

Theory of transverse runaway of hot electrons

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(Submitted 8 June 1984)

Zh. Eksp. Teor. Fiz. **87**, 1834–1841 (November 1984)

A theory of transverse runaway is developed. It is shown that it is related to the dependence of the heating field E on the scattering mechanism. General conditions for the development of the transverse runaway are identified and they are analyzed to find the specific combinations of the scattering mechanisms which cause this effect to appear.

The problem of the dependence of the nonequilibrium distribution function on the scattering mechanisms has been considered on many occasions. In all the published treatments the heating field E is assumed to be given and the attention has been concentrated entirely on a corresponding dependence of the heating function representing the deviation of the nonequilibrium distribution function from equilibrium. It is true even of the work of Levinson,¹ who was the first to introduce the concept of runaway of hot carriers in semiconductors. Levinson provided a classification of the possible types of runaway on the basis of an asymptotic behavior of the heating function. However, it has been shown later² that there is a new type of runaway of hot electrons associated with the presence of any nonzero transverse magnetic field. This is called the transverse runaway.²

We shall develop a theory of the transverse runaway. We shall show that it is related to the dependence of the heating field (consisting of the applied and Hall fields) on the scattering mechanisms. This dependence in turn is governed by the nature of the distribution function, which itself depends on the heating field. The problem is therefore self-consistent; the formation of the distribution function of hot carriers is governed not only by the dependence of the heating function on the scattering mechanisms, but also by the dependence of the heating field on these mechanisms. We shall consider the conditions under which the influence of a field E on the formation of the distribution function is the dominant one. We shall also obtain a general condition for the development of the transverse runaway.

To the best of our knowledge, the problem has not been formulated in this way before. The approach to this problem reveals a number of new effects.

We shall consider a semiconductor subjected to strong electric E and magnetic H fields. The isotropic parts of the nonequilibrium distribution function of hot electrons for the quasielastic energy dissipation by interaction with phonons of different types and of momentum by interaction with phonons or defects can be represented as follows:

$$f_0(x) \propto \exp \left\{ - \int \frac{dx}{1 + E^2 \theta(x)} \right\}, \quad (1)$$

where

$$\theta(x) = \frac{x^{(t+s)/2}}{E_0^2} \frac{1 + \eta x^t \cos^2 \beta}{1 + \eta x^t} \quad (2)$$

is the heating function; moreover,

$$E_0 = \frac{3^{1/2} kT}{e(l_0 \bar{l}_0)^{1/2}}, \quad \eta = \left(\frac{H}{H_0} \right)^2;$$

$$H_0 = \frac{(2mc^2 kT)^{1/2}}{el_0}, \quad x = \frac{\varepsilon}{kT};$$

β is the angle between \mathbf{E} and \mathbf{H} ; kT is the thermal energy; e and m are the charge and the effective mass of an electron. We shall assume that the energy dependences of the momentum (l) and energy (\bar{l}) mean free paths can be represented by

$$l(x) = l_0 x^{(t+1)/2}, \quad \bar{l}(x) = \bar{l}_0 x^{(s+1)/2}.$$

The values of t and s for all the known scattering mechanisms are listed in Ref. 3.

It is well known that the total current density is the sum of three currents:

$$\mathbf{j} = -en \left\{ \mu_1 \mathbf{E} + \mu_2 \frac{[\mathbf{E} \times \mathbf{H}]}{H} + \mu_3 \frac{\mathbf{H}(\mathbf{E} \cdot \mathbf{H})}{H^2} \right\}, \quad (3)$$

where n is the electron density. The mobility coefficients can be described in terms of $f_0(x)$ as follows:

$$\frac{\mu_1}{\mu_0} = \frac{A}{D}, \quad \frac{\mu_2}{\mu_0} = \frac{B_0}{D}, \quad \frac{\mu_3}{\mu_0} = \frac{C}{D}, \quad (4)$$

where

$$D = \left[\Gamma \left(\frac{t+5}{2} \right) / \Gamma \left(\frac{3}{2} \right) \right] \int_0^\infty x^{1/2} f_0(x) dx, \quad (5)$$

$$A = \int_0^\infty \left(- \frac{\partial f_0}{\partial x} \right) \frac{x^{(t+3)/2}}{1 + \eta x^t} dx, \quad (6)$$

$$B_0 = \sqrt{\eta} \int_0^\infty \left(- \frac{\partial f_0}{\partial x} \right) \frac{x^{(2t+3)/2}}{1 + \eta x^t} dx, \quad (7)$$

$$C = \eta \int_0^\infty \left(- \frac{\partial f_0}{\partial x} \right) \frac{x^{(3t+3)/2}}{1 + \eta x^t} dx, \quad (8)$$

μ_0 is the mobility in "zero" electric field, and $\Gamma(t)$ is the gamma function.

