Current damping in superconducting junctions with nonequilibrium electron distribution functions

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The effect of the nonequilibrium electron distribution functions in the superconductors on the properties of a tunneling junction is investigated. The state of nonequilibrium gives rise to a sharp increase in the effective temperature of the junction and an increase in the decay probability for the metastable current state. When the state of nonequilibrium is at a level higher than the threshold level, the effective temperature is negative. In this case phase and voltage oscillations arise in the junction at a current value lower than the critical value.

1. INTRODUCTION

The current states of a superconducting tunneling junction are metastable. At not too low temperatures the lifetime Γ^{-1} of such a state is determined by the thermal fluctuations, and is proportional to

$$\Gamma^{-1}\propto\exp(\delta U/T), \tag{1}$$

where T is the junction temperature and δU is the height of the potential barrier between neighboring minima:

$$\delta U=\frac{J_c}{e}\left[\left(1-\left(\frac{J}{J_c}\right)^2\right)^{1/2}-\frac{J}{J_c}\arccos\left(\frac{J}{J_c}\right)\right]. \tag{2}$$

In Eq. (2) J is the strength of the current flowing through the junction and J_c is the critical current of the junction.

In a junction with a small J_c value the lifetime Γ^{-1} of the current state is short when $T\sim T_c$, but can be fairly long at low temperatures.

At low temperatures it is easy to make the electron distribution function in a superconductor a nonequilibrium one. Such a disequilibrium state can be produced by a high-frequency field, by means of tunneling injection of excitations through an auxiliary junction, or by irradiating the junction by fast particles. In this case the lifetime of the current state of the junction can be greatly shortened. The temperature T in the formula (1) should in this case be replaced by an effective temperature T^* that depends on the nonequilibrium electron distribution function. The determination of this dependence is the object of the present paper.

Usually, the starting point of an investigation of the nonequilibrium situation is the kinetic equation. In the present case we must first of all derive such an equation. A characteristic of the case under consideration is the fact that we have a collective variable—the phase difference 2φ between the order parameters of the two superconductors—as the only distinct degree of freedom. The kinetic equation for the distribution function of this generalized coordinate is derived in two limiting cases: the case of low viscosity and the case in which the quasiclassical approximation is applicable.

For the investigation of the equilibrium situation the approach with imaginary time turns out to be convenient.[1–4] In the nonequilibrium case we shall use the method of functional integration in real time.[5]

2. EFFECTIVE ACTION

In superconductors separated by an impenetrable barrier, the physical quantities do not depend on the phases of the order parameters of the individual superconductors. A finite barrier penetrability leads to the appearance of a superconducting current that depends on the phase difference between the order parameters of the two superconductors. For the purpose of describing the quantum and thermal fluctuations we must treat the phase difference as a dynamical variable. The probability W_{if} for $(i\to f)$ transition between different states can be written in the form of a path integral over this dynamic variable[5]:

$$W_{if}=\int D\varphi\,\exp\{iA[\varphi]\};$$
$$A[\varphi]=-i\ln W[\varphi]+\int dt\left[\frac{C}{2e^2}\left(\frac{\partial\varphi}{\partial t}\right)^2+\frac{J\varphi}{e}\right], \tag{3}$$

where C is the capacitance of the junction. The quantity $W[\varphi]$ is equal to

$$\ln W[\varphi]=-\frac{1}{2}\iint dt dt_1\sum_{\mu\nu}|T_{\mu\nu}|^2\{\exp[i(\varphi(t)-\varphi(t_1))]\langle\hat{T}_c a_\nu(t)a_\nu^+(t_1)\rangle\langle\hat{T}_c a_\mu(t_1)a_\mu^+(t)\rangle$$
$$+\exp[-i(\varphi(t)-\varphi(t_1))]\langle\hat{T}_c a_\nu^+(t)a_\nu(t_1)\rangle\langle\hat{T}_c a_\mu^+(t_1)a_\mu(t)\rangle$$
$$-\exp[i(\varphi(t)+\varphi(t_1))]\langle\hat{T}_c a_\nu(t)a_\nu(t_1)\rangle\langle\hat{T}_c a_\mu^+(t_1)a_\mu^+(t)\rangle$$
$$-\exp[-i(\varphi(t)+\varphi(t_1))]\langle\hat{T}_c a_\nu^+(t)a_\nu^+(t_1)\rangle\langle\hat{T}_c a_\mu(t_1)a_\mu(t)\rangle\}, \tag{4}$$

where $T_{\mu\nu}$ is the transition matrix element connecting the states ν and μ of the left and the right superconductors, and $\hat{T}_c$ is the operator effecting the ordering on the Keldysh contour going traced the initial moment of time t_i to the final moment of time t_f and back; the integration in (3) and (4) is performed over this contour.

The Green functions in the formula (4) depend only on the energies of the states ν and μ, and possess a sharp maximum in the vicinity of the Fermi surface. The matrix ele-

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ment $|T_{\mu\nu}|^2$, averaged over the states at the Fermi surface, can be expressed in terms of the resistance R_N of the junction in the normal state. The summation over μ and ν gives rise to Green functions integrated over the energy variable.

It is convenient to go over from the integration over the Keldysh contour to integration from the instant t_i to the instant t_f, and introduce for this purpose the matrix Green functions

$$\hat{G}_s(t,t') = \begin{vmatrix} \hat{g}; & \hat{g}^< \\ -\hat{g}^>; & -\tilde{g} \end{vmatrix} = \frac{1}{2} \begin{vmatrix} \hat{g}^K+\hat{g}^R+\hat{g}^A; & \hat{g}^K-\hat{g}^R+\hat{g}^A \\ -(\hat{g}^K+\hat{g}^R-\hat{g}^A); & -(\hat{g}^K-\hat{g}^R-\hat{g}^A) \end{vmatrix}_{(t,t')}. \tag{5}$$

Each of the functions $\hat{g}$ is a (2×2) matrix in the Gor'kov sense:

$$\hat{g}(t,t') = \begin{vmatrix} g; & F \\ -F^+; & -\check{g} \end{vmatrix}_{(t,t')}.$$

Let us also introduce the (4×4) diagonal matrix $\hat{\varphi}(t)$ with elements

$$|\varphi_1(t); -\varphi_1(t); \varphi_2(t); -\varphi_2(t)|. \tag{6}$$

In this notation the formula (4) for $W[\varphi]$ assumes the form

$$W[\varphi] = \exp\left\{\frac{\pi}{4R_N e^2}\int_i^f\int dt dt_1\, \mathrm{Sp}[\exp(i\hat{\varphi}(t))\,\hat{G}^{(L)}(t,t_1) \times \exp(-i\hat{\varphi}(t_1))\,\hat{G}^{(R)}(t_1,t)]\right\}. \tag{7}$$

In the formula (7) the superscripts L and R denote the left and right superconductors respectively.

