# Quantum theory of dislocation absorption of sound in solid helium

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The absorption of sound by dislocations is analyzed in the delocalized kink model. The cases of "pinned" and "free" dislocations are considered. The spectrum and temperature dependence of the logarithmic damping factor are derived for various dislocation distributions in the crystal. The estimated absolute values of the damping factor agree with experimental results.

An anomalously large absorption of energy from sound waves is observed experimentally in helium crystals.[1-3] Tsymbalenko[3] found that the amplitude-independent damping factor of bending vibrations of a rod reached $R \approx 0.4$. All the experimental results which have been obtained have been derived in the classical Granato-Lücke theory,[4] but the applicability of this theory to such quantum-mechanical entities as $^4$He has not been directly analyzed.

The influence of quantum effects on the absorption of sound due to defects in helium crystals was first studied by Meĭerovich.[5] He calculated the component of the internal friction which arises from the presence of delocalized point defects in the crystal. The estimated absolute values of this component of the friction, however, are several orders of magnitude smaller than the values observed experimentally, so that point defects can be eliminated from the posible explanations for the anomalously high damping factor. Another explanation, and apparently the most likely one, is based on dislocations. We show below that the quantum-mechanical structural features of dislocations give rise to a spectrum and a temperature dependence of the sound absorption which differ substantially from the predictions of the Granato-Lücke theory.

**1.** Dislocations in crystals contain kinks because of either the geometric arrangements of the dislocations or thermal fluctuations. In a quantum crystal, as Andreev[6] has pointed out, each kink on an infinitely long dislocation is a quasiparticle for which quasimomentum is a good quantum number. The values of the kink energy $\varepsilon$ fill a continuous band:

$$\varepsilon = \varepsilon_0 - J\cos ka, \tag{1}$$

where $\varepsilon_0$ is the "classical" energy of the kink, $J$ is the width of the energy band, $k$ is the quasimomentum, and $a$ is the interatomic distance.

For dislocations in a real crystal there are always pinning points, which are dislocation branch points, regions of attachment to an impurity, etc. If the height of the potential barrier at these points exceeds the widths of the energy band of the kink, the kink is elastically reflected from the barrier and localizes between neighboring pinning points. Let us assume, for example, that the length of a dislocation segment between pinning points is $l$. It is easy to see that in this case $l/a$ states arise with an energy given by (1) with $k_j = (\pi/l)j$, where $j = 1, \ldots, l/a$. The external sound field induces transitions between these states, so that energy is absorbed from the sound wave.

To calculate the energy absorption we start from the kinetic equation for the kink density matrix $\hat{\rho}$, in the $\tau$ approximation:

$$\frac{\partial\hat{\rho}}{\partial t} + \frac{i}{\hbar}[\hat{H}_0 + \hat{H}_1, \rho] + \frac{\hat{\rho}-\hat{\rho}_0}{\tau} = 0. \tag{2}$$

Here $\hat{H}_0$ is the unperturbed Hamiltonian of the kink, and $H_1$ is the Hamiltonian of the perturbation. In the absence of a perturbation, the equilibrium density matrix $\rho_0$ is diagonal in the states of the Hamiltonian $\hat{H}_0$; i.e.,

$$\hat{\rho}_0 = \frac{\exp(-\hat{H}_0/T)}{\mathrm{Sp}\exp(-\hat{H}_0/T)}.$$

The propagation of a periodic sound wave through the crystal is described by off-diagonal terms in the perturbation Hamiltonian. Most of the absorption is caused by the matrix elements $H_{j,j+1} = H_{j+1,j} = V_j\cos\omega t$ representing transitions between nearest levels ($V_j$ is real). Taking the Fourier transform of Eq. (2), we find the following equations for the off-diagonal elements of the density matrix in the linear approximation:

$$\begin{aligned}
&\left\{i\omega + \frac{1}{\tau} + \frac{i}{\hbar}(\varepsilon_j - \varepsilon_{j-1})\right\}\delta\rho_{j,j-1} = \frac{i}{\hbar}H_{j,j-1}(n_j - n_{j-1}),\\
&\left\{i\omega + \frac{1}{\tau} - \frac{i}{\hbar}(\varepsilon_j - \varepsilon_{j-1})\right\}\delta\rho_{j-1,j} = \frac{i}{\hbar}H_{j-1,j}(n_{j-1} - n_j).
\end{aligned} \tag{3}$$