All three components of the total (heating) field E differ from zero; E_x is the applied field, E_y is the Hall field, and E_z is known as the longitudinal-transverse galvanomagnetic field. Selecting now different experimentally attainable boundary conditions (and not specifying them so far), we can express E_y and E_z in terms of E_x using the mobility coefficients of Eq. (4). In turn, these coefficients depend on E .

Consequently, we obtain the following complex equation for the heating field:

$$E^2 = E_x^2 \{1 + \Phi_1[t, s, E^{\varphi_1(t, s)}, H^{\varphi_2(t, s)}, \beta] + \Phi_2[t, s, E^{\varphi_3(t, s)}, H^{\varphi_4(t, s)}, \beta]\}. \quad (9)$$

The actual forms of the functions $\Phi_{1,2}$ and φ_{1-4} depend on the boundary conditions. Hence, it is clear that the heating field is a function of the parameters that govern the scattering mechanism (t, s), as well as a function of the fields (E_x, H) and of the angle β . It would be of interest to answer the following questions: 1) is there a combination of the scattering mechanisms for which a solution of Eq. (9) representing the heating field E becomes infinite when considered as a function of one of the parameters E_x, H , and β when the other two are fixed? 2) if such a combination does exist, how the heating function for such scattering mechanisms behaves in the asymptotic limit $x \rightarrow \infty$?

If $E \rightarrow \infty$, it follows from Eq. (1) that the distribution function is not normalizable. Consequently, hot carriers exhibit the transverse runaway effect. It is also obvious that there may be a transverse runaway threshold both in respect of the applied electric field and in respect of H and β . If for the scattering mechanisms corresponding to $E \rightarrow \infty$ the heating function $\theta(x)$ also tends to infinity for $x \rightarrow \infty$, then the nature of the runaway effect (nonself-sustained partly restrained scattering mechanisms¹ or transverse runaway) can be found by investigating the rate of rise of $\theta(x)$ for $x \rightarrow \infty$ and of E^2 for $E_x \rightarrow E_x^*$ (or $H \rightarrow H^*$ and $\beta \rightarrow \beta^*$).

We shall investigate the behavior in crossed fields under constant current conditions. When the relevant boundary conditions are used (open-circuited Hall contacts) and it is assumed that $\Phi_2 = 0$, Eq. (9) becomes

$$(\alpha/\alpha_x - 1)^{1/2} = \eta^{1/2} \Phi(\alpha, \eta), \quad (10)$$

where the following notation is used for convenience in further analysis:

$$\alpha = (E/E_0)^2, \quad \alpha_x = (E_x/E_0)^2, \quad \Phi(\alpha, \eta) = B_0/\eta^{1/2} A. \quad (11)$$

The conditions for the appearance of the transverse runaway expressed in terms of the applied electric and magnetic fields are of the form $\partial\alpha/\partial\alpha_x = \infty$ for $\eta_x = \text{const} \neq 0$ and $\partial\alpha/\partial\eta = \infty$ for $\alpha_x = \text{const}$, where

$$\frac{\partial\alpha}{\partial\alpha_x} = \frac{\alpha}{\alpha_x} \left(1 - 2\eta\alpha_x\Phi \frac{\partial\Phi}{\partial\alpha}\right)^{-1}, \quad (12)$$

$$\frac{\partial\alpha}{\partial\eta} = \alpha_x\Phi \left(\Phi + 2\eta \frac{\partial\Phi}{\partial\eta}\right) \left(1 - 2\eta\alpha_x\Phi \frac{\partial\Phi}{\partial\alpha}\right)^{-1}. \quad (13)$$

