanism of the tilt effect in the interaction between electrons and sound in metals located in weak magnetic fields

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A theory is proposed of the tilt effect in the absorption and velocity dispersion of high-frequency sound in weak magnetic fields, in which the electron cyclotron radius is much greater than the sound wavelength. The mechanism and the expression of the tilt effect in this case differs appreciably from those in strong fields. Due to the three-dimensional character of the electron motion, the effect is determined by the transverse conductivity (with respect to the magnetic field vector) rather than by the longitudinal conductivity. Asymptotic formulas are derived for the angular dependence of the absorption and velocity dispersion. Numerical calculations are carried out for a finite relaxation frequency, and the results are analyzed. The theory is in qualitative agreement with the experiments.

1. The tilt effect is one in which the absorption and velocity dispersion of high-frequency sound in metals change sharply as a function of the angle between the magnetic field **H** and the direction of propagation of the sound wave. The physical reason for this phenomenon is connected with the jump-like onset of resonant collision-free absorption of sound by the conduction electrons. There appear on the Fermi surface electron states for which the condition of in-phase motion of electrons with the field is satisfied:

$$\omega = qv\sin\varphi. \tag{1}$$

In this equation, ω and $\mathbf{q}$ are the frequency and the wave vector of sound, respectively, v is the Fermi velocity of the electrons, $\pi/2-\varphi$ is the angle between the vectors $\mathbf{q}$ and $\mathbf{H}$. At $\varphi<\varphi_c$ ($\sin\varphi_c=\omega/qv$) the sound absorption in metals is a collision phenomenon and is proportional to the frequency of scattering of the electrons ν. At $\varphi>\varphi_c$ the resonance interaction (1) will dominate in comparison with the collisional absorption if

$$\omega \gg \nu. \tag{2}$$

In recent times, the tilt effect has been studied experimentally and theoretically[1–4] in the region of strong magnetic fields, when

$$qR \ll 1 \tag{3}$$

(R is the cyclotron radius) and the motion of the electrons has a quasi-one-dimensional character. Therefore, the point of view has been taken by many investigators that the tilt effect exists only in strong magnetic fields. Such a point of view has recently been demolished in a publication[5] in which the observation of the tilt effect has been reported in single tungsten crystals in the region of weak magnetic fields, when

$$qR \gg 1. \tag{4}$$

Only the deformation mechanism of interaction of the electrons with the lattice was considered in the theoretical section of Ref. 5, and electromagnetic fields accompanying the sound in the metal were not taken into account. This approximation validly describes the kinematic nature of the features of the tilt effect in a weak field but is not subject to rigorous analysis and serves only as an illustration of the possibility of the existence, in principle, of the observed phenomenon.

The purpose of the present work is the theoretical investigation of the mechanism and features of the tilt effect in weak fields. It should be emphasized that the kinematic nature of the tilt effect, which is connected with the keying in of collision-free interaction of electrons with phonons at $\varphi=\varphi_c$, is the same in both weak and strong fields. However, the appearance of these features turns out to be significantly different, since the motion of the electrons in weak fields is three-dimensional and the characteristics of the tilt effect are determined by the transverse, and not the longitudinal (relative to the vector **H**), components of the kinetic coefficients.

In strong fields, the collision-free interaction is due to the entire region of electron states on the Fermi surface, and not only to electrons with maximal velocity along the direction of the vector **H** near a turning point. In this case, the damping and change in sound velocity do not contain the small parameter s/v (s is the speed of sound, i.e., the tilt effect in the region of strong magnetic fields turns out to be essentially a non-adiabatic phenomenon.[4] In contrast to this, under conditions of strong spatial inhomogeneity, the resonant interaction of electrons with sound takes place not over the entire electron orbit, but only on a small portion, where the condition (1) is satisfied. This means that a relatively small number of electrons take part in the resonance. Therefore, a power of the small parameter $1/qR$ appears in the damping and sound velocity dispersion, while the features on the curves of the angular dependence of the damping, $\Gamma(\varphi)$ and $s(\varphi)$ are accentuated.