The energy absorbed per unit time per unit volume is

$$\overline{W} = \overline{\mathrm{Sp}(\dot{\hat{H}}_1\delta\hat{\rho})},$$

where the superior bar means a time average. Writing the sum of diagonal elements, and taking the time dependence of the matrix elements into account explicitly, we find the following expressions for the energy dissipation:

$$\begin{aligned}
W = \sum_{j=2}^{l/a} &\frac{\omega|V_j|^2}{4\hbar}(n_{j-1} - n_j)(e^{i\omega t} - e^{-i\omega t})\\
&\times\left\{e^{i\omega t}\left[i\omega + \frac{1}{\tau} + \frac{i}{\hbar}(\varepsilon_j - \varepsilon_{j-1})\right]\right.^{-1}\\
&\left. + e^{-i\omega t}\left[-i\omega + \frac{1}{\tau} + \frac{i}{\hbar}(\varepsilon_j - \varepsilon_{j-1})\right]^{-1} - \text{c.c.}\right\}.
\end{aligned} \tag{4}$$

Averaging over the time is an elementary procedure; it results in the following final expression in the physically interesting case in which the reciprocal of the relaxation time is smaller than the distance between energy levels:

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$$\overline{W}=\sum_{j=2}^{l/a}\frac{|V_j|^2}{2\hbar}(n_j-n_{j-1})\,\omega\tau\left[\left(\frac{\varepsilon_j-\varepsilon_{j-1}}{\hbar}-\omega\right)^2\tau^2+1\right]^{-1}. \quad (5)$$

To apply this expression we need to express the transition matrix elements in terms of the characteristics of the crystal, sum over all possible transitions, and average the various positions of the dislocations. We write the wave functions of kinks in unperturbed states as follows in the gas approximation:

$$\psi_j=\left(\frac{2}{l}\right)^{1/2}\sum_n \sin\left(\frac{\pi n}{l}j\right)w_0(x-na),$$

where $w_0(x)$ is the wave function of a localized kink. The matrix element $V_{jm}$ for a transition between kink states $j$ and $m$ is

$$V_{jm}=\frac{8ljm}{\pi^2(j^2-m^2)^2}(\mathbf{fn}),\ \text{for odd } (j-m),$$
$$V_{jm}=0,\ \text{for even } (j-m). \quad (6)$$

Here **f** is the force acting on the kink, and the unit vector **n** specifies the direction of the dislocations. In accordance with the suggestion above, the absorption is caused primarily by transitions between adjacent levels. In the case of shear stresses at a kink, a nonzero force acts even if the strain in the crystal is uniform:

$$f_i=\varepsilon_{ikl}\sigma_{kj}'b_jl_l,$$

where $\sigma_{ik}'=\sigma_{ik}-\frac{1}{3}\delta_{ik}\sigma_{ll}$ is the stress deviator, $b_j$ is the Burgers vector, and the vector $l_l$ is a measure of the length and direction of the kink. We are interested here in waves which are long on the atomic scale. Clearly, the processes of interest occur particularly intensely during the propagation of a transverse sound wave. This is the case which we consider below.

In a cubic latice, dislocations can lie along three equivalent directions. Considering only single kinks and dislocations (for which the height and the Burgers vector, respectively, are equal to the interatomic distance); taking an average over all possible **n**, **b**, and **l**; and noting that for the case of shear the deviator is identical to the stress tensor, we find the following expression for the square matrix element:

$$|V_j|^2=4l^2a^4|\sigma_{ik}|^2/3\pi^4. \quad (7)$$

We then substitute (7) into the absorbed energy (5). Using expression (1) for the kink energy, and noting that the number of levels is large ($\sim 10^4$), we switch from a discrete summation to an integration. We describe the kinks by means of Boltzmann statistics. As a result we find the absorbed energy as a function of the frequency of the incident wave, expressed parametrically:

$$\overline{W}=\frac{4a^5lJ|\sigma_{ik}|^2\,\omega\tau}{3\pi^3\hbar T}\exp\frac{\mu-\varepsilon_0}{T}\int_0^{\pi}\exp\left\{\frac{J}{T}\cos x\right\}$$
$$\times\left[\left(\frac{2\pi aJ}{l}\sin x-\omega\right)^2\tau^2+1\right]^{-1}dx, \quad (8)$$

where $\mu$ is the chemical potential of the kinks.

