Spin waves and long-range magnetic correlations in spin-polarized quantum gases and quantum liquids

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The macroscopic quantum phenomena that occur in spin-polarized Boltzmann gases of the $^3He\uparrow$ and $H\uparrow$ types are considered. The thermodynamic functions for arbitrary values of the degree of polarization of the gas are computed. The equations of weakly-dissipative spin dynamics that allow for the collective effects are derived from first principles. The spectrum and the conditions for the existence of weakly-damped spin waves in the gas are determined on the basis of these equations. The results agree with the experimentally observed phenomena. It is shown that the quantum collective effects lead to the existence in the gas of long-range spin correlations that fall off at large distances according to the power law r^{-1}. The collective spin modes and the correlation functions in liquid $^3He\uparrow$ and $^3He\uparrow$–^4He solutions are investigated. It is concluded on the basis of the experimental data that the spin waves are suppressed in the solution at concentrations of the order of 1–3%. The contribution of the magnons to the thermodynamics, the magnon Bose condensation, high-temperature ferromagnetism, and the dissipative helical structures in quantum liquids are discussed.

1. INTRODUCTION

Quantum gases are remarkable objects, in which essentially quantum effects can clearly manifest themselves even in the classical temperature region. We shall use the epithet "quantum" to designate those gases for which the mean de Broglie wavelength Λ of the particles of the system is significantly greater than the atomic dimensions r_0. Indeed, as the temperature T is lowered, we shall, if the gas does not condense before, get into the extremely interesting region

$$\varepsilon_d \ll T \ll \hbar^2/mr_0^2, \tag{1.1}$$

where ε_d is the quantum-degeneracy temperature and m is the gas-particle mass. The existence of the region (1.1) is guaranteed by the smallness of the gas parameter $Nr_0^3 \ll 1$, which is the natural parameter for systems with a short-range particle-particle interaction potential (here N is the number of gas molecules in a unit volume). The condition (1.1) is equivalent to the following hierarchy of characteristic lengths in the system:

$$N^{-1/3} \gg \Lambda \gg r_0, \quad \Lambda = \hbar/mv_T = \hbar(mT)^{1/2}, \tag{1.2}$$

which indicates that we are dealing with a quantum gas, and not with a quantum liquid. Nevertheless, although the gas molecules obey the Boltzmann statistics, in the region (1.1) the scale of the particle delocalization turns out to be greater than the particle dimensions, i.e., $\Lambda \gg r_0$, so that we can expect the system to exhibit qualitatively new—in comparison with the classical gas—and essentially quantum properties.

One such property that is nontrivial and extremely interesting is the possibility, predicted by the present author in Ref. 1, of propagation of weakly-damped magnetization oscillations in spin-polarized quantum gases. The characteristics of the macroscopic quantum phenomena that occur in Boltzmann systems were studied in greater detail in the particular cases of nondegenerate semimagnetic semiconductors and some other objects in later papers.[2] In 1982 the collisional absorption of spin waves was also computed by Lhuillier and Laloë[3] on the basis of spin-dynamics equations of the type of the Leggett equations[4] for a Fermi liquid. Meyerovich's paper[5] is also devoted to the formulation of the macroscopic equations of motion of the magnetic moment in spin-polarized Fermi systems.

The overwhelming majority of gases condense quite long before the condition (1.1) begins to be fulfilled, and the principal claimants for the ability of exhibiting quantum effects are the gaseous isotopes of hydrogen and helium (the traditional objects of investigation in low-temperature physics), which possess appreciable saturated-vapor pressure even in the temperature region (1.1). Spin-polarized atomic hydrogen $H\uparrow$ does not condense at all even as $T \to 0$. As follows from the results obtained in Refs. 1 and 2, for the existence of weakly-damped spin waves in a Boltzmann quantum gas to be possible the degree of spin polarization must be sufficiently high:

$$1 \geqslant \alpha \gg r_0/\Lambda. \tag{1.3}$$

On account of (1.3), the most natural candidate for the observation of spin oscillations in it is spin-polarized atomic hydrogen $H\uparrow$ in which $\alpha \approx 1$. Recently, Johnson *et al.*[6] experimentally confirmed for the first time with the aid of nuclear magnetic resonance the existence of spin waves in the Boltzmann gaseous $H\uparrow$ at $T < 0.8$ °K in the case when $N \approx 10^{16}$ cm^{-3} and $H = 7.7$ T, where H is the magnetic field. In Levy and Ruckenstein's paper[6b] a quasiparticle approach to the description of the properties of gaseous $H\uparrow$ is developed, and a quantitative interpretation of the experimental data reported in Ref. 6a is given. Recently, a high degree of spin polarization was achieved in gaseous ^3He as well with the aid of optical pumping. In the experiments performed by the Paris group[7] the value $\alpha \approx 0.7$ was obtained at room temperature and the value $\alpha \approx 0.25$ at $T = 4.2$ °K. These α values gave us reason to hope that collective magnetic phenom-

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ena would be detected in the gaseous phase of $^3He\uparrow$. Let us emphasize that the period of time during which the gas remains magnetically polarized after the optical pump has been switched off turns out to be extremely long (the depolarization time has been found[8] to be more than two days at $T = 4.2\ ^\circ K$), which is very convenient for the experimental investigation of the polarized quasiequilibrium state of $^3He\uparrow$. Though highly damped, the collective spin modes in gaseous $^3He\uparrow$ have been experimentally detected by Nacher *et al.*[9] (the first researchers to do so) in the $2 \leqslant T \leqslant 6\ ^\circ K$ $10^{16} \leqslant N \leqslant 10^{18}\ cm^{-3}$ regions under conditions when $\alpha = 0.3$.

The most surprising characteristic of the Boltzmann gases is the fact that there can propagate in them undamped high-frequency magnetization oscillations for which $\omega\tau \gg 1$, where ω is the wave frequency and τ is the characteristic time interval between two-particle collisions. Indeed, the description of the scattering processes in a classical gas with a short-range particle-particle interaction potential amounts to the fact that all the changes in the states of the particles occur only at the instant of collision, and that between two collisions, i.e., on the mean free path, a molecule of the gas moves freely without any external influence. In classical gases, when the weak long-range van der Waals forces are neglected, all the spatial (equal-time) correlation functions decrease sharply over distances of the order of the atomic dimensions r_0. For this reason it is certainly impossible for any macroscopic superstructure or collective mode to exist in such systems. But in spin-polarized quantum gases the very existence of undamped high-frequency ($\omega\tau \gg 1$) magnetic-moment oscillations indicates the presence of long-range spatial correlation between the spins of the gas molecules. This phenomenon is explained by the effects of the nondissipative quantum refraction that occurs during the mutual scattering of the particles under conditions when $\Lambda \gg r_0$. In fact, in the case of the propagation of a beam of low-energy particles through a rarefied medium of scattering centers the contribution of the interaction to the real part of the index of refraction of the beam is significantly greater than the corresponding contribution to the imaginary part,[10] which describes the beam dispersion. The correction to the real part of the refractive index indicates the existence of some correction to the self-energy of a beam particle. This correction is linear in the forward-scattering amplitude, is a functional of the distribution function for the entire ensemble of scattering centers, and can be considered to be the result of the existence of a distinctive self-consistent quantum-mechanical field of the Fermi-liquid type. It is the presence of such a field that makes the appearance of quantum collective effects possible even in the classical temperature region.

Also of considerable interest is the investigation of the long-wavelength spin fluctuations in spin-polarized quantum Fermi liquids. A consistent description of the spin collective modes on the basis of the Landau theory of the Fermi liquid[11] has been constructed by Silin[12] in the case of liquid $^3He\uparrow$ and by Meyerovich and the present author[13] for the case of dilute degenerate $^3He\uparrow$–^4He solutions. Recent investigations by Corruccini *et al.*,[14] Sholtz,[15] Owers-Bradley *et al.*,[16] and Gully and Mullin[17] furnish convincing experimental proof of the existence of collective spin waves in normal liquid $^3He\uparrow$ and superfluid $^3He\uparrow$–^4He solutions. The contribution of the magnons to the thermodynamics of liquid $^3He\uparrow$ turns out to be quite large. Under certain conditions the competition between the magnon and fermion contributions to the free energy gives rise to a situation in which spontaneous ferromagnetic ordering in liquid ^3He becomes possible at finite temperatures.[18]

In the present paper we investigate the thermodynamics of spin-polarized quantum gases. We formulate the kinetic equation with allowance for the quantum collective effects, and compute the generalized susceptibility and the dynamical magnetic form factor for polarized gases, liquid $^3He\uparrow$, and superfluid $^3He\uparrow$–He II mixtures. The spatial spin correlation function and the spectrum of the magnetization oscillations in the high-frequency regime are analyzed in detail for the above-enumerated cases. Under certain conditions spatially inhomogeneous phases and superstructures can exist in spin-polarized quantum gases and quantum liquids. The possibility of the occurrence of high-temperature equilibrium magnetic order in liquid ^3He and of magnon Bose condensation in liquid $^3He\uparrow$ is discussed.

2. THE THERMODYNAMIC FUNCTIONS OF SPIN-POLARIZED GASES

We shall, for definiteness, consider a system of spin-$\frac{1}{2}$ particles. Certain aspects of the effect of magnetic polarization on the thermodynamic properties of systems are discussed in Refs. 19 and 20. The thermodynamic functions of dilute $^3He\uparrow$–^4He solutions are computed in Ref. 13 in the limit of very low temperatures. In the present paper we derive for the macroscopic characteristics of spin polarized Boltzmann gases expressions that are valid at arbitrary temperatures. We shall first consider a spin-polarized gas in the absence of an external magnetic field, e.g., gaseous $^3He\uparrow$ polarized by means of optical pumping.

