Generation of high harmonics of laser radiation in plasma during electron wavebreaking

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This paper was stimulated by experiments in which several tens of harmonics were observed in a plasma target exposed to powerful CO_2 laser radiation. High-order harmonics are naturally generated when wavebreaking-type discontinuities are periodically produced. The universal characteristic features observed as a result of the wavebreaking phenomenon are investigated. The spectra of harmonics produced at the time of formation of a discontinuity, during collisions between two discontinuities, and during Coulomb and collective "friction" between electrons and ions, are examined. The theoretical spectra that most closely resemble experimental distributions are those obtained by considering the radiation emitted during the interaction between electron wavebreaking and small-scale perturbations in ion density.

1. INTRODUCTION

The theory of weak nonlinearity predicts an exponential fall in intensity with increasing order number of the heating-radiation harmonic,[1] so that experiments demonstrating the heating of targets by CO_2-laser harmonics[2–4] up to the 46th harmonic[4] ($q = 5\times 10^{15}$ W/cm^2) have not had a satisfactory theoretical interpretation. The difficulties in this theory are due to the fact that a finite nonlinearity has to be considered. One of the natural manifestations of electron oscillations with finite nonlinearity is the appearance of wavebreaking-type discontinuities. The motion near the wavebreaking point occurs practically in the ballistic state, so that we have been able to provide[5] an analytic description of the emission of harmonics for two processes connected with wavebreaking. In the present paper, our aim has been to give a more detailed description of discontinuities and to examine the emission of harmonics due to other processes that have turned out to be significant. Our account is largely confined to the hydrodynamic approximation that is valid for $\tilde{v} \gg v_{Te}$, where $\tilde{v}$ is the characteristic electron oscillation velocity. Allowance for the finite thermal velocity v_{Te} of electrons requires a kinetic treatment and leads to an exponential reduction in the intensity I_N with increasing number N of the harmonic for N greater than some $N_{\rm crit}$.

The second part of this paper is devoted to questions relating to singularities that occur during wavebreaking. For inertial motion (ballistic approximation), for which $\mathbf{x} = \mathbf{a} + \mathbf{v}t$, we have investigated the singularities associated with the first appearance of discontinuity, namely, fold and crease discontinuities, which are well known in the theory of discontinuities of differentiable mappings. Only oscillations of sufficiently high intensity are found to break when interactions between electrons and ions are taken into account. When this is so, critical wavebreaking in the nonballistic case results in a singularity accompanied by a discontinuity in the dependence of the particle velocity on spatial coordinates, which has been seen in numerical experiments.[6]

The structural stability of the above discontinuities is exceedingly important for the analytic description of a finite nonlinearity. Structural instability is understood to refer to the persistence of singularities of this type under a small change in the initial and boundary conditions, i.e., this is a concept that is different from that introduced in the theory of singularities of differentiable mappings.[7] Structural stability of discontinuities ensures that the character of the latter does not depend on the wavebreaking conditions and is universal for homogeneous and inhomogeneous plasmas, both in the case of the breaking of free oscillations and in the case of an external periodic field acting on the plasma. This enables us to obtain a complete solution for the generation of harmonics of incident radiation without introducing the assumption of small aplitude, which is critical to the theory of weak nonlinearity.

In the third section, we shall calculate the spectra of harmonics in certain processes connected with periodic electron wavebreaking. In ballistic wavebreaking, the electron density becomes infinite at certain points or on certain surfaces, and gives rise to electrostatic field singularities. In the next approximation, electrons are accelerated by this field and radiate. Each type of singularity or combinations of them determine the particular elementary process of emission of radiation with a particular (usually of the power-law) spectrum of harmonics. These processes include quadrupole radiation emitted at the instant of appearance of the singularity, and dipole radiation emitted during the collision between a broken electron wave and an ion density step broken by an ion wave, a Langmuir caviton, or a short-wave ion-acoustic perturbation of a general form. The same section will examine the kinetic limits for the validity of the hydrodynamic description of wavebreaking.

The harmonic generation mechanism discussed in Sec. 4 is unrelated to wavebreaking but is close to it in principle: it is based on the well-known singularity in collision frequency $\nu_{ei} \sim v^{-3}$, where v is the oscillating part of the electron velocity in the Langmuir or electromagnetic wave. The emission of high harmonics occurs as the velocity v passes through zero, when the electron experiences a strong "frictional" force. The frequency ν_{ei} is very low in the CO_2-laser corona, so that we shall consider the collective emission due to scat-

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tering of electrons by ion waves.

Section 5 discusses an alternative mechanism for harmonic generation based on the well-known effect of steepening of the target density profile under the influence of forces due to light pressure. The plasma density behind the critical surface may rise by several hundred times as compared with n_c at short distances, and this provides us with the possibility of effective excitation of tens of Langmuir resonances, followed by the re-emission of Langmuir waves into transverse waves. There is a number of very different mechanisms of resonance excitation. We shall consider Čerenkov emission of Langmuir waves by the broken electron wavefront in homogeneous plasma. Interest in this process is dictated by the fact that its effectiveness has proved to be sufficiently high.