2. We now proceed to the derivation of the dispersion equation. The complete set of equations describing the propagation of sound waves in metals, as is well known, consists in the equations of elasticity theory for the field of displace-

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ments $u(r,t)$, the Maxwell equations for the electromagnetic fields accompanying the sound wave, and the kinetic equation for the conduction electron distribution function. In the case of longitudinal sound ($\mathbf{u}\|\mathbf{q}$, $u\sim\exp(i\mathbf{qr}-i\omega t)$ the equation of elasticity, after substitution in it of the distribution function, can be represented in the form

$$\omega^2\rho u=Kq^2u-iq\{\varkappa\mathbf{E}'+\delta q\omega u\}-\frac{1}{c}[\mathbf{j}\times\mathbf{H}]\frac{\mathbf{q}}{q}. \tag{5}$$

Here ρ is the density of the crystal, $K=\rho s_0^2$ is the longitudinal adiabatic elastic modulus, s_0 is the velocity of longitudinal sound,

$$\mathbf{E}'=\mathbf{E}-\frac{i\omega}{c}[\mathbf{u}\times\mathbf{H}], \tag{6}$$

$\mathbf{E}$ is the electric field, which satisfies the Maxwell equation

$$[\mathbf{q}\times[\mathbf{q}\times\mathbf{E}]]+\frac{4\pi i\omega}{c^2}\mathbf{j}=0. \tag{7}$$

The density of the electric field $\mathbf{j}$ is expressed in terms of $\mathbf{E}'$ and u by the formula

$$\mathbf{j}=\hat{\sigma}\mathbf{E}'+\eta q\omega u. \tag{8}$$

The kinetic coefficients σ (the conductivity tensor), η, $\varkappa$, δ in Eqs. (5)–(8) can be represented in the form

$$\sigma_{ik}=e^2\hat{L}v_i(\tau)v_k(\tau_1),\quad \delta=\hat{L}\Lambda(\tau)\Lambda(\tau_1),$$
$$\eta_i=-e\hat{L}v_i(\tau)\Lambda(\tau_1),\quad \varkappa_i=-e\hat{L}\Lambda(\tau)v_i(\tau_1), \tag{9}$$

where e is the absolute value of the electronic charge. The action of the operator $\hat{L}$ is determined by the relation

$$\hat{L}A(\tau)B(\tau_1)\equiv\frac{2}{(2\pi\hbar)^3}\sum\int\limits_{p_{\min}}^{p_{\max}}\frac{mdp_z}{\Omega}\int\limits_0^{2\pi}d\tau A(\tau)$$
$$\times\int\limits_{-\infty}^{\tau}d\tau_1B(\tau_1)\exp\left[\int\limits_{\tau}^{\tau_1}d\tau_2\frac{\nu-i\omega+i\mathbf{qv}}{\Omega}\right].$$

The symbol Σ denotes summation over the different groups of quasiparticles, p_z is the projection of the electron momentum on the vector $\mathbf{H}$, $p_{\max}$ and $p_{\min}$ are the maximal and minimal values of p_z on the Fermi surface, $\Omega=eH/mc$ is the cyclotron frequency, m is the cyclotron mass. The quantity Λ represents the longitudinal (relative to the vector $\mathbf{q}$) component of the deformation potential which vanishes in averaging over the Fermi surface.

The kinetic coefficients (9) are easily calculated in the set of coordinates xyz, in which the z axis is parallel to the field $\mathbf{H}$, while the x axis is perpendicular to the vectors $\mathbf{q}$ and $\mathbf{H}$. The kinetic coefficients were found under the condition

$$|\nu-i\omega|\ll\Omega, \tag{10}$$

consistent with the inequality (4). All the quantities (9) are conveniently represented in the form of an expansion in the small parameter $|\nu-i\omega|/\Omega$. It turns out that the components σ_{iy}, η_y and $\varkappa_y$ are small, at least by the factor $(\Omega/|\nu-i\omega|)^2$ as a consequence of the fact that the vector $\mathbf{q}$ is almost parallel to the y axis. For this reason, the component of the current j_y in (7) can be set equal to zero, while, $j_z=0$ because of the condition of electric neutrality. Consequently, the set of Maxwell equations reduces to only two equations for the determination of the components E_x and E_z. After their solution and substitution of the result in the equation of elasticity (5), we can find the dispersion equation for longitudinal sound in a metal:

$$\omega^2-q^2s_0^2=-\frac{i\omega q^2}{\rho}\Psi(qR,\varphi). \tag{11}$$

The function Ψ describes the interaction of electrons with sound and has the form