The Keldysh function $\hat{g}^K$ can be expressed in terms of the Green functions $\hat{g}^R$ and $\hat{g}^A$ with the aid of two distribution functions,[6] $f^{(1)}$ and $f^{(2)}$. Usually, even in the nonequilibrium case the Green functions $\hat{g}$ depend only on the time difference, and for the Fourier transforms we have

$$\hat{g}^K(\varepsilon) = \hat{g}^R(\varepsilon)\,(f^{(1)}(\varepsilon)+\tau_z f^{(2)}(\varepsilon)) - (f^{(1)}(\varepsilon)+\tau_z f^{(2)}(\varepsilon))\,\hat{g}^A(\varepsilon), \qquad \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{8}$$

Here $f^{(1)}(\varepsilon)$ is an odd, and $f^{(2)}(\varepsilon)$ an even, function of ε; they are connected with the occupation numbers $n(\varepsilon)$ by the relations

$$f^{(1)}(\varepsilon) = -n(\varepsilon)+n(-\varepsilon), \qquad f^{(2)}(\varepsilon)\,\frac{g^R(\varepsilon)-g^A(\varepsilon)}{2} = 1-[n(\varepsilon)+n(-\varepsilon)]. \tag{9}$$

The Green functions $g^{R,A}(\varepsilon)$ are little affected by the state of nonequilibrium, and can be replaced by their equilibrium values. Because of the long energy relaxation time, the distribution functions $f^{(1,2)}$ can change greatly even in the case of weak external influence.

The effective action given by the formulas (3) and (7) can be considered to be the action for a single particle with coordinate φ. The capacitance C/e^2 plays the role of particle mass and $\ln W$ describes the retarded potential and the transitions between the states in this potential. For a sufficiently large capacitance, when the oscillation frequency of the particle is low compared to Δ, the effects of the retardation and the transition probabilities are small. The effective potential in this case is equal to

$$U[\varphi] = -\left\{\frac{J-J_a}{e}\varphi + \frac{J_c}{2e}\cos(2\varphi+\alpha)\right\}, \tag{10}$$

where J_a, J_c, and α will be found below.

For current strengths J close to J_c the motion of the "particle" turns out to be slow even when the capacitance is small. In this case it is essential to renormalize the capacitance C, whose renormalized value can be found by separating from $\ln W$ the term proportional to $(\partial\varphi/\partial t)^2$. For identical superconductors on the left and right of the barrier

$$C^* = C + 3\pi/32R_N\Delta. \tag{11}$$

The C^* values for the case in which the superconductors are different is given in Ref. 5. In deriving the formula (11) we assumed that the current strength is close to the critical value and that the phase φ is close to the extremal value $\pi/4$. Furthermore, we took the electron distribution function in the superconductors at zero temperature. The degree of accuracy aimed at here would have been exceeded if we had allowed for the temperature dependence of C^*, since the contribution of such corrections is small compared to the contribution of the dissipative terms in $\ln W$.

For current strengths not close to the critical value, the effective capacitance C^* depends also on the phase φ. But in this case the adiabatic approximation holds only when the unrenormalized capacitance C is large and the renormalization effects are small.

3. THE CRITICAL CURRENT OF A NONEQUILIBRIUM JUNCTION

In the adiabatic approximation the phases φ_1 and φ_2 are slowly varying functions of the time. Furthermore, the phase φ_1 is close to φ_2. In this approximation the effective action $A[\varphi]$ can be written in the form

$$A_0[\varphi] = \frac{m^*}{2}\int dt\left[\left(\frac{\partial\varphi_1}{\partial t}\right)^2 - \left(\frac{\partial\varphi_2}{\partial t}\right)^2\right] - \int dt[U(\varphi_1)-U(\varphi_2)], \tag{12}$$

where $m^* = C^*/e^2$.

Evaluating the integral in the formula (7) in the adiabatic approximation, we obtain for $U(\varphi)$ the expression (10), where

$$\begin{aligned} J_a &= \frac{1}{8R_N e}\int d\varepsilon[g_L^R(\varepsilon)-g_L^A(\varepsilon)][g_R^R(\varepsilon)-g_R^A(\varepsilon)] \times [f_L^{(2)}(\varepsilon)-f_R^{(2)}(\varepsilon)]; \\ J_c\cos(2\varphi+\alpha) &= \frac{i}{8R_N e}\left\{\cos 2\varphi\int d\varepsilon[[F_L^R(\varepsilon)+F_L^A(\varepsilon)][F_R^R(\varepsilon) - F_R^A(\varepsilon)]f_R^{(1)}(\varepsilon)+[F_L^R(\varepsilon)-F_L^A(\varepsilon)] \times [F_R^R(\varepsilon)+F_R^A(\varepsilon)]f_L^{(1)}(\varepsilon)] \right. \\ &\left. +i\sin 2\varphi\int d\varepsilon[F_L^R(\varepsilon)+F_L^A(\varepsilon)] \times [F_R^R(\varepsilon)+F_R^A(\varepsilon)][f_R^{(2)}(\varepsilon)-f_L^{(2)}(\varepsilon)]\right\}. \end{aligned} \tag{13}$$

In superconductors without magnetic impurities

$$F^R(\varepsilon)=-[F^A(\varepsilon)]^*=\frac{\Delta}{[(\varepsilon+i\nu)^2-\Delta^2]^{1/2}},$$

$$g^R(\varepsilon)=-[g^A(\varepsilon)]^*=\frac{\varepsilon}{[(\varepsilon+i\nu)^2-\Delta^2]^{1/2}}.$$

In this approximation the angle α is small. For identical superconductors

$$J_c=\frac{\pi\Delta}{4R_Ne}[f_L^{(1)}(\Delta)+f_R^{(1)}(\Delta)]. \tag{14}$$

The quantity J_a is nonzero only in nonequilibrium superconductors, in which the even part of the distribution function $f_2(\varepsilon)$ is nonzero. In such nonequilibrium junctions the strength of the "critical" current depends on the direction, and is equal to $J_c \pm J_a$. If the junction is included in a superconducting circuit with a large inductance, then the total current J flowing through the junction, which is equal to $J=J_a+J_c\sin 2\varphi$, changes little when nonequilibrium excitations are produced in the superconductor. But now it is equal to the sum of the superconducting current and the current J_a produced by the nonequilibrium excitations. Therefore, there will occur a significant change in the phase φ and, consequently, in the magnitude of the magnetic flux in the circuit. In particular, to the ground state corresponds the magnetic flux

$$\Phi=\frac{\varphi}{e}=\frac{1}{2e}\arcsin\left(\frac{J_a}{J_c}\right). \tag{15}$$