Owing to the presence of a natural small parameter, $Nr_0^3 \ll 1$, all the thermodynamic functions of the gas can be represented in the form of the corresponding virial expansions, which are equivalent to the functional series expansion in powers of the distribution function for the ideal gas. In the case of pair collisions the contribution of the interaction between the particles to the free energy of the gas can be represented in the following manner:

$$F_{int} = \sum_{\mathbf{p},\mathbf{p}'} \Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')\, n_{\beta\alpha}^{(0)}(\mathbf{p})\, n_{\nu\mu}^{(0)}(\mathbf{p}'), \tag{2.1}$$

where α, β, μ, and ν are the spin indices (summation over repeated indices is implied), and we have introduced the single-particle polarization density matrix $n_{\alpha\beta}^{(0)}(\mathbf{p})$ for ideal gases, which has in the Boltzmann temperature region the form

$$n_{\alpha\beta}^{(0)}(\mathbf{p}) = (\delta_{\alpha\beta} + \alpha\boldsymbol{\sigma}_{\alpha\beta}\mathfrak{M})\, n_0(\mathbf{p}), \qquad \alpha = (N_+ - N_-)/N. \tag{2.2}$$

Here $\mathfrak{M}$ is the unit vector in the direction of the spin polarization, the $\boldsymbol{\sigma}_{\alpha\beta}$ are the Pauli matrices, N_+ and N_- are the numbers per unit volume of gas particles with spins oriented along and oppositely to $\mathfrak{M}$, so that $N_+ + N_- = N$, and the Maxwellian distribution function

$$n_0(\mathbf{p}) = {}^1/_2 N (2\pi\hbar^2/mT)^{3/2} \exp(-p^2/2mT). \tag{2.3}$$

In the nonrelativistic approximation the interaction potential for the gas particles does not depend on their spins. Furthermore, in the second-virial coefficient approximation used the interaction function Φ from (2.1) is, contrary to what obtains in, say, the case of a dense Fermi liquid, not a functional of the density matrix, and is determined only by the amplitude of the two-particle scattering in a vacuum. Therefore, the function Φ does not depend on the degree α of polarization, and coincides with the value in the unpolarized gas. The spin dependence of the interaction function is determined solely by the exchange effects. Thus, in the exchange approximation the function Φ for an arbitrary value of the degree of polarization can be represented in the traditional form:

$$\Phi_{\alpha\beta,\,\mu\nu}(\mathbf{p},\,\mathbf{p}')=\psi(\mathbf{p},\,\mathbf{p}')\delta_{\alpha\beta}\delta_{\mu\nu}+\zeta(\mathbf{p},\,\mathbf{p}')\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}. \qquad (2.4)$$

Substituting (2.4) into (2.1), we obtain

$$F_{int}=4\sum_{\mathbf{p},\mathbf{p}'}\{\psi(\mathbf{p},\mathbf{p}')+\alpha^2\zeta(\mathbf{p},\mathbf{p}')\}n_0(\mathbf{p})n_0(\mathbf{p}'). \qquad (2.5)$$

Thus, the interaction-governed dependence of the corrections to the thermodynamics of the gas on the degree of polarization reduces to a quadratic function in the case of arbitrary (not necessarily small!) values of α. The next problem is to carry out a macroscopic computation of the functions ψ and ζ.

With the aid of a direct virial expansion of the partition function by the Beth-Uhlenbeck method,[21,22] we can obtain an expression for the thermodynamic potential Ω:

$$\Omega_{int}^{(\pm)}=\sum_{\mathbf{P},\mathbf{q}}\exp\left\{\frac{2\mu}{T}-\frac{P^2}{4mT}-\frac{q^2}{2mT}\right\}A_\pm(\mathbf{q}),$$

$$A_\pm(\mathbf{q})=-\frac{4\pi\hbar^2}{m}\left\{\operatorname{Re}f_\pm(0,q)+\frac{mT}{\hbar}\right.$$

$$\left.\times\left[\operatorname{Re}f_\pm(\theta,q)\frac{\partial}{\partial q}\operatorname{Im}f_\pm(\theta,q)-\operatorname{Im}f_\pm(\theta,q)\frac{\partial}{\partial q}\operatorname{Re}f_\pm(\theta,q)\right]\right\}, \qquad (2.6)$$

where $\Omega_{\text{int}}^{(+)}$ and $\Omega_{\text{int}}^{(-)}$ are the contributions to the thermodynamic potential from the interaction between the particles in the triplet and singlet states, $\mathbf{P}$ and $\mathbf{q}$ are the momenta of the center of mass and the relative motion of the particles, μ is the chemical potential of the gas, and $f_+(\theta,q)$ and $f_-(\theta,q)$ are the values of the amplitude of the scattering of two particles in the triplet and singlet states. According to the theorem on small corrections, $\Omega_{\text{int}}=F_{\text{int}}$. Substituting into the formulas (2.1) and (2.4) for the unpolarized state of the gas the eigenvalues of the operator $\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}$ for the triplet and singlet scattering, and comparing with (2.6), we obtain

$$\psi(\mathbf{p},\,\mathbf{p}')={}^1/_4[3A_+(\mathbf{q})+A_-(\mathbf{q})],$$

$$\zeta(\mathbf{p},\,\mathbf{p}')={}^1/_4[A_+(\mathbf{q})-A_-(\mathbf{q})],\quad 2\mathbf{q}=\mathbf{p}-\mathbf{p}'. \qquad (2.7)$$

Since the functions ψ and ζ depend only on $\mathbf{q}$, we can go over to the variables $\mathbf{p}$ and $\mathbf{q}$ in the expression (2.5), and perform the integration over the center-of-mass variables, which yields

$$F_{int}=N^2[X(T)+\alpha^2Y(T)],$$

$$X(T)=(\pi mT)^{-3/2}\int\psi(\mathbf{q})\exp(-q^2/mT)\,d^3q, \qquad (2.8)$$

$$Y(T)=(\pi mT)^{-3/2}\int\zeta(\mathbf{q})\exp(-q^2/mT)\,d^3q.$$

The total free energy of the spin-polarized gas is determined by the sum of F_{int} from (2.8) and the term corresponding to the contribution of the ideal Boltzmann gas, with allowance made for the quantum-mechanical corrections quadratic in N_+ and N_-. Finally, we have

$$F(\alpha)=F(0)+\frac{NT}{2}\left[\frac{2}{(18\pi)^{1/2}}\left(\frac{\varepsilon_d}{T}\right)^{3/2}\alpha^2+\ln(1-\alpha^2)\right.$$

$$\left.+\alpha\ln\frac{1+\alpha}{1-\alpha}\right]+Y(T)N^2\alpha^2,$$

$$F(0)=-NT\ln\left[\frac{2e}{N}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right]$$

$$+\frac{NT}{(18\pi)^{1/2}}\left(\frac{\varepsilon_d}{T}\right)^{3/2}+X(T)N^2,\quad \varepsilon_d=\frac{(3\pi^2N)^{2/3}\hbar^2}{2m}. \qquad (2.9)$$

The expressions (2.9) allow us to find any thermodynamic characteristic of the system. Thus, for the pressure P we obtain the expression

$$P(\alpha)=NT\left\{1+(18\pi)^{-1/2}(\varepsilon_d/T)^{3/2}(1+\alpha^2)\right.$$

$$\left.+\frac{N}{T}[X(T)+\alpha^2Y(T)]\right\}. \qquad (2.10)$$

When the condition (1.1) is fulfilled, the interaction between the gas particles reduces essentially to s scattering:

$$f_{\alpha\beta,\,\mu\nu}=-(a/2)(1-iqa/\hbar)(\delta_{\alpha\beta}\delta_{\mu\nu}-\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}),\quad |a|\sim r_0. \qquad (2.11)$$

Substituting (2.11) into (2.6)–(2.10), we find

$$P(\alpha)=NT\left\{1+\frac{1}{(18\pi)^{1/2}}\left(\frac{\varepsilon_d}{T}\right)^{3/2}(1+\alpha^2)\right.$$

$$\left.+\pi(Na^3)(1-\alpha^2)\left[\frac{\Delta}{T}+1\right]\right\},$$

$$\Delta\equiv\frac{\hbar^2}{ma^2}. \qquad (2.12)$$

As can be seen from the expressions (2.10) and (2.12), the correction to the pressure, which arises as a result of the magnetic polarization of the gas, is of the order of the virial correction in the unpolarized gas, multiplied by α^2, so that for sufficiently large values of α the observation of the magnetomechanical effect is certainly experimentally feasible at the present time. In the limiting case $T\ll\hbar^2/mr_0^2$ and $\alpha\to 1$ we obtain

$$[P(1)-NT]/[P(0)-NT]=2. \qquad (2.13)$$

3. THE KINETIC EQUATION AND THE COLLECTIVE EFFECTS IN A GAS

We shall be interested in those virial corrections in the kinematic part of the kinetic equation, which make a nonzero contribution to the hydrodynamic equations for the imperfect gas. It is obvious that such corrections exist. Indeed, by differentiating the pressure (2.10), we can obtain the virial expansion of the sound velocity in the gas. But if we wish to compute the velocity of sound with the aid of the kinetic equation, then we should integrate this equation over phase space, thereby obtaining the continuity equation. Furthermore, we should also integrate the kinetic equation, after multiplying it by the momentum, in order to obtain the Euler equation. As a result of the two integrations, the collision integral will vanish because of the laws of conservation of particle number and total momentum. Consequently, the left-hand side of the kinetic equation should contain terms that do not vanish in the indicated integrations, and yield the virial corrections to the velocity of sound in the ideal gas. Below we shall call these terms kinematic virial corrections in the kinetic equation.