$$\Psi=\delta-\frac{\eta_z^2}{\sigma_{zz}}+i\frac{(H\cos\varphi)^2}{4\pi\omega}+\left[\sigma_{xx}+i\frac{q^2c^2}{4\pi\omega}+\frac{\sigma_{zx}^2}{\sigma_{zz}}\right]^{-1}$$
$$\times\left\{\left[\frac{\sigma_{zx}^2}{\sigma_{zz}}\eta_z\varkappa_z-\varkappa_x\eta_x+\frac{\sigma_{zx}}{\sigma_{zz}}(\eta_x\varkappa_z-\varkappa_x\eta_z)\right]\right.$$
$$\left.-\frac{Hcq\cos\varphi}{4\pi\omega}\left[(\eta_x-\varkappa_x)+\frac{\sigma_{zx}}{\sigma_{zz}}(\varkappa_z+\eta_z)\right]+\left(\frac{Hcq\cos\varphi}{4\pi\omega}\right)^2\right\}. \tag{12}$$

It is seen from Eqs. (12) and (9) that the term δ is due to the direct action of the electrons with the sound, while the other terms describe the effect due to the electromagnetic field (6) that accompanies the propagation of the sound wave.

3. The dispersion relations (11) and (12) have a general character and are valid upon satisfaction of the inequalities (4) and (10) for an arbitrary, multiconnected Fermi surface in both compensated and uncompensated metals. We now carry out further calculations for the simplest model of a metal with isotropic square law dispersion.

For the model of the Fermi surface considered, the deformation potential tensor is equal to

$$\Lambda_{ik}=\lambda(3n_in_k-\delta_{ik}), \tag{13}$$

where the coefficient λ is of the order of the Fermi energy ε_F, n_i are the components of the unit vectors of the velocity of the electron. With accuracy to terms that are small in the parameter $|\nu-i\omega|/\Omega$ the kinetic coefficients (9) can be written in the form

$$\sigma_{xx}=-\frac{i\sigma_H}{q_zR}\int\limits_{-1}^{1}dt\frac{1-t^2}{t-\beta}J_1^2[z(1-t^2)^{1/2}],$$
$$\sigma_{xz}=-\sigma_{zx}=-\frac{\sigma_H}{2q_zR}\frac{d}{dz}[g(z)+\beta f(z)],$$
$$\sigma_{zz}=-\frac{i\sigma_H\beta}{q_zR}[g(z)+\beta f(z)],$$
$$\delta=-i\frac{4\pi mp}{(2\pi\hbar)^3\Omega}\frac{\lambda^2}{q_zR}f(z),$$
$$\eta_x=-\varkappa_x=\frac{4\pi mpev}{(2\pi\hbar)^3\Omega}\frac{\lambda}{2q_zR}\frac{d}{dz}f(z), \tag{14}$$

$$\eta_z=\varkappa_z=i\frac{4\pi mpev}{(2\pi\hbar)^3\Omega}\frac{\lambda}{q_zR}[g(z)+\beta f(z)].$$

Here we have introduced the following notation:

$$g(z)\equiv\int_{-1}^{1}dtJ_0^2[z(1-t^2)^{1/2}],\qquad f(z)\equiv\int_{-1}^{1}\frac{dt}{t-\beta}J_0^2[z(1-t^2)^{1/2}],$$

$$\sigma_H=\frac{3Ne^2}{2m\Omega},\qquad q_zR=qR\sin\varphi,$$

$$z=qR\cos\varphi,\qquad \beta=\frac{\sin\varphi_c}{\sin\varphi}\left(1+\frac{i\nu}{\omega}\right), \tag{15}$$

$J_n(x)$ is the Bessel function, p is the Fermi momentum, N is the concentration of electrons. The remaining terms of the kinetic coefficients are negligibly small and we shall not write them down.

It is seen from Eqs. (14) and (15) that the kinetic coefficient contains a singular integral ($f(z)$, which is due to the quantity $(t-\beta)^{-1}$ in the integrand of (15), and which changes strongly in the vicinity of the point $t_0=\sin\varphi_c/\sin\varphi$ with width $\Delta t\approx t_0\nu/\omega$. The presence of such a singularity also describes the tilt effect, upon satisfaction of the condition (2). In fact, at $\varphi<\varphi_c$, the singular point t_0 turns out to be outside the interval of integration and terms with $f(z)$ turn out to be small in comparison with the nonsingular terms. At $\varphi\gg\varphi_c$ the singular terms increase strongly, which leads to a strong angular dependence in the spectrum and to damping of the sound at $\varphi\sim\varphi_c$.