The paper concludes with a discussion of the results and a comparison between theory and some experimental data.

2. ELECTRON WAVEBREAKING

It is well known (see, for example, Ref. 8) that a nonlinearity in the hydrodynamic equations in which thermal motion and interactions between particles have been neglected leads to a multistream-type discontinuity in a finite interval of time. Let us examine the singularities that occur during ballistic wavebreaking. Of course, in the ballistic approximation, no emission is produced. Emission will occur only when the electrostatic field of the discontinuities is taken into account (see Sec. 3). We shall need to find the density distribution in discontinuities. We shall transform from the Euler variables $(\mathbf{x}, t)$ to the Lagrange variables $(\mathbf{a}, t)$. The condition for free motion can be written in the form

$$\mathbf{x}=\mathbf{a}+\mathbf{v}(\mathbf{a})t, \tag{1}$$

where $\mathbf{v}(\mathbf{a})$ is the velocity distribution in space for $t=0$. The velocity distribution at arbitrary time has the form $\mathbf{v}(\mathbf{a}(\mathbf{x}, t))$, where $\mathbf{a}(\mathbf{x}, t)$ is the solution of (1). The velocity near a singular point is, in general, a multivalued function of $\mathbf{x}$. We note, however, that $\mathbf{x}(\mathbf{a})$ given by (1) remains a single-valued and analytic function of $\mathbf{a}$ that can be expanded in powers of $\mathbf{a}-\mathbf{a}_0$ about the singular point $\mathbf{a}_0$:

$$\begin{aligned} x_i={}&x_{0i}+tv_k{}^i(\mathbf{a}_0)(a_k-a_{0k})+{}^1/_2 tv_{kl}{}^i(\mathbf{a}_0)(a_k-a_{0k})(a_l-a_{0l})\\ &+{}^1/_6 tv_{klm}^i(\mathbf{a}_0)(a_k-a_{0k})(a_l-a_{0l})(a_m-a_{0m})+\ldots \end{aligned} \tag{2}$$

A singularity will occur at points at which the Jacobian

$$\begin{aligned} \frac{\partial(\mathbf{x})}{\partial(\mathbf{a})}&=\det\|\delta_{ik}+tv_k{}^i(\mathbf{a})\|=\det\|\delta_{ik}+tv_k{}^i(\mathbf{a}_0)\|\\ &+u_i(\mathbf{a}_0,t)(a_i-a_{0i})+u_{ik}(\mathbf{a}_0,t)(a_i-a_{0i})(a_k-a_{0k})+\ldots, \end{aligned} \tag{3}$$

vanishes, where u_i and u_{ik} are certain functions of $\mathbf{a}$ and t.

By suitably choosing the four quantities $(\mathbf{a}_0, t)$, we can ensure that the four functions $\det\|\delta_{ik}+tv_k{}^i(\mathbf{a}_0)\|$ and $u_i(\mathbf{a}_0, t)$ will vanish. This singularity corresponds to the onset of wavebreaking. It occurs at a certain instant of time $t=t_c$ and at a certain point. When $t=t_c$, the flow is still of the single-stream type the Jacobian (3) does not change sign, i.e., the quadratic form $u_i(\mathbf{a}_0, t_c)$ is sign-definite. In the neighborhood of the point $(\mathbf{a}_0, t_c)$, the first term of the expansion for the Jacobian has the form

$$\partial(\mathbf{x})/\partial(\mathbf{a})=\alpha(t-t_c)+u_{ik}(a_i-a_{0i})(a_k-a_{0k})+\ldots.$$

Without loss of generality, we may conclude from this that, when $t<t_c$, there will not be any singularities anywhere: for $t=t_c$ we have the only singular point $\mathbf{a}=\mathbf{a}_0$; for $t>t_c$ the singular points lie on an ellipsoid in $\mathbf{a}$-space of dimension of the order of $(t-t_c)^{1/2}$, the interior of which corresponds to three-stream flow. The transformation (2), which is degenerate (for $t=t_c$) in the linear approximation in $\mathbf{a}-\mathbf{a}_0$, transforms an ellipsoid into an ellipse on a certain degenerate plane in $\mathbf{x}$-space, and inclusion of terms of the order of $(a-a_0)^3$ and $(t-t_c)(a-a_0)$ leads to a "thickening" of the ellipse to the state where it takes the form of a "plate" of diameter $\propto(t-t_c)^{1/2}$, thickness $\propto(t-t_c)^{3/2}$, and sharp semicubic edge. In terms of the normal cylindrical coordinates (r, z), the three-stream region is described by

$$\left(\frac{r}{R}\right)^2+\left(\frac{z}{b}\right)^{2/3}\leqslant\frac{t-t_c}{t_0},\quad 0<t-t_c\ll t_0, \tag{4}$$

where R and b are, respectively, the characteristic transverse and logitudinal dimensions of the initial velocity perturbation and $t_0\sim b/v$ is the characteristic wavebreaking time. This picture corresponds to the first appearance of the singularity and its development immediately thereafter.