It is convenient to derive the kinetic equation with allowance for the collective kinematic corrections with the aid of the Bogolyubov method.[23]

In the second-quantization representation the Hamiltonian of the system has the traditional form

$$\hat{H}=\sum_{12}\varepsilon_{12}\hat{a}_1^+\hat{a}_2+\left(\frac{1}{2}\right)\sum_{1234}\langle 12|U|34\rangle\hat{a}_1^+\hat{a}_2^+\hat{a}_4\hat{a}_3, \quad (3.1)$$

where ε_{ik} is the single-particle energy matrix, the $\langle ik|U|lm\rangle$ are the pair-interaction matrix elements, and $\hat{a}_i^+$ and $\hat{a}_k$ are the Fermi creation and annihilation operators with the usual anticommutation relations:

$$\hat{a}_i\hat{a}_k^++\hat{a}_k^+\hat{a}_i=\delta_{ik}\quad \hat{a}_i\hat{a}_k+\hat{a}_k\hat{a}_i=0,\quad \hat{a}_i^+\hat{a}_k^++\hat{a}_k^+\hat{a}_i^+=0. \quad (3.2)$$

In the Heisenberg representation the equation of motion for the density operator $\hat{n}_{ki}=\hat{a}_i^+\hat{a}_k$ has the form

$$\partial\hat{n}_{ki}/\partial t=(i/\hbar)[\hat{H},\hat{n}_{ki}]. \quad (3.3)$$

It follows from the requirement that the wave function of a system of identical fermions be antisymmetric, i.e., essentially from the relations (3.2), that

$$\langle 12|U|34\rangle=-\langle 21|U|34\rangle=-\langle 12|U|43\rangle. \quad (3.4)$$

Let us define the single-particle density matrix with the aid of the usual relations

$$n_{ik}=\langle\hat{n}_{ik}\rangle=\langle\hat{a}_k^+\hat{a}_i\rangle. \quad (3.5)$$

Let us assume that the interaction between the particles of the system is not strong, so that the condition for the applicability of perturbation theory is fulfilled. Let us average the Liouville equation (3.3) with the Hamiltonian (3.1) with allowance made for the relations (3.4). Expanding the equation obtained as a result of the averaging in powers of the weak interaction, and using the Wick theorem, we finally obtain

$$\partial n_{21}/\partial t+(i/\hbar)[\tilde{\varepsilon},n]_{21}=\mathrm{St}\,n_{21}, \quad (3.6)$$

where $\tilde{\varepsilon}_{21}$ is the energy matrix for a single particle in a self-consistent field of the Fermi-liquid type:

$$\tilde{\varepsilon}_{21}=\varepsilon_{21}+2\sum_{45}\langle 24|U|15\rangle n_{54}, \quad (3.7)$$

the commutator of the matrices $\tilde{\varepsilon}_{ik}$ and n_{ik} is given by the standard formula

$$[\tilde{\varepsilon},n]_{21}=\sum_{3}(\tilde{\varepsilon}_{23}n_{31}-\tilde{\varepsilon}_{31}n_{23}), \quad (3.8)$$

and $\mathrm{St}\,n_{21}$ is the collision integral, which is quadratic in the interaction, and whose specific structure will not be of interest to us here. Let us, in order to emphasize the analogy with the equations of the theory of the Fermi liquid, note that the following equality obtains:

$$\tilde{\varepsilon}_{21}=\delta E/\delta n_{12}. \quad (3.9)$$

Indeed, we have for the total energy E of the system the expression

$$E=\langle\hat{H}\rangle=\sum_{12}\varepsilon_{12}n_{21}+\frac{1}{2}\sum_{1234}\langle 12|U|34\rangle(n_{31}n_{42}-n_{41}n_{32}). \quad (3.10)$$

Varying E from (3.10) according to the formula (3.9), we arrive at the expression (3.7). Equations of the type (3.6) were obtained for a Boson gas in Ref. 24.

Next, going over from the density matrix $n_{21}\equiv n_{\mathbf{p}_1\beta;\mathbf{p}_1\alpha}$ in the momentum representation to the mixed Wigner distribution function

$$n_{\alpha\beta}(\mathbf{p})=\sum_{\mathbf{k}}n_{\mathbf{p}-\hbar\mathbf{k}/2,\alpha;\mathbf{p}+\hbar\mathbf{k}/2,\beta}e^{-i\mathbf{kr}}, \quad (3.11)$$

we arrive in the quasiclassical approximation, at the following Fermi-liquid-type kinetic equation:

$$\frac{\partial n_{\alpha\beta}}{\partial t}+\frac{i}{\hbar}[\tilde{\varepsilon},n]_{\alpha\beta}+\frac{1}{2}\left(\frac{\partial\tilde{\varepsilon}_{\alpha\gamma}}{\partial\mathbf{p}}\nabla n_{\gamma\beta}+\nabla n_{\alpha\gamma}\frac{\partial\tilde{\varepsilon}_{\gamma\beta}}{\partial\mathbf{p}}\right)$$
$$-\frac{1}{2}\left(\nabla\tilde{\varepsilon}_{\alpha\gamma}\frac{\partial n_{\gamma\beta}}{\partial\mathbf{p}}+\frac{\partial n_{\alpha\gamma}}{\partial\mathbf{p}}\nabla\varepsilon_{\gamma\beta}\right)=\mathrm{St}\,n_{\alpha\beta}, \quad (3.12)$$

where $[\tilde{\varepsilon},n]_{\alpha\beta}$ is the commutator in spin space, while for the renormalized self-energy $\tilde{\varepsilon}_{\alpha\beta}$ we have from the relations (3.7) and (3.9) the expression

$$\tilde{\varepsilon}_{\alpha\beta}=\frac{p^2}{2m}\delta_{\alpha\beta}+\sum_{\mathbf{p}'}\left\{\left[U(0)-\frac{1}{2}U(\mathbf{p}-\mathbf{p}')\right]\delta_{\alpha\beta}\delta_{\mu\nu}\right.$$
$$\left.-\frac{1}{2}U(\mathbf{p}-\mathbf{p}')\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}\right\}n_{\nu\mu}(\mathbf{p}'),\quad U(\mathbf{p})=\int U(r)e^{-i\mathbf{pr}/\hbar}\,d^3r. \quad (3.13)$$

In real gases the interparticle interaction is by no means weak. Nevertheless, it turns out that we can use the perturbation-theory results to derive the kinematic virial corrections in the kinetic equation in the temperature region (1.1) with the aid of Fermi's renormalization method.[10] Let us, in conformity with this method, introduce an effective complex potential $\tilde{U}(r)$ satisfying the condition for the applicability of perturbation theory:

$$|\tilde{U}(r)|\ll\hbar^2/m\tilde{r}_0^2,\quad \tilde{r}_0\sim|f_\pm| \quad (3.14)$$

and normalized in such a way that the forward-scattering amplitude, computed for the pseudopotential $\tilde{U}(r)$ in the Born approximation and with allowance for the identity of the colliding particles, coincides with the true zero-angle scattering amplitude $f_{\alpha\beta,\mu\nu}(0,q)$, which cannot be obtained by perturbation theory at all. These renormalization relations have the form

$$\frac{m}{\hbar q}\int\tilde{U}(r)\sin\left(\frac{2qr}{\hbar}\right)r\,dr\equiv-W(q)=f_{\alpha\beta,\mu\nu}(0,q)\boldsymbol{\sigma}_{\beta\alpha}\boldsymbol{\sigma}_{\nu\mu}, \quad (3.15)$$
$$2W(0)=f_{\alpha\beta,\mu\nu}(0,q)(\delta_{\beta\alpha}\delta_{\nu\mu}-\boldsymbol{\sigma}_{\beta\alpha}\boldsymbol{\sigma}_{\nu\mu}),\quad 2\mathbf{q}=\mathbf{p}-\mathbf{p}'.$$

The subsequent procedure consists in the application of perturbation theory to the pseudopotential $\widetilde{U}(r)$, and, as long as the results can be expressed only in terms of combinations of the form (3.15), i.e., in terms of the true scattering amplitude, the use of perturbation theory can be considered to be justified. Substituting the potential $\widetilde{U}(r)$ into (3.13), and using the relations (3.15), we obtain

$$\tilde{\varepsilon}_{\alpha\beta}=\frac{p^2}{2m}\delta_{\alpha\beta}-\frac{4\pi\hbar^2}{m}\sum_{\mathbf{p}'}f_{\alpha\beta,\mu\nu}(0,q)\,n_{\nu\mu}(\mathbf{p}'). \qquad (3.16)$$

We have, in accordance with the optical theorem for scattering, that

$$\operatorname{Im} f_{\alpha\beta,\mu\nu}(0,q)=(q/4\pi\hbar)\,\sigma_{\alpha\beta,\mu\nu}, \qquad (3.17)$$

where $\sigma_{\alpha\beta,\mu\nu}$ is the total cross section for two-particle scattering. Therefore, when $r_0 \ll \Lambda$, we find that $|\operatorname{Re} f_{\alpha\beta,\mu\nu}(0,q)| \gg |\operatorname{Im} f_{\alpha\beta,\mu\nu}(0,q)|$, so that, using the expression (2.11), we find in the leading approximation that

$$\tilde{\varepsilon}_{\alpha\beta}=\frac{p^2}{2m}\delta_{\alpha\beta}+g\sum_{\mathbf{p}'}(\delta_{\alpha\beta}\delta_{\mu\nu}-\sigma_{\alpha\beta}\sigma_{\mu\nu})\,n_{\nu\mu}(\mathbf{p}'),\; g\equiv\frac{2\pi a\hbar^2}{m}. \qquad (3.18)$$

Since we retain the gradient terms in the left member of the kinetic equation (3.12), we should, in principle, retain on the right-hand side of the equation the nonlocal gradient corrections in the collision integral:

$$\operatorname{St} n_{\alpha\beta}\infty\delta n_{\alpha\beta}Nr_0^2v_T(1+k\Lambda), \qquad (3.19)$$

where k^{-1} is the characteristic scale of the spatial inhomogeneity. In the temperature region (1.1) the nonlocal corrections in (3.19) turn out to be significantly smaller (by a factor of the order of $r_0/\Lambda \ll 1$) than the kinematic self-consistent corrections with $\tilde{\varepsilon}_{\alpha\beta}$ given by (3.18). Therefore, allowance for the quantum-mechanical self-consistent field in the Boltzmann equation with a local collision integral at the given temperatures is quite justified. At higher temperatures, i.e., for $T \gg \hbar^2/mr_0^2$, the pseudopotential method is ineffective, and the nonlocal corrections to $\operatorname{St} n_{\alpha\beta}$, which are proportional to $Nr_0^2v_Tkr_0\delta n_{\alpha\beta}$, become greater than, or comparable with, the kinematic virial corrections, so that we shall be exceeding the accuracy if we allow for the self-consistent field in the kinetic equation with a local collision integral at high temperatures. (The term $\nabla n_{\alpha\gamma}\partial\varepsilon_{\gamma\beta}/\partial\mathbf{p}$ on the left-hand side of the Boltzmann equation is significantly greater at any temperature than the nonlocal corrections in $\operatorname{St} n_{\alpha\beta}$ because of the smallness of the gas parameter $Nr_0^3 \ll 1$.) In computing the velocity of sound, however, we can take the kinematic virial corrections into account at arbitrary temperatures because of the fact that the exact collision integral vanishes when the appropriate integrations are performed in the course of the derivation of the macroscopic hydrodynamic equations.

