Radiative decay of two-dimensional plasmons

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The emission of electromagnetic waves by a two-dimensional electron plasma in the system consisting of a metal diffraction grating on a metal-insulator-semiconductor structure (an open-resonator system) is analyzed. {The effect has been observed experimentally by Tsui *et al.* [Solid State Commun. **35**, 875 (1980)] and Höpfel *et al.* [Surf. Sci. **113**, 118 (1982)].} A model is proposed which allows an approximate solution while retaining the basic features of the actual experimental situation (the diffraction problem cannot be solved exactly). The radiative width of the plasma resonance and the corrections to the dispersion law for two-dimensional plasmons are calculated.

Experiments on the resonant absorption of electromagnetic radiation by two-dimensional plasmons are being carried out in open-resonator systems.[1] A periodic array of metal stripes (a diffraction grating) on the field electrode of a metal-insulator-semiconductor system modulates the transparency of this electrode with respect to the incident electromagnetic wave. The modulation period determines the momentum of the plasmon which is excited.

Once the plasma wave has been excited, the same periodic array of electrodes acts as a transmitting antenna; i.e., the plasma waves in a structure of this type experience an additional damping from radiative decay. This effect has been seen directly[2] as emission at the plasma frequency during the heating of electrons. Heitman *et al.*[3] have noted that the width of the plasma resonance is greater than would follow from estimates of the electron collision time, and they wondered whether this effect might be linked to radiative decay of the plasmons.

Since the earliest experiments on two-dimensional plasmons, the literature has remained silent on an important question: Why does the observed dispersion law for the plasma waves agree well with the law for a closed resonator (i.e., for a system with a solid metal electrode)? It would appear that there is no small parameter in this problem. The length of the plasma wave is exactly equal to the period of the diffraction grating, and the dimensions of the transparent and opaque regions of the grating are not very different (in the earliest experiments, they were nearly equal to each other). We are thus dealing with waves in a periodic structure in a situation in which the wave vector is equal to the reciprocal-lattice vector, so that the continuum approximation is completely inapplicable.

We show below, for the particular case of a system with a weak spatial modulation, that distortions in the dispersion law for plasma waves have an additional small parameter, which is unrelated to the weakness of the modulation. This "extra" small parameter is determined by the ratio of the surface electron densities in the plasma layer and in the opaque parts of the diffraction grating. This parameter evidently remains small in the real situation, in which the modulation depth is not small, and thus resolved the contradiction.

We will calculate the radiative damping of plasmons which results from their conversion into electromagnetic radiation, and we will show that the conversion coefficient contains only a kinematic small parameter, the ratio of the phase velocity of the plasmon to the velocity of light. In this system the imaginary increment in the frequency is thus far larger than the corrections to the real part of the frequency. This circumstance is crucial to an understanding of the experiments of Tsui *et al.* and Höpfel *et al.*[2] on the electromagnetic emission from a two-dimensional plasma.

Figure 1 shows the model of the system. There is a two-dimensional plasma (e.g., the inversion layer of a metal-insulator-semiconductor structure) in plane P ($z = \Delta$), and there is a periodic structure of metal electrodes in plane M ($z = 0$). We may assume that there is also a two-dimensional plasma in plane M, with an equilibrium density $N(y)$ which is a periodic function, $N(y+a) = N(y)$. Here ε_f and ε_s are the dielectric constants of the insulator and the semiconductor, respectively.

A more natural approach, of course, would be to consider a diffraction problem, with appropriate boundary conditions imposed on the fields at the system of metal stripes. In that approach, however, even the simpler, purely electro-

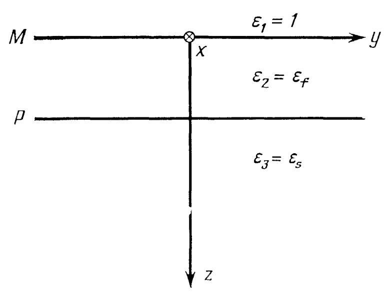

FIG. 1.