If we demand that only the zero-order term in the expansion for the Jacobian (3) must vanish, we find another singularity that is almost independent of time and lies on the surface in $\mathbf{a}$-space defined by the equation $\det\|\delta_{ik}+tv_k{}^i(\mathbf{a})\|=0$. Its map in $\mathbf{x}$-space is a surface moving with velocity of the order of v. This singularity is quasione-dimensional and corresponds to the broken wave front.

The above simple discontinuities are well known in the theory of singularities of differentiable mappings.[7] Thus, the broken wave front and the smooth surface of the plate are "fold singularities." Lines with singularities of a more complicated form may lie on a fold: the sharp edge of a plate is a "crease"-type singularity. Folds and creases are structurally stable discontinuities in the sense defined in Ref. 7, i.e., discontinuities of this kind are nonremovable under small changes in the mapping of three-dimensional space onto itself. The first-wavebreaking singularity is removable by a small change in time but, in the course of its motion, the system necessarily passes through this type of state, with its completely determined physical consequences (for example, radiation). We shall therefore refer to this instability as structurally stable (in a wide sense of this phase). We note that Arnol'd *et al.*[7] have pointed out the importance of unstable (in the narrow sense) discontinuities in applications.

For us, structural stability is important because the above discontinuities appear "routinely" in all differentiable motions $\mathbf{a}\to\mathbf{x}$ of three-dimensional space that are not forbidden by violation of mutual single-valuedness, e.g., ballistic motion of a continuous medium. We shall show below that, if electrons and ions interact, the discontinuities examined above will remain and, as a discontinuity is approached, the ratio of electrostatic and kinetic energies tends to zero, i.e., the ballistic condition is more readily satisfied, and the discontinuity has the same character as in the ballistic case, although discontinuities in the higher derivatives of the mapping $\mathbf{x}(\mathbf{a})$ appear after wavebreaking. Any smooth plasma-inhomogeneity profile, or external fields that vary smoothly in space and time, will not affect the differentiability of the motion $\mathbf{x}(\mathbf{a})$, so that wavebreaking singularities are universal in character, subject only to the condition that the wavebreaking itself takes place. This enables us to apply the results obtained for discontinuities to ballistic motion. In particular, we shall be interested in electron-density singularities where, in terms of Lagrangian coordinates, the density has the form

$$n(\mathbf{a},\ t)=n_0(\mathbf{a})\,|\partial(\mathbf{x})/\partial(\mathbf{a})\,|^{-1}. \tag{5}$$

Using expansions (2) and (3), we can readily show from (5) that, for the first wavebreaking singularity, the density tends to infinity at the singular point in accordance with the expression

$$n(t)\sim n_0 t_0/|t-t_c|, \tag{6}$$

whereas, for the fold, we have a density discontinuity of the form

$$n(x,\ t)\sim n_0[b/(vt-x)]^{1/2}\theta(vt-x), \tag{7}$$

where the x axis is perpendicular to the quasiplane wave front.

The electrostatic interaction does not forbid electron wavebreaking but leads to a threshold condition for the energy of the oscillations. The simplest way to verify this is to consider free one-dimensional Langmuir oscillations in cold homogeneous plasma. It is well known that the corresponding equations in terms of the Lagrange variables $(\mathbf{a}, t)$ preserve linearity right up to the moment of wavebreaking (they are nonlinear in Euler coordinates). For the problem with initial conditions

$$v(a,0)=\tilde{v}(a),\quad n_e(a,0)=n_0(a)$$

the solution for the density is

$$n_e(a,t)=n_0(a)\left[1+\frac{1}{\omega_p}\frac{d\tilde{v}(a)}{da}\sin\omega_p t+\left(\frac{n_0(a)}{n_i}-1\right)(1-\cos\omega_p t)\right]^{-1}. \tag{8}$$

where n_i = const is the ion density and $\omega_p=(4\pi n_i e^2/m)^{1/2}$. Equation (8) shows that, if the condition

$$\left(\frac{1}{\omega_p}\frac{d\tilde{v}(a)}{da}\right)^2\geqslant 2\frac{n_0(a)}{n_i}-1 \tag{9}$$

is satisfied, the density will become infinite at a certain time, i.e., wavebreaking will take place.