The structure of the function $\Psi(qR,\varphi)$ is such that the very large singular term δ and η_z^2/σ_{zz} compensate one another. This means that the contribution of the vortex terms of the electric field E_z to a significant amount wipe out the contribution of the direct deformation interaction. A similar compensation was established previously in case of the tilt effect, in the region of strong magnetic fields[4] and in a number of other cases (see, for example, Ref. 6). As a result of such compensation a small factor $(qR)^{-1}$ appears in the expression (12) for the function Ψ (the term $H^2/4\pi\omega$ is less than the quantity $\delta-\eta_z^2/\sigma_{zz}$, by at least a factor of $qR\gg1$). After taking the asymptote (at $z\gg1$) in the nonsingular terms, which contain the function $g(z)$, and a series of transformations, we can represent the dispersion equation (11) in the following form:

$$\omega^2-q^2s^2=\frac{3N\lambda^2}{4\rho s^2\varepsilon_F}\frac{\omega^2}{(1+i\nu/\omega)q_yR}\left\{1+\frac{\sin(2q_yR-\pi/4)}{(\pi q_yR)^{1/2}}\right.$$
$$-\frac{\Omega}{\pi(\nu-i\omega)(q_yR)^2}\left[\cos\left(2q_yR-\frac{\pi}{4}\right)\right.$$
$$\left.+\frac{2}{3}\frac{\varepsilon_F}{\lambda}\frac{q^2c^2(\pi q_yR)^{1/2}}{\omega_p^2}\left(1+i\frac{\nu}{\omega}\right)\right]^2$$
$$\left.\times\frac{\sigma_H}{\sigma_{xx}+iq^2c^2/4\pi\omega+\sigma_{xz}^2/\sigma_{zz}}\right\}, \tag{16}$$

where $\omega_p^2=4\pi Ne^2/m$. It is seen that the right side of the dispersion equation (16), in contrast to the case of a strong magnetic field, actually turns out to be proportional to the small parameter $1/qR$. The second term in the curly brackets of (16) describes the oscillations of the geometric resonance of electrons on the central cross section of the Fermi surface. These oscillations can appear in the angular dependence of the spectrum and the sound damping in the range of angles $\varphi\gtrsim(qR)^{-1/2}\approx\varphi_c(\Omega/\omega\varphi_c)^{1/2}$. It follows from the latter estimate that Pippard oscillations as a function of the tilt angle take place in the range of angles $\varphi\gg\varphi_c$. It is also seen from Eq. (16) that singular terms in the region of the characteristic discrepancy values $\varphi\sim\varphi_c$ are present only in the last term in the curly brackets of (16).

The last term on the right side of the dispersion equation (16) admits of further simplification. Analysis shows that the conditions (2), (4) and (10) are sufficient for us to neglect the quantity $|\sigma_{xz}|^2$ in comparison with $|\sigma_{xx}\sigma_{zz}|$. If we take this circumstance into account, then it turns out that the conductivity will no longer enter into the dispersion equation. The tilt effect in the absorption and velocity of high-frequency sound is determined by the transverse (relative to the vector **H**) conductivity σ_{xx}, in contrast with the case of a strong magnetic field, in which the effect is on the whole due to the quantity σ_{zz}. If the sound frequency is not too large, then we cannot take into account the quantity $q^2c^2/4\pi\omega$ in comparison with $|\sigma_{xx}|$. For this, it is sufficient that the following inequalities be satisfied:

$$q^2c^2\ll\omega_p^2,\qquad \omega^3/\omega_0^2\ll\Omega, \tag{17}$$

where $\omega_0^2=3\omega_p^2S^3/2c^2v$. For typical metals, the conditions (17) provide an upper bound to the sound frequency of the order of 10^{11} rad/s, while the magnetic field should be no less than 10 kOe. In principle, it is not difficult to analyze the opposite limiting case, when $q^2c^2\gg4\pi\omega|\sigma_{xx}|$. Here the tilt effect will exist at lower frequencies and much weaker magnetic fields, while the denominator in the dispersion equation (16) should be expanded in σ_{xx}. However, we shall not concern ourselves here with the investigation of this limiting case. Thus, we represent the dispersion equation finally in the form

$$\omega-qs_0=\frac{3N\lambda^2}{8\rho s^2\varepsilon_F}\frac{\omega}{(1+i\nu/\omega)qR}\left\{1+\frac{\sin(2qR-\pi/4)}{(\pi qR)^{1/2}}\right.$$
$$-\frac{\Omega}{\pi(\nu-i\omega)(qR)^2}\left[\cos\left(2qR-\frac{\pi}{4}\right)\right.$$
$$\left.\left.+\frac{2}{3}\frac{\varepsilon_F}{\lambda}\frac{q^2c^2(\pi qR)^{1/2}}{\omega_p^2}\left(1+i\frac{\nu}{\omega}\right)\right]^2\frac{\sigma_H}{\sigma_{xx}}\right\}. \tag{18}$$

Here q_y is replaced by q because of the smallness of the angles φ and φ_c.