It can be shown that, in virtue of specific circumstances, the kinetic equation with a local collision integral can also be used to describe the collective spin modes at arbitrary temperatures when allowance is made for the kinematic virial corrections only in the spin commutator $[\tilde{\varepsilon}, n]_{\alpha\beta}$. In this case all the gradient terms, except the term $\nabla n_{\alpha\gamma}\partial\varepsilon_{\gamma\beta}/\partial\mathbf{p}$, are dropped in the Boltzmann equation, and the role of the interaction-governed self-consistent correction to the gas-particle energy is played by the variational derivative $\delta E_{\rm int}/\delta n_{\beta\alpha}(\mathbf{p})$, where $E_{\rm int}=F_{\rm int}$ is given by the formulas (2.1), (2.4), (2.6), and (2.7). Similar equations are used in Ref. (20) to describe the magnetic resonance in binary gas mixtures. But at $T \gtrsim \hbar^2/mr_0^2$, because of the violation of the condition (1.3), the spin waves are always strongly damped. In the present paper we shall investigate only weakly damped spin waves, since it is precisely these waves that are of greatest interest. Therefore, we shall limit ourselves here to the derivation and solution of the kinetic equation valid at low temperatures, i.e., at $T \ll \hbar^2/mr_0^2$. Notice that, even when the condition (1.1) is fulfilled, we would be exceeding the accuracy if we allowed for the terms of the next order in $r_0/\Lambda \ll 1$ in the scattering amplitude $f_{\alpha\beta,\mu\nu}$, i.e., if we allowed for the scatterings with higher momenta, without making allowance for the damping because of the violation of the inequality (1.3) for the corresponding terms.

The analogy between the kinetic equations for a quantum Boltzmann gas and the equations of the theory of the Fermi liquid does not only lie in the relation (3.9) and in the fact that the correction to the particle energy (3.16) can be expressed in terms of the zero-angle scattering amplitude, but also has a more profound physical meaning. As $T\to 0$, all the results for a gas of Fermi particles naturally go over into the corresponding Galitzkiĭ results[25] for a tenuous Fermi liquid. But in a low-density Fermi liquid the excitation-damping constant is proportional to the square of the gas parameter, and is small even in the region far from the Fermi surface, and extending right up to the highest momenta $p \lesssim \hbar/r_0$. Therefore, even in the Boltzmann region extending right down to temperatures $T\sim\hbar^2/mr_0^2$ we have well-defined long-lived quasiparticles with the energy spectrum (3.18). [The problem of proving the existence of kinematic gradient corrections in the left member of the kinetic equaton is in a sense simpler in the case of a Fermi liquid than in the case of a gas, since in the former case the right-hand side of the equation, i.e., the collision integral, always contains the small factor $(T/\varepsilon_d)^2\to 0$.] Nevertheless, we must not wholly identify the cooperative properties of the quantum Boltzmann gas with the high-temperature "echoes" of the properties of the Fermi liquid as a system of fermions. Indeed, it can be seen from the very method of deriving the kinetic equation (3.12) that in the region (1.1), where the quantum-degeneracy effects are negligible, an equation of the type (3.6)–(3.8), (3.12) is valid and can be derived in a similar fashion for particles with an arbitrary spin value, regardless of whether they are fermions or bosons.

4. THE SPIN-WAVE SPECTRUM AND THE CORRELATION FUNCTIONS

It is convenient to use the generalized paramagnetic susceptibility to investigate the correlation properties of a spin-polarized gas. Let us introduce the effective external field $\mathscr{H}(\mathbf{r},t)\propto\exp(i\mathbf{kr}-i\omega t)$, and seek the linear response of the system to this perturbation in the form

$$n_{\alpha\beta}(\mathbf{p})-n_{\alpha\beta}^{(0)}(\mathbf{p})\equiv\delta n_{\alpha\beta}(\mathbf{p})=\lambda(\mathbf{p})\,\sigma_{\alpha\beta},\quad \lambda(\mathbf{p})\infty e^{i\mathbf{kr}-i\omega t}. \qquad (4.1)$$

In this case the renormalized gas-particle energy (3.18) in the external magnetic field is equal to

$$\tilde{\varepsilon}_{\alpha\beta}=\left(\frac{p^2}{2m}+gN\right)\delta_{\alpha\beta}-\left[gN\alpha\mathfrak{M}+2g\sum_{\mathbf{p}}\lambda(\mathbf{p})-\beta\vec{\mathscr{H}}\right]\sigma_{\alpha\beta}. \quad (4.2)$$

Substituting (4.2) into the kinetic equation (3.12), and linearizing it in the small deviations λ (**p**) and $\vec{\mathscr{H}}$, we obtain

$$(\omega-\mathbf{kv})\lambda(\mathbf{p})+\left\{\mathbf{kv}\left(\frac{\partial n_+}{\partial\varepsilon}+\frac{\partial n_-}{\partial\varepsilon}\right)\mathbf{F}(\lambda,\vec{\mathscr{H}})+\frac{2i}{\hbar}(n_+-n_-)[\mathfrak{M}\mathbf{F}(\lambda,\vec{\mathscr{H}})]\right\}=\mathrm{St}\,\lambda, \quad (4.3)$$

$$\mathbf{F}(\lambda,\vec{\mathscr{H}})=g\sum_{\mathbf{p}}\lambda(\mathbf{p})+\frac{\beta\vec{\mathscr{H}}}{2},\qquad \mathbf{v}=\frac{\mathbf{p}}{m},$$

where β is the elemental magnetic moment of a particle, and n_+ and n_- are the equilibrium occupation numbers for states with opposite spin orientations. Choosing the z axis in the direction of the vector $\mathfrak{M}$, and going over to the circular variables $\lambda_\pm=\lambda_x\pm i\lambda_y$, $\mathscr{H}_\pm=\mathscr{H}_x\pm i\mathscr{H}_y$, we arrive at an equation describing the transverse-magnetization dynamics in a spin-polarized quantum gas:

$$(\omega-\Omega_{int}-\mathbf{kv})\lambda_-(\mathbf{p})-\left[\mathbf{kv}\left(\frac{\partial n_+}{\partial\varepsilon}+\frac{\partial n_-}{\partial\varepsilon}\right)+\frac{2(n_+-n_-)}{\hbar}\right]\times\left[g\sum_{\mathbf{p}}\lambda_-(\mathbf{p})+\frac{1}{2}\beta\mathscr{H}_-\right]=\mathrm{St}\,\lambda_-,$$

$$\Omega_{int}=-2gN\alpha/\hbar=-4\pi a\hbar N\alpha/m. \quad (4.4)$$

The equation for λ_+ is obtained from (4.4) through the substitution $\omega\to-\omega$ and $\mathbf{k}\to-\mathbf{k}$. Since in the exchange approximation the absolute value of the magnetization is conserved, in the case of homogeneous distributions with $\mathbf{k}=0$ the integration of Eq. (4.4) over momentum space leads to the vanishing of the collision integral $\mathrm{St}\lambda_-$, so that the collision-governed relaxation time of the transverse spin waves with small **k** is clearly dependent on **k**, the dependence being such that $\tau_{\mathrm{coll}}(\mathbf{k}\to 0)\to\infty$. The possibility of neglecting the collision integral in the kinetic equation for small **k** is predicated on the fulfillment of the inequality

$$\hbar^{-1}|[\varepsilon,n]_{\alpha\beta}|\gg|\mathrm{St}\,n_{\alpha\beta}|, \quad (4.5)$$

which is equivalent to the condition $|\Omega_{\mathrm{int}}|\tau\gg1$, where $\tau^{-1}\sim Nr_0^2v_T$. The substitution of Ω_{int} from (4.4) again leads to the criterion (1.3), which was obtained earlier[1] from the condition for the existence of a region in which both the Landau damping and the collisional absorption of spin waves are weak. A similar criterion arises automatically in the formulation of the macroscopic equations of spin dynamics.[3,5]

In the collisionless regime, i.e., when the condition (1.3) is fulfilled, we find after integrating Eq. (4.4) the magnetic moment $\delta M_\pm=\delta M_x\pm i\delta M_y$ induced by the external magnetic field:

$$\delta M_\pm=\chi_\pm\mathscr{H}_\pm,$$

$$\chi_-(\omega,\mathbf{k})=\chi_+^*(-\omega,-\mathbf{k})=\beta^2\frac{R(\tilde{\omega},\mathbf{k})+Q(\tilde{\omega},\mathbf{k})}{1-gR(\tilde{\omega},\mathbf{k})-gQ(\tilde{\omega},\mathbf{k})},$$

$$\tilde{\omega}=\omega-\Omega_{int}, \quad (4.6)$$

where the functions R and Q are given by the expressions

$$R(\omega,\mathbf{k})=\sum_p\left(\frac{\partial n_+}{\partial\varepsilon}+\frac{\partial n_-}{\partial\varepsilon}\right)\frac{\mathbf{kv}}{\omega-\mathbf{kv}},$$

$$Q(\omega,\mathbf{k})=\frac{2}{\hbar}\sum_p\frac{n_+-n_-}{\omega-\mathbf{kv}}. \quad (4.7)$$