We shall now show that, as we approach the singularity, the condition for the validity of the ballistic approximation becomes less stringent. For example, in the case of one-dimensional wavebreaking, we have a density singularity $n(x, t_c)\propto x^{-2/3}$ and, consequently, a potential discontinuity $\varphi\propto x^{4/3}$. On the other hand, the kinetic energy of the particle is $v^2\propto x^{2/3}$. As $x\to 0$, it frequently exceeds the potential energy. For a fold, $\varphi\propto x^{3/2}$, $v^2\propto x$, and the ballistic approximation is also valid.

The existence of the threshold condition (9) leads to the appearance of a new nonballistic singularity to which we shall refer as the critical singularity. This appears when the energy of the oscillations becomes exactly equal to the threshold value [this corresponds to the equality sign in (9)]. The system will assume this state either as a result of a slow increase in the amplitude of the weak external pump, or when the wavelength b decreases because of plasma inhomogeneity. This wavebreaking was examined in Ref. 6 in the one-dimensional case. The exponents corresponding to this singularity can be found analytically. Flow is possible because electrons do not affect one another up to the point of wavebreaking in the one-dimensional case, whereas, in the ballistic case, the Euler coordinate x remains an analytic function of the Lagrange variables a, t, as before. The expansion for this function is

$$x=b_1(a_0,t)(a-a_0)+b_2(a_0,t)(a-a_0)^2+b_3(a_0,t)(a-a_0)^3+\dots.$$

The vanishing of b_1 and b_2 corresponds to the critical discontinuity. The electron density is then

$$n=n_0|dx/da|^{-1}\infty x^{-2/3} \tag{10}$$

and becomes infinite in the same way as in the ballistic case. The function $v(x)$ can be found by equating the kinetic and electrostatic energies, $v^2\sim\varphi\propto x^{4/3}$, whence

$$v\infty x^{2/3} \tag{11}$$

(the velocity was higher, $v\propto x^{1/3}$, in the ballistic case). The function $v\propto x^{2/3}$ is a good representation of the edges obtained numerically and shown in Figs. 2 and 3 of Ref. 6. Expressions (10) and (11) can also be obtained as a result of a rigorous solution of the equations for electron oscillations in homogeneous plasma.

Finally, let us consider the wavebreaking conditions for electrons in a target illuminated by laser radiation. We note that the electron trajectories cannot cross in the field of a purely transverse plane electromagnetic wave in vacuum. In fact, an exact solution[9] can be obtained for the motion of a charge in the wave, and it follows from this solution that the amplitude of the longitudinal displacement of a charge performing helical motion in the magnetic field of the wave is always less than a quarter of the wavelength for any wave intensity. Hence it follows that wavebreaking processes can occur only in the region of the plasma corona with density of the order of the critical value n_c, in which the light wave is strongly distorted and a longitudinal electric field appears. Wavebreaking occurs when the displacement of electrons becomes greater than a characteristic dimension. If the plasma is cold, the breaking of Langmuir oscillations will necessarily occur as a consequence of the reduction in the characteristic dimension (wavelength) due to plasma inhomogeneity. When there is a steep density jump on a scale of the order of the Debye length, wavebreaking is unavoidable for $\tilde{v}\gg v_{Te}$. In the numerical experiments reported in Ref. 6 and 10 (in which $v_{Te}=0$), the breaking of the plasma resonance occurred not later than after a few periods of external field oscillations, and then recurred periodically.[6] Here, we must stipulate that there is a sufficiently fast transfer of electrons from the cold core to the resonance region, which replaces electrons that were accelerated during wavebreaking,[11] and this ensures that the condition $\tilde{v} \gg v_{Te}$ is always, satisfied.

3. RADIATION ACCOMPANYING WAVEBREAKING

Let us now consider the radiation emitted when electrons are accelerated near singular points. Each type of singularity can be placed in correspondence with its own particular elementary emission process, where the radiation produced as a result of this decreases with frequency in accordance with a power law in which the exponent is determined solely by the nature of the discontinuity. A well-known example of this kind is the transition radiation emitted by a charge crossing a jump in permittivity. Its frequency dependence is ω^{-4} (Ref. 12). We shall consider the stronger collective emission.

We begin by considering the radiation emitted at the instant of the first electron wavebreaking in homogeneous plasma. The time t will be measured from the time t_c at which the singularity appears. The density singularity given by (6) produces a burst of electrostatic field $E \approx 2\pi e n_0 b (t/t_0)^{1/2}\theta(t)$ which gives rise to radiation by plasma electrons located in the region around the plate, the size of this region being of the order of the plate diameter $l \sim R(t/t_0)^{1/2}$. Since, in this process, the electrons exchange momentum only with one another, the center of the charge is at rest, and we have quadrupole radiation. In the ballistic approximation, the quadrupole moment $D_{ik} = \Sigma\, e x_i\, x_k$ is given by the expression

$$\dddot{D}_{ik} = \frac{e^2}{m}\int n(\mathbf{x},t)\,[3v_iE_k + (v_it + x_i)\dot{E}_k]\,d^3\mathbf{x} + (i \leftrightarrow k),$$

which corresponds to the estimate

$$\dddot{D}(t) \sim e n_0 R^4 \omega_{pe}^2 \tilde{v}\,(t/t_0)^{3/2}\theta(t), \quad t \ll t_0.$$