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magnetic, problem of the diffraction of a plane wave by the periodic array of stripes can be solved only in the case $t/a = 1/2$, where t is the width of the transparent regions of the structure.[4] (Experimentally, an effort is made to keep t/a small—usually about 0.15 ± 0.2—in order to increase the amplitudes of the spatial harmonics of the field which excites the plasma.) The model which we are using allows us to construct an approximate solution, while retaining the basic features of the actual experimental situation.

We must solve the system of Maxwell's equations and the equation of motion of the plasma. A distinctive feature of this problem is that the spectrum of plasma waves and the spectrum of the external force acting on the plasma are strictly related, since the two spectra are determined by the same periodic function, $N(y)$. The spatial Fourier harmonics of the external force are nonzero only for wave numbers $2\pi n/a$ $(n = 0, \pm 1, \pm 2, \ldots)$, and the gaps in the plasma-wave spectrum evidently lie at plasmon quasimomenta $n\pi/a$. It follows that plasmons with frequencies near band gaps of even index are observed in experiments on the resonant absorption of electromagnetic waves. In a reduced-band model they would correspond to the center of the Brillouin zone. We should thus seek solutions of Maxwell's equations in which the fields are periodic functions of y with a period a.

For this geometry of the system (Fig. 1), the nonvanishing field components are the electric components E_y and E_z and the magnetic component H_x. For the latter we have

$$H_x^{(1)}=\sum_n D_n \exp(iqyn+\varkappa_{n1}z), \qquad z<0;$$

$$H_x^{(2)}=\sum_n (A_n \exp(\varkappa_{n2}z)+B_n \exp(-\varkappa_{n2}z))\exp(iqyn),$$

$0<z<\Delta;$

$$H_x^{(3)}=\sum_n C_n \exp(iqyn-\varkappa_{n3}z), \qquad z>\Delta. \tag{1}$$

Here

$$q=2\pi/a, \qquad \varkappa_{nj}\equiv(q^2n^2-\varepsilon_j k^2)^{1/2} \qquad (j=1,\,2,\,3),$$

$k = \omega/c$, ω is the frequency, and c is the velocity of light in vacuum.

The tangential component of the electric field is given by

$$E_y^{(j)}=\frac{ic}{\omega\varepsilon_j}\frac{\partial H_x^{(j)}}{\partial z}. \tag{2}$$

The boundary conditions at $z = 0$ are

$$E_y^{(1)}=E_y^{(2)}, \quad H_x^{(2)}-H_x^{(1)}=\frac{4\pi}{c}j_s, \quad j_s=\frac{ie^2N(y)}{m\omega}E_y, \tag{3}$$

and those at $z = \Delta$ are

$$E_y^{(2)}=E_y^{(3)}, \quad H_x^{(3)}-H_x^{(2)}=\frac{4\pi}{c}\frac{ie^2N_s}{m^*\omega}E_y. \tag{3'}$$

The surface current densities in (3) and (3′) are written in the a cold collisionless plasma approximation; N_s is the equilibrium density of the two-dimensional plasma in the plane $z = \Delta$; e is the electron charge; and m and m^* are the effective masses of the carriers in the metal and the inversion channel, respectively.

Using (1) and (2), we can derive an equation for the coefficients D_n from system (3), (3′). The existence of three spatial regions with different dielectric constants makes this equation extremely complicated in general, but it can be simplified substantially by making use of the two obvious small parameters $k/q \ll 1$ and $k\Delta \ll 1$ (typical experimental values are $k/q < 10^{-2}$ and $k\Delta \sim 10^{-3}$). We then have

$$D_n\alpha_n\frac{F_n}{M_n}-\frac{4\pi e^2}{\omega^2 m}\sum_l N_{n-l}\varkappa_{l1}D_l=0. \tag{4}$$