It follows from the above expressions that, firstly, the threshold values of the applied electric and magnetic fields are

$$\alpha_x^* = \left(2\eta\Phi \frac{\partial\Phi}{\partial\alpha}\right)^{-1}, \quad (14)$$

$$\eta^* = \left(2\alpha_x\Phi \frac{\partial\Phi}{\partial\alpha}\right)^{-1}, \quad (15)$$

which are inversely proportional to η in Eq. (14) and to α_x in

Eq. (15), which is expected on physical grounds. Secondly, the transverse runaway does not appear for the scattering mechanism such that the function Φ is either independent of the electric field (i.e., $\partial\Phi/\partial\alpha = 0$) or is a falling function of α (i.e., $\partial\Phi/\partial\alpha < 0$). Moreover, the magnetic field threshold exists if the numerator of Eq. (13) differs from zero. This requirement leads to the condition $\Phi \neq \text{const} \cdot \eta^{-1/2}$ when $\alpha \rightarrow \infty$. Consequently, the condition for the existence of the magnetic field threshold is determined by the explicit form of the dependence of Φ on η . If $\Phi \propto \eta^{-1/2}$, then there is no magnetic field threshold and it follows automatically from Eq. (14) that there is no dependence of α_x^* on the magnetic field.

We can see that the necessary conditions for the transverse runaway are the presence of any nonzero magnetic field and the existence of such scattering mechanisms that Φ is a growing function of α . The actual combination of the scattering mechanisms can be identified by finding first the nature of the dependence of Φ on α that ensures the transverse runaway. With this in mind, we shall substitute Eq. (14) into Eq. (10) and bear in mind that we then have $\alpha \rightarrow \infty$. We find that the inverse runaway appears if the asymptotic relationship is $\Phi \propto \alpha^{1/2}$ in the limit $\alpha \rightarrow \infty$. The same conclusion follows also from the requirement of the finite nature of the threshold fields of Eqs. (14) and (15), which is ensured if $\Phi \cdot \partial\Phi/\partial\alpha = \text{const}$ when $\alpha \rightarrow \infty$, which gives the asymptote just quoted.

Therefore, if we allow for the dependence $\Phi \propto \alpha^{1/2}$, $\Phi \neq \text{const} \cdot \eta^{-1/2}$ and use the general form of the distribution function, we can identify such a combination of the energy and momentum dissipation mechanisms for which there are transverse runaway thresholds in respect of the applied electric and magnetic fields.

Using Eqs. (1), (6), (7), and (11), we find that $\Phi(\alpha)$ is described by

$$\Phi(\alpha) = J_{(2t+3)/2}(\alpha) / J_{(t+3)/2}(\alpha), \quad (16)$$

where

$$J_k = \int_0^\infty x^k \frac{G(\alpha, x)}{F(\alpha, x)} dx,$$

$$G(\alpha, x) = \exp\left\{-\int \frac{1+\eta x^t}{F(\alpha, x)} dx\right\},$$

$$F(\alpha, x) = 1 + \eta x^t + \alpha x^{(t+s)/2}.$$

We shall investigate the asymptote of the function Φ at high values of α . We shall be interested in such combinations of t and s for which the main contribution to the integrals in Eq. (16) in the limit $\alpha \rightarrow \infty$ is made by large values of x , since the runaway occurs only in such a situation.

We shall establish first of all the values of t and s for which both integrals in Eq. (16) converge. It is found that $\Phi(\alpha)$ exists for those t and s which satisfy the condition

$$2+t-s > 0. \quad (17)$$

Considering now the asymptote $\Phi(\alpha)$ in the limit $\alpha \rightarrow \infty$,

we find that the main contribution to Φ comes from large values of x for the scattering mechanisms such that

$$5+t-s > 0, \quad 5-s > 0. \quad (18)$$

Therefore, the combination of t and s ensuring the transverse runaway should be sought among those t and s which make the inequalities (17) and (18) mutually compatible. In other words, it means that in the (t,s) plane the required combination corresponds to those points which lie in the region shown shaded in Fig. 1.