Assuming that this singularity appears in each period $2\pi/\omega_0$ of the incident radiation, let us expand the function that pulsates in this way into a Fourier series. The high Fourier harmonics ($N\omega_0 \gg t_0^{-1}$) are wholly determined by the behavior of the function in the small neighborhood of the singular point ($t \ll t_0$), and the spectrum corresponding to a singularity of the form $f(t) = t^q\,\theta(t)$ is

$$f_{N\omega_0} = i^{q+1}\omega_0\Gamma(q+1)/(N\omega_0)^{q+1}.$$

It is then a relatively simple matter to estimate the intensity of radiation emitted into the Nth harmonic in this process:

$$I_{N\omega_0} \sim \frac{1}{c^5}|\dddot{D}_{N\omega_0}|^2 \sim m n_0 \left(\frac{\tilde{v}}{c}\right)^5 \frac{R^8}{b^3}\frac{\omega_{pe}^6}{\omega_0^3}\frac{1}{N^5}$$
$$\times(\omega_0 t_0)^{-1} \ll N \ll \omega_0 t_0 \left(\frac{\lambda_0}{R}\right)^2. \qquad (12)$$

The upper bound imposed on N appears as a result of the requirement that the multipolarity parameter is $l/\lambda \ll 1$, where $l = R(t/t_0)^{1/2}$, $\lambda = \lambda_0/N$, and λ_0 is the wavelength of the laser radiation. If the dimension l of the radiating zone is greater than the wavelength, the intensity of the emitted radiation is sharply reduced by interference.

Expression (12) for the radiation emitted by a single plate predicts a rapid reduction in intensity with increasing number of harmonic, which is of the power type but still very rapid. Experiment shows that the spectrum is almost N independent and has a sharp cutoff for $N > N_{\text{crit}}$. The spectra emitted in processes with stronger singularities fall off more slowly with increasing N. For example, an inhomogeneity in the ion background leads to a momentum exchange between electrons and ions and, consequently, to the appearance of dipole emission.[13] The variation in the dipole moment can be calculated from the formula

$$\ddot{\mathbf{d}} = \frac{e^2}{m}\int \varphi_e \nabla n_i\, d^3\mathbf{x}, \qquad (13)$$

where φ_e is the potential due to electrons and n_i is the ion density.

Let us now consider the radiation emitted when an electron singularity, such as a fold, collides with an ion-density perturbation created during a Langmuir collapse.[14] In order not to have to deal with the evolution of sound after the collapse, we shall suppose that it is rapidly damped out. We then find that $n_i(\mathbf{x})$ is an even function and $\int n_i(\mathbf{x})\,d^3\mathbf{x} = 0$, i.e., from the standpoint of quasi-one-dimensional long-wave perturbations, n_i is similar to the second derivative of a δ function. We shall suppose that the electron singularity is one-dimensional and will omit numerical factors. The electron-density singularity (7) produces a potential singularity of the form

$$\varphi_e = (\tilde{v}t - x)^{3/2}\theta(\tilde{v}t - x).$$

Substituting φ_e and $n_i = \delta''(x)$ in (13), we find that $d(t) = t^{-3/2}\,\theta(t)$, which yields

$$I_N = \text{const}\, N, \quad 1 \ll N \ll \tilde{v}/\omega_0 L, \qquad (14)$$

where L is the dimension of the caviton. When $N \gg \tilde{v}/\omega_0 L$, the intensity falls exponentially.

The collision between a broken electron wave and an ion-density step (approximating a steep plasma density profile) or a broken ion wave can be considered in an analogous manner. The resulting radiation spectra depend on N as N^{-7} and N^{-6}, respectively.

It is interesting to consider the radiation emitted during periodic electron wavebreaking in plasma with short-wave ion-density perturbations of a general form. The emission spectrum does not then reduce to the power-type expression but depends on the spatial spectrum of ion fluctuations, $w(\mathbf{k})$, which we shall introduce through the phase average

$$\langle n_{\mathbf{k}} n_{\mathbf{k}'}^{*}\rangle = n_0^2 w(\mathbf{k})\,\delta(\mathbf{k} - \mathbf{k}'), \qquad (15)$$

where $n_{\mathbf{k}}$ is the spatial Fourier component of the ion-density perturbation.