4. In this section, we shall write down and discuss the results of the calculations of the relative damping Γ and the change in the sound velocity $\Delta s/s_0$ as functions of the angle φ:

$$\Gamma=-\mathrm{Im}\,\omega/\omega,\qquad \Delta s/s_0=(s-s_0)/s_0. \tag{19}$$

We consider the collision-free regime, in which ν can approach zero. In this limiting case, the real part of the conductivity σ_{xx} is easily obtained from (14), if we replace Im $(t-\beta)^{-1}$ in the integrand by $\pi\delta(t-x^{-1})$, where

$$x=\sin\varphi/\sin\varphi_c, \tag{20}$$

and then take the asymptote of the Bessel function $J_1[qR(1-x^{-2})^{1/2}]$ at large values of its argument. Here we shall not consider the very narrow region of angles φ directly adjacent to the critical angle φ_c, where this asymptote is inapplicable. The imaginary part of σ_{xx} is due to the contribution of all the values of t in the interval (14) and its asymptotic expression is obtained directly from (14) by the replacement of the function $J_1[z(1-t^2)^{1/2}]$ by its asymptote. Since the right side of (18) contains the small factor (qR), the expressions for Γ and $\Delta s/s_0$ have the form

$$\Gamma=\frac{A}{8\pi(qR)^2}\left[\cos\left(2qR-\frac{\pi}{4}\right)+\frac{2}{3}\frac{\varepsilon_F}{\lambda}\frac{q^2c^2(\pi qR)^{1/2}}{\omega_p^2}\right]^2 \times x^2\frac{(x^2-1)^{1/2}\sin^2[qR(1-x^{-2})^{1/2}-\pi/4]}{(x^2-1)\sin^4[qR(1-x^{-2})^{1/2}-\pi/4]+1/16}\theta(x-1), \quad (21)$$

$$\frac{\Delta s}{s_0}=\frac{A}{2qR}\left\{1+\frac{\sin(2qR-\pi/4)}{(\pi qR)^{1/2}}-\frac{x^2}{16\pi qR}\left[\cos(2qR-\pi/4)+\frac{2}{3}\frac{\varepsilon_F}{\lambda}\frac{q^2c^2(\pi qR)^{1/2}}{\omega_p^2}\right]^2 \times\left[(x^2-1)\sin^4[qR(1-x^{-2})^{1/2}-\pi/4]\theta(x-1)+1/16\right]^{-1}\right\}, \quad (22)$$

where $A=3N\lambda^2/\rho s^2\varepsilon_F$ is dimensionless constant of order unity. The unit step function $\theta(x-1)$ describes the turning on of the collision-free interaction at angles $\varphi>\varphi_c$.

A singular discrepancy effect in regions of weak fields is the oscillation dependence of Γ and $\Delta s/s_0$ on the angle φ, which is characterized by the presence of the factor $\sin^2[qR(1-x^{-2})^{1/2}-\pi/4]$ in Eqs. (21) and (22). These oscillations are due to electrons belonging to noncentral cross sections of the Fermi surface with a radius equal to $R(1-\varphi_c^2/\varphi^2)^{1/2}$. The character of the oscillations and their number in the interval $\varphi-\varphi_c$, which is of the order of several times the value of φ_c, depend significantly on the parameter qR, i.e., on the magnetic field H. In fact, oscillations of the collision-free absorption, upon change in x (the angle φ), are possible only under the condition that the argument of the sine in (21) can reach the values $\pi, 2\pi, \ldots$. More precisely, the quantity $qR(1-x^{-2})^{1/2}$ should, as x increases in the region $x>1(\varphi>\varphi_c)$, take on values μ_{1n} that are the roots of the first Bessel function $J_1(\mu)$. If $qR<\mu_{11}\approx 5\pi/4$, then oscillations of Γ are absent: if $\mu_{11}<qR<\mu_{12}$ there is a single oscillation; if $\mu_{12}<qR<\mu_{13}$ there are two oscillations, and so on. On the curve of the dependence of Γ on φ, the maxima of the oscillations are shifted to the direction of smaller φ upon increase in the parameter qR. Moreover, the amplitude of the oscillations of the absorption and sound velocity change as functions of H (i.e., qR) at a fixed φ in rather complicated fashion because of the imposition of oscillations from the central and noncentral cross sections on the Fermi surface in (21) and (22). We emphasize that the extrema on the angular dependence $\Gamma(\varphi)$ are not localized at the point $\varphi=\varphi_c$, but are located the further from φ_c the smaller the value of qR.