It is easy to evaluate this integral in the limit in which the relaxation time is much longer than the reciprocal of the characteristic frequency $\omega_0=2\pi a\,J/\hbar l$. The integrand tends toward a $\delta$-function, so that we find

$$\overline{W}=\frac{2a^4l^2|\sigma_{ik}|^2\omega_0^2\omega}{3\pi^3T}\exp\frac{\mu-\varepsilon_0}{T}$$
$$\times\exp\left\{\frac{J}{\omega_0T}(\omega_0^2-\omega^2)^{1/2}\right\}(\omega_0^2-\omega^2)^{-1/2} \quad (9)$$

if $\omega<\omega_0$ or $\overline{W}=0$ if $\omega>\omega_0$. A singularity appears in the absorption spectrum at the resonant frequency $\omega_0$ in this approximation, as it should. This frequency corresponds to the inflection point in the quasiparticle dispersion law. This singularity is removed by taking into account the finite relaxation time. After a simple but lengthy integration, and after expanding the integrand near the singularity, we find

$$\overline{W}=\frac{8a^5l|\sigma_{ik}|^2J\omega(\omega_0\tau)^{1/2}}{3\pi^3\hbar T\omega_0}\exp\frac{\mu-\varepsilon_0}{T}\Phi((\omega_0-\omega)\tau), \quad (10)$$

where

$$\Phi(\xi)=[\xi+(1+\xi^2)^{1/2}]^{1/2}(1+\xi^2)^{-1/2}.$$

Assuming that the width of the energy band of the kink is on the order of 1 K, we find a resonant frequency $\omega_0\sim 10^7$ Hz for $l\sim 10^4$ cm.

**2.** To calculate the scale time for relaxation to a thermodynamic equilibrium in the ensemble of kinks, we note that the number of thermal kinks is not constant; it must be determined from the condition minimizing the thermodynamic potential. As we will see below, however, the relaxation time for an equilibrium in terms of the number of particles is far longer than the time for relaxation to an equilibrium distribution function for a given number of kinks. The latter processes occur in transitions of quasiparticles between levels accompanied by the emission or absorption of transverse phonons; they also domiante the absorption of the sound at frequencies $\omega\sim 10^7$ Hz.

The vector operator representing the displacement of the atoms for a phonon is given by the standard expression[7] in the second quantization representation:

$$\hat{\mathbf{u}}=(\hbar/2\rho\omega_0)^{1/2}\mathbf{e}(\hat{a}_k+\hat{a}_{-k}^+). \quad (11)$$

Here $\hat{a}_k^+$ and $\hat{a}_k$ are the phonon creation and annihilation operators, and the unit vector **e** specifies the polarization direction. To evaluate the matrix elements ($V_k$) for the emission of a phonon with a momentum **k**, we construct a coordinate system at the dislocation. We direct the vector **n**, along the $z$ axis, while **b** and **l** run along the $x$ axis. The component $\hat{\sigma}_{xy}^+$ component of the phonon stress tensor then contributes to $V_k$. The interaction operator is

$$V_k=\langle\psi_i|z|\psi_{i-1}\rangle a^2\hat{\sigma}_{xy}^+. \quad (12)$$

The calculations for all other possible arrangements of the vectors **n**, **b**, and **l** are completely analogous, and they lead to the result found above. Using (6) and (11), we find from (12)

$$V_k=\left(\frac{2\hbar}{\rho\omega_0}\right)^{1/2}\frac{Ga^2l}{\pi}(k_xl_y+k_yl_x)(\hat{a}_k^++\hat{a}_{-k}) \quad (13)$$

$G$ is the shear modulus). The phonon absorption operator is given by the Hermitian-adjoint expression. To calculate the total probability ($w$) for the emission and absorption of a phonon per unit time, per unit volume, we need to average the square matrix element over all directions of the wave vector and all polarizations of the phonon. We use the familiar expression

$$w=\frac{1}{4\pi^2}\int\frac{|V_k|^2}{\hbar^2}\frac{\omega_0^2}{c^3}do, \tag{14}$$

where $do$ is the element of solid angle. Substituting (13) into (14), we find

$$\left.\begin{matrix}w_e\\ w_a\end{matrix}\right\}=\frac{4\omega_0{}^3G^2a^4l^2}{5\pi^5c^5\hbar\rho}\begin{cases}n_k+1,\\ n_k\end{cases} \tag{15}$$

where $w_e$ and $w_a$ are the phonon emission and absorption probabilities, and $n_k$ is the phonon distribution function. Here we have used the usual formula for taking the average of unit vectors **e** perpendicular to the vector **k**:

$$\overline{e_ie_j}={}^1/_2(\delta_{ij}-k_ik_j/k^2).$$

Solving the simple kinetic equation, we find the relaxation time to be

$$\tau=\tau_1=\frac{5\pi^5c^5\hbar\rho}{4\omega_0{}^3G^2a^4l^2}\operatorname{th}\frac{\hbar\omega_0}{2T}. \tag{16}$$