The nonequilibrium distribution function $f^{(2)}$ can be produced through, for example, the injection of quasiparticles from an auxiliary tunneling junction with a sufficiently high voltage potential across it. The distribution function $f^{(2)}$ in this case is found in Ref. 7 (see also Refs. 8–10). In the notation of Ref. 7 the current J_a is equal to (pumping into the right-hand superconductor, in which the order parameter Δ has a large value)

$$J_a=\Delta_L\Delta_R^{1/4}(w/\gamma)^{1/2}(eV-\Delta_R)^{3/4}/2^{5/4}R_Ne(\Delta_R^2-\Delta_L^2)^{1/2}. \tag{16}$$

In the case of fast injection of the excitations there arise magnetic-flux oscillations that relax into the new equilibrium configuration (15). The relaxation time is determined by the viscosity, which will be found below.

4. PERTURBATION THEORY

In the adiabatic approximation the Josephson junction is eqivalent to a quantum particle with mass $m^*=C^*/e^2$ moving in the potential field $U(\varphi)$. In the zeroth approximation in the adiabaticity parameter, such a particle can occupy any quantum level for an infinitely long time. Allowance for the next terms leads to the appearance of a finite probability for transition between the states and the establishment of a distribution function $N(E)$ for these levels. Let us find the probabilities for the transitions $i\to f$ between the states under the assumption that these probabilities are small. The transitions between the states occur because of the presence in the effective action of terms that contain φ_1 and φ_2 at the same time. Assuming these terms to be small, and dropping them from the argument of the exponential function, we obtain

$$W_{if}=-\frac{\pi}{4R_Ne^2}\int dt\int dt_1\int D\varphi_1 D\varphi_2\exp(iA_0[\varphi])$$
$$\times\mathrm{Sp}\{\exp(i\varphi_1(t)\tau_z)\hat g_L^<(t,t_1)\exp(-i\varphi_2(t_1)\tau_z)\hat g_R^>(t_1,t)$$
$$+\exp(i\varphi_2(t)\tau_z)\hat g_L^>(t,t_1)\exp(-i\varphi_1(t_1)\tau_z)\hat g_R^<(t_1,t)\}. \tag{17}$$

The functional integral in the formula (17) is computed with the effective action $A_0[\varphi]$, which corresponds to the motion of a particle in the potential field $U(\varphi)$. Therefore, it can be expressed in the usual manner in terms of the matrix elements of the quantum-mechanical problem. The Green functions $\hat g^{<}$ depend on the time difference $t-t_2$. Therefore, for sufficiently long periods W_{if} is proportional to the transition time t_f-t_i, i.e., $W_{if}=(t_f-t_i)\mathscr{W}_{if}$. Expressing the Green functions $\hat g^{<}$ in terms of the distribution functions $f^{(1,2)}$ in accordance with the formulas (5) and (8), we obtain

$$\mathscr{W}_{if}=\frac{1}{2R_Ne^2}\int d\varepsilon\{-|\langle f|\exp(i\varphi)|i\rangle|^2\rho_L(\varepsilon)\rho_R(\varepsilon-\omega)$$
$$\times[(f_L^{(1)}(\varepsilon)-1)(f_R^{(1)}(\varepsilon-\omega)+1)+f_L^{(2)}(\varepsilon)f_R^{(2)}(\varepsilon-\omega)]$$
$$+{}^1/_2[\langle f|\exp(i\varphi)|i\rangle^2+\langle f|\exp(-i\varphi)|i\rangle^2]$$
$$\times[\mathscr{F}_L^-(\varepsilon)\mathscr{F}_R^-(\varepsilon-\omega)$$
$$\times(f_L^{(1)}(\varepsilon)-1)(f_R^{(1)}(\varepsilon-\omega)+1)$$
$$-\mathscr{F}_L^+(\varepsilon)\mathscr{F}_R^+(\varepsilon-\omega)f_L^{(2)}(\varepsilon)f_R^{(2)}(\varepsilon-\omega)]$$
$$+{}^1/_2[\langle f|\exp(i\varphi)|i\rangle^2$$
$$-\langle f|\exp(-i\varphi)|i\rangle^2][\mathscr{F}_L^-(\varepsilon)\mathscr{F}_R^+(\varepsilon-\omega)$$
$$\times(f_L^{(1)}(\varepsilon)-1)f_R^{(2)}(\varepsilon-\omega)$$
$$-\mathscr{F}_L^+(\varepsilon)\mathscr{F}_R^-(\varepsilon-\omega)f_L^{(2)}(\varepsilon)(f_R^{(1)}(\varepsilon-\omega)+1)]\} \tag{18}$$

where $\omega=E_f-E_i$ and

$$\rho(\varepsilon)=(g^R(\varepsilon)-g^A(\varepsilon))/2,\quad \mathscr{F}^-(\varepsilon)=(F^R(\varepsilon)-F^A(\varepsilon))/2,$$
$$\mathscr{F}^+(\varepsilon)=(F^R(\varepsilon)+F^A(\varepsilon))/2. \tag{19}$$

In deriving the formula (18) we took into account the fact that $f^{(1)}(\varepsilon)$ and $\mathscr{F}^-(\varepsilon)$ are odd functions of ε, while $f^{(2)}(\varepsilon)$, $\rho(\varepsilon)$, and $\mathscr{F}^+(\varepsilon)$ are even functions. In superconductors without magnetic impurities

$$\rho(\varepsilon)=\frac{|\varepsilon|\theta(|\varepsilon|-\Delta)}{(\varepsilon^2-\Delta^2)^{1/2}},\quad \mathscr{F}^-(\varepsilon)=\frac{\Delta\rho(\varepsilon)}{\varepsilon},$$
$$\mathscr{F}^+(\varepsilon)=\frac{-i\Delta\theta(\Delta-|\varepsilon|)}{(\Delta^2-\varepsilon^2)^{1/2}}. \tag{20}$$

In this case the quantity $f^{(2)}(\varepsilon)\mathscr{F}^+(\varepsilon)$ is small, and the last three terms in the formula (18) vanish.