According to (13), the Nth harmonic of the dipole moment is given by

$$\ddot{\mathbf{d}}_{N\omega_0} = \frac{e^2\omega_0}{2\pi m}\int \varphi(x,t)\,\nabla n_i(\mathbf{x})\,e^{iN\omega_0 t}\,d^3\mathbf{x}\,dt, \qquad (16)$$

where the integral with respect to time is evaluated between zero and $2\pi/\omega_0$. Substituting $n_i(\mathbf{x}) = \int n_{\mathbf{k}}\, e^{i\mathbf{k}\mathbf{x}}\,d\mathbf{k}$, $\varphi(x,t) = 2\pi e n_0 b^{1/2}(\tilde{v}t - x)^{3/2}\,\theta(\tilde{v}t - x)$, in (16), and using (15), we find that the intensity of the Nth harmonic for $N \gg 1$ is given by

$$I_{N\omega_0}=\frac{4}{3c^3}|\ddot{\mathbf{d}}_{N\omega_0}|^2=\frac{3\pi^2}{16}\frac{\omega_{pe}^6}{\omega_0^2c^3}mn_0\tilde{v}^2Sb\frac{1}{N^3}w\left(N\frac{\omega_0}{\tilde{v}}\right)$$

where S is the cross-sectional area of the laser beam incident on the target. The spectrum of sound was taken to be isotropic: $w(\mathbf{k}) = w(k)$. It follows that the emission of harmonics with $N\gg1$ per unit illuminated area can be estimated from $(\omega_0\sim\omega_{pe})$

$$q_{N\omega_0}\equiv\frac{I_{N\omega_0}}{S}\sim mn_0\tilde{v}^2\,\omega_0 b\left(\frac{\omega_0}{c}\right)^3\frac{1}{N^3}w\left(N\frac{\omega_0}{\tilde{v}}\right). \qquad (17)$$

The efficiency of re-emission into the harmonics can be estimated from the formula

$$\frac{q_{N\omega_0}}{q}\gtrsim\frac{k_t^4 b}{k_s^3}\left(\frac{\delta n_i}{n_0}\right)^2, \qquad (18)$$

where $k_t\equiv\omega_0/c$, k_s is the characteristic (maximum) wavenumber in the spectrum of ion perturbations, and δn_i is the variation of ion density. The maximum number of harmonics that can give rise to this process can be estimated from

$$N_{max}\sim k_s\tilde{v}/\omega_0. \qquad (19)$$

The emission spectra obtained in the hydrodynamic approximation are valid in a range that depends on the thermal spread of the particles. To take into account the low thermal velocity $v_T\ll\tilde{v}$, we shall use instead of (1) the one-dimensional transport equation for the distribution function $f(v, x, t)$ of the interacting particles (because the main wavebreaking singularities are quasi-one-dimensional):

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}=0.$$

Its solution expressed in terms of the initial distribution function has the form

$$f(v, x, t)=f(v, x-vt, 0).$$

If we assume that the initial distribution function is Maxwellian, with ordered velocity $\tilde{v}(x)$, the particle density is given by

$$n(x,t)=A\int\exp\left\{-\left[\frac{v-\tilde{v}(x-vt)}{v_T}\right]^2\right\}dv,\quad A\sim\frac{n_0}{v_T}. \qquad (20)$$

When there is a thermal spread, the singularity is smeared out and the density at the singular points does not become infinite but is bounded (for $v_T\ll\tilde{v}$) by some large but finite quantity. The density is a maximum at points at which the functions v and $\tilde{v}(x-vt)$ have second- or third-order points of contact. It is readily seen that the former case corresponds to a fold-type singularity. We then have $v-\tilde{v}(x-vt)\sim v^2/\tilde{v}$ and (20) gives

$$n_{max}\sim n_0(\tilde{v}/v_T)^{1/2}. \qquad (21)$$

The case of a cubic point of contact occurs at the single point (x, t) and corresponds to the instant of breaking. We then have $v-\tilde{v}(x-vt)\sim v^3/\tilde{v}^2$ and

$$n_{max}\sim n_0(\tilde{v}/v_T)^{2/3}. \qquad (22)$$

Comparison of (22) with (6) leads to the conclusion that the hydrodynamic description of breaking is valid at times that are not too close to the time of formal onset of discontinuity, i.e., for $|t|\gg t_0(v_T/\tilde{v})^{2/3}$, which corresponds to frequencies $N\omega_0\ll t_0^{-1}(\tilde{v}/v_T)^{2/3}$. Hence, we find that the range of validity in N for (12) is

$$\frac{1}{\omega_0 t_0}\ll N\ll\min\left\{\omega_0 t_0\left(\frac{\lambda_0}{R}\right)^2,\ \frac{1}{\omega_0 t_0}\left(\frac{\tilde{v}}{v_T}\right)^{2/3}\right\}. \qquad (23)$$

Similarly, comparison of (21) with (7) shows that, for processes involving the participation of a fold [the spectra (14) and (17)]

$$1\ll N\ll\frac{1}{\omega_0 t_0}\frac{\tilde{v}}{v_T}. \qquad (24)$$

For N exceeding the right-hand sides of inequalities (23) and (24), the radiation intensity decreases exponentially with increasing N. We note that $\omega_0 t_0\sim1$ when the wave amplitude exceeds the value corresponding to breaking by an amount of the order of unity.