Substituting numerical values into this expression, we find $\tau_1\sim 10^{-6}$ s. We can now immediately estimate the relaxation time $\tau_n$ in terms of the number of particles. The interaction operator, which reduces the number of kinks, differs from (12) only in that the wave function of the kink in the matrix element is replaced by the wave function of a kink of the opposite sign. However, these two wave functions are identical. In this case we thus have (16), where $2\varepsilon_0$—the formation energy of two kinks—appears in the argument of the hyperbolic tangent. As a result we find $\tau_n\gg\tau_1$.

Substituting the relaxation time (16) into expression (10) for the energy dissipation, we find

$$\overline{W}=\frac{2a^3l|\sigma_{ik}|^2J\omega}{3\pi T\omega_0}\left(\frac{5c\hbar}{al\rho TJ}\right)^{1/2}\times\exp\left(\frac{\mu-\varepsilon_0}{T}\right)\Phi((\omega_0-\omega)\tau). \tag{17}$$

Here we have used the condition $\hbar\omega_0<T$, which holds down to $10^{-4}$ K.

From the experimental standpoint the quantity of most interest is the logarithmic damping factor, rather than the absorbed energy. We assume that a standing transverse sound wave has been excited in the crystal. Using the expression

$$E^1/_2\int\sigma_{ik}u_{ik}d^3x$$

for the elastic energy of the wave, we find from (17) an expression for the logarithmic damping factor:

$$R=\frac{4c^2a^2l^2\hbar}{3\pi T}\left(\frac{5c\hbar\rho}{alTJ}\right)^{1/2}\times\exp\frac{\mu-\varepsilon_0}{T}\Phi((\omega_0-\omega)\tau). \tag{18}$$

In different crystals, the dislocations will have different length distributions. We first assume that the lengths of the dislocations are spread over a narrow interval near the mean value $L$. We introduce $\Lambda$, the total length of the dislocations per unit volume, and $D$, the number of geometric kinks per unit length of a dislocation. At sufficiently low temperatures, the number of thermal kinks is small, and geometric kinks dominate the absorption. The chemical potential appears implicitly in the expression for the number of particles,

$$D=\frac{1}{a}I_0\left(\frac{J}{T}\right)\exp\frac{\mu-\varepsilon_0}{T}, \tag{19}$$

where $I_0(x)$ is the Bessel function of imaginary argument of index zero. Substituting (19) into (18), and multiplying by the total number of dislocations, we then find

$$R=\frac{4c^2a^2\hbar\Lambda D}{3\pi T}\left(\frac{5c\hbar\rho La}{TJ}\right)^{1/2}I_0{}^{-1}\left(\frac{J}{T}\right)\Phi((\omega_0-\omega)\tau). \tag{20}$$

The number of thermal kinks is found by minimizing the thermodynamic potential. Substituting $\mu=0$ into (18), we find

$$R=\frac{4c^2\hbar\Lambda L}{3\pi T}\left(\frac{5c\hbar\rho La}{TJ}\right)^{1/2}e^{-\varepsilon_0/T}\Phi((\omega_0-\omega)\tau). \tag{21}$$

We now consider the case in which the lengths of the dislocation segments differ greatly from the mean value. For randomly distributed impurities, for example, the number $N(l)dl$ of dislocations with lengths between $l$ and $l+dl$ is[4]

$$N(l)=(\Lambda/L^2)\exp(-l/L). \tag{22}$$

For the absorbed energy we have the average

$$\overline{W}=\int_0^\infty N(l)\overline{W}(l)dl,$$

where $\overline{W}(l)$ is given by (9). In this case, we can write the final results for the logarithmic damping factor in the case in which thermal or geometric kinks are predominant. For thermal kinks we have

$$R=\frac{32\pi^2a^7c^2\rho J^3\Lambda}{3\hbar^3T\omega^3L^2}e^{-\varepsilon_0/T}\Psi\left(\frac{2\pi aJ}{\hbar\omega L}\right), \tag{23}$$

where

$$\Psi(\xi)=I_0(\xi)-L_0(\xi)+\xi[L_1(\xi)-I_1(\xi)].$$

Here $I_n(x)$ is the Bessel function of imaginary argument of index $n$, and $L_n(x)$ is the Struve function of index $n$. Here we have used Eq. 3.387(3) from Ref. 8 for the definite integral. For geometric kinks we should replace the number of thermal kinks, $(\Lambda/a)e^{-\varepsilon_0/T}$, in (23) by the number of geometric kinks, $aD\Lambda/LI_0(J/T)$, as in (20) and (21). It is easy to see that the damping factor in (23) has a maximum at frequencies $\omega\sim\omega_0=2\pi aJ/\hbar L$.