For the majority of the numbers i the matrix elements in the formula (18) can be expressed in terms of the Fourier components of the quantity $\exp(\pm i\varphi)$ on the classical trajectory:

$$\langle f|\exp(\pm i\varphi)|i\rangle=\frac{\omega}{2\pi}\oint\exp(\pm i\varphi(t))\exp(-i\omega(f-i)t)\,dt, \tag{21}$$

where $\oint$ denotes integration over a period of the motion and ω is the frequency of the classical periodic motion. For the low-lying energy levels we can use the oscillator approximation. In this case the only nonzero transition matrix elements are the ones connecting neighboring levels:

$$\langle j-1|\exp(\pm i\varphi)|j\rangle=\langle j|\exp(\pm i\varphi)|j-1\rangle = (\pm i)\exp(\pm i\varphi_0)\left(\frac{je^2}{2C^*\Omega}\right)^{1/2}, \quad (22)$$

where φ_0 is the minimum point of the potential $U(\varphi)$ (the formula (10)) and Ω is the frequency of the oscillations about the minimum:

$$\Omega^2=\frac{e^2}{C^*}\left(\frac{\partial^2 U}{\partial\varphi^2}\right)_{\varphi_0}. \quad (23)$$

5. THE KINETIC EQUATION

The distribution function N_i, which is equal to the probability of finding the quantum particle in the ith state, satisfies the equation

$$-\partial N_i/\partial t=\sum_j (N_j\mathscr{W}_{ij}-N_i\mathscr{W}_{ji}). \quad (24)$$

For an equilibrium electron distribution, when $f^{(2)}=0$ and $f^{(1)}(\varepsilon)=\tanh(\varepsilon/2T)$, it follows from the formula (18) that the transition probabilities $\mathscr{W}_{ij}$ satisfy the relation

$$\mathscr{W}_{ij}=\mathscr{W}_{ji}\exp[-(E_j-E_i)/T]. \quad (25)$$

From this it follows that Eq. (24) possesses in this case the steady-state solution

$$N_i=\exp(-E_i/T). \quad (26)$$

In the case of a nonequilibrium electron distribution the transition probabilities $\mathscr{W}_{ij}$ are not connected by a simple relation of the form (25). But in this case also a steady-state solution to Eq. (24) can be sought in the form

$$N_i=\exp\left(-\int^{E_i} dE_i/T^*(E_i)\right). \quad (27)$$

The quantity $T^*(E)$ plays the role of an effective temperature, and varies little over energy ranges of the order of the level spacing. The matrix elements $\mathscr{W}_{ij}$ decrease rapidly as the difference $|i-j|$ increases. Therefore, from the formulas (24) and (26) we obtain the following equation for the effective temperature:

$$\sum_j \{\mathscr{W}_{ij}-\mathscr{W}_{ji}\exp[-(E_j-E_i)/T^*(E_i)]\}=0. \quad (28)$$

6. HIGH EFFECTIVE TEMPERATURE

Let us consider the important particular case when the effective temperature is high compared to the characteristic frequency $\omega_{ij}\sim\Omega$ at which the drop in the transition probability $\mathscr{W}_{ij}$ occurs. When this condition is fulfilled, the distribution function varies slowly, and Eq. (24) reduces to the differential equation

$$-\frac{1}{\omega(E)}\frac{\partial N(E)}{\partial t}+\frac{\partial}{\partial E}\left[\frac{AN(E)}{\omega(E)}+\frac{B}{\omega(E)}\frac{\partial N(E)}{\partial E}\right]=0, \quad (29)$$

where $\omega(E)$ is the distance between nearest levels, which is equal to the frequency of the classical motion in the potential well $U(\varphi)$,

$$A=\sum_j (E_j-E_i)\mathscr{W}_{ji}, \qquad B=\frac{1}{2}\sum_j (E_j-E_i)^2\mathscr{W}_{ji}. \quad (30)$$

In the steady state case the solution to Eq. (29) has the form (27), with the effective temperature

$$T^*(E)=B/A. \quad (31)$$

The expression (31) for T^* can also be obtained from the formula (28). Let us assume that the electron distribution function varies little at energies $\varepsilon\sim\Omega$. Using for the matrix elements the formula (21), we obtain for the effective temperature [Eq. (31)], in the case of identical superconductors on the left and right of the barrier, the expression

$$T^*(E)=\left\{\oint dt\left(\frac{\partial\varphi}{\partial t}\right)^2\int d\varepsilon[\rho^2(\varepsilon)+\cos 2\varphi(\mathscr{F}^-(\varepsilon))^2] \times(1-f_L^{(1)}(\varepsilon)f_R^{(1)}(\varepsilon))\right\} \left\{\oint dt\left(\frac{\partial\varphi}{\partial t}\right)^2\int d\varepsilon[\rho^2(\varepsilon)+\cos 2\varphi(\mathscr{F}^-(\varepsilon))^2]\right| \times\left(\frac{\partial f_L^{(1)}(\varepsilon)}{\partial\varepsilon}+\frac{\partial f_R^{(1)}(\varepsilon)}{\partial\varepsilon}\right)\Bigg\}, \quad (32)$$

where $\varphi(t)$ is the solution to the classical equation of motion with energy E. In (32) we have dropped the terms proportional to the product $f_L^{(2)}(\varepsilon)f_R^{(2)}(\varepsilon)$, which are usually small. Equation (32) can be simplified further if the current strength J is close to the critical value, and $\cos 2\varphi=0$, or if the correction to the distribution function $f^{(1)}$ is concentrated in the vicinity of the threshold in the narrow energy region where $|\mathscr{F}^-(\varepsilon)|=\rho(\varepsilon)$. In both of these cases the time integrals in the numerator and denominator of (32) cancel out, so that

$$T^*=\int d\varepsilon\rho^2(\varepsilon)\,(1-f_L^{(1)}(\varepsilon)f_R^{(1)}(\varepsilon))\Big/ \int d\varepsilon\rho^2(\varepsilon)\left(\frac{\partial f_L^{(1)}(\varepsilon)}{\partial\varepsilon}+\frac{\partial f_R^{(1)}(\varepsilon)}{\partial\varepsilon}\right). \quad (33)$$

If only one superconductor is pumped, and the correction to the distribution function $f^{(1)}$ has the form of a step of width $eV-\Delta$ concentrated in the neighborhood of the gap in the superconductor, then

$$T^*=(eV-\Delta)\ln\left(\frac{eV-\Delta}{\Omega}\right). \quad (34)$$

In deriving (34) we found the numerator in the expression (33) for T^* with logarithmic accuracy. If the correction to the distribution function does not have the form of a step, then the denominator of (33) also contains a large logarithm. In this case

$$T^*=[1-f_L^{(1)}(\Delta)f_R^{(1)}(\Delta)]\Big/\left[\frac{\partial f_L^{(1)}(\varepsilon)}{\partial\varepsilon}+\frac{\partial f_R^{(1)}(\varepsilon)}{\partial\varepsilon}\right]_{\varepsilon=\Delta}. \quad (35)$$

The case of superconductors with different order-parameter values $\Delta_{L,R}$ will be considered below.