Here the N_n are the Fourier components of N_y;

$$\alpha_{n\neq0}=\{\varepsilon_f[\varepsilon_s \operatorname{cth}(q\Delta|n|)+\varepsilon_f]+\varepsilon_f \operatorname{cth}(q\Delta|n|)+\varepsilon_s\}$$
$$\times[\varepsilon_s+\varepsilon_f \operatorname{cth}(q\Delta|n|)]^{-1}, \quad \alpha_0=\varepsilon_s+1; \quad F_n=1-\omega_n^2/\omega^2;$$

$$M_n=1-\overline{\omega}_n^2/\omega^2; \quad \varkappa_{l1}\approx\begin{cases} q|l|, & l\neq0\\ -ik, & l=0\end{cases},$$

where ω_n and $\overline{\omega}_n$ are the frequencies of the two-dimensional plasmons with momentum $q|n|$ in a system without a metal electrode and in a system with an ideally conducting solid electrode, respectively:

$$\omega_n^2=\frac{4\pi e^2N_sq|n|[\varepsilon_f \operatorname{cth}(q\Delta|n|)+1]}{m^*\{\varepsilon_f[\varepsilon_s \operatorname{cth}(q\Delta|n|)+\varepsilon_f]+\varepsilon_f \operatorname{cth}(q\Delta|n|)+\varepsilon_s\}}$$

$$\overline{\omega}_n^2=\frac{4\pi e^2N_sq|n|}{m^*[\varepsilon_f \operatorname{cth}(q\Delta|n|)+\varepsilon_s]}. \tag{5}$$

Assuming that the modulation of $N(y)$ is relatively weak ($N_{\pm1}, N_{\pm2}, \ldots \ll N_0$), we solve system (4) by the weak-coupling method. As mentioned above, the fundamental plasma resonance corresponds to a plasmon momentum of $\pm 2\pi/a$; i.e., the coefficients D_1 and D_{-1} are important in system (4). The relationship between them is determined by the Fourier components $N_{\pm2}$. Since we wish to retain effects involving the emission of electromagnetic waves, we must also retain the coefficient D_0, since only $\varkappa_{01}$ among all the quantities $\varkappa_{l1}$ is purely imaginary and thus corresponds to radiation. The relationship between D_0 and $D_{\pm1}$ is determined by the Fourier components $N_{\pm1}$ and has the additional small parameter $k/q = a\omega/2\pi c$ (which does not figure in the relationship $D_1 \leftrightarrow D_{-1}$). Three equations thus remain from system (4) and lead to the dispersion relation

$$\left[\frac{F_1}{M_1}\frac{\omega^2 m\alpha_1}{4\pi e^2N_0q}-1+i\frac{4\pi e^2N_1k}{\omega^2(\varepsilon_s+1)m}\frac{N_{-1}}{N_0}\right]^2$$
$$=\left[\frac{N_2}{N_0}-i\frac{4\pi e^2N_1k}{\omega^2(\varepsilon_s+1)m}\frac{N_1}{N_0}\right]\left[\frac{N_{-2}}{N_0}-i\frac{4\pi e^2N_{-1}k}{\omega^2(\varepsilon_s+1)m}\frac{N_{-1}}{N_0}\right]. \tag{6}$$

We first ignore the radiative loss; i.e., we omit from (6) the terms with the factor $k = \omega/c$. Furthermore, we assume $N_0 \gg N_s$, in accordance with the experimental situation. The roots of the dispersion relation in which we are interested are thus close to the zeros of M_1, and we find two values of the plasmon frequency corresponding to the momentum $2\pi/a$:

$$\omega_\pm^2=\bar\omega_1^2\left[1-\frac{\alpha_1 m}{4\pi e^2N_0q}(\omega_1^2-\bar\omega_1^2)\left(1\mp\left|\frac{N_2}{N_0}\right|\right)\right]. \tag{7}$$

The size of the gap in the plasmon frequency spectrum is

$$\omega_+-\omega_-\approx\bar\omega_1\frac{\alpha_1 m(\omega_1^2-\bar\omega_1^2)}{4\pi e^2N_0q}\left|\frac{N_2}{N_0}\right|\sim\bar\omega_1\frac{N_s}{N_0}\left|\frac{N_2}{N_0}\right|. \tag{8}$$