We have carried out numerical calculations of the angular dependence of the absorption and sound velocity according to the exact dispersion equation (18) without use of the asymptote of the conductivity σ_{xx}. The results are shown in Figs. 1–4. Figure 1 illustrates the monotonic dependence of $\Gamma(\varphi)$ and $\Delta s(\varphi)$. The monotonic character is due to the fact that the parameter $qR=4.5$ is not large enough and therefore the extrema of $\Gamma(\varphi)$ are shifted toward $\varphi>2\varphi_c$; furthermore, the quantity ω/ν is of the order of or greater than $(qR)^2$, so that the collision-free approximation (21), (22) turns out to be inadequate. The dependences of $\Gamma(\varphi)$ at large values of qR are shown in Figs. 2 and 3, and it is seen that there are oscillations of the absorption with angle—these are more

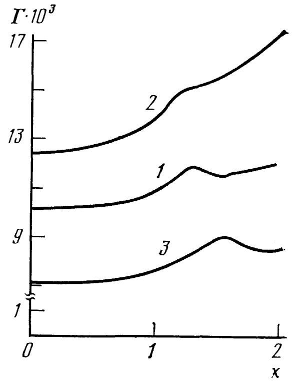

FIG. 2. Dependence of Γ on φ/φ_c at $\omega/\nu=6$ and different values of qR: 1—6.0, 2—7.5, 3—9.0.

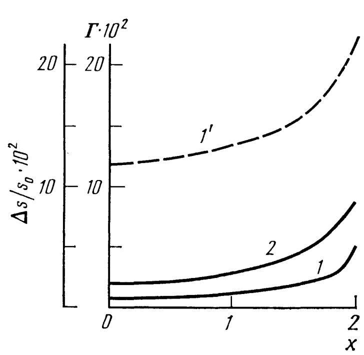

FIG. 1. Dependence of the relative damping (continuous curves) and the change in the sound velocity (dashed curve, scale to the left) on the tilt angle ($z=\varphi/\varphi_c$). The value of $qR=4.5$ for all curves; 1, 1′—$\omega/\nu=20$, 2—$\omega/\nu=6$.

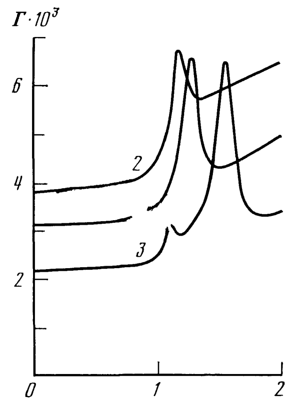

FIG. 3. Dependence of Γ on φ/φ_c at $\omega/\nu=20$ and different values of qR: 1—6.0, 2—7.5, 3—9.0.

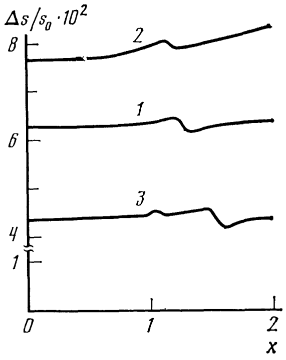

FIG. 4. Dependence of $\Delta s/s_0$ on φ/φ_c at $\omega/v = 20$ and different values of qR: 1—6.0, 1—7.5, 3—9.0.

pronounced the greater the value of ω/v. Finally, the dependence of $\Delta s/s_0$ on the angle φ is shown in Fig. 4. The tilt effect in the sound velocity turns out to be much weaker than in the absorption. This is connected with the fact that here oscillations from the noncentral cross section are present only in the denominator of the last term of Eq. (22). We note that there is a nonmonotonic dependence on qR in all the drawings, which is a consequence of the Pippard oscillations in Γ and $\Delta s/s_0$. So far as a comparison of the results of our work with experiments in tungsten[5] is concerned, such a comparison would be premature at the present time because of the tentative character and incompleteness of the experimental results.

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