7. THE LANGEVIN EQUATION

For the motion in the quasiclassical region, we derive, following Schmid,[11] the Langevin equation for the phase φ. In this case the phases $\varphi_{1,2}$ at one and the same instant t in the "forward" and "backward" motions in time are close:

$$\varphi_{1,2}=\varphi\pm y/2.$$

Expanding the effective action $A[\varphi]$ up to terms of second order in y, and evaluating the Gaussian functional integral with respect to y, we obtain for φ the Langevin equation

$$\frac{\delta A[\varphi]}{\delta y}\Big|_{y=0}=\xi, \tag{36}$$

where ξ is a random quantity with a Gaussian distribution and with a correlator:

$$\langle\xi(t)\xi(t')\rangle=K(t,t')=-i\frac{\delta^2 A}{\delta y(t)\delta y(t')}. \tag{37}$$

Using Eqs. (3) and (7) for the action $A[\varphi]$, we obtain the expression

$$K(t,t')=-\frac{\pi}{8R_N e^2}\mathrm{Sp}\{\tau_z\exp(i\varphi(t)\tau_z)\times[\hat g_L^<(t,t_1)\exp(-i\varphi(t_1)\tau_z)\tau_z\hat g_R^>(t_1,t)+\hat g_L^>(t,t_1)\exp(-i\varphi(t_1)\tau_z)\tau_z\hat g_R^<(t_1,t)]+(t\rightleftarrows t_1)\}. \tag{38}$$

In the region of high temperatures, where T is much greater than the characteristic frequency of variation of the phase φ, the correlator $K(t,t')$ is a δ-function of the time difference $t-t'$, i.e., the noise is converted into white noise:

$$K(t,t')=K_0(\omega)\delta(t-t'),\quad \omega=eV=\partial\varphi/\partial t;$$

$$K_0(\omega)=\frac{1}{2R_N e^2}\int_{-\infty}^{\infty}d\varepsilon\{\rho_L(\varepsilon)\rho_R(\varepsilon-\omega)[1-f_L^{(1)}(\varepsilon)f_R^{(1)}(\varepsilon-\omega)-f_L^{(2)}(\varepsilon)f_R^{(2)}(\varepsilon-\omega)]$$

$$+\cos 2\varphi(t)[\mathcal{F}_L^-(\varepsilon)\mathcal{F}_R^-(\varepsilon-\omega)[1-f_L^{(1)}(\varepsilon)f_R^{(1)}(\varepsilon-\omega)]+\mathcal{F}_L^+(\varepsilon)\mathcal{F}_R^+(\varepsilon-\omega)f_L^{(2)}(\varepsilon)f_R^{(2)}(\varepsilon-\omega)]$$

$$+i\sin 2\varphi(t)[\mathcal{F}_L^+(\varepsilon)\mathcal{F}_R^-(\varepsilon-\omega)f_L^{(2)}(\varepsilon)f_R^{(1)}(\varepsilon-\omega)-\mathcal{F}_L^-(\varepsilon)\mathcal{F}_R^+(\varepsilon-\omega)f_L^{(1)}(\varepsilon)f_R^{(2)}(\varepsilon-\omega)]\}. \tag{39}$$

For example, for a junction with different superconductors, in the case when the superconductor with the higher order-parameter value Δ_R is pumped,

$$K_0(\omega)=\frac{\Delta_R+\Delta_L\cos 2\varphi}{R_N e^2(\Delta_R^2-\Delta_L^2)^{1/2}}\left\{(2\pi T\Delta_R)^{1/2}\exp\left(-\frac{\Delta_R}{T}\right)\mathrm{ch}\left(\frac{\omega}{T}\right)+\int_{\Delta_R}^{\infty}\frac{d\varepsilon\,\varepsilon}{(\varepsilon^2-\Delta_R^2)^{1/2}}[1-f_R^{(1)}(\varepsilon)]\right\}. \tag{39a}$$

In the effective-temperature problem under consideration in this paper the voltage drop V across the junction is equal to zero. We have nevertheless cited a more general expression, (39), for the correlator $K_0(\omega)$, i.e., an expression that is valid for a finite, slowly varying voltage potential across the junction, since this expression may be useful in other problems, e.g., in the problem of the emission-line width of a Josephson junction.[12] The formula (39) describes thermal noise ($eV\ll T$) and shot noise ($eV\gg T$).

In this approximation Eq. (36) has the form

$$m^*\frac{\partial^2\varphi}{\partial t^2}-\frac{J}{e}+I_1(\omega)\sin 2\varphi+I_2(\omega)\cos 2\varphi+I_3(\omega)=\xi, \tag{40}$$

where J is the total current flowing through the junction,

$$I_1(\omega)=\frac{i}{2R_N e^2}\int_{-\infty}^{\infty}d\varepsilon\{\mathcal{F}_L^+(\varepsilon-\omega)\mathcal{F}_R^-(\varepsilon)[f_R^{(1)}(\varepsilon)+f_L^{(2)}(\varepsilon-\omega)]+\mathcal{F}_L^-(\varepsilon-\omega)\mathcal{F}_R^+(\varepsilon)[f_L^{(1)}(\varepsilon-\omega)+f_R^{(2)}(\varepsilon)]\},$$

$$I_2(\omega)=\frac{1}{2R_N e^2}\int d\varepsilon\{\mathcal{F}_L^-(\varepsilon)\mathcal{F}_R^-(\varepsilon-\omega)[f_L^{(1)}(\varepsilon)-f_R^{(1)}(\varepsilon-\omega)]-\mathcal{F}_L^+(\varepsilon)\mathcal{F}_R^+(\varepsilon-\omega)[f_L^{(2)}(\varepsilon)-f_R^{(2)}(\varepsilon-\omega)]\}, \tag{41}$$

$$I_3(\omega)=\frac{1}{2R_N e^2}\int d\varepsilon\rho_L(\varepsilon)\rho_R(\varepsilon-\omega)\times[f_L^{(1)}(\varepsilon)+f_L^{(2)}(\varepsilon)-f_R^{(1)}(\varepsilon-\omega)-f_R^{(2)}(\varepsilon-\omega)].$$