A calculation of the asymptote $\Phi(\alpha)$ for $\alpha \gg 1$ is quite difficult, because it requires an analysis of a large number of cases. It is therefore convenient to split the range of values of the parameters t and s shown shaded in Fig. 1 into regions I, II, III, and line IV, as shown in Fig. 2. This splitting is carried out as follows:

$$\text{I. } t > 0, \quad t+s > 2; \quad \text{II. } t > 0, \quad t+s < 2;$$

$$\text{III. } t < 0; \quad \text{IV. } t > 0, \quad t+s=2.$$

The following asymptotic relationships are obtained for these regions:

I) $\alpha \gg 1$,

$$\Phi = \left(1 + \frac{t-s}{2}\right)^{t/(2+t-s)} \frac{\Gamma_1}{\Gamma_3} \left(\frac{\alpha}{\eta}\right)^{t/(2+t-s)}. \quad (19)$$

IIa) $1 \ll \alpha \ll \eta^{(t+s-2)/2t}$,

$$\Phi = \left(1 - \frac{t+s}{2}\right)^{t/(2-t-s)} \frac{\Gamma_2}{\Gamma_4} \alpha^{t/(2-t-s)}, \quad (20a)$$

IIb) $\alpha \gg \eta^{(t+s-2)/2t}$,

$$\Phi = \left(1 + \frac{t-s}{2}\right)^{t/(2+t-s)} \frac{\Gamma_1}{\Gamma_3} \left(\frac{\alpha}{\eta}\right)^{t/(2+t-s)}. \quad (20b)$$

IIIa) $\alpha \gg 1, \alpha \gg \eta^{(s+t-2)/2|t|}$,

$$\Phi = \left(1 + \frac{t-s}{2}\right)^{t/(2+t-s)} \frac{\Gamma_1}{\Gamma_3} \left(\frac{\alpha}{\eta}\right)^{t/(2+t-s)}, \quad (21a)$$

IIIb) $1 \ll \alpha \ll \eta^{(s+t-2)/2|t|}$,

$$\Phi = \left(1 - \frac{t+s}{2}\right)^{t/(2-t-s)} \frac{\Gamma_2}{\Gamma_4} \alpha^{t/(2-t-s)}. \quad (21b)$$

IV) $\alpha \gg 1$,

$$\Phi = t^{1/2} \frac{\Gamma_5}{\Gamma_6} \left(\frac{\alpha}{\eta}\right)^{1/2}. \quad (22)$$

Here,

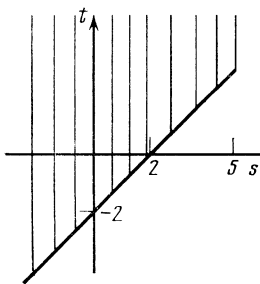


FIG. 1. The shaded region represents the numerical values of t and s that ensures the transverse runaway effect.

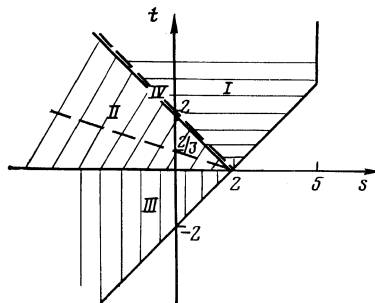


FIG. 2. Splitting of the (t,s) plane into regions for which the asymptotes $\Phi(\alpha)$ are calculated.

$$\Gamma_1 = \Gamma\left(\frac{5+t-s}{2+t-s}\right), \quad \Gamma_2 = \Gamma\left(\frac{5+t-s}{2-t-s}\right), \quad \Gamma_3 = \Gamma\left(\frac{5-s}{2+t-s}\right),$$

$$\Gamma_4 = \Gamma\left(\frac{5-s}{2-t-s}\right), \quad \Gamma_5 = \Gamma\left(\frac{3+t-s}{2t}\right), \quad \Gamma_6 = \Gamma\left(\frac{3-s}{2t}\right).$$

Using this asymptotic expressions for Φ , we find that the dependence $\Phi \propto \alpha^{1/2}$ is obtained only in the region II for $3t+s=2$ and in the region IV for any values of t and s (dashed lines in Fig. 2).

We thus find that the transverse runaway develops for the scattering mechanisms which satisfy the following conditions: 1) $t > 0, 3t+s=2$; in this case we find from Eq. (20a) that Φ does not depend exclusively on η , in other words there are thresholds in respect of the electric and magnetic fields; 2) $t > 0, t+s=2$; in this case we have $\Phi \propto (\alpha/\eta)^{1/2}$, which means that there is a threshold only in respect to the applied electric field. The latter conclusion is confirmed by the results of Ref. 2, where the case is considered.

