Stability of steady-state stimulated Brillouin scattering

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The conditions under which a temporal instability (generation) can occur during stimulated Brillouin scattering are derived by a new approach. These conditions differ from the customary Kroll-Bobroff conditions. The requirements which would be imposed on an experiment carried out to observe the generation regime are quite stringent.

It is generally believed that stimulated Brillouin scattering (or "stimulated Mandel'shtam-Brillouin scattering") in a sample without a resonator can occur in two regimes: amplification or generation. This conclusion was reached in theoretical work by Kroll[1] and Bobroff[2] and is reflected, for example, in the review by Starunov and Fabelinskiĭ.[3] In a real experimental situation it is frequently necessary to take into account the depletion of the incident light wave. A distinction should be made between steady-state and time-varying regimes. In the present paper we wish to draw a general picture of the process. It turns out that the generation regime is possible only if the experimental conditions meet some stringent requirements.

In the generation regime the amplitude of the scattered light and of the sound increase without bound and do not reach steady-state levels. This process is stopped only because of the depletion of the incident light wave. We wish to determine the implications of the generation regime in a study of how the scattered light intensity I_1 depends on the incident intensity I_0. In the amplification regime, I_1 reaches a steady-state value after a long time, and it increases exponentially with increasing I_0. When the generation threshold is reached, I_1 should increase abruptly to a level on the order of I_0.

When depletion of the incident wave is taken into account, a deviation from an exponential dependence of I_1 on I_0 also occurs in the amplification regime.[4] We will show that it is possible to arrange a situation in which this process becomes important at intensities I_0 below the generation threshold. Consequently, the generation threshold derived in Refs. 1 and 2 without allowance for the depletion does not apply in this situation. Since Tang[4] considered only the steady-state situation, we have checked the temporal stability of the solutions of Ref. 4 with respect to small perturbations. It is found that perturbations which are introduced decay over time regardless of the value of I_0, i.e., no new thresholds of any sort arise in the theory incorporating depletion of the incident light.

The complete system of equations is[3]

$$\frac{\partial E_0}{\partial x}+\alpha_1 E_0=-aE_1u,\qquad a=\frac{\varepsilon^{3/2}p\omega q}{4c},\tag{1}$$

$$\frac{\partial E_1}{\partial x}+\alpha_1 E_1=-aE_0u,\tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{1}{w}\frac{\partial u}{\partial t}+\alpha_2(u-u_0)=bE_0E_1,\qquad b=\frac{\varepsilon^2 p}{32\pi\rho w^2},\tag{3}$$

$$\begin{matrix} u(0,t)=u_0, & E_1(L,t)=0, \\ u(x,0)=u_0, & E_1(x,0)=0, \end{matrix}\quad E_0(0,t)=\mathscr{E}.\tag{4}$$

We consider the case of backscattering, in which the scattered light wave emerges from the sample in the direction opposite that of the incident wave. We denote by E_0 (x, t) and E_1 (x, t) the electric field amplitudes in the incident and scattered light waves; u (x, t) is the displacement amplitude in the sound wave; ε is the dielectric constant; p is the photoelastic constant; ρ is the density; w is the sound velocity; c is the velocity of light; q is the sound wave vector; and α_1 and α_2 are the light and sound damping rates. We assume $\alpha_2 \gg \alpha_1$. We do not distinguish between the frequencies of the incident and scattered light, denoting both by ω. We have discarded some time derivatives in Eqs. (1) and (2) which are small in terms of the parameter $L \ll ct$, where L is the length of the sample, and t is the time scale of the problem.

The point $x = 0$ is at the input end of the sample. Since the scattered light is produced in the sample and emerges from the sample in the direction opposite that of the incident light, we have $E_1 = 0$ at the rear of the sample. The quantity u_0 is determined by the thermal noise level in the sample. Our boundary conditions, which we believe correspond to the experimental situation, are not the same as those in Refs. 1 and 2. Since a change in these conditions changes the solution, our first step here is to construct a solution under conditions (4) and to determine the generation threshold in the absence of depletion of the incident wave, i.e., under the condition $E_1^2 \ll \mathscr{E}^2$. We do not consider Eq. (1) in this step.