In the case when the phase φ varies slowly Eq. (40) has the form

$$m^*\frac{\partial^2\varphi}{\partial t^2}+\frac{\partial U(\varphi)}{\partial\varphi}+\eta\frac{\partial\varphi}{\partial t}=\xi, \tag{42}$$

where the potential energy $U(\varphi)$ is given by (20) and η is the coefficient of viscosity:

$$\eta=\frac{\partial I_3(\omega)}{\partial\omega}+\cos 2\varphi\frac{\partial I_2(\omega)}{\partial\omega}+\sin 2\varphi\frac{\partial I_1(\omega)}{\partial\omega}. \tag{43}$$

In the case of weak viscosity the equation (29) of diffusion in energy terms can be obtained from Eq. (42). The coefficients A and B [the formulas in (30)] are connected with the coefficient of viscosity η and the kernel K_0 by the relations

$$A(E)=\frac{\omega(E)}{2\pi}\oint dt\left(\frac{\partial\varphi}{\partial t}\right)^2\eta(\varphi),\qquad B=\frac{\omega(E)}{4\pi}\oint dt\left(\frac{\partial\varphi}{\partial t}\right)^2 K_0(\varphi). \tag{44}$$

Thus, from the Langevin equation we obtain for the effective temperature T^* the same expression obtained from the high-T^* perturbation theory [Eq. (31)]. In particular, in the case of identical superconductors we obtain for the effective temperature the formulas (32) and (33).

8. NEGATIVE TEMPERATURES

In the case when the superconductors on the left and

right of the barrier are different and the superconductor with the higher order-parameter value (say, the right-hand superconductor: $\Delta_R > \Delta_L$) is pumped, the integrals $I_{2,3}$ and the coefficient of viscosity η are equal to

$$I_2 = \frac{\Delta_L(2\pi T\Delta_R)^{1/2}}{R_N e^2(\Delta_R^2-\Delta_L^2)^{1/2}} \exp\left(-\frac{\Delta_R}{T}\right)\mathrm{sh}\left(\frac{\omega}{T}\right) - \frac{\omega\Delta_L\Delta_R\tilde{N}}{R_N e^2(\Delta_R^2-\Delta_L^2)^{1/2}},$$

$$I_3 = \frac{\Delta_R(2\pi T\Delta_R)^{1/2}}{R_N e^2(\Delta_R^2-\Delta_L^2)^{1/2}} \exp\left(-\frac{\Delta_R}{T}\right)\mathrm{sh}\left(\frac{\omega}{T}\right) - \frac{\omega\Delta_L^2\tilde{N}}{R_N e^2(\Delta_R^2-\Delta_L^2)^{1/2}},$$

$$\eta(\omega=0) = \frac{\Delta_R}{R_N e^2(\Delta_R^2-\Delta_L^2)^{1/2}} \times\left[\left(\frac{2\pi\Delta_R}{T}\right)^{1/2}\exp\left(-\frac{\Delta_R}{T}\right)\left(1+\frac{\Delta_L}{\Delta_R}\cos 2\varphi\right) - \frac{\Delta_L\tilde{N}}{\Delta_R^2-\Delta_L^2}\left(\cos 2\varphi+\frac{\Delta_L}{\Delta_R}\right)\right],$$

$$\tilde{N} = \int_{\Delta_R}^{\infty}\frac{d\varepsilon\varepsilon}{(\varepsilon^2-\Delta_R^2)^{1/2}}[1-f_R^{(1)}(\varepsilon)]. \qquad (45)$$

At low temperatures the viscosity coefficient η can vanish and become negative even under conditions of weak pumping. At the pumping level at which the viscosity coefficient η vanishes, the static solution to Eq. (40) with $J<J_c$ is unstable. The question of the loss of stability under conditions of high-frequency irradiation of the junction was first considered by Aronov and Spivak.[13] They investigated the contribution of only the first term of the formula (43) to the viscosity. Notice that the second term of the formula (43) becomes negative at a lower pumping level. As a result there arise a number of interesting effects: the instability threshold depends on the strength of the current flowing through the junction, and increases with increasing current strength. Another effect, which occurs at $J=0$, is that the negative temperature first arises for the state with minimum energy. Therefore, the energy distribution function for the contact has a sharp peak in the vicinity of the point E_{cr} at which $T(E_{cr})=\infty$. The higher-energy states have positive temperature, and the phase φ exists in a separate potential well for an exponentially long time. From the formula (44) we find the equation for E_{cr}:

$$\int_{\varphi_1}^{\varphi_2} d\varphi[E_{cr}-U(\varphi)]^{1/2}\eta(\varphi)=0,$$

where $\varphi_{1,2}$ are the turning points.

In the vicinity of this energy the distribution function is equal to

$$N(E)\propto\exp[-(E-E_{cr})^2\beta];$$

here

$$\beta = \frac{\partial}{\partial E}\left(\frac{1}{T^*}\right) = \frac{1}{B}\left(\frac{\partial A}{\partial E}\right)_{cr},$$

$$\left(\frac{\partial A}{\partial E}\right)_{cr} = \frac{\omega(E_{cr})}{\pi(2m^*)^{1/2}}\int_{\varphi_1}^{\varphi_2}\frac{d\varphi\eta(\varphi)}{[E_{cr}-U(\varphi)]^{1/2}},$$

$$\omega(E) = \frac{2^{1/2}\pi}{(m^*)^{1/2}}\left\{\int_{\varphi_1}^{\varphi_2}\frac{d\varphi}{[E-U(\varphi)]^{1/2}}\right\}^{-1}.$$

The noise spectrum possesses a sharp peak at the frequency $\omega(E_{cr})$ of the classical motion:

$$I(\omega)\propto\exp[-\beta(\omega-\omega(E_{cr}))^2/(\partial\omega/\partial E)^2].$$

At sufficiently high pumping levels the effective temperature can, depending on the current strength and the value of the ratio Δ_R/Δ_L, be negative at all energy values $E<\delta U$. In this case there arises across the junction a high voltage is close to $eV=\Delta_L+\Delta_R$.

9. LOW EFFECTIVE TEMPERATURES

Usually, the capacitance of a Josephson junction is fairly high. Therefore, the level spacing Ω in the potential $U(\varphi)$ is small, and the equation (29) of diffusion in energy terms is applicable in a broad range of temperatures $T\gg\Omega$. Low-capacitance junctions have been developed in the last few years, so that the temperature region $T\lesssim\Omega$ has become accessible. In this temperature region, the diffusion approximation is inapplicable, and, to determine the effective temperature $T^*(E)$, we must use the general equation (28).