We see from (8) that the relative size ("smallness") of this gap results from not only the weak-coupling parameter N_2/N_0 but also the ratio N_s/N_0. Here N_0 is the average surface density of electrons in the metal diffraction grating on the field electrode of the metal-insulator-semiconductor structure. For the typical values $N_s\sim10^{12}$-10^{13} cm^{-2} the ratio N_s/N_0 would be no greater than 10^{-3}. For the case of a square modulation of the transparency of the field electrode we would have

$$N_n/N_0=a\sin(n\pi t/a)/n\pi(a-t),$$

and if $t\ll a$ we would have $N_2/N_0\sim t/a$.

This analysis thus shows why the dispersion law found in experiments carried out to observe plasmons corresponds quite accurately to the situation with a solid, ideally conducting electrode ($\omega=\omega_n$) despite the presence of a diffraction grating.

To find the radiative decay of the plasmons, $\Gamma_R=-\mathrm{Im}\omega$, we now assume for simplicity that the function $N(y)$ is symmetric with respect to the center of a metal stripe of the grating. The upper branch ω_+ then damps according to

$$\frac{\Gamma_R}{\omega_+}=\frac{k}{q}\left(\frac{N_1}{N_0}\right)^2\frac{\alpha_1}{\varepsilon_s+1}\frac{\omega_1^2-\bar\omega_1^2}{\omega_+^2}. \tag{9}$$

In the region $q\Delta\ll1$ the radiative loss naturally increases with decreasing Δ:

$$\frac{\Gamma_R}{\omega_+}=\frac{k}{q}\left(\frac{N_1}{N_0}\right)^2\frac{1}{q\Delta}\frac{\varepsilon_f}{\varepsilon_s+1}. \tag{10}$$

Expression (10) states that radiative damping makes an extremely small contribution to the overall width of the plasma resonance.

For the experimental conditions of Ref. 3 ($\Delta\sim5\cdot10^{-6}$ cm, $a\sim5\cdot10^{-5}$ cm, $t/a\sim0.25$, and $k=2\pi\cdot10^2$ cm^{-1}) we find $\Gamma_R/\omega\sim0.3\cdot10^{-3}$, which is two orders of magnitude smaller than the collisional damping. The additional broadening of the resonance observed in Ref. 3 was evidently caused by other factors.

In the opposite case, $q\Delta\gg1$, the damping becomes exponentially small: $\Gamma_R\propto\exp(-2q\Delta)$. This result follows from (5) and (9): The difference $\omega_1^2-\bar\omega_1^2$ approaches zero, so that the field of the first harmonic of the plasma wave falls off exponentially with increasing distance between the inversion layer and the grating.

The solution corresponding to the lower edge of the gap, $\omega=\omega_-$, does not experience a radiative damping in the present case of a symmetric function $N(y)$. This result is understood easily by considering the spatial distribution of the field E_y. In the first case ($\omega=\omega_+$) the field E_y has an antinode at the center of a metal stripe, while in the second case ($\omega=\omega_-$) it has a node there.

Expressions (7) and (9) give the frequency and intensity, respectively, of the spontaneous emission by a two-dimensional plasma which was observed in Refs. 2. The quantity $2\Gamma_R$ is the fraction of the energy of the given plasma mode (q,ω) which is radiated per unit time. Interestingly, under the usual condition $q\Delta\ll1$ [expression (10)] this quantity does not depend on the geometric parameters of the structure (a and Δ) and is determined exclusively by the surface density N_s and the modulation depth:

$$\Gamma_R=\frac{4\pi e^2N_s}{mc(\varepsilon_s+1)}\left(\frac{N_1}{N_0}\right)^2.$$

This functional dependence could be tested easily in experiments with metal-insulator-semiconductor structures.

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Translated by Dave Parsons