4. EMISSION DURING "FRICTION" BETWEEN ELECTRONS AND IONS

Since dipole emission is due to momentum exchange between electrons and ions, it is natural to consider momentum transfer in Coulomb collisions with frequency $\nu_{ei}=Av^{-3}$. Assuming that the thermal velocities are small and the velocity $\mathbf{v}$ is due to oscillations in the field of a linearly polarized electromagnetic wave, the momentum transfer to a single ion is given by

$$\dot{\mathbf{p}}=mA\mathbf{v}/v^3,\quad \mathbf{v}=\mathbf{v}_0\sin\omega_0 t. \qquad (25)$$

The asymptotic behavior of $\dot{p}$ at high frequencies (important for $N\gg1$) is determined by the singularity t^{-2}, which immediately yields $\mathbf{p}_\omega\propto\omega$ and

$$I_N=\text{const}\,N^2,\quad 1\ll N\ll\tilde{v}/v_{Te}. \qquad (26)$$

The ratio ν_{ei}/ω_{pe} is small in the experiments reported in Ref. 4, so that we shall consider stronger "friction" with ion-density fluctuations due to ion-sound-type oscillations. When these oscillations have short enough wavelengths, we find that, as shown by Zavoĭskiĭ and Rudakov,[15] there is complete analogy with Coulomb collisions, and the role of the ion charge is taken by the charge of the ion perturbation, i.e., $q\sim e\delta n\lambda^3$. As a result, we obtain

$$\nu_{\rm turb}\sim\omega_{pe}(\delta n/n_0)^2(v_{Te}/\tilde{v})^3. \qquad (27)$$

The radiation intensity emitted per unit volume is then given by

$$I_N=BN^2,\quad B\sim\frac{e^{10}n_0^6\lambda^9}{c^3\tilde{v}^4}\left(\frac{\delta n}{n_0}\right)^4,\quad 1\ll N\ll\left(\frac{\tilde{v}}{\lambda\omega_0}\right)^{1/2}. \qquad (28)$$

Inclusion of long-wave ion-density perturbations gives rise to spectra that grow less rapidly.

5. ČERENKOV EXCITATION OF LANGMUIR WAVES DURING WAVEBREAKING

The appearance of high harmonics was ascribed in Ref. 4 to the presence near the plasma resonance point of a region in which the plasma density was several times greater than the critical value. No mechanism was proposed in Ref. 4 for the excitation of oscillations in the region $n \gg n_c$, so that we now note one effective process for the excitation of plasma resonances. This is the Čerenkov emission of Langmuir waves by a breaking electron wave front. Consider the one-dimensional linear problem of the response of cold plasma to the "extraneous current" of a fold

$$j(x,t) = en_0\tilde{v}[b/(\tilde{v}t-x)]^{1/2}\theta(\tilde{v}t-x). \tag{29}$$

The Fourier component of this current

$$j_{k\omega} = \frac{en_0 b^{1/2}}{2\pi^{1/2}}\left(\frac{i}{k}\right)^{1/2}\delta\left(k-\frac{\omega}{\tilde{v}}\right)$$

gives the Fourier transform of the electric field

$$E_{k\omega} = -\frac{4\pi i}{\omega\varepsilon(\omega)}j_{k\omega}, \quad \varepsilon(\omega) = 1-\frac{\omega_{pe}^2}{\omega^2}+i0\,\mathrm{sgn}\,\omega,$$

and hence

$$E(x,t) = \int E_{k\omega}e^{ikx-i\omega t}\,dk\,d\omega$$

$$= -(2\pi^3)^{1/2}en_0\left(\frac{b\tilde{v}}{\omega_{pe}}\right)^{1/2}\left[\cos\omega_{pe}\left(t-\frac{x}{\tilde{v}}\right)+\sin\omega_{pe}\left(t-\frac{x}{\tilde{v}}\right)\right]. \tag{30}$$

The rate of energy loss by the fold is given by

$$W = -\int_{-\infty}^{\infty} jE\,dx = 2\pi^2\frac{(en_0\tilde{v})^2 b}{\omega_{pe}}.$$

The ratio of this power to the characteristic energy of the broken wave $m\tilde{v}^2 n_0 b$ gives the following expression for the frequency corresponding to the fold lifetime:

$$1/\tau \sim \omega_{p0}^2/\omega_{pe}, \tag{31}$$

where ω_{p0} is the plasma frequency calculated from the characteristic density of the fold n_0, and ω_{pe} is the local plasma frequency ($\omega_{pe} \gg \omega_{p0}$). The result given by (31) indicates that the efficiency of Čerenkov excitation of longitudinal waves by the electron singularity is very high. The fold may lose energy in a time greater than or of the order of the period of the Langmuir oscillations. When the fold passes periodically through the region of a high concentration gradient, the maximum amplitude of the Langmuir oscillations is established at points where ω_{pe} is a multiple of ω_0. To calculate the steady-state Langmuir field, we must know the mechanisms that restrict the field at the resonances.[6] Moreover, the spectrum of harmonics emitted by the plasma depends on the nature of the plasma density distribution $n_0(x)$ and the coefficient of the transformation from Langmuir into electromagnetic waves. This coefficient may be quite large since $\tilde{v}/c$ is not very small.