For the low-lying levels the phase φ is close to the external value φ_0; therefore, we can replace the potential $U(\varphi)$ by the oscillator potential, and use for the matrix elements the formula (22). In this approximation Eq. (28) can be easily solved, and for the effective temperature $T^*(E)$ we obtain the expression

$$\exp(\Omega/T^*(i)) = \mathscr{W}_{i,i-1}/\mathscr{W}_{i-1,i} = \mathscr{W}_{i+1,i}/\mathscr{W}_{i,i+1} = \frac{K_0(\Omega,\varphi_0)+\tilde{I}_3(\Omega)+\cos 2\varphi_0\tilde{I}_2(\Omega)+\sin 2\varphi_0\tilde{I}_1(\Omega)}{K_0(\Omega,\varphi_0)-\tilde{I}_3(\Omega)-\cos 2\varphi_0\tilde{I}_2(\Omega)-\sin 2\varphi_0\tilde{I}_1(\Omega)},$$
$$\tilde{I}_k(\Omega) = {}^1/_2[I_k(\Omega)-I_k(-\Omega)], \quad k=1,2,3, \qquad (46)$$

where the quantities K_0 and I_k are given by the formulas (39), (39a), (41), and (45). When T and T^* are much higher than Ω, the formula (46) goes over into the expression (31) for T^*.

For the high-lying levels the oscillator approximation is inapplicable, but the matrix elements can be found from the quasiclassical formula (21). If the current strength J is close to the critical value, the phase φ is close to $\pi/4$, and the potential $U(\varphi)$ has the form of a cubical parabola. In this case the matrix elements

$$\langle f|\varphi|i\rangle = -\frac{\pi^2 n(\varphi_2-\varphi_3)}{2k^2K^2(k)\,\mathrm{sh}(n\tilde{\alpha})}. \qquad (47)$$

Here

$$n=f-i, \quad k=\left(\frac{\varphi_2-\varphi_3}{\varphi_1-\varphi_3}\right)^{1/2}, \quad \tilde{\alpha}=\frac{\pi K'(k)}{K(k)}$$

where $\varphi_3<\varphi_2<\varphi_1$ are the roots of the equation $E_i-U(\varphi)=0$, while $K(k)$ and $K'(k)$ are complete elliptic integrals. Equation (28) for the effective temperature assumes the form

$$\sum_{n=-\infty}^{\infty}\frac{n^3}{\mathrm{sh}^2 n\tilde{\alpha}}\{K_0(n\omega,\pi/4)-I_3(n\omega)-I_4(n\omega)\}\times\left(1-\exp\left(\frac{n\omega}{T^*}\right)\right)=0,$$

$$\omega=\frac{\pi}{K(k)}\left[\frac{eJ_c}{3C^*}(\varphi_1-\varphi_3)\right]^{1/2}, \tag{48}$$

where ω is the frequency of the classical motion in the potential field $U(\varphi)$ of a particle with energy E_i. When the energy E_i is close to the height of the potential barrier, the frequency ω and the coefficient $\tilde{\alpha}$ are close to zero. For such energies the sum over n in the formula (48) can be replaced by an integral. In this case the level spacing ω is small, but the transition to the high-temperature limit occurs only when $T^* \gg \omega/\tilde{\alpha} \sim \Omega$.

10. LIFETIME OF THE METASTABLE CURRENT STATE

The qualitative picture of the decay of the metastable state in the nonequilibrium case will look the same as in the equilibrium case. There exist two broad regions, depending on the magnitude of the effective temperature T^*: the classical region of high T^* values, in which the decay occurs as a result of above-the-barrier transitions, and the low-temperature region, where the decay is due to quantum-mechanical tunneling. Each of these regions can, depending on the magnitude of the viscosity, be divided into two regions: regions of strong and weak viscosity. In the high-temperature region we have with exponential accuracy a decay probability proportional to the distribution function at the top of the barrier. In the case of weak viscosity, when $\eta/m^* < \Omega$, the distribution function is given by the formula (27). In the case of strong viscosity, when $\eta/m^* > \Omega$, we can obtain from the Langevin equation (36), (40) a diffusion equation in terms of the coordinate φ. The solution to this equation has the form

$$N(\varphi) \propto \exp\left\{-2\int d\varphi \frac{\partial U(\varphi)}{\partial \varphi}\frac{\eta(\varphi)}{K_0(\varphi)}\right\}. \tag{49}$$

In those cases when the effective temperature $T^*(\varphi)=K_0(\varphi)/2\eta(\varphi)$ does not depend on φ, we obtain for $N(\varphi)$ the standard Arrhenius formula with T^* given by the formulas (33) and (34). The pre-exponential factor in the decay probability for high temperatures and arbitrary viscosity can be found in much the same way as has been done in the equilibrium case.[14-17]

The strong-viscosity limit in the nonequilibrium case has not been investigated in the low-temperature region. In the weak-viscosity region $\eta/m^* < \Omega$ the decay probability

$$\gamma=\sum_i \gamma(E_i)N(E_i)\Big/\sum_i N(E_i). \tag{50}$$

Here $\gamma(E)$ is the probability for tunneling decay from the state with energy E:

$$\gamma(E)=\frac{\omega(E)}{2\pi}\exp(-w(E)),$$

$$w(E)=2\int d\varphi[2m^*(U(\varphi)-E)]^{1/2}, \tag{51}$$

where $\omega(E)$ is the frequency of the classical motion of a particle with energy E. The sum over the states in the numerator in the formula (50) can be replaced by an integral. At low temperatures we can use for $N(E)$ the formula (27) and compute the integral in the numerator in the formula (50) by the method of steepest descent.

In the classical case of high temperatures the weak-viscosity region ($\eta/m^* < \Omega$) splits into two subregions: the "medium-viscosity" subregion, where the energy $\delta E = 2\pi A/\omega(E)$ lost by the particle in a period during the motion in the vicinity of the top of the potential barrier is large compared to T^*, and the very-weak-viscosity subregion, where the opposite limit obtains. The pre-exponential factor in the expression for the decay probability in the high-temperature region has been found in the limiting cases by Kramers[14] and in the intermediate case by Mel'nikov.[17] In the case of medium viscosity the transition from the classical to the quantum regime occurs at

$$T^*=\Omega/2\pi=(-w')^{-1} \tag{52}$$

(the derivative w' is taken at an energy δU equal to the height of the potential barrier). This transition is investigated in Ref. 18, and can be obtained from the formula (50) through integration over E up to δU with the distribution function given by the formula (27). The transition from the classical to the quantum regime in the limiting case of very weak viscosity needs to be investigated further even at equilibrium. In the case of very weak viscosity the transition from the classical to the quantum regime occurs at a temperature lower than the temperature obtained from the formula (51). For such weak viscosity the distribution function $N(E)$ in the important energy region becomes depleted because of the quantum tunneling processes, which must now be taken into account in Eq. (24):

$$\partial N_i/\partial t+\sum_j (N_i\mathscr{W}_{ij}-N_j\mathscr{W}_{ji})+\gamma_i N_i=0. \tag{53}$$