All these expressions have been derived in the gas approximation. Strictly speaking, at a high kink density we would have to take into account the interaction between kinks, which would give rise to collective excitations and which would redetermine the quasienergy spectrum. Ignoring this complication, we derive a result which is correct in

order of magnitude. The scale value of $\Lambda$ for a helium crystal is $\Lambda \sim 10^7$ cm$^{-2}$. The mean length of a dislocation segment is $L \sim 10^{-4}$ cm. With $T \sim J \sim 1$ K and $\omega \sim 10^7$ Hz, we then find $R \sim 10^{-1}$ from (21), in agreement with the results of Ref. 3. The absorption of longitudinal sound, which interacts more weakly (by a factor $ka \sim 10^{-4}$) with dislocations, was measured in Refs. 1 and 2. The absolute values of the damping factor are small by a factor of the same order. An interesting dependence was found in Ref. 2. As the temperature is lowered in the vicinity of 1 K, the damping factor for the sound increases. The crystal studied there was apparently one describable by (20), in which dislocations contain a large number of geometric kinks. Study of the frequency dependence is interesting, especially at low temperatures. Data on the resonant frequency were given in Ref. 1. Over the temperature interval 1.62–1.78 K the absorption peak lies at 20 MHz.

In addition to the direct emission and absorption of phonons, processes which are second order in the interaction amplitude occur, but these can be ignored at low temperatures, as we will see below. The matrix element for the scattering of a phonon with an initial momentum $\mathbf{k}$ and a final momentum $\mathbf{k}'$, accompanied by a transition of a kink from state $i$ to state $i+2$, is

$$V_2 = \frac{V_k^* V_{k'}}{\varepsilon_i + \omega_k - \varepsilon_{i+1}} + \frac{V_k^* V_{k'}}{\varepsilon_i - \omega_{k'} - \varepsilon_{i+1}},$$

where the frequencies satisfy the identity $\varepsilon_{i+2} - \varepsilon_i = \omega_k - \omega k'$. To evaluate the probability for the process, $w_2$, we integrate over all possible initial momenta $\mathbf{k}$; taking energy conservation into account, we find

$$w_2 = \int\limits_{\omega_k > \varepsilon_{i+2} - \varepsilon_i} \frac{|V_2|^2 \omega_k^2 \omega_{k'}^2}{\hbar^2 c^6} d\omega_k \, do \, do'. \tag{24}$$

Substituting matrix elements $V_k$ from (13) into (24), we then find a simple estimate of the relaxation time for the second-order processes:

$$\tau_2^{-1} \sim \int\limits_{\omega_k > \varepsilon_{i+2} - \varepsilon_i} \frac{\rho^2 (ca^2 l)^4 \omega_k^2 \omega_{k'}^2}{\hbar^2 c^6} \times [(N_{k'}+1)N_k - (N_k+1)N_{k'}] d\omega_k \sim \frac{\rho^2 a^4 l^4 c^2 \omega_0^3 T^5}{\Theta^7}, \tag{25}$$

where $\Theta$ is the Debye temperature. Comparison of (25) with the single-phonon relaxation time in (16) yields $\tau_2/\tau_1 \sim (\Theta/T)^3 \omega_0 \tau_1$; i.e., the single-phonon processes dominate in the range of applicability of the resonant-relaxation theory ($\omega_0 \tau > 1$), as stated above.

**3.** The results derived above are valid when the reciprocal of the relaxation time is smaller than the energy distance between adjacent levels: $\omega_0 \tau > 1$. Using (16), we find that this condition is equivalent to the condition

$$\Theta a / Tl > 10^{-3} (\Theta / a^3 \rho c^2)(\Theta / J) \sim 10^{-3};$$

if this inequality is violated, the kinks on the dislocations should be treated semiclassically. At $T \sim 1$ K, the semiclassical region corresponds to $l > 10^{-4}$ cm. As will be shown below, dislocations of this type contribute to the absorption of the sound at frequencies $\omega < 10^6$ Hz.