The term $\alpha_2 u_0$ is not usually written in Eq. (3). This modification of Eq. (3) cannot change the rate of the exponential growth of the amplitudes above the threshold for stimulated Brillouin scattering (the amplification threshold). In the absence of a pump E_0, the modified version of Eq. (3) leaves u at the thermal noise level u_0, in agreement with conditions (4).

Laplace time transforms are convenient for solving system (2)–(4). After transforms are taken, Eqs. (2) and (3) become a system of ordinary differential equations. The solution of this system after the transforms are inverted consists to two parts: a steady-state part and an unsteady, time-varying part. The known steady-state part cannot be identified in the transient solutions of Refs. 1 and 2, apparently because of the initial and boundary conditions adopted there. Above the amplification threshold, $abE_0^2 > \alpha_1\alpha_2$, we find the familiar result for the steady-state part:

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$$u=u_0\frac{\alpha_1\alpha_2}{\alpha_1\alpha_2-abE_0^2}+u_0\frac{abE_0^2}{abE_0^2+\alpha_1\alpha_2}\exp\left[\left(-\alpha_1+\frac{abE_0^2}{\alpha_2}\right)(L-x)\right]. \quad (5)$$

Expression (5) is written for the case $\alpha_2^2 \gg abE_0^2$, so that the expression for the displacement can be simplified. In the Laplace transform of the solution, the steady-state contribution corresponds to a pole $s = 0$, where s is the parameter of the Laplace time transforms. The time-varying part corresponds to poles of s on the negative part of the real axis. If a pole goes into the region $s > 0$ at some pump energy, then the decay of the time-varying part of the solution gives way to an increase, implying a transition to the generation regime.

The position of the pole under the condition $abE_0^2 L^2 < 1$ is

$$s/2w=-(\alpha_1+\alpha_2)/2-[abE_0^2+y^2/L^2]^{1/2}, \quad (6)$$

where y is a nonzero solution of the transcendental equation

$$\text{th}\, y=y\,[y^2+abE_0^2L^2]^{-1/2}. \quad (7)$$

Under the condition $abE_0^2L^2 > 1$ we have

$$s/2w=-(\alpha_1+\alpha_2)/2+[abE_0^2-y^2/L^2]^{1/2}, \quad (8)$$

$$\text{tg}\, y=y\,[abE_0^2L^2-y^2]^{-1/2}. \quad (9)$$

The pole in (6) corresponds to a solution which decays exponentially over time, while in (8) we could have a situation with $s > 0$, corresponding to exponential growth. A necessary condition here is

$$abE_0^2>[(\alpha_1+\alpha_2)/2]^2+y^2/L^2. \quad (10)$$

The generation condition from Refs. 1 and 2 differs from (10) in that it lacks the second term on the right side.

At which amplitudes E_0 can the pump wave be assumed given? In other words, at which amplitudes can the depletion of the pump wave be ignored? According to (5), this is possible if

$$abE_0^2L/\alpha_2<1. \quad (11)$$

In the opposite case, the amplitudes u and, correspondingly, E_1 begin to grow exponentially. On the other hand, a violation of condition (11) is not sufficient for growth of the intensity of the sound wave from a low thermal-noise level to the levels determined by the nonlinear solution[4] of system (1)–(3). A necessary condition for this growth is

$$abE_0^2L/\alpha_2>R, \quad (12)$$

where the number R is on the order of 15–30. This number can be described parametrically as $\ln(\alpha_2/au_0)$.

Comparing conditions (10) and (12), we first consider the case $\alpha_2L > 1$; this case corresponds to the usual experimental situation. In this case, only the first term on the right side of (10) is important. A comparison shows that under the conditions

$$\alpha_2L>4R\gg1 \quad (13)$$

the depletion of the incident wave becomes important at amplitudes at which temporal generation has not yet begun. In this situation it is not correct to derive the condition for the generation threshold on the basis of a given amplitude of the incident wave. Under the condition

$$4R>\alpha L>1 \quad (14)$$

a generation regime is possible. A comparison of (10) and (12) in the case $\alpha_2L < 1$ leads to two analogous situations.