In the case when the distribution functions n_+ and n_- are Maxwellian, the functions R and Q from (4.7) can be expressed in terms of the error function of the complex variable:

$$R(\omega,\mathbf{k})=\frac{N}{T}\left\{1+i\pi^{1/2}\frac{\omega}{2^{1/2}kv_T}\times\exp\left(-\frac{\omega^2}{2k^2v_T^2}\right)\mathrm{erfc}\left(-\frac{i\omega}{2^{1/2}kv_T}\right)\right\},$$

$$Q(\omega,\mathbf{k})=\frac{2\alpha N}{\hbar\omega}\left[1-\frac{T}{N}R(\omega,\mathbf{k})\right],\qquad \mathrm{erfc}\,z=\frac{2}{\pi^{1/2}}\int_z^\infty e^{-t^2}dt. \quad (4.8)$$

The pole of the generalized magnetic susceptibility (4.6) determines the law of transverse-spin-wave dispersion:

$$1-gR(\omega-\Omega_{int},\mathbf{k})-gQ(\omega-\Omega_{int},\mathbf{k})\equiv D(\omega,\mathbf{k})=0. \quad (4.9)$$

In the long-wavelength region $kv_T\ll|\Omega_{\mathrm{int}}|$, $|\omega|\ll|\Omega_{\mathrm{int}}|$ the dominant contribution to $D(\omega,\mathbf{k})$ is made by the function Q, and, after a simple algebra, we find from (4.8) and (4.9) in the approximation linear in $r_0/\Lambda\ll1$ that

$$\omega'\equiv\mathrm{Re}\,\omega=-(kv_T)^2/\Omega_{int},$$

$$\omega''\equiv\mathrm{Im}\,\omega=-\left(\frac{\pi}{2}\right)^{1/2}\frac{\Omega_{int}^2}{kv_T}\exp\left(-\frac{\Omega_{int}^2}{2k^2v_T^2}-1\right),$$

$$|\omega''|\ll|\omega'|. \quad (4.10)$$

The condition, expressed by the inequality $kv_T\ll|\Omega_{\mathrm{int}}|$, of applicability of (4.10) ensures the weakness of the collisionless, Landau-damping-related spin-wave absorption. Let us note that it is impossible to determine the region of applicability of (4.10) in the scheme of the Leggett equations,[3,5] since collisionless damping does not figure there at all. Also because of Landau damping, spin waves, like, incidentally, any zero-sound-type oscillations governed by a linear dispersion law, cannot propagate in the short-wavelength region $kv_T\gg|\Omega_{\mathrm{int}}|$.

It is easy to verify that the elements of the generalized susceptibility matrix $\chi_{ik}(\omega,\mathbf{k})$, defined by the usual relation

$$\delta M_i(\omega,\mathbf{k})=\chi_{ik}(\omega,\mathbf{k})\mathscr{H}_k(\omega,\mathbf{k}),\quad i,k=x,y, \quad (4.11)$$

can be represented in terms of $\chi_\pm(\omega,\mathbf{k})$ as follows:

$$\chi_{xx}=\chi_{yy}=(\chi_++\chi_-)/2,\quad \chi_{yx}=-\chi_{xy}=-i(\chi_+-\chi_-)/2. \quad (4.12)$$

The dynamical magnetic form factor

$$S_{ik}(\omega,\mathbf{k})=\int d^3r\int_{-\infty}^{\infty}dt\,e^{i(\omega t-\mathbf{kr})}S_{ik}(t,r),$$

$$S_{ik}(t,r)=\langle\delta M_i(t_1,\mathbf{r}_1)\delta M_k(t_2,\mathbf{r}_2)\rangle,\quad t\equiv t_1-t_2,\quad r=|\mathbf{r}_1-\mathbf{r}_2| \quad (4.13)$$

of the system can be expressed with the aid of the dissipation–fluctuation theorem in terms of χ_{ik}:

$$S_{ik}(\omega,\mathbf{k})=2\hbar\,\mathrm{Im}\,\chi_{ik}(\omega,\mathbf{k})\,(1-e^{-\hbar\omega/T})^{-1}. \quad (4.14)$$

In the long-wavelength region $kv_T \ll |\Omega_{\rm int}|$ we find from (4.6) and (4.8) that

$$\operatorname{Im}\chi_-(\omega,\mathbf{k})=\beta^2\frac{g\Omega_{int}Z}{[\omega-\omega'(k)]^2+(Zg\Omega_{int})^2}\frac{\Omega_{int}}{g},$$
$$Z=\left(\frac{\pi}{2}\right)^{1/2}\frac{N}{T}\left(2\alpha\frac{T}{\hbar\tilde{\omega}}-1\right)\frac{\tilde{\omega}}{kv_T}\exp\left(-\frac{\tilde{\omega}^2}{2k^2v_T^2}\right)\ll 1, \tag{4.15}$$

where $\omega'(k)$ is the spin-wave spectrum given by the formula (4.10). With the aid of the well-known relation

$$\lim_{\gamma\to 0}\frac{\gamma}{x^2+\gamma^2}=\pi\delta(x) \tag{4.16}$$

we find from (4.12), (4.14), and (4.15) that

$$S_{xx}(\omega,\mathbf{k})=S_{yy}(\omega,\mathbf{k})$$
$$=2\pi\frac{\beta^2N\alpha}{1-e^{-\hbar\omega/T}}\left[\delta\left(\omega-\frac{k^2v_T^2}{|\Omega_{int}|}\right)-\delta\left(\omega+\frac{k^2v_T^2}{|\Omega_{int}|}\right)\right]. \tag{4.17}$$

The presence of δ-function terms in (4.17) corresponds to a contribution by the magnons, which constitute the collective Bose branch of the long-wavelength elementary excitations in the gas.

The static structure factor

$$S_{ik}(k)=\int e^{i\mathbf{kr}}S_{ik}(0,r)\,d^3r=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{ik}(\omega,\mathbf{k})\,d\omega \tag{4.18}$$

determines the purely spatial correlation of the transverse spin fluctuations in the system. It follows from the obvious inequality

$$N|a|^3\hbar^2/ma^2\ll\varepsilon_d\ll T\ll\hbar^2/ma^2 \tag{4.19}$$

that the inequality $\hbar|\Omega_{\rm int}|\ll T$ always holds in a quantum gas; therefore, when $k\ll|\Omega_{\rm int}|/v_T$, the performance of the trivial integration in the formulas (4.18), (4.17) leads to the result

$$S_{xx}(k)=\beta^2N\alpha\operatorname{cth}\frac{\hbar k^2v_T^2}{2|\Omega_{int}|T}=2\pi|a|\left(\frac{2\beta N\alpha}{k}\right)^2=S_{yy}(k). \tag{4.20}$$

Carrying out direct computations with (4.15), (4.12), and (4.14), or carrying out the integration in the formula

$$S_{yx}(\omega,\mathbf{k})=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1-\exp(-\hbar\omega'/T)}{1-\exp(-\hbar\omega/T)}\frac{S_{xx}(\omega',\mathbf{k})}{\omega'-\omega}d\omega', \tag{4.21}$$

which can be obtained from the Kramers-Krönig formulas, we find

$$S_{yx}(\omega,\mathbf{k})=-S_{xy}(\omega,\mathbf{k})$$
$$=\frac{2\beta^2N\alpha}{1-e^{-\hbar\omega/T}}\left[\frac{1}{\omega+(k^2v_T^2/|\Omega_{int}|)}+\frac{1}{\omega-(k^2v_T^2/|\Omega_{int}|)}\right] \tag{4.22}$$

and in the case when $\hbar|\omega|/T\ll 1$ we obtain the natural result $S_{yx}(k)=-S_{xy}(k)=0$ with the same accuracy with which the final expression, (4.20), for $S_{ii}(k)$ was obtained.

The Fourier inversion of the formula (4.20) for $S_{ik}(k)$ allows us to find the correlation function $S_{ik}(r)$ in the coordinate representation:

$$S_{ik}(r)=2|a|(\beta N\alpha)^2r^{-1}\delta_{ik}, \tag{4.23}$$

which falls off with distance according to the law that obtains in a cubic ferromagnet with localized spins. The final answers for the correlation function (4.20), (4.23) and for the magnon energy $\hbar\omega'(k)$ given by (4.10) do not contain the Planck constant, which is a reflection of the fact that, in spite of the essentially quantum nature of the cause of spin waves in a gas, in the statistical sense these waves behave like classical fluctuations, and can be considered to be the spatially inhomogeneous precession of the macroscopic magnetic moment. The correlation between the spins of different particles over macroscopic distances results in the existence of a definite macroscopic inhomogeneity energy. Let us represent the corresponding change that occurs in the free energy as a result of the transverse-magnetization fluctuations in the form

$$\Delta F=\frac{1}{2}\int\varphi(r)\,\delta\mathbf{M}(\mathbf{r}_1)\,\delta\mathbf{M}(\mathbf{r}_2)\,d^3r_1\,d^3r_2,\qquad r=|\mathbf{r}_1-\mathbf{r}_2|. \tag{4.24}$$

Within the framework of the theory of classical fluctuations,[22] it is not difficult to relate the magnitude of the Fourier transform of the function $\varphi(r)$ and the static form factor:

$$S_{ik}(k)=\delta_{ik}T/\varphi(k), \tag{4.25}$$

whence it immediately follows that

$$\varphi(k)=\frac{T}{2\pi|a|}\left(\frac{k}{2\beta N\alpha}\right)^2. \tag{4.26}$$

Usually, the free energy due to the slow variation of the direction of the vector $\mathbf{M}$ along the system is represented in the form of an expansion in a series in powers of the magnetization gradients:

$$\Delta F=\frac{1}{2}A\delta_{ik}\int\frac{\partial\mathbf{M}}{\partial r_i}\frac{\partial\mathbf{M}}{\partial r_k}d^3r. \tag{4.27}$$

Comparing (4.27), (4.24), and (4.26), we find

$$A=(T/2\pi|a|)(2\beta N\alpha)^{-2}. \tag{4.28}$$

The prescription of the coefficient A from (4.27), (4.28) allows us to fully describe the long-wave magnetization oscillations in a purely phenomenological manner. Indeed, it is easy to verify that the substitution of ΔF from (4.27) with the coefficient A given by (4.28) into the Landau-Lifshitz equation

$$\frac{\partial\mathbf{M}}{\partial t}=-\frac{2\beta}{\hbar}\left[\frac{\delta}{\delta\mathbf{M}}\Delta F,\mathbf{M}\right] \tag{4.29}$$

reproduces the spectrum $\omega'(k)$ given by (4.10). This implies that an essentially hydrodynamic situation obtains in the long-wavelength region $kv_T\ll|\Omega_{\rm int}|$ despite the "high-frequency" character of the spin oscillations.