The probability γ_i for tunneling decay depends exponentially on the energy. Therefore, when $\gamma_i < \mathscr{W}_{ij}$ the last term is insignificant, and $N(E)$ has the form (27). More precisely, the boundary E_c of the region above which the distribution function falls off rapidly can be found from the condition

$$\gamma(E_c)=\sum_j \mathscr{W}_{ij}, \qquad E_c=\delta U+\frac{1}{w'}\ln\left[\omega(E_c)/2\pi\sum_j \mathscr{W}_{ij}\right], \tag{54}$$

where the quantity $w(E)$ is given by the formula (51). The transition from the classical to the quantum decay regime occurs in a narrow neighborhood of the temperature

$$T_0^{-1}=-w'(E_c). \tag{55}$$

In the neighborhood of the temperature T_0 the summation over the energies in the numerator in the formula (50) is performed over a broad range of energies around E_c. In this region $N(E)$ is given by the formula (27) when $E < E_c$ and is small when $E > E_c$, while

$$w(E)=w(E_c)+w'(E_c)(E-E_c)+w''(E_c)(E-E_c)^2/2. \tag{56}$$

As a result, for γ we obtain the expression[19]

$$\gamma=\frac{2\pi}{\omega(E_c)}\gamma(E_c)\operatorname{sh}\left(\frac{\Omega}{2T^*(0)}\right)\exp\left(-\int_0^{E_c}\frac{dE}{T^*(E)}\right)$$
$$\times\left[1+\Phi\left(\frac{w'(E_c)+T^{-1}(E_c)}{(2w''(E_c))^{1/2}}\right)\right]$$
$$\times\frac{\exp\left((\omega'(E_c)+T^{-1}(E_c))/2w''(E_c)\right)}{(2\pi w''(E_c))^{1/2}}, \tag{57}$$

where

$$\Phi(x)=\frac{2}{\pi^{1/2}}\int_0^x dt\,e^{-t^2}$$

is the error integral.

If we add to the diffusion equation (29) a term describing the tunneling, which depends exponentially on the energy, then for the decay probability we obtain[19]

$$\gamma=\frac{2B}{T^*(E_c)\omega(E_c)}\operatorname{sh}\left(\frac{\Omega}{2T^*(0)}\right)2^{-2\nu}\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}$$
$$\times\exp\left(-\int_0^{E_c}\frac{dE}{T^*(E)}\right) \tag{58}$$
$$\nu=-(T^*(E_c)w'(E_c))^{-1}.$$

At high temperatures the formula (58) goes over into the Kramers formula, and joins onto the formula (57) as $T^*\to T_0$. In the intermediate region there appears in the formula (58) an additional factor of the order of unity as a result of the inapplicability of the diffusion approximation. We can find this factor if we go over in Eq. (53) from the sum over the states to an integral over the energy. In the quasiclassical approximation the matrix elements $\mathscr{W}_{ij}$ decrease rapidly as the difference $|i-j|$ increases, and depend smoothly on the state number i. Equation (53) in this approximation can be solved by a method similar to the one expounded in Ref. 17. As a result we obtain

$$\gamma=\frac{2\gamma(E_c)}{\omega(E_c)}\operatorname{sh}\left(\frac{\Omega}{2T^*(0)}\right)\mathcal{N}, \tag{59}$$

where the energy E_c is given by the formula (54), while

$$\mathcal{N}=\left[(\pi/w')\exp\left(-\int_0^{E_c}dE/T^*(E)\right)\right][\sin(\pi/T^*(E_c)w')]^{-1}$$
$$\times\exp\left\{\frac{i}{2w'}\int_{-1/2T-i\infty}^{-1/2T+i\infty}d\tau\ln\left[\frac{\mathcal{J}(0)-\mathcal{J}(\tau)}{\mathcal{J}(0)}\right]\right. \tag{60}$$
$$\left.\times\left[\operatorname{ctg}\left(\frac{\pi\tau}{w'}\right)-\operatorname{ctg}\left(\frac{\pi(\tau+1/T^*)}{w'}\right)\right]\right\}$$
$$\mathcal{J}(t)=\int_{-\infty}^{\infty}dz\exp(-tz)\mathscr{W}_{ij},\quad z=E_j-E_i.$$

When the temperature T^* is close to $(-w')^{-1}$, the last factor in the formula (60) is equal to unity. As a result the decay probability

$$\gamma=-\frac{1}{w'+1/T^*(E_c)}\exp\left(-\int_0^{E_c}\frac{dE}{T^*(E)}\right)\frac{2\gamma(E_c)}{\omega(E_c)}\operatorname{sh}\left(\frac{\Omega}{2T^*(0)}\right). \tag{61}$$

When the temperature T^* is not too close to $(-w')^{-1}$, the expression (61) goes over into the formula (57).

At high temperatures the important values of τ in the formula (60) are of the order of $1/T^*$. For such small values of the quantity τ we easily find that

$$\mathcal{J}(\tau)=\mathcal{J}(0)+\omega(E_c)B\tau(\tau+1/T^*(E_c)), \tag{62}$$

where the quantity B is given by the formulas (30) and (44). Substituting this value of $\mathcal{J}(\tau)$ into the formula (60), and evaluating the τ integral, we obtain

$$\gamma=\frac{2B}{\omega(E_c)T^*}\operatorname{sh}\left(\frac{\Omega}{2T^*(0)}\right)\exp\left(-\int_0^{E_c}\frac{dE}{T^*(E)}\right). \tag{63}$$

The expression (63) coincides with the formula (58) when $T^*\gg(-w')^{-1}$ and with Kramer's result.[14]

CONCLUSION

One of the methods that can be used to observe the effects considered in this paper consists in the establishment of a state of nonequilibrium with the aid of an auxiliary tunneling junction. The application of a voltage potential ($eV>\Delta$) across the auxiliary junction leads to a sharp increase in the effective temperature and a decrease in the lifetime of the metastable current state. The same effect is produced by the irradiation of the junction by a high-frequency field. The effective temperature is, generally speaking, a smooth function of the energy. At a certain pumping level the effective temperature becomes negative in some energy region. In this case the voltage potential across the Josephson junction undergoes oscillations.

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