In this approximation, the boundary conditions clearly do not impose any further restrictions on the energy quantization, and we should describe the one-dimensional gas of kinks by a semiclassical kinetic equation. In this sense, the dislocations may be regarded as free. This problem was solved in Ref. 5 for a three-dimensional gas of vacancy quasiparticles. The kinetic equation for a gas of kinks can be written as follows in the $\tau$ approximation for the case with a spatially uniform external force:

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \frac{\partial \varepsilon}{\partial \mathbf{p}} + \frac{\partial n}{\partial \mathbf{p}} \mathbf{f} \cos \omega t + \frac{n - n_0}{\tau} = 0. \tag{26}$$

Noting that the absorbed energy is determined by the average over the ensemble of the product $\mathbf{fv}$, where $\mathbf{v} = \partial \varepsilon / \partial \mathbf{p}$ is the quasiparticle velocity, we find[5]

$$\overline{W} = -\frac{\tau}{1+\omega^2\tau^2} \int \frac{dp}{2\pi\hbar} \frac{\partial n}{\partial \varepsilon} \left( \mathbf{f} \frac{\partial \varepsilon}{\partial \mathbf{p}} \right)^2. \tag{27}$$

Substituting the kink energy spectrum (1) for Boltzman statistics into (25), and carrying out the straightforward integration, we find

$$\overline{W} = \frac{aJ^2 f^2}{4T\hbar^2} \exp \frac{\mu - \varepsilon_0}{T} \frac{\tau}{1+\omega^2\tau^2} \left[ I_0\left(\frac{J}{T}\right) + I_2\left(\frac{J}{T}\right) \right]. \tag{28}$$

The procedure from this point on is the same as in Section 1. We substitute in the external force $\mathbf{f}$ and take an average over the positions of the dislocations. The final result in terms of the logarithmic damping factor is, for the case of thermal kinks,

$$R = \frac{\pi a^5 c^2 \rho J^2 \Lambda_1}{3\omega T \hbar^2} e^{-\varepsilon_0/T} \frac{\tau}{1+\omega^2\tau^2} \left\{ I_0\left(\frac{J}{T}\right) + I_2\left(\frac{J}{T}\right) \right\}. \tag{29}$$

Here $\Lambda_1$ is the total length of the dislocation segments for which the condition $Rl/\Theta a > 10^3$ holds. For geometric kinks we should replace $e^{-\varepsilon_0/T}$ by $D_1 a I_0^{-1}(J/T)$, where $D_1$ is the number of geometric kinks per unit length of a dislocation, as described above.

To evaluate the damping factor in (27), we use a previously obtained estimate[6,9] of the transport relaxation time:

$$\tau \sim \tau_3 \approx 2.5 \frac{m^* a^3 p}{\hbar} \left( \frac{\Theta}{8T} \right)^9. \tag{30}$$

Here $m^*$ is the mass of a kink, and $p$ is a typical value of the quasimomentum. The relaxation time (30) is found from an analysis of two-phonon processes, as in the derivation of (16). The different powers of the temperature can be explained on the basis that the quasicontinuous kink spectrum causes (30) to be dominated by processes with a small energy transfer, for which the cross section is $\sigma \sim \omega^4$—very different from the scattering cross section in(16) because of the threshold for the energy of an absorbed phonon.

Substituting the values of the parameters in estimates (20)–(23) into (29) and (30), we find $R \sim 10^{-4} \Lambda_1$ at frequencies $\omega \sim 10^4$ Hz. However, it does not seem possible to evaluate $\Lambda_1$ in anything approaching a reliable manner, since this quantity depends exponentially on the large ratio $l/L$, and

the deviation from (22) in a real crystal may be significant in the tail of the distribution.

The scattering of a dislocation by a boundary changes the effective relaxation time. In the case $\omega < v/l$ we can use the estimate $\tau \sim l/v$, where $v$ is the average kink velocity. If a crystal contains extended dislocation segments, there will be a nonzero absorption of energy, even if the shear strain in the crystal has a time-independent gradient.

From the standpoint of structural features in the sound absorption spectrum, we are interested in frequencies on the order of 10 MHz, where the absorption should reach a maximum due to the resonant mechanism, as follows from Section 1 and 2. It should be noted that the results derived here apply only to delocalized kinks, so that the condition $J > \hbar/\tau$ must hold. This condition seems to apply to helium up to the Debye temperature.

We note in conclusion that the expressions derived here can, in an independent study of the structure of a dislocation forest, be used to calculate the exact values of the quantities characterizing the kink spectrum.

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