The expression (4.20) for $S_{ik}(k)$ is valid for small values of the wave vector $kv_T\ll|\Omega_{\rm int}|$, so that the formula (4.23) is also suitable for the description of spin correlations over fairly large distances $r\gg r_{\rm int}\equiv v_T/|\Omega_{\rm int}|$. The characteristic parameter $r_{\rm int}$ can be interpreted as a distinctive correlation scale in the self-consistent quantum field, i.e., as a scale characterizing the difference between the correlation properties of a quantum gas and point correlations, which are characteristic of a classical gas. Thus, the function $\varphi(k)$ can be expanded in even powers of $kr_{\rm int}$, which corresponds to the exponential decrease of the function $\varphi(r)$ at large distances $r \gg r_{\rm int}$. In this case, on account of the natural—for a gas—inequalities

$$(|\Omega_{int}|/v_T)N^{-1/3} \sim (\varepsilon_d/T)(|a|/\Lambda) \ll 1, \quad 1 \gg kN^{-1/3} \gg k\Lambda, \quad (4.30)$$

the sphere of radius $r_{\rm int}$ (the correlation zone) always contains a macroscopically large number of particles: $Nr_{\rm int}^3 \gg 1$, which qualitatively accounts for the long-range spin correlations in the system.

When the condition (1.3) is fulfilled, small imaginary corrections to the spectrum (4.10) can also be easily obtained; these due to the collision-governed spin-wave absorption:

$$\omega = -\frac{(kv_T)^2}{\Omega_{int}}\left(1 + \frac{i}{\Omega_{int}\tau}\right), \quad (4.31)$$

where τ is the relaxation time, which has a gas-kinetic order of magnitude: $\tau \sim (Nr_0^2 v_T)^{-1}$. In this case, we find for the imaginary part of the generalized susceptibility, and, hence, for the absorption line shape obtainable in magnetic-resonance experiments,

$$\Delta\omega \sim k^2 \frac{T}{\alpha^2}\frac{mv_T}{\hbar^2 N} \propto \frac{T^{3/2}k^2}{N\alpha^2},$$
$$I(0) \sim \frac{\beta^2 N\alpha}{\hbar\Delta\omega} \propto \frac{\alpha^3 N^2}{T^{3/2}} k^{-2}, \quad (4.32)$$

i.e., as k increases, the linewidth increases and the peak intensity $I(0)$ decreases, while as T decreases, or α increases, the lines narrow down and become more intense and the number of observable lines increases. It is precisely these dependences that have been observed in experiments.[6]

In conclusion, let us note that the presence of a constant external magnetic field $\mathbf{H}$ leads to the appearance of an additional term $\delta\varepsilon_{\alpha\beta} = -\beta\boldsymbol{\sigma}_{\alpha\beta}\cdot\mathbf{H}$ in the expression for the particle energy, the appearance of a gap in the spin-wave spectrum:

$$\omega = -2\beta H/\hbar - (kv_T)^2/\Omega_{int}, \quad (4.33)$$

and the screening of the spin correlations at very large distances:

$$S_{ik}(r) = \delta_{ik}(2|a|/r)(\beta N\alpha)^2 e^{-r/r_H}, \quad \beta H \ll T, \quad a<0,$$
$$S_{ik}(r) = \delta_{ik}(2|a|/r)(\beta N\alpha)^2 \cos(r/r_H), \quad a>0, \quad (4.34)$$

where the magnetic correlation length r_H is given by the relation

$$r_H{}^2 = \frac{1}{4\pi\alpha}\frac{T}{2\beta H}\frac{1}{N^{1/3}|a|}N^{-2/3} \gg N^{-2/3}. \quad (4.35)$$

5. LIQUID ^{3}He↑ AND THE ^{3}He↑–He II SOLUTION

The collective oscillations in liquid ^{3}He and degenerate ^{3}He–^{4}He solutions are described by the kinetic equation of the Landau theory of the Fermi liquid.[11] For small values of the degree of polarization, i.e., for $\alpha \ll 1$, it is possible to obtain an exact analytic solution for the transverse-spin-wave spectrum[18]:

$$\omega = \frac{\hbar k^2}{2m^*\alpha}B(Z), \quad B(Z) = \frac{(1+Z_0)(1+Z_1/3)}{Z_0 - Z_1/3}, \quad (5.1)$$

where Z_0 and Z_1 are the first harmonics of the spin part of the local Fermi-liquid function and m^* is the effective Fermi-excitation mass. In principle, the nonlocal character of the Fermi-liquid interaction also gives rise to quadratic—in k—corrections to the magnon spectrum, which are, however, important only in highly polarized Fermi liquids, and can be neglected when $\alpha \ll 1$. Indeed, it can easily be verified with the aid of the kinetic equation that the first nonlocal correction

$$\delta\varepsilon_{\alpha\beta} = \sum_{\mathbf{p}'} \varphi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')\nabla^2\delta n_{\nu\mu}(\mathbf{p}'), \quad \varphi_{\alpha\beta,\mu\nu} = \eta\delta_{\alpha\beta}\delta_{\mu\nu} + \xi\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu} \quad (5.2)$$

to the single-particle excitation energy leads to the appearance of the following term in the spectrum (5.1):

$$\delta\omega = -\frac{2}{\hbar}k^2 N\alpha \int \xi(\theta)\frac{do}{4\pi} = -\frac{4}{3\hbar}\varepsilon_F k^2 Z_0 \alpha, \quad (5.3)$$

which is always smaller than (5.1) by a factor of the order of $\alpha \ll 1$.

At $T \to 0$ the spin fluctuations in a degenerate Fermi system are quantum fluctuations, so that the classical relations (4.25) and (4.28) are no longer applicable. As pointed out in Ref. 18, we do not exceed the accuracy of the theory of the Fermi liquid if we allow for the contribution of the transverse spin waves to the thermodynamics of the polarized system. Let us represent the Hamiltonian corresponding to this contribution in the form

$$\hat{H} = \frac{1}{2}\int \varphi(\mathbf{r}_1 - \mathbf{r}_2)\delta\hat{\mathbf{M}}(\mathbf{r}_1, t)\delta\hat{\mathbf{M}}(\mathbf{r}_2, t)\, d^3r_1\, d^3r_2, \quad (5.4)$$

where $\delta\hat{\mathbf{M}}(\mathbf{r},t)$ is the Heisenberg operator for the slight deflection of the magnetization vector from the equilibrium direction of $\mathfrak{M}$. Averaging the Hamiltonian (5.4) for $T=0$ over the ground state, we find the zero-point oscillation energy of the magnon field:

$$\sum_{\mathbf{k}} \varphi(k)\langle|\delta\hat{\mathbf{M}}_{\mathbf{k}}|^2\rangle = A\sum_{\mathbf{k}} k^2[S_{xx}(k) + S_{yy}(k)] = \hbar\omega(k),$$
$$\delta\hat{\mathbf{M}}_k = \int \delta\hat{\mathbf{M}}(\mathbf{r},t)e^{-i\mathbf{k}\mathbf{r}}\, d^3r. \quad (5.5)$$

A similar approach was developed earlier in the hydrodynamics of He II by Pitaevskiĭ.[26] It follows from the phenomenological spin-wave theory based on the Landau-Lifshitz equations and from the relation (5.5) that in the small-k region

$$S_{xx}(k) = S_{yy}(k) = \hbar\omega(k)/Ak^2 = \beta^2 N\alpha. \quad (5.6)$$

In the collisionless regime we have for the generalized susceptibility the expression

$$\mathrm{Im}\,\chi_-(\omega, \mathbf{k}) = B\delta(\omega - bk^2) \quad b = \hbar B(Z)/2m^*\alpha. \quad (5.7)$$

The coefficient B in (5.7) is not a function of the temperature, since in the kinetic equation for the Fermi liquid, with the aid of which the expression (5.7) was derived, the temperature is contained only in the collision term, which we neglect. The temperature corrections in the kinematic part of the kinetic equation can lead only to a weak dependence of the spin-wave spectrum on T, i.e., to the renormalization of

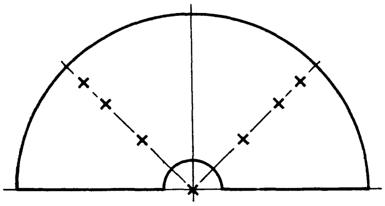

FIG. 1.

the quantity b. Computing the static form factor with the aid of the formulas (4.12), (4.14), (4.18), and (5.7), and normalizing the quantity B by the value of $S_{ik}(k)$ from (5.6) for $T=0$, we obtain

$$S_{ik}(k)=\delta_{ik}\beta^2N\alpha\operatorname{cth}\frac{\hbar bk^2}{2T},$$

$$S_{ik}(r)=\left(\frac{\beta}{2\pi}\right)^2\frac{N\alpha}{ir}\delta_{ik}\int_{-\infty}^{\infty}e^{ikr}\operatorname{cth}\left(\frac{\hbar bk^2}{2T}\right)k\,dk$$

$$=\left(\frac{\beta}{2\pi}\right)^2\frac{N\alpha}{ir}\int_{-\infty}^{\infty}f(k)\,dk\,\delta_{ik}. \tag{5.8}$$

The function $f(k)$ has poles of order one at $k=0$ and at the points $k=k_l$:

$$k_l=\begin{cases}\pm(\pi T/\hbar b)^{1/2}|l|^{1/2}(1+i), & l=1,2,3,\dots,\\ \pm(\pi T/\hbar b)^{1/2}|l|^{1/2}(-1+i), & l=-1,-2,-3,\dots.\end{cases} \tag{5.9}$$

Therefore, choosing the integration contour as shown in Fig. 1, we find

$$S_{ik}(r)=\delta_{ik}\frac{\beta^2N\alpha}{4\pi r}\Big[\operatorname{Res}f(k)\,|_{k=0}+2\sum_l\operatorname{Res}f(k)\,|_{k=k_l}\Big], \tag{5.10}$$

$$S_{ik}(r)=\delta_{ik}\frac{\beta^2N\alpha}{2\pi r}\frac{T}{\hbar b}$$

$$\times\left\{1+2\sum_{l=1}^{\infty}\exp\left[-\left(\frac{\pi Tl}{\hbar b}\right)^{1/2}r\right]\cos\left(\frac{\pi Tl}{\hbar b}\right)^{1/2}r\right\}.$$

Basically, the law of decrease of the correlation function $S_{ik}(r)$ at large distances is determined, as before, by the power-law dependence r^{-1}.