An analysis of the asymptotic behavior of the heating function readily shows that the scattering mechanisms 1) and 2) are the retaining scattering mechanisms, in accordance with the Levinson classification.

We shall consider below the galvanomagnetic properties in the case when $t > 0, 3t+s=2$. Among the known scattering mechanisms these conditions are satisfied by the following³: $t = +1$ (momentum dissipated by dipoles, and by polarization optical and acoustic phonons considered in the approximations of high and low temperatures) and $s = -1$ (energy dissipated by deformation acoustic phonons in the approximations of high and low temperatures).

If $t = 1$ and $s = -1$, the distribution function is [see Eqs. (1) and (2)]

$$f_0(x) \propto \exp\left\{-x + \frac{\alpha}{\eta} \ln\left(1 + \frac{\eta x}{1+\alpha}\right)\right\}. \quad (23)$$

When an allowance is made for Eq. (23), it is found that A , B_0 , and D [see Eqs. (5)-(7)] are described by

$$A = \frac{1}{1+\alpha} \int_0^{\infty} x^2 \left(1 + \frac{\eta x}{1+\alpha}\right)^{\alpha/\eta-1} e^{-x} dx,$$

$$B_0 = \frac{1}{1+\alpha} \sqrt{\eta} \int_0^{\infty} x^{3/2} \left(1 + \frac{\eta x}{1+\alpha}\right)^{\alpha/\eta-1} e^{-x} dx, \quad (24)$$

$$D = \frac{\Gamma(3)}{\Gamma(3/2)} \int_0^{\infty} x^{1/2} \left(1 + \frac{\eta x}{1+\alpha}\right)^{\alpha/\eta} e^{-x} dx.$$

It is clear from Eq. (24) that the galvanomagnetic properties can be obtained in an analytic form only in various combinations, among which we can use the approximation of a weak magnetic field governed by the condition $\eta x / (1 + \alpha) \ll 1$. In this approximation a solution of Eq. (10) is

$$E = E_x \left[1 + B^2 \left(\frac{H}{H_0} \right) \right]^{1/2} \left[1 - B^2 \left(\frac{H}{H_0} \right) \left(\frac{E_x}{E_0} \right)^2 \right]^{-1/2},$$

$$B^2 \left(\frac{H}{H_0} \right) = 2.76 \left(\frac{H}{H_0} \right)^2.$$

If $H = \text{const}$, then for $E_x \rightarrow 0.6E_0H_0/H$ we can expect the transverse runaway. However, if $E_x = \text{const}$, the transverse runaway occurs for $H \rightarrow 0.6E_0H_0/E_x$.

Expressing the galvanomagnetic properties in terms of the applied field, we find that the Hall angle, Hall field, magnetoresistance, and Hall coefficient are given by

$$\text{tg } \theta = B \left(\frac{H}{H_0} \right) \left[\frac{1 + (E_x/E_0)^2}{1 - B^2 (H/H_0) (E_x/E_0)^2} \right]^{1/2},$$

$$E_y = B \left(\frac{H}{H_0} \right) \left[\frac{1 + (E_x/E_0)^2}{1 - B^2 (H/H_0) (E_x/E_0)^2} \right]^{1/2} E_x,$$

$$\rho = \frac{1}{1 + B^2 (H/H_0)} \frac{[1 - B^2 (H/H_0) (E_x/E_0)^2]^{1/2}}{[1 + (E_x/E_0)^2]^{1/2}},$$

$$\frac{R}{R_0} = \frac{B (H/H_0)}{1 + B^2 (H/H_0)} \left[1 - B^2 \left(\frac{H}{H_0} \right) \left(\frac{E_x}{E_0} \right)^2 \right],$$

where

$$\rho_0 = -(en\mu_0)^{-1}, \quad R_0 = -(en\mu_0 H_0)^{-1}.$$

The current-voltage characteristic is

$$\frac{J_x}{J_0} = [1 + B^2 (H/H_0)] \frac{[1 + (E_x/E_0)^2]^{1/2}}{[1 - B^2 (H/H_0) (E_x/E_0)^2]^{1/2}} \frac{E_x}{E_0},$$

where $J_0 = -en\mu_0 E_0$.