It should be noted that a separate analysis of the amplification and generation regimes is meaningful if

$$\alpha_2wT\gg1, \quad (15)$$

where T is the length of the laser pulse. Condition (15) means that in the amplification regime, before and end of the light pulse, the stimulated Brillouin scattering reaches a steady-state limit. In the opposite case of a time-varying stimulated Brillouin scattering, the process evolves over time until the end of the laser pulse, and the distinction between amplification and generation regimes is not meaningful. In this case it would be meaningless to single out a temporal asymptotic part of the solution, (5).

Observation of the generation regime will thus require the simultaneous satisfaction of the several conditions discussed above:

$$4R>4abE_0^2L/\alpha_2>\alpha_2L>1 \quad (16)$$

and condition (15). These conditions impose stringent requirements on such as experiment.

Several analogous conditions arise in the case $\alpha_2L < 1$:

$$R>abE_0^2L/\alpha_2>y^2/\alpha_2L>1. \quad (17)$$

Here y is a root of Eq. (9). Again, an important point is that these conditions must be compatible with condition (15).

We now ask whether a temporal instability occurs at any value of $\mathscr{E}$ in the solution of the complete system of equations for the stimulated Brillouin scattering, (1)–(4), for the situation (13), in which the depletion of the incident wave sets in before the temporal instability in a system with a given pump. To answer this question, we examine the temporal stability of the steady-state solution of Eqs. (1)–(3) derived by Tang.[4] Here it is convenient to introduce the new function

$$\varphi=a\int_0^x u(x',t)\,dx'. \quad (18)$$

System (1)–(3) then reduces to the nonlinear equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{1}{w}\frac{\partial^2\varphi}{\partial x\,\partial t}+\alpha_2\frac{\partial\varphi}{\partial x}=ab\,(C^2e^{-2\varphi}-D^2e^{2\varphi}), \quad (19)$$

$$E_0=Ce^{-\varphi}+De^{\varphi},\quad E_1=Ce^{-\varphi}-De^{\varphi}. \quad (20)$$

Comparing the first and third terms of Eq. (19) over distances on the order of L, we can discard the second derivative with respect to the coordinate which is of order $\alpha_2L\gg1$. This simplification is not legitimate in a small neighborhood $x \lesssim 1/\alpha_2$ of the input end of the sample. In this neighborhood the sound grows sharply from the thermal-noise level to levels determined by the nonlinear solution of the system of equations. This neighborhood makes only a small contribution to the interaction of the light and the sound. We wish to stress that the solution in the region $x \gg 1/\alpha_2$, i.e., in the greater part of the sample, is determined by the growth in the intensity of the scattered light as it propagates away from the rear of the sample to the front, and the region $x \lesssim 1/\alpha_2$ is actually the region in which this solution is joined with the boundary condition at $x = 0$.

Let us examine the nonlinear solution and temporal stability of the simplified version of Eq. (19). It is simple to find a steady-state solution of this equation:

$$e^{-\varphi} = \left[\frac{D}{C} \operatorname{cth} \frac{2aCD}{\alpha_2} (x - x_0) \right]^{1/2}, \qquad (21)$$

where the constants are to be determined from the boundary conditions. It can be shown that substitution of (21) into (20) leads to the solution derived by Tang.[4]

To analyze the stability of the solution of Eq. (19), we linearize it with respect to a small deviation $\delta\varphi(x, t)$ from solution (21). After taking Laplace transforms in variable t, we can write the linearized equation in the form

$$\left(\alpha_2 + \frac{s}{w} \right) \frac{d}{dx} \delta\varphi(x, s) = -2abCD \left[\operatorname{th} \frac{2abCD}{\alpha_2} (x - x_0) + \operatorname{cth} \frac{2abCD}{\alpha_2} (x - x_0) \right] \delta\varphi(x, s) + \frac{d\eta(x)}{dx}, \qquad (22)$$

where $\eta(x) = \delta\varphi(x, t = 0)$. A solution of the first-order equation (22) can be written. When inverse Laplace transforms are taken, it is found that the entire solution is proportional to $\exp(-w\alpha_2 t)$, and the remainder of the time dependence is weaker.

In summary, solution (21) is stable with respect to small deviations regardless of the amplitude of the incident light wave. Consequently, a generation regime cannot occur at all under conditions (13).

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