The magnitude of the collision-governed spin-wave absorption is determined by the value of the parameter $\Omega_{\text{int}}\tau$, which can be computed exactly with the aid of the Leggett equations[4]:

$$\Omega_{int}\tau=\frac{4}{3}\alpha\frac{\varepsilon_F}{\hbar}\tau\frac{Z_0-Z_1/3}{1+Z_1/3}=\frac{2\alpha D_0m^*}{\hbar B(Z)},\quad\varepsilon_F=\frac{m^*v_F^2}{2}, \tag{5.11}$$

where D_0 is the coefficient of spin diffusion. If $|\Omega_{\text{int}}|\tau\gg1$, a condition which is easily fulfilled because of the presence in τ of the large factor $(\varepsilon_F/T)^2\gg1$, then the collisional damping is weak, and the spin-wave spectrum contains only a small imaginary correction:

$$\omega=bk^2[1-i(\Omega_{int}\tau)^{-1}]. \tag{5.12}$$

The condition for weak collisionless damping, $kv_F\ll|\Omega_{\text{int}}|$, determines the regions of wave vectors and distances $r\gg r_{\text{int}}\equiv v_F/|\Omega_{\text{int}}|$ where the expressions (5.1) and (5.10) are valid.

The contribution of the magnons with the dispersion law (5.1) to the thermodynamics of the Fermi liquid is cut off at high frequencies $|\omega|\ll|\Omega_{\text{int}}|$. Therefore, such a contribution will have its maximum value at $T\lesssim\hbar|\Omega_{\text{int}}|$. If the polarization of the spins of the system is produced by an external magnetic field, then the spin-fluctuation spectrum contains a gap.

$$\omega=\Omega_H+bk^2,\quad\alpha=3\hbar\Omega_H/4\varepsilon_F(1+Z_0),\quad\hbar\Omega_H=2\beta H, \tag{5.13}$$

and the inequality $T\lesssim\hbar|\Omega_{\text{int}}|$ reduces to the relation $T\lesssim\hbar\Omega_H/B(Z)$. If $B(Z)\gtrsim1$, then we shall always have $bk^2\ll\Omega_H$, and the contribution of the spin waves to the thermodynamics when $T\ll\hbar\Omega_H/B(Z)$ and $b>0$ turns out to be exponentially small:

$$E_M=\frac{1}{2\pi^2}\int\frac{\hbar bk^4\,dk}{\exp[(\hbar bk^2+\hbar\Omega_H)/T]-1}$$

$$=\frac{3}{16}\exp\left(-\frac{\hbar\Omega_H}{T}\right)T\left(\frac{T}{\pi\hbar b}\right)^{3/2} \tag{5.14}$$

When $B(Z)\ll1$ and $b>0$, the magnon contribution to the total Fermi-liquid energy in the region of sufficiently low temperatures, specifically, in the region $T\ll\hbar\Omega_H$, is also given by the formula (5.14), i.e., turns out to be significantly smaller than the quadratic—in $\beta H/\varepsilon_F$—Fermi corrections. The situation is entirely different in the temperature region $\hbar\Omega_H\ll T\ll\hbar\Omega_H/B(Z)$, where the dominant contributions to the thermodynamics that arise as a result of the presence of the external magnetic field are made precisely by the spin waves, and not by the Fermi excitations. With the aid of the formulas obtained in Ref. 18, we obtain for the specific heat, after substituting the value of b from (5.7) and (5.1), the expression

$$C_V=\frac{\pi^2}{2}N\frac{T}{\varepsilon_F}\left\{1+\frac{15}{32\pi^2}\Gamma\left(\frac{5}{2}\right)\zeta\left(\frac{5}{2}\right)\right.$$

$$\left.\times\left(\frac{T}{\varepsilon_F}\right)^{1/2}\left[\frac{3\hbar\Omega_H}{\varepsilon_FB(Z)(1+Z_0)}\right]^{3/2}\right\}. \tag{5.15}$$

The case $b<0$, which is realized in liquid ^{3}He, will be considered at the end of the section.

If, on the other hand, we are dealing with the quasi-equilibrium spin-polarized state of the Fermi liquid in the absence of an external field (e.g., liquid ^{3}He↑ with a lifetime of the order of 5 min, obtained by rapidly melting the polarized crystalline phase[27]), then the thermodynamics of the spin waves has an entirely different character. Indeed, since we are considering times shorter than the relativistic relaxation time of the absolute value of the magnetization, and the probability for decay of a spin wave into two Fermi excitations with parallel spins is exponentially small, the number of magnons in the system is conserved: it is temperature independent, and is equal to the number of spin waves that were excited at the initial moment of time during the creation of the polarized state. In this case the system of magnons is characterized by a nonzero chemical potential μ, which is given by the normalization condition

$$\frac{1}{(2\pi)^3}\int\frac{d^3k}{\exp[(\hbar bk^2-\mu)/T]-1}=N_m, \tag{5.16}$$

where N_m is the fixed number of magnons. The temperature $T = T_0$, at which μ vanishes:

$$T_0=\frac{2\pi}{\zeta^{2/3}(3/2)}\frac{\hbar^2N_m^{2/3}}{M},\qquad M=\frac{\hbar}{2b}=m^*\frac{\alpha}{B(Z)},\tag{5.17}$$

determines the critical point of the Bose-Einstein condensation in an ideal magnon gas, which, under the assumption that the degeneracy has been lifted and that there exists only one condensate, implies the appearance of spontaneous transverse magnetization. In the region $T_0 \ll T \ll \hbar\Omega_{\rm int}/B(Z)$ the contribution of the spin waves to the thermodynamics reduces to a temperature-independent term in the expression for the specific heat:

$$C_V={}^3/_2N_m+{}^1/_2\pi^2N(T/\varepsilon_F).\tag{5.18}$$

In the region $T < T_0$, $T \ll \hbar\Omega_{\rm int}/B(Z)$ we find for the thermodynamic functions of a spin-polarized Fermi liquid the expressions

$$E_M=\frac{3}{2}\frac{\zeta(5/2)}{\zeta(3/2)}N_mT\left(\frac{T}{T_0}\right)^{3/2},\qquad C_V=\frac{\pi^2}{2}N\frac{T}{\varepsilon_F}+\frac{5}{2}\frac{E_M}{T},\tag{5.19}$$

$$N_m^{(0)}=N_m\left[1-\left(\frac{T}{T_0}\right)^{3/2}\right],\quad \Delta\left(\frac{\partial C_V}{\partial T}\right)=-\frac{27}{16\pi}\zeta^2\left(\frac{3}{2}\right)\frac{N_m}{T_0},$$

where $N_m^{(0)}$ is the number of magnons in the condensate and $\Delta(\partial C_V/\partial T)$ is the jump in the thermodynamic derivative at the phase transition point.

In Ref. 18 the possibility of the occurrence of an equilibrium ferromagnetic order at finite temperatures in a Fermi liquid possessing a large paramagnetic susceptibility, i.e., in the case $|1 + Z_0| \ll 1$, is considered. In liquid ^{3}He, in which $1 + Z_0 \approx 0.3$, the temperature region where this effect could manifest itself falls on the boundary of the region of applicability of the theory of the Fermi liquid, so that there is, of course, no absolute certainty that a high-temperature ferromagnetic phase occurs in liquid ^{3}He. Appreciable anomalies in the magnetic properties of liquid ^{3}He at the corresponding temperatures were experimentally observed in Ref. 28. On the other hand, no appreciable deviations were detected by Sen and Archie[29] in their measurements of the specific heat of ^{3}He in a magnetic field. We must, in searching for the indicated phase, bear in mind the following important circumstance not noted in Ref. 18. The system's maximum geometrical dimension d for which the spatial distribution of the magnetization in the system is still homogeneous can be estimated from the obvious relation[30]

$$M^2/\chi\sim AM^2/d^2.\tag{5.20}$$

If the system has a dimension greater than d, a spatially inhomogeneous distribution is realized in it, with a possible partial, or even complete cancellation of the magnetic moment. The specific picture of the magnetization distribution depends on the boundary conditions and the shape of the sample. Let us emphasize that we are not talking about the domain structure, since the dimension of a domain wall under these conditions can be significantly greater than the dimension of the system. Substituting A from the results of Ref. 18, or from the formula (5.6), into (5.20), we obtain

$$d\sim(N^{-1/3}/\alpha)[(1+Z_1/3)/(Z_0-Z_1/3)]^{1/2}\gg N^{-1/3}.\tag{5.21}$$

Although the parameter d is of macroscopic scale, estimates show that it is nonetheless significantly smaller than the geometrical dimensions of standard experimental cells, which must thus exhibit macroscopic magnetic inhomogeneities. But the formulas of Ref. 18 imply a homogeneous magnetization along the system, i.e., a micromagnetism of the samples, for example, in pores of diameter $l \lesssim d$.

All the expressions obtained above describe degenerate concentrated ^{3}He–^{4}He solutions as well. In the case of weak solutions the harmonics Z_0 and Z_1 can be computed in their explicit form with the aid of the method used in Ref. 31:

$$Z_0=-2\lambda\left[1+\frac{4\lambda}{3}(1-\ln 2)\right],$$

$$\frac{Z_1}{3}=-\frac{8\lambda^2}{15}(2+\ln 2),\qquad \lambda\equiv\left(\frac{3N}{\pi}\right)^{1/3}a.\tag{5.22}$$

We can obtain in the leading approximation in λ results that are valid at arbitrary values of α. Noting that, in the case of a tenuous ensemble of impurity ^{3}He quasiparticles with $N|a|^3 \ll 1$, the relation

$$|Q(\tilde\omega,\mathbf{k})|\sim N\alpha/\hbar|\Omega_{int}|\gg kv_FN/\varepsilon_F|\Omega_{int}|\sim|R(\tilde\omega,\mathbf{k})|,$$
$$kv_F\ll|\Omega_{int}|,\tag{5.23}$$

is always fulfilled, we obtain from Eq. (4.9) with the Fermi distribution functions

$$n_\pm(\varepsilon)=\theta(\varepsilon)\theta(\varepsilon_F^{(\pm)}-\varepsilon)-\frac{\pi^2}{6}T^2\frac{\partial}{\partial\varepsilon}\delta(\varepsilon-\varepsilon_F^{(\pm)}),\tag{5.24}$$

where $\theta(\varepsilon)$ is the Heavyside function, the expression (cf. Ref. 13)

$$\omega=-\frac{3}{40}\frac{\hbar k^2}{m^*\lambda}\frac{(1+\alpha)^{5/3}-(1-\alpha)^{5/3}}{\alpha^2}\times\left\{1+\frac{5\pi^2}{8}\left(\frac{T}{\varepsilon_F}\right)^2\frac{(1+\alpha)^{1/3}-(1-\alpha)^{1/3}}{(1+\alpha)^{5/3}-(1-\alpha)^{5/3}}\right\}.\tag{5.25}$$

The generalized susceptibility in the vicinity of its pole $\omega = bk^2$ has the natural form:

$$\operatorname{Im}\chi_-(\omega,\mathbf{k})=\lim_{\gamma\to0}\beta^2\operatorname{Im}\left(\frac{\partial D}{\partial\omega}\right)^{-1}\frac{R+Q}{\omega-bk^2-i\gamma}=\frac{2\pi\beta^2N\alpha}{\hbar}\delta(\omega-bk^2).\tag{5.26}$$

In their explicit form, Im R and Im Q can be computed with the aid of the standard rule for bypassing the poles:

$$\int\frac{f(\mathbf{p})\,d\Gamma}{\omega-\mathbf{kv}-i\delta}=\int\frac{f(\mathbf{p})\,d\Gamma}{\omega-\mathbf{kv}}+i\pi\int f(\mathbf{p})\delta(\omega-\mathbf{kv})\,d\Gamma,\tag{5.27}$$

and turn out to be equal to

$$\begin{aligned}&\operatorname{Im}Q(\omega,\mathbf{k})=(m^2/2\pi\hbar^4k)[\varkappa_+\theta(\varkappa_+)-\varkappa_-\theta(\varkappa_-)],\\&\varkappa_\pm=\varepsilon_F(1\pm\alpha)^{2/3}-(m\omega^2/2k^2),\\&\operatorname{Im}R(\omega,\mathbf{k})=-(m^2\omega/4\pi\hbar^3k)[\theta(\eta_+)+\theta(\eta_-)],\\&\eta_\pm=kv_F(1\pm\alpha)^{1/3}-\omega.\end{aligned}\tag{5.28}$$

The experimental spin-wave spectrum data reported in Ref. 16 yield the values $Z_0 \approx 0.08$ and $Z_1 \approx 0.34$ for a 5% solution, so that $|Z_0| < |Z_1/3|$, $B(Z) < 0$. On the other hand, at sufficiently low concentrations we have $Z_0 \approx -2\lambda > 0$, $|Z_0| \gg |Z_1/3|$, $B(Z) > 0$. Therefore, there exists a critical concentration value x_c at which $Z_0 - (Z_1/3) = 0$, while the function $B(Z)$ changes sign at the discontinuity point: $B(Z) = \pm\infty$ when $x = x_c \pm 0$, i.e., $|\Omega_{\text{int}}|\tau = 0$, and an oscillating spin-wave solution does not, in general, exist: only purely diffusive spreading of the magnetization occurs. Estimates show that the concentration region around x_c where $|\Omega_{\text{int}}|\tau \ll 1$ and the propagation of weakly damped spin waves turns out to be impossible lies in the 1–3% range. The difficulty in experimentally observing spin waves at $x = 0.01$ is noted in Ref. 16. When the concentration in the ^{3}He–^{4}He solution is lowered further, there again appear spin waves, whose observation certainly lies within the limits of experimental possibilities. Thus, using the expressions for the line widths and intensities obtainable in NMR measurements, i.e., the expressions

$$\Delta\omega = [\hbar B(Z)k]^2/(2m^*\alpha)^2 D_0, \quad I(0) \equiv I_0 = 2\beta^2 N\alpha/\hbar\Delta\omega, \tag{5.29}$$

the formulas of the theory of dilute solutions,[31] and the experimental data reported in Ref. 16, we find the relative characteristics of the lines in solutions with $x = 0.05$ and $x = 0.005$:

$$\begin{aligned} &\Delta\omega(x=0.5\%)/\Delta\omega(x=5\%) \approx 0.5, \\ &I_0(x=0.5\%)/I_0(x=5\%) = 0.2. \end{aligned} \tag{5.30}$$

The condition $|\Omega_{\text{int}}|\tau \gg 1$ for weak damping of the spin waves in the region $x < x_c$ is equivalent to the inequality

$$10\alpha N|a|^3\hbar^2/m^* a^2 T \gg 1, \tag{5.31}$$

which, under the conditions of the experiment reported in Ref. 16, is satisfied at all reasonable concentrations $x_c > x \gg 5.8\times10^{-9}$ when $\Omega_H = 925$ kHz and $T = 0.3$ mK and at not too low concentrations $x_c > x \gg 5.8\times10^{-3}$ when $T = 3$ mK. Since the function $B(Z)$ changes sign in the region $x < x_c$, the shift of the resonance frequency relative to Ω_H will also have the sign opposite to the one that was observed at $x = 0.05$, i.e., in the region $x > x_c$. The determination of a by measuring the resonance-frequency shift in dilute solutions will allow a more accurate estimate of the critical temperature T_c of the transition of the ^{3}He in a ^{3}He–^{4}He solution into the superfluid state.[32] Let us emphasize that the detection of zero-sound-type longitudinal spin waves with a linear—in k—dispersion law is impossible in the region of temperatures where the impurity ^{3}He atoms form a nonsuperfluid system, since the criterion for their observation,[13] $T \ll \varepsilon_F \exp(-1/|\lambda|)$, contradicts the condition $T > T_c \sim \varepsilon_F \exp(-/|\lambda|)$.

The spin waves in nondegenerate dilute solutions are described by the formulas for quantum Boltzmann gases with m replaced by m^* (Ref. 13). If the polarization of the solution is effected with the aid of an external magnetic field, then the condition (1.3) for weak damping of the spin oscillations, i.e., the condition

$$1 \geqslant \text{th}(\beta H/T) \gg |a|/\Lambda, \quad T \gg \varepsilon_F, \tag{5.32}$$

implies the use of sufficiently high, but practically attainable, fields $\beta H/\varepsilon_F \gg N^{1/3}|a|$. In this case the temperature region where the propagation of weakly damped magnetization oscillations is possible in the nondegenerate solutions is given by the relation

$$\varepsilon_F[\beta H/\varepsilon_F(N^{1/3}|a|)]^{2/3} \gg T \gg \varepsilon_F. \tag{5.33}$$

An extremely interesting situation arises in the case when the quadratic—in k—term in the spin-wave spectrum in an external magnetic field has the negative sign:

$$\omega = \Omega_H + bk^2, \quad b < 0. \tag{5.34}$$

Such a situation can be realized if we polarize the system in a certain direction $\mathfrak{M}$ with the aid of some dynamical method, e.g., by optical puming, by melting the magnetically-ordered crystalline phase, or through injection of a polarized beam, and then apply a weak external magnetic field oriented in a compensating manner ($\mathbf{H}\uparrow\downarrow\mathfrak{M}$) if $b > 0$, or parallel to $\mathfrak{M}$ ($\mathbf{H}\uparrow\uparrow\mathfrak{M}$) in the case $b < 0$. In that case, if the field intensity is not too high, i.e., if $\Omega_H/|b| \equiv k_H^2 \ll |\Omega_{\text{int}}|^2/v_F^2$ for a Fermi liquid and $k_H^2 \ll |\Omega_{\text{int}}|^2/v_T^2$ in the case of a quantum Boltzmann gas, then the spin-wave frequency will vanish at $k = k_H$ in the wave-vector region where undamped magnetization oscillations still occur.

This does not, however, imply the onset of thermodynamic instability, since we are talking about a polarized quasi-equilibrium state, and over the long relativistic longitudinal-relaxation time, during which the true thermodynamic equilibrium sets in, the dynamically induced polarization vanishes completely, and the system goes over into a new state with $\mathfrak{M}\uparrow\uparrow\mathbf{H}$ and an α value determined by the external field. Nevertheless, this implies the possibility of the existence of a dissipative helicoidal superstructure with a spatial period $2\pi/k_H$ and a lifetime

$$t = \tau(|\Omega_{int}|/|b|k_H^2) = \tau(|\Omega_{int}|/\Omega_H) \gg \tau, \tag{5.35}$$

which turns out in the case when $\varepsilon_F \gg T \to 0$ to be arbitrarily long, since under these conditions $\tau \propto (\varepsilon_F/T)^2$. If $b < 0$, and the polarization of the system is effected through the application of an external magnetic field, so that the absolute values of α and b are themselves determined by the value of H, then in the overwhelming majority of cases the spin waves begin to attenuate strongly before their frequency vanishes, i.e., $k_H^2 \gtrsim |\Omega_{\text{int}}|^2/v_F^2$ for a Fermi liquid and $k_H^2 \gtrsim |\Omega_{\text{int}}|^2/v_T^2$ in the case of a nondegenerate gas. In certain cases with special relations between the Fermi-liquid harmonics, there can, in principle, exist the situation in which $k_H^2 \ll |\Omega_{\text{int}}|^2/v_F^2$ even when the system is polarized by an external magnetic field, which may indicate the thermodynamic instability of the homogeneously magnetized state.

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