The general form of the dependence $E = f(E_x, H)$ was determined by solving Eq. (10) on a computer for various values of the magnetic field. We can see from Figs. 3 and 4 that for each value of magnetic field there is a characteristic threshold value of the applied electric field. An increase in the magnetic field reduces the threshold value of E_x and there is a tendency for the curves to merge forming one dependence. Hence, we may conclude that in the asymptotic limit at high values of magnetic fields the field E_x^* is independent of the magnetic field.

Having calculated the average energy, we can then find explicitly the condition that the magnetic field is weak:

$$H \ll H_0. \quad (25)$$

We shall now obtain estimates for the following scattering mechanisms: we shall assume that the momentum is dissipated by interaction with dipole centers and the energy by interaction with the deformation potential of acoustic phonons.

When we use the results of Refs. 4-6, we find that the conditions for the predominance of the momentum dissipation by dipole center over the dissipation of the momentum by ions and phonons is

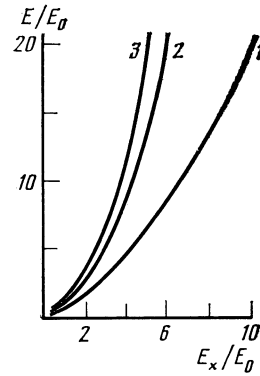


FIG. 3. Dependence of E on E_x in a weak magnetic field: 1) $\eta = 0.01$; 2) $\eta = 0.1$; 3) $\eta = 0.2$.

$$C_0 = 1, \quad N_D \gg \left(\frac{m}{m_0} \right)^{3/2} T^{3/2} \cdot 10^{15} \text{ cm}^{-3},$$

where $C_0 = N_A/N_D$ and N_A and N_D are the acceptor and donor concentrations.

In the case of Ge ($m = 0.2m_0$) at $T = 4.2$ K the effect should be observed when $N_D \gg 9 \times 10^{14} \text{ cm}^{-3}$, and at $T = 77$ K it should be observed for $N_D \gg 7 \times 10^{16} \text{ cm}^{-3}$. In the case of InSb ($m = 0.013m_0$) at the same temperatures the effect should be observed for $N_D \gg 10^{13}$ and $N_D \gg 10^{15} \text{ cm}^{-3}$, respectively.

When the average electron energy rises, the scattering by dipoles changes to the scattering by ions. This condition limits the applied electric field from above in accordance with the condition

$$\left(\frac{E_x}{E_0} \right)^2 < (A_0 - 1) \left[1 + A_0 B^2 \left(\frac{H}{H_0} \right) \right]^{-1}, \quad (26)$$

where

$$A_0 = 4\pi^2 \hbar^2 / 3mkT d^2, \quad d \propto N_D^{-1/2}.$$

In applied fields which satisfy an inequality which is the opposite of Eq. (26) the main scattering mechanisms are the dissipation of momentum by ions ($t = 3$) and the dissipation of energy by the deformation potential of acoustic phonons.

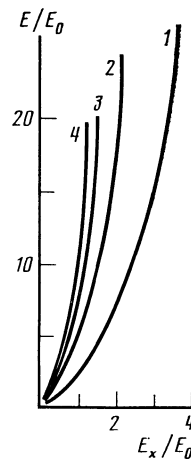


FIG. 4. Dependences of E on E_x in moderate and strong magnetic fields: 1) $\eta = 1$; 2) $\eta = 5$; 3) $\eta = 10$; 4) $\eta = 20$.

However, it is known from Ref. 2 that in this case the transverse runaway occurs and the threshold applied electric field is independent of the magnetic field.

In the limit $E_x \rightarrow E_x^*$ the transverse field rises strongly. In this field an electron may acquire an energy sufficient for the transverse breakdown. The increase in the energy continues until a restraining scattering mechanism becomes activated. If this mechanism is sufficiently weak, i.e., if it begins to act in the range of high energies, then in some semiconductors the transverse runaway effect may give rise to a transverse breakdown.

It therefore follows that the transverse runaway effects can give rise to a transverse breakdown in a fairly weak magnetic field.

The authors are grateful for the numerous discussions and valuable comments of V. L. Bonch-Bruevich and I. B. Levinson.

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Translated by A. Tybulewicz