6. DISCUSSION

Other mechanisms for the generation of high harmonics have been proposed in addition to the above wavebreaking processes. The authors of Ref. 4 had already noted the presence near resonance of a region with high plasma density, but did not propose any mechanisms for wave excitation in this region. The Čerenkov emission of Langmuir waves by an electron fold, discussed in Sec. 5, seems to us to be an effective mechanism in this context.

The radiation emitted by an electron crossing a steep density jump has also been considered (see, for example, Ref. 16). This single-particle emission is too weak as compared with collective emission, and cannot explain the large coefficients of transformation into harmonics, which can be[4] of the order of 10^{-4}. The zero exponent of the power function representing the fall in the radiation intensity in Ref. 16 is due to an error and, when this is corrected, the result becomes $I_N \sim N^{-4}$, as should be the case for transition radiation.[12] We also note that, when there is a sharp density jump, a Langmuir wave of arbitrary amplitude will always be broken in a time of the order of the period of the oscillations, and the collective effects that we have examined come into play.

The authors of Ref. 10 use numerical methods to consider the emission of $N = 2$–30 harmonics. They say nothing about the mechanism responsible for the generation of these harmonics. In our view, this is connected with wavebreaking which undoubtedly occurred in these calculations (the authors themselves mention this). Unfortunately, the equations used in Ref. 10 are valid only up to the wavebreaking point even though they were used after this point as well.

All the harmonic generation mechanisms proposed here and in other papers demand that the condition $\tilde{v} \gg v_{Te}$ be satisfied. The condition $\tilde{v} \gtrsim v_{\rm ph}$ ($v_{\rm ph}$ is the phase velocity) is not absolute for waves with such large amplitudes, and wavebreaking is quite natural. If we suppose that, at plasma resonance, the longitudinal field is of the order of the external transverse field, and the characteristic temperature of the corona is $T_e = 1$ keV, the oscillation velocity becomes comparable with the thermal velocity even for intensities $q \simeq 5\times10^{13}$ W/cm^2 (for $\lambda = 10.6\ \mu$m). This figure becomes much lower when we allow for the possibility of field enhancement at resonance.

The number of radiated harmonics is determined by the characteristic frequencies of the processes occurring on wavebreaking. For example, at the time of appearance of the singularity, the characteristic time is $t_0 = b/\tilde{v}$. If we suppose that $b \sim r_D$, the frequency corresponding to this motion is given by $N\omega_0 \sim \omega_0\tilde{v}/v_{Te}$. For processes involving the participation of a fold, the characteristic number of harmonics is given by (19). If we substitute $k_s \sim r_D^{-1}$ into this expression, the result is the same, namely,

$$N_{max} \sim \tilde{v}/v_{Te}, \tag{32}$$

and this is in qualitative agreement with the experiment reported in Ref. 4 if we take $\tilde{v}/v_{Te} \approx 30$. We note that the electron temperature in the corona can be even lower than the temperature of the core because of adiabatic cooling as the electrons flow out of the dense core plasma into the more tenuous corona, in the opposite direction to that of electrons accelerated by wavebreaking. This means that the condition $\tilde{v} \gg v_{Te}$ is not too stringent.

The number of harmonic-generating processes occurring during wavebreaking is too large to enable us to use existing experimental data to identify reliably the dominant mechanism. The observed weak dependence of intensity on the number of the harmonic can result from a collision of a fold with a collapsing caviton, or with an ion-density-perturbation $w(k) \propto k^3$, but this can also be produced by the process discussed in Sec. 4.

Wavebreaking processes may play an important role not only in the laser corona but elsewhere as well. We note that the generation of harmonics that fall off slowly with harmonic number has also been observed in laboratory experiments in the radiofrequency range during the excitation of oblique Langmuir wave in magnetic fields.[17]

Our principal conclusions are as follows.

1. The character of the singularities that occur during wavebreaking can be examined analytically despite the fact that the amplitude is finite.

2. The generation of the higher harmonics observed experimentally[4] is most likely due to the crossing of electron trajectories (wavebreaking).

3. If the second of the above two conclusions is valid, wavebreaking is an important mechanism for the absorption of powerful electromagnetic waves as observed experimentally,[4] with all the consequences for the hydrodynamics of targets that